The axion potential in quark matter

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A vanilla introduction to the QCD axion

$$\mathcal{L}_{\text{odd}} \propto \theta \tilde{F} \cdot F$$

$$\tilde{F} \equiv \varepsilon_{\mu\nu\rho\sigma} F_a^{\rho\sigma}$$

$$\mathcal{L}_{\text{axion}} \propto \frac{A}{f_a} \tilde{F} \cdot F$$

$$\frac{\langle A \rangle}{f_a} = -\theta$$

$$A = \langle A \rangle + a$$

QCD-axion

$$\mathcal{L}_{\text{odd}} + \mathcal{L}_{\text{axion}} \propto \frac{a}{f_a} \tilde{F} \cdot F$$

axion decay constant

Peccei and Quinn (1977)

The model

Lagrangian density

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \hat{\mu}\gamma_0 - m_0\right)q + \bar{e}\left(i\partial\!\!\!/ + \mu_e\gamma_0\right)e + \mathcal{L}_{\text{int}}$$

Chemical potential matrix

$$\hat{\mu} = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix} \otimes \mathbf{1}_c$$

$$\mu_u = \mu - \frac{2}{3}\mu_e, \quad \mu_d = \mu + \frac{1}{3}\mu_e$$

$$\mu_d = \mu_u + \mu_e$$

Strong interaction

$$\frac{a}{f_a}\tilde{F}\cdot F$$

$$\mathcal{L}_{\text{int}} = G_1\left[(\bar{q}\tau_a q)(\bar{q}\tau_a q) + (\bar{q}\tau_a i\gamma_5 q)(\bar{q}\tau_a i\gamma_5 q)\right]$$

$$+8G_2\left[e^{i\frac{a}{f_a}}\det(\bar{q}_R q_L) + e^{-i\frac{a}{f_a}}\det(\bar{q}_L q_R)\right]$$

$U(1)_A$-preserving

$U(1)_A$-breaking

Coupling of a to quarks

A little remark

$$\mathcal{L} \propto \theta \tilde{F} \cdot F$$

QCD at finite θ

$$\mathcal{L}_{\text{int}} = G_1 \left[(\bar{q}\tau_a q)(\bar{q}\tau_a q) + (\bar{q}\tau_a i\gamma_5 q)(\bar{q}\tau_a i\gamma_5 q) \right] + 8G_2 \left[e^{i\theta} \det(\bar{q}_R q_L) + e^{-i\theta} \det(\bar{q}_L q_R) \right]$$

The 1-loop thermodynamic potential

$$\Omega = \Omega_{\rm mf} + \Omega_{1-\rm loop} + \Omega_e$$

$$\Omega_{\rm mf} = -G_2(\eta^2 - \sigma^2)\cos(a/f_a) + G_1(\eta^2 + \sigma^2) - 2G_2\sigma\eta\sin(a/f_a)$$

$$\sigma = \langle \bar{q}q \rangle, \; \eta = \langle \bar{\eta} i\gamma_5 \eta \rangle$$

$$\Omega_{1-\rm loop} = -4N_c \sum_{f=u,d} \int \frac{d^3p}{(2\pi)^3} \left[\frac{E_p}{2} + \frac{1}{2\beta} \log(1 + e^{-\beta(E_p - \mu_f)})(1 + e^{-\beta(E_p + \mu_f)}) \right]$$

$$\alpha_0 = -2\left[G_1 + G_2\cos(a/f_a)\right]\sigma + 2G_2\eta\sin(a/f_a)$$

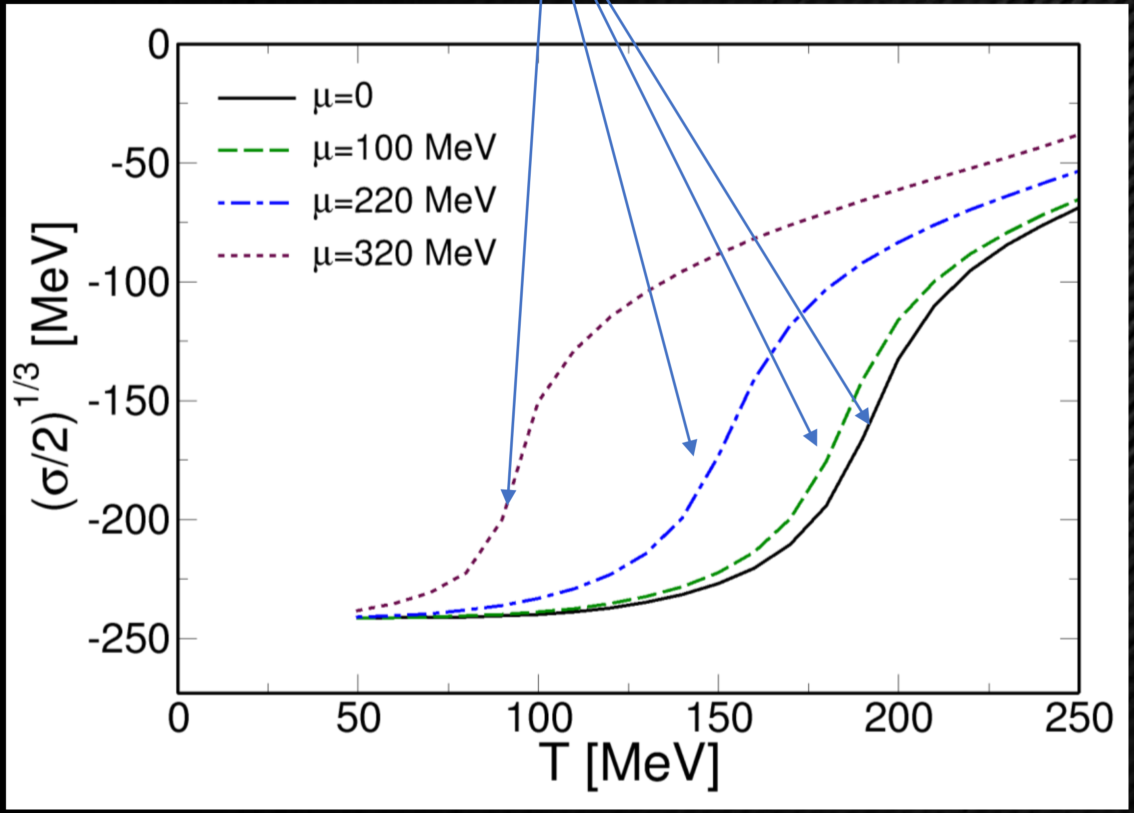
$$E_p = \sqrt{p^2 + \Delta^2}, \quad \Delta^2 = (m_0 + \alpha_0)^2 + \beta_0^2$$

