Extra dimensions, compactification and the Higgs mass controversy

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Contribution to QCD @ Work - Lecce

Effective Field Theories

- Fundamental theory ... $\searrow$ energy ... effective descriptions
- Standard Model and QFTs: **not** fundamental

...But...

QFT **unavoidable** low-energy description

Weinberg

- Contain an ultimate UV scale Λ
- $E > \Lambda$: UV completion, NEW DOFs take over and leave imprint at Λ
- $E < \Lambda$: QFT valid as an effective description

Naturalness Problem

But in EFT framework...

- Scalar (Higgs) masses receive contributions from quantum fluctuations $\Delta m^2 \sim \Lambda^2$

Quadratic sensitivity to the highest scale available in EFT range of validity

Physically: Severe sensitivity to left-over of the UV completion

Unification

Extra dimensions can lead to unification

- $5D$ Gravity $\rightarrow$ S^1 compactification $\rightarrow$ $4D$ Einstein-Maxwell Kaluza, Klein
- Extra dimensions small: effectively hidden

- String Theory: boost interest in extra dimensions

Compactification

Compactification: Simplest example $\mathcal{M}^4 \times S^1$

- Decompose fields $\Phi(x, y)$ in eigenfunctions of S^1
- Require periodicity $\Phi(x, y + 2\pi R) = U\Phi(x, y)$, U global symmetry
- Integrate out y: $S_{5D} \to S_{4D}$

$\Rightarrow$ Generate a tower of effective 4D fields

$$\Psi(x, y + 2\pi R) = e^{2i\pi q_\Psi}\Psi(x, y) \;\Rightarrow\; \Psi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \psi_n(x) e^{i(n+q_\Psi)\frac{y}{R}}$$

each one with effective 4D mass

$$m_{4D}^2 = m_{5D}^2 + \left(\frac{n}{R} + q_\Psi\right)^2$$

Hosotani mechanism

5D gauge theory

- 5D gauge field $A_M \Longrightarrow$ 4D gauge field A_μ + scalar A_5
- Identify Higgs ϕ = zero mode of A_5
- Coupling of matter to A_5 : $(\partial_5 - igqA_5)^2 \Rightarrow$ NO gauging away

Key feature: coupling to ϕ in shifts of p_5 $\left(= \frac{n}{R}\right)$

VEV for ϕ → SSB of higher dim. gauge symmetry

Typical effective 4D quadratic operator

$$M^2(\phi) = \left(\frac{n}{R} + \boxed{q(\phi)}\right)^2$$

Scherk-Schwarz mechanism

5D SUSY theory w/ different R-charge for bosons and fermions

$$\Psi(x, y+2\pi R) = e^{2i\pi R q_\Psi}\Psi(x,y) \;\Rightarrow\; \Psi(x,y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{+\infty}\psi_n(x)e^{i(\frac{n}{R}+q_\Psi)y}$$

$$\Phi(x, y+2\pi R) = e^{2i\pi R q_\Phi}\Phi(x,y) \;\Rightarrow\; \Phi(x,y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{+\infty}\phi_n(x)e^{i(\frac{n}{R}+q_\Phi)y}$$

- Complete tower = 5D theory → SUSY restored
- Effective 4D theory w/ only finite number of modes → SUSY broken
- Identify Higgs = zero mode of Φ

Effective 4D quadratic operator

$$M^2(\phi) = \boxed{m^2(\phi)} + \left(\frac{n}{R}+q\right)^2$$

UV-insensitive Higgs

About 20 years ago: in these two classes Higgs UV insensitive

Compact extra dimensions provide a solution to naturalness!

Compact expression for both classes: $M^2(\phi) = m^2(\phi) + \left(\frac{n}{R} + q(\phi)\right)^2$

For each tower:

- Power UV sensitivity through $m(\phi) \Longrightarrow$ canceled by SUSY/gauge
- No UV sensitivity through $q(\phi)$

$$V_{1l}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

How is this obtained?