$$\beta_0 = -2\left[G_1 - G_2\sin(a/f_a)\right]\eta + 2G_2\sigma\sin(a/f_a).$$

$$\Omega_e = -2T\frac{4\pi}{8\pi^3}\left(\frac{7\pi^4}{180}T^3 + \frac{\pi^2\mu_e^2 T}{6} + \frac{\mu_e^4}{12T}\right)$$

Chiral condensate

Chiral crossover



$$(\sigma/2)^{1/3} = (\langle \bar{u}u + \bar{d}d \rangle /2)^{1/3}$$

Gap equations

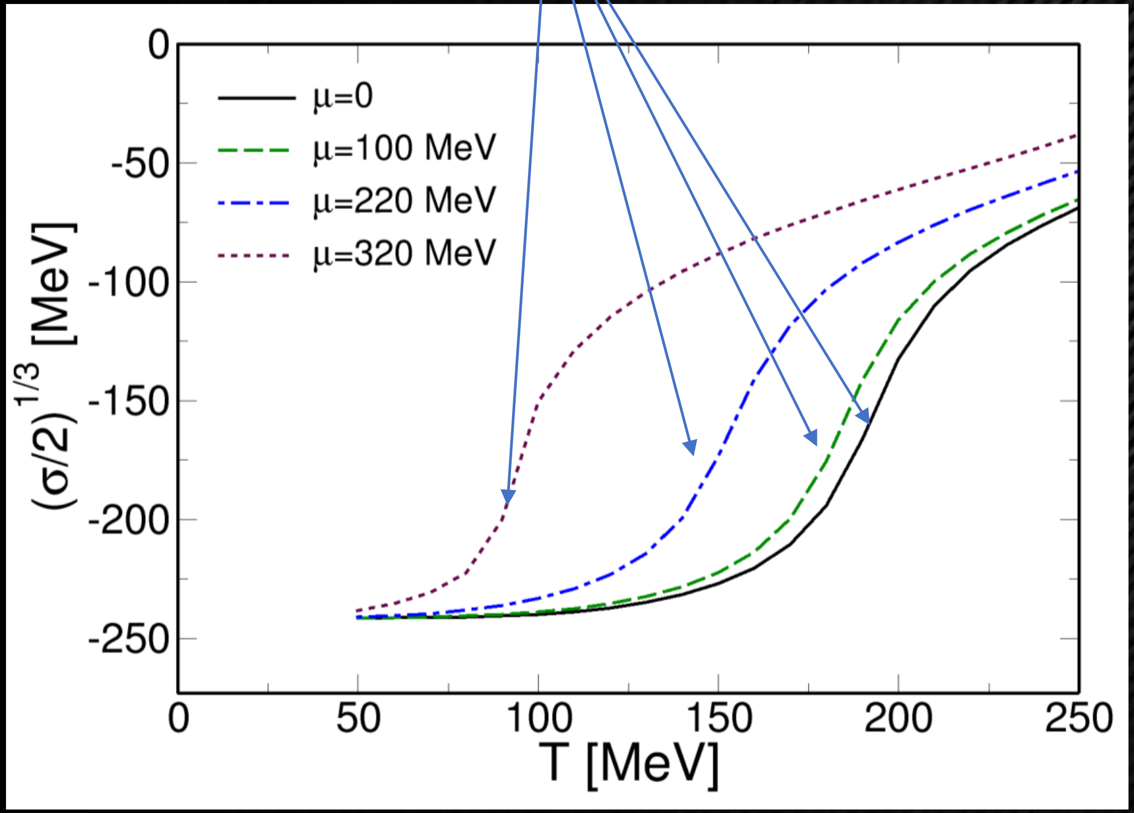
$$\frac{\partial \Omega}{\partial \sigma} = 0, \qquad \frac{\partial \Omega}{\partial \eta} = 0$$

Electrical neutrality

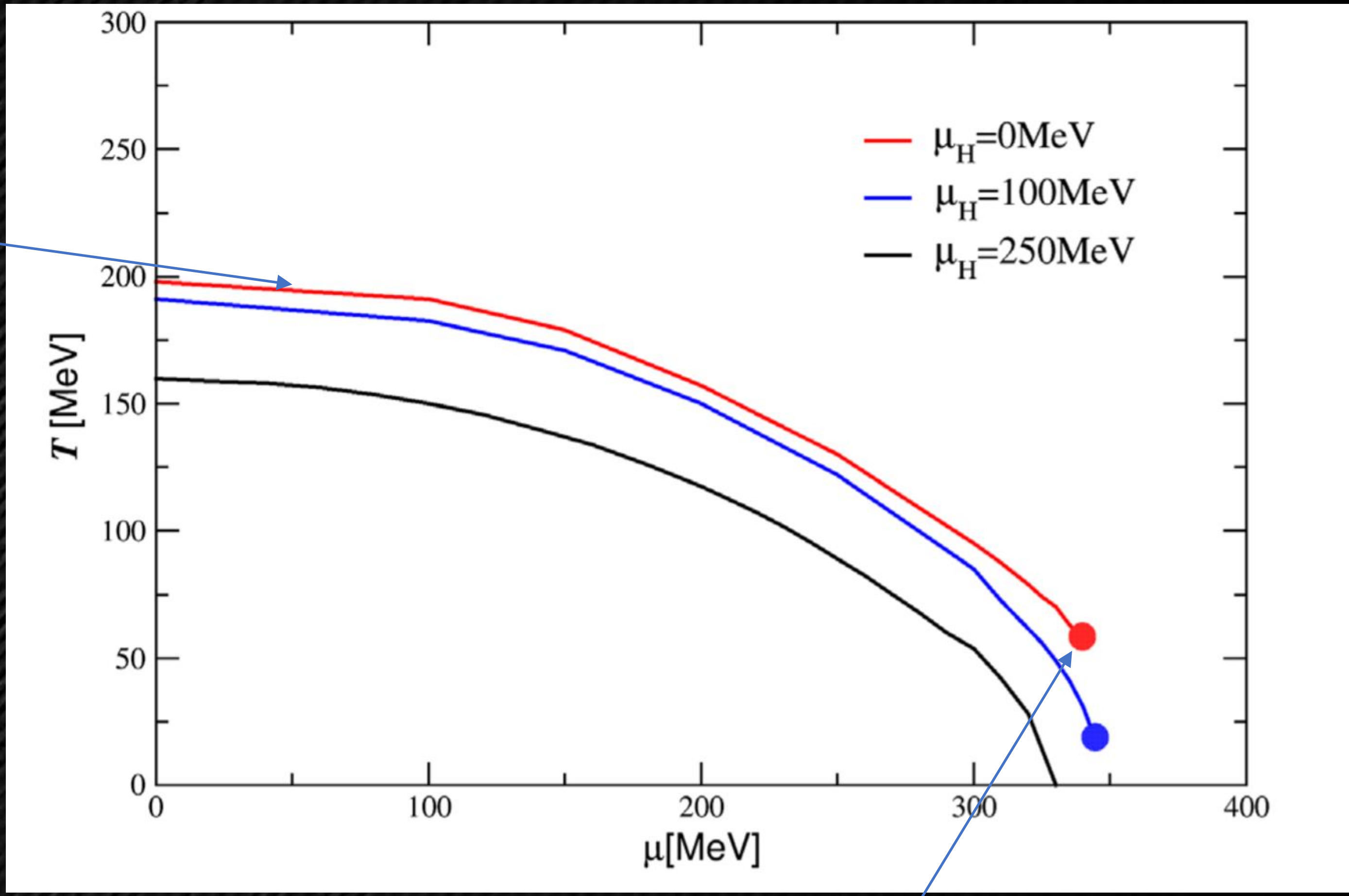
$$\frac{\partial \Omega}{\partial \mu_e} = 0$$