The KK regularization

By analogy with finite temperature

- Proper time — Antoniadis, Quiros/ Antoniadis, Dimopoulos, Dvali/ Antoniadis, Dimopoulos, Pomarol, Quiros
  1. Integrate d^4p, use infinite sum to do Poisson resummation
  2. Integral over s convergent
- 4D hard cut-off Λ — Delgado, Pomarol, Quiros
  1. perform infinite sum first "à la Kapusta-book"
  2. Integrate d^4p up to Λ: w/ SUSY or gauge no need for Λ

Invoking a non-locality of the theory

Barbieri, Hall, Nomura/ Arkani-Hamed, Hall, Nomura, Smith, Weiner / Masiero, Scrucca, Serone, Silvestrini

- Infinite sum **must** be performed first: "KK regularization"

Hard cut-off for sum (L) & integral (Λ): $L/R \gg \Lambda$ — Ghilencea, Nilles / Kim

Thick brane: proper time & infinite sum first — Delgado, von Ghersdoff, John, Quiros

Pauli-Villars: always infinite sum first — Contino, Pilo

Stringy: find justification in string embedding — Ghilencea, Nilles, Stieberger / Antoniadis, Benakli, Quiros

Regularizing sum and integral

- Integral cut-off Λ, sum cut-off $L = \zeta R \Lambda$

$$V_{1l}(\phi) = \frac{2m^2 \tan^{-1}\zeta + \zeta\left(\zeta^2 \log\frac{\zeta^2}{\zeta^2+1} + 1\right)\left(m^2 + 3q^2\right)}{48\pi^2} R\Lambda^3$$
$$+ \frac{\zeta^2\left(m^2 + 3q^2\right) + \zeta^2\left(\zeta^2+1\right)\left(m^2+3q^2\right)\log\frac{\zeta^2}{\zeta^2+1} + m^2 + q^2}{32\pi^2\left(\zeta^2+1\right)} \Lambda^2$$
$$+ \frac{\zeta\, m^2\left(6q^2R^2+1\right)\left(\zeta^2+1\right) + \zeta\, q^2\left(q^2R^2+1\right)\left(3\zeta^2+5\right)}{96\pi^2\left(\zeta^2+1\right)^2} \frac{\Lambda}{R}$$
$$+ \frac{\zeta \log\frac{\zeta^2}{\zeta^2+1}\left(3R^2\left(m^2+q^2\right)^2 + m^2 + 3q^2\right) - 3m^4R^2\tan^{-1}\zeta}{96\pi^2} \frac{\Lambda}{R}$$
$$+ \frac{3\left(\zeta^2+1\right)^2 m^4 + 6\left(\zeta^4+4\zeta^2+3\right) m^2 q^2 + \left(3\zeta^4+6\zeta^2+11\right) q^4}{192\pi^2\left(\zeta^2+1\right)^3}$$
$$+ \frac{16\pi m^5 R + 15\log\left(\frac{\zeta^2}{\zeta^2+1}\right)\left(m^2+q^2\right)^2}{960\pi^2} + \cdots + R_2$$

Regularizing sum and integral

$$V_{1l}^{\text{SUSY}}(\phi) = \frac{q_B^2 - q_F^2}{16\pi^2}\left[2\zeta R\left(\frac{m^2(\phi)}{\zeta^2+1} + m^2(\phi)\log\left(\frac{\zeta^2}{\zeta^2+1}\right)\right)\Lambda + m^2(\phi)\log\left(\frac{\zeta^2}{\zeta^2+1}\right) + \frac{\left(\zeta^4+4\zeta^2+3\right)m^2(\phi)}{\left(\zeta^2+1\right)^3}\right] + \cdots + R_2$$