Phase diagram

Chiral crossover



$$(\sigma/2)^{1/3} = (\langle \bar{u}u + \bar{d}d \rangle /2)^{1/3}$$

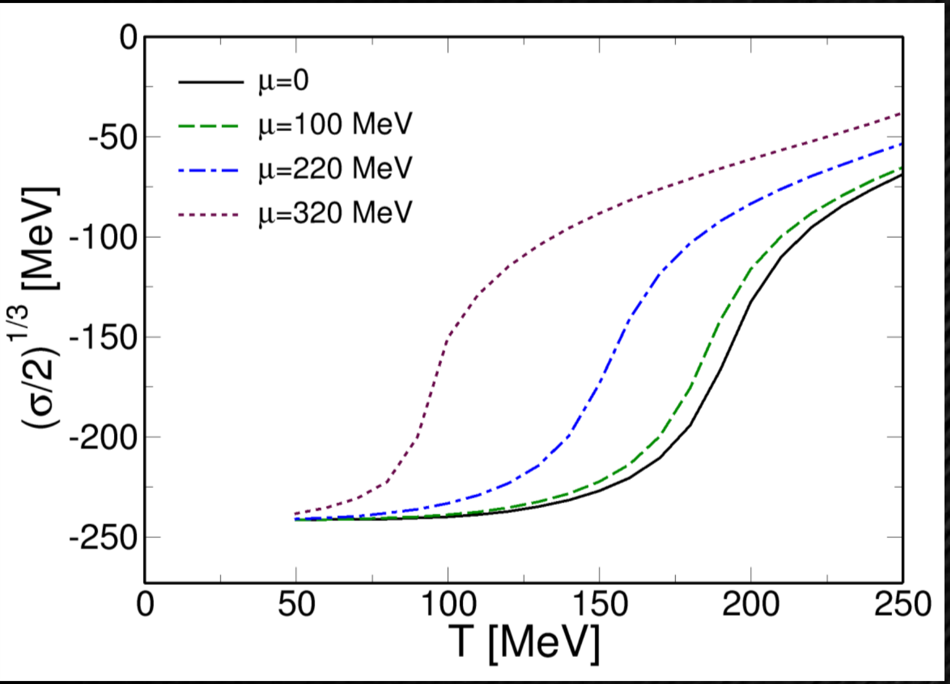


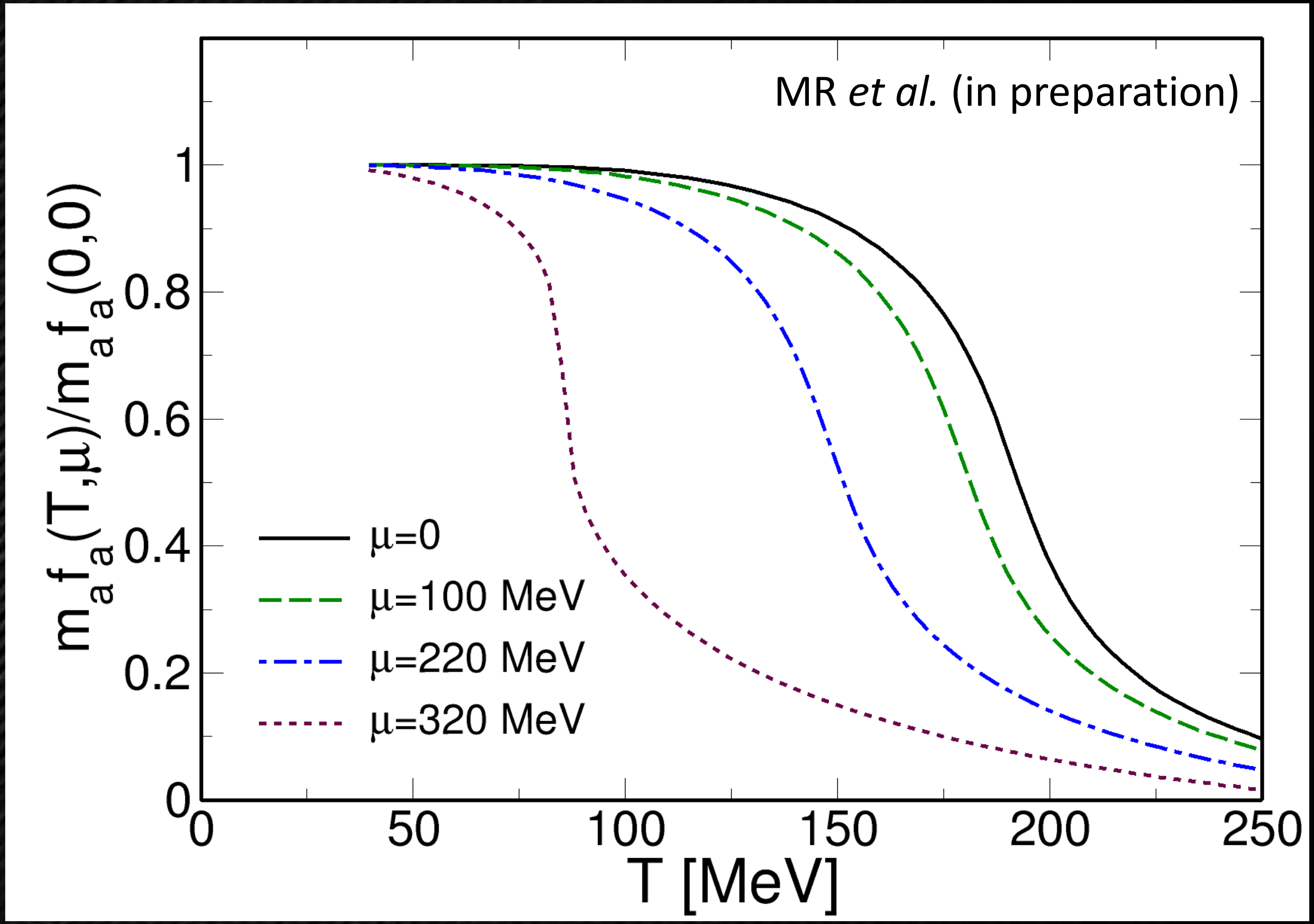
M.R. *et al.* (2021)