$$V_{1l}^{\text{gauge}}(\phi) = \frac{\zeta^3\log\frac{\zeta^2}{\zeta^2+1}+\zeta}{16\pi^2}q^2(\phi)R\Lambda^3 + \frac{3\zeta^2+3\zeta^2\left(\zeta^2+1\right)\log\frac{\zeta^2}{\zeta^2+1}+1}{32\pi^2\left(\zeta^2+1\right)}q^2(\phi)\Lambda^2 + \frac{\zeta\left(3\zeta^2+5\right)+3\zeta\left(\zeta^2+1\right)^2\log\frac{\zeta^2}{\zeta^2+1}}{96\pi^2\left(\zeta^2+1\right)^2}\left(R^2q^4(\phi)+q^2(\phi)\right)\frac{\Lambda}{R} + \left(\frac{3\zeta^4+6\zeta^2+11}{192\pi^2\left(\zeta^2+1\right)^3} + \frac{\log\frac{\zeta^2}{\zeta^2+1}}{64\pi^2}\right)q^4(\phi) + \cdots + R_2.$$

Complete anisotropy vs isotropy

Complete Anisotropy:

- $\zeta \gg R\Lambda, 1$ expansion $\Rightarrow$ KK regularization result!
- Non-trivial cancellation between terms $\Rightarrow$ q UV sensitivity canceled

Large cut-offs hierarchy cancels q UV power sensitivity

Maximal isotropy: spherical $5D$ cut-off $p^2 + \frac{n^2}{R^2} \leq \Lambda^2$

$$V_{1l}(\phi) = \frac{5m^2 + 3q^2}{180\pi^2} R\Lambda^3 - \frac{35m^4 + 14m^2q^2 + 3q^4}{840\pi^2} R\Lambda + \frac{m^5 R}{60\pi} + R_2 + \dots$$

Rest R_2 always give the periodic oscillatory q dependence of KK

The secret liaison of proper time, thick brane & PV

Pedagogical example: tadpole diagram

- Proper time:

$$\sum_{n=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)^4}\int_{\frac{1}{\Lambda^2}}^{\infty}ds\,e^{-s\left(p^2+m^2+\left(\frac{n}{R}+q\right)^2\right)}=\sum_{n=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{e^{-\frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{\Lambda^2}}}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$$

- Thick brane:

$$\sum_{n=-\infty}^{\infty}\int^{(\Lambda)}\frac{d^4p}{(2\pi)^4}\frac{e^{-\frac{\left(\frac{n}{R}+q\right)^2}{\Lambda^2}}}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$$

- Pauli-Villars:

$$\sum_{n=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{(\Lambda R)^8}{\left[(\Lambda R)^4+p^2+(\frac{n}{R}+q)^2\right]^2}\frac{1}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$$

They all allow for infinite summation! ...but most importantly ...

Allowing the infinite sum: smooth cut-off regularization

$$V_{1l}(\phi) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \log \left(\frac{p^2 + m^2(\phi) + \left(\frac{n}{R} + q(\phi)\right)^2}{p^2 + \frac{n^2}{R^2}} \right) e^{-\frac{p^2 + \frac{n^2}{R^2}}{\Lambda^2}}$$

Numerical result totally in agreement with spherical hard cut-off:

Cubic and linear power-like UV sensitivity through $q(\phi)$

Same for $\sum_n \Leftrightarrow \int dp_5$; in KK $\sum \Leftrightarrow \int$ no dependence on q at all!

. . . so . . . most importantly . . .

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Deformation (regularization) include q, not just p and p_5 (n/R)!

Technically:

- $p_5 \to p_5' = p_5 + q$ no dependence
- $n/R \to k/R = n/R + q$ only if $qR \in \mathbb{N} \implies$ Periodic q dependence!

The various KK justifications . . .