Critical endpoint

The axion mass

$$\Omega \approx \Omega(a=0) + \frac{m_a^2}{2}a^2 + \frac{\lambda_a}{4!}a^4$$





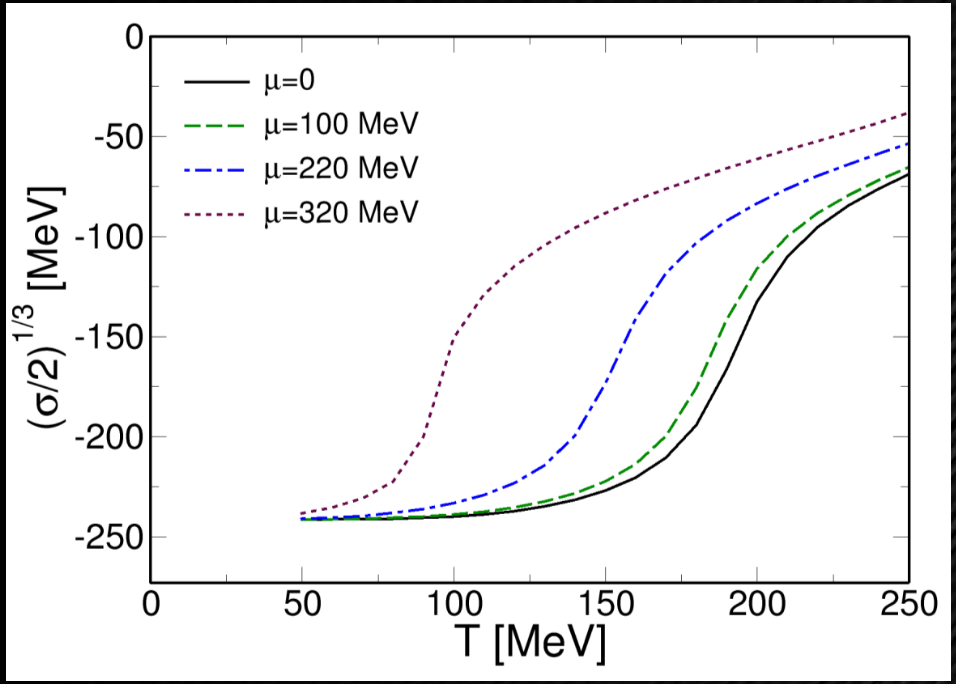
$T = \mu = 0$

$m_a f_a = 6.38 \times 10^3 \ \mathrm{MeV}^2$

Axion mass is very sensitive to the phase transition.