- Criticism towards hard cut-off: sum must not be cut
  1. Preserve $5D$ SUSY or gauge Barbieri, Hall, Nomura/ Arkani-Hamed, Hall, Nomura, Smith, Weiner
  2. Similarities w/ string calculation Ghilencea, Nilles, Stieberger/ Antoniadis, Benakli, Quiros
- Common lore: infinite sum is revealing an embedding structure
  1. Non-locality of SUSY/gauge breaking
     Barbieri, Hall, Nomura/ Arkani-Hamed, Hall, Nomura, Smith, Weiner / Masiero, Scrucca, Serone, Silvestrini
  2. String states and modular invariance
     Ghilencea, Nilles, Stieberger / Antoniadis, Benakli, Quiros

...and why they do not work

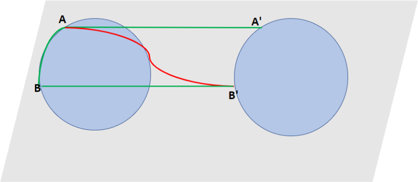
- Non-locality

  "KK regularizations" give same q suppression if $\sum \Leftrightarrow \int$:

  Nothing to do with the compactness and non locality

  Equivalent to solving the dynamics for separate $1+4\,D$

  Arkani-Hamed, Hall, Nomura, Smith, Weiner



- String

  Similarity of formal expression ...

  Of course, string $\rightarrow$ finite...but ...In EFT domain, EFT methods!

True question: How to piece together?

Conclusions

- Old establishment: Compactifications where $m^2(\phi)$ UV-sensitivity cancel $\rightarrow$ solution to naturalness
- Compactness (intrinsic non-locality) of extra dimension thought responsible

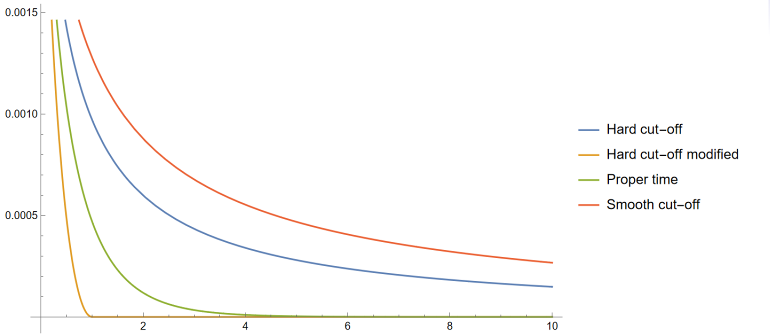
. . . We have seen . . .

- Due to the incorrect way regularization is implemented (NO Wilson)
- $q(\phi)$ UV-sensitivity is thrown away a priori
- Same happens if $\sum \Leftrightarrow \int$: nothing to do with compactification
- Non-locality seems to be there only if you artificially decouple $4D$ and $1D$ dynamics

As long as we have no explanation for large cut-off hierarchy and/or reason why only deformation including q should be considered . . .

Unfortunately, not a solution to naturalness

Thank you for your attention!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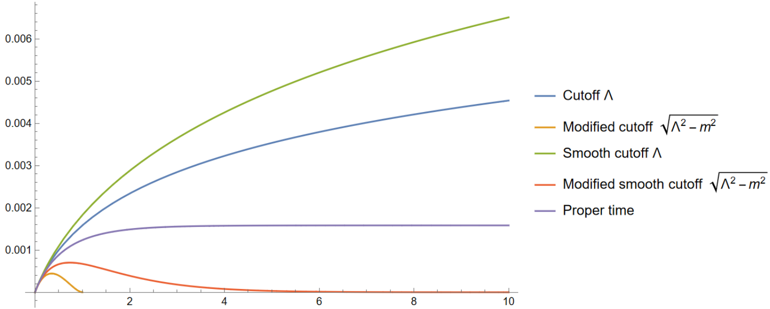
0.0010
0.0005
2
4
6
8
10
Hard cut–off
Hard cut–off modified
Proper time
Smooth cut–off


0.006
0.005
0.004
0.003
0.002
0.001
2
4
6
8
10
Cutoff Λ
Modified cutoff $\sqrt{\Lambda^2 - m^2}$
Smooth cutoff Λ
Modified smooth cutoff $\sqrt{\Lambda^2 - m^2}$
Proper time