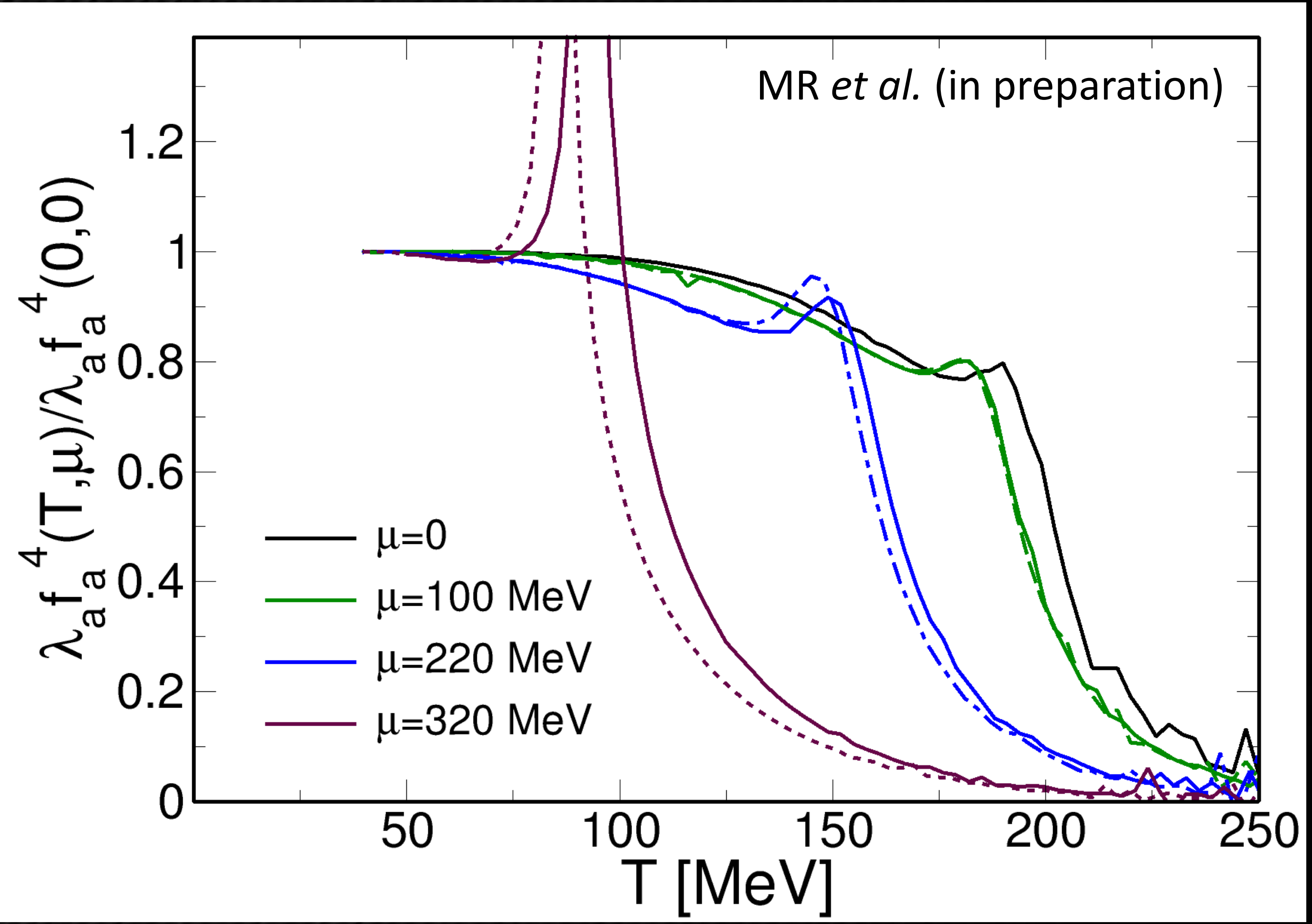
The axion self-coupling

$$\Omega \approx \Omega(a=0) + \frac{m_a^2}{2}a^2 + \frac{\lambda_a}{4!}a^4$$



$$T = \mu = 0$$

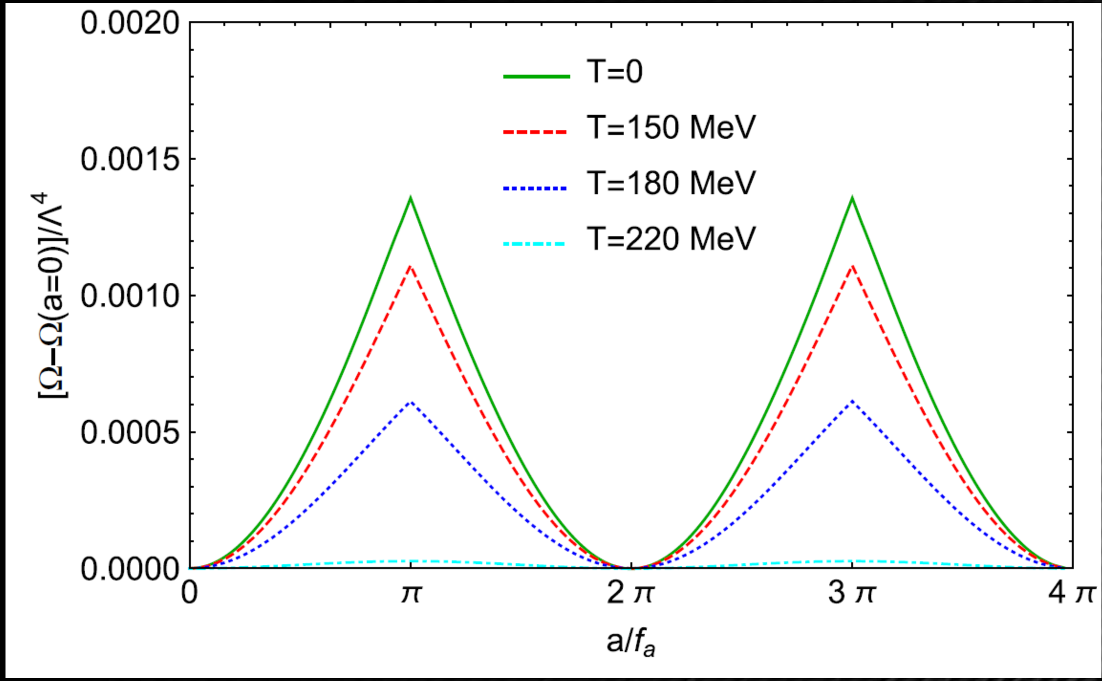
$$\lambda_a f_a^4 = -(55.63\ \mathrm{MeV})^4$$



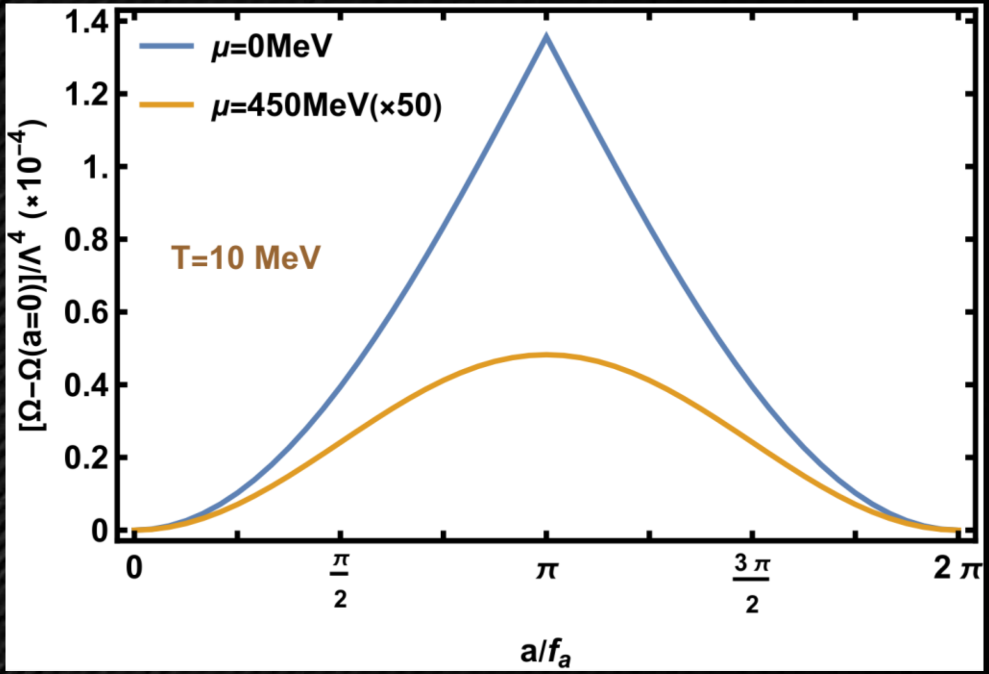
In agreement with Abhishek *et al.* (2021)

Axion coupling gets enhanced in the critical region

The axion potential



Lu *et al.* (2019)



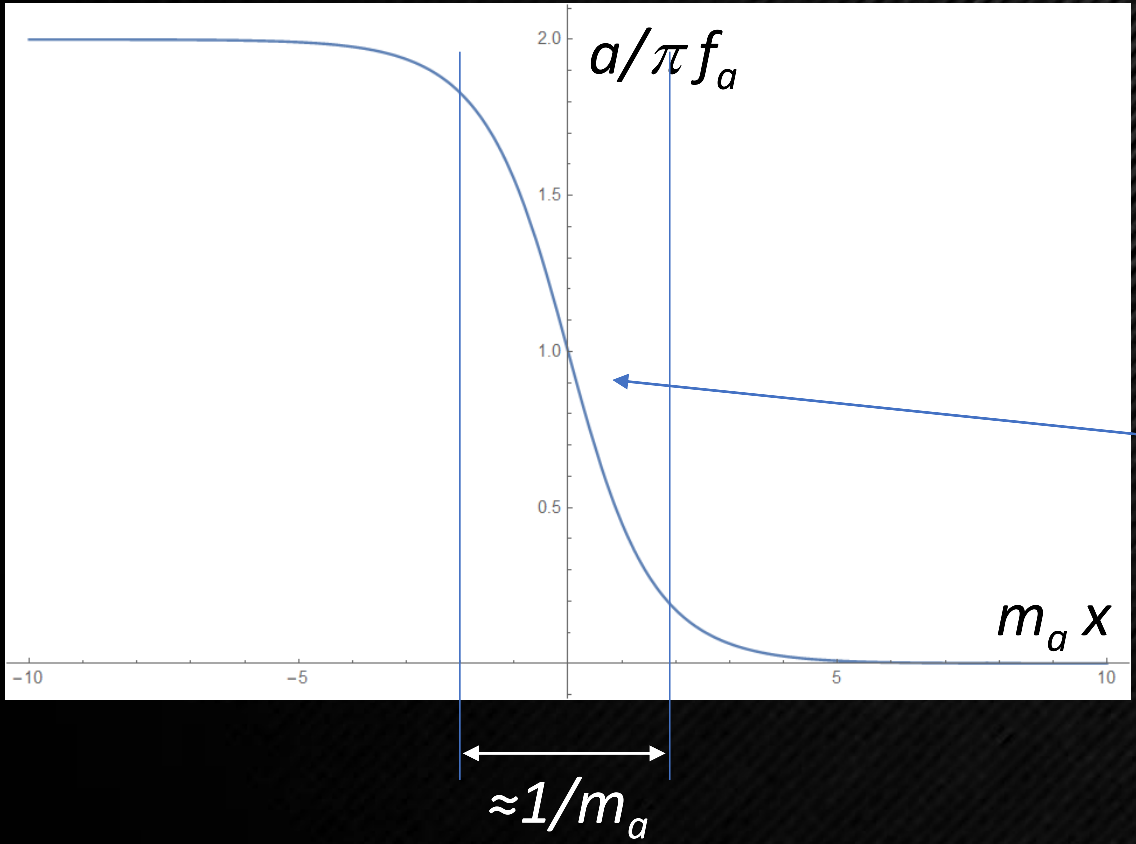
Chiral restoration implies a substantial decrease of the free energy barrier:

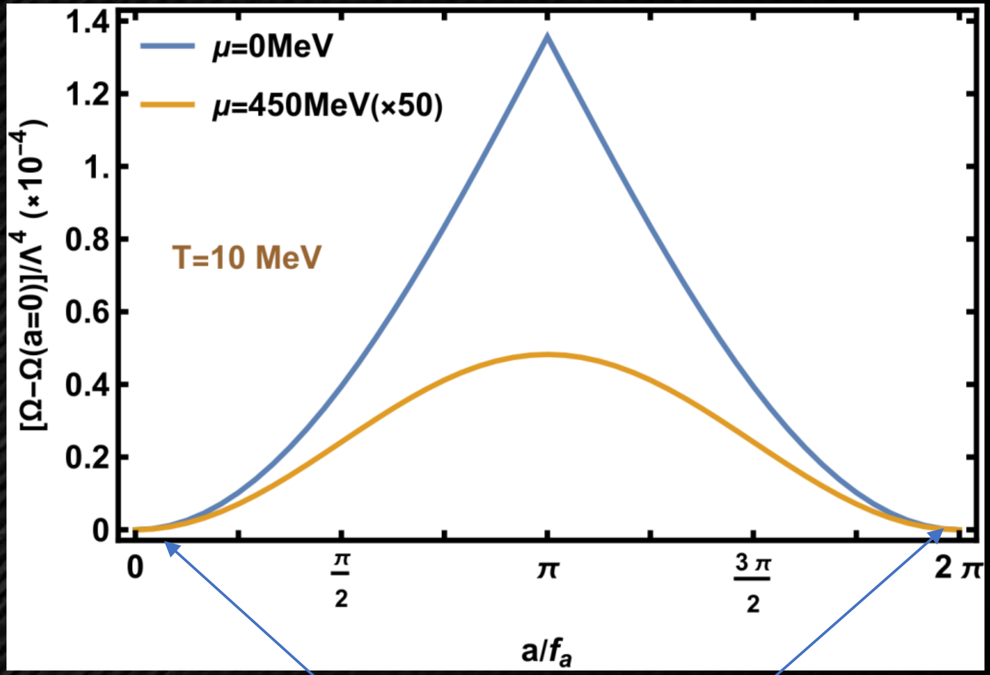
Energy cost to form field configurations connecting two adjacent vacua is lowered.

The axion walls

$$\mathcal{L} = \frac{1}{2}\partial^\mu a \partial_\mu a - V(a/f_a)$$

$$V(\theta) = \Omega(\theta) - \Omega(0)$$



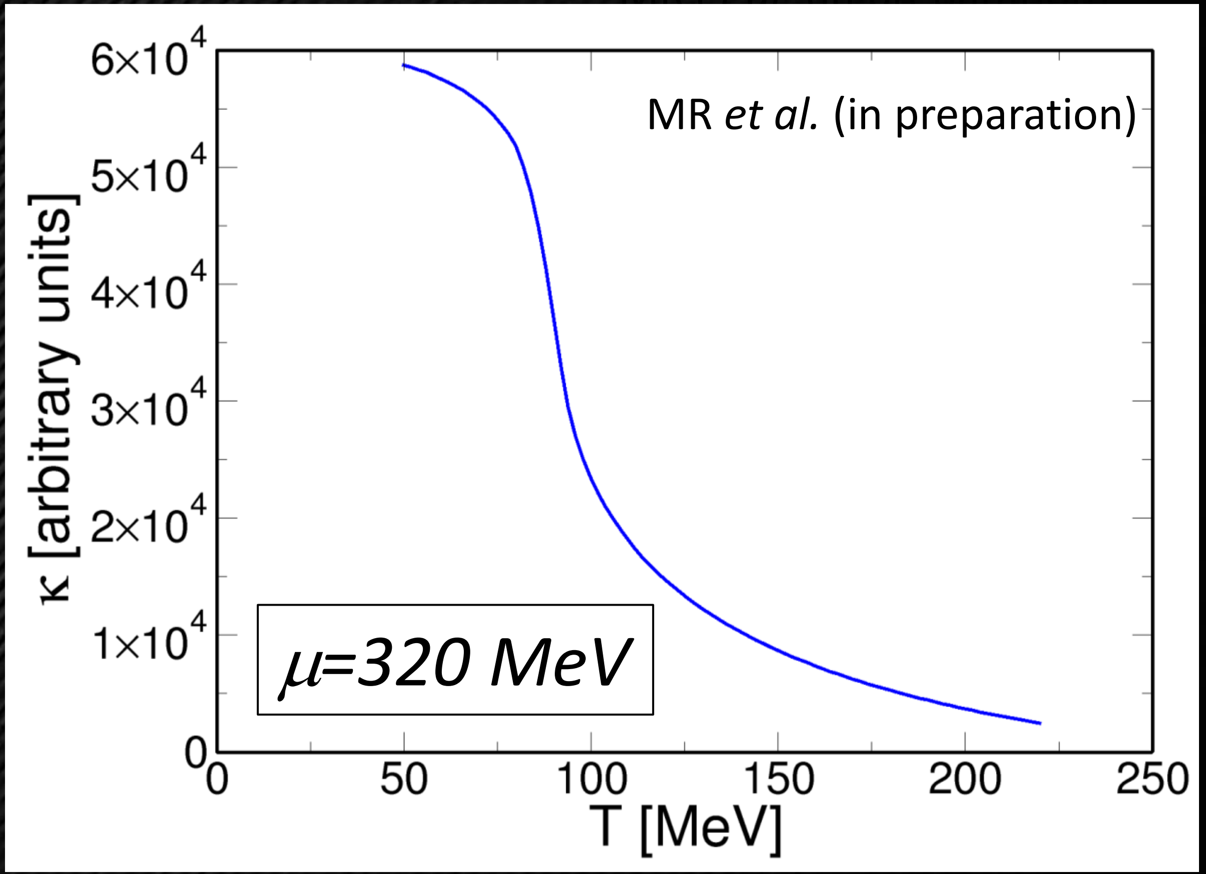


Field configuration that interpolates between two adjacent vacua

The axion wall surface tension

Surface tension

$$\kappa \equiv \frac{E}{L^2} = \int_{-\infty}^{+\infty} dx \left[\frac{1}{2} \left(\frac{da}{dx} \right)^2 + V(a/f_a) \right]$$



Restoration of chiral symmetry lowers the κ of the walls.

The axion wall surface tension

$$\kappa \equiv \frac{E}{L^2} = \int_{-\infty}^{+\infty} dx \left[\frac{1}{2} \left(\frac{da}{dx} \right)^2 + V(a/f_a) \right]$$

MR *et al.* (in preparation)

Free energy cost to add one wall $\sim L^2$

Free energy of bulk quark matter $\sim \mu^4 L^3, T^4 L^3, \mu^2 T^2 L^3$

Ratio of the two $\sim 1/L$

In the thermodynamic limit, the free energy cost of adding one wall to the bulk quark matter is zero:

Axion walls might be abundant in quark matter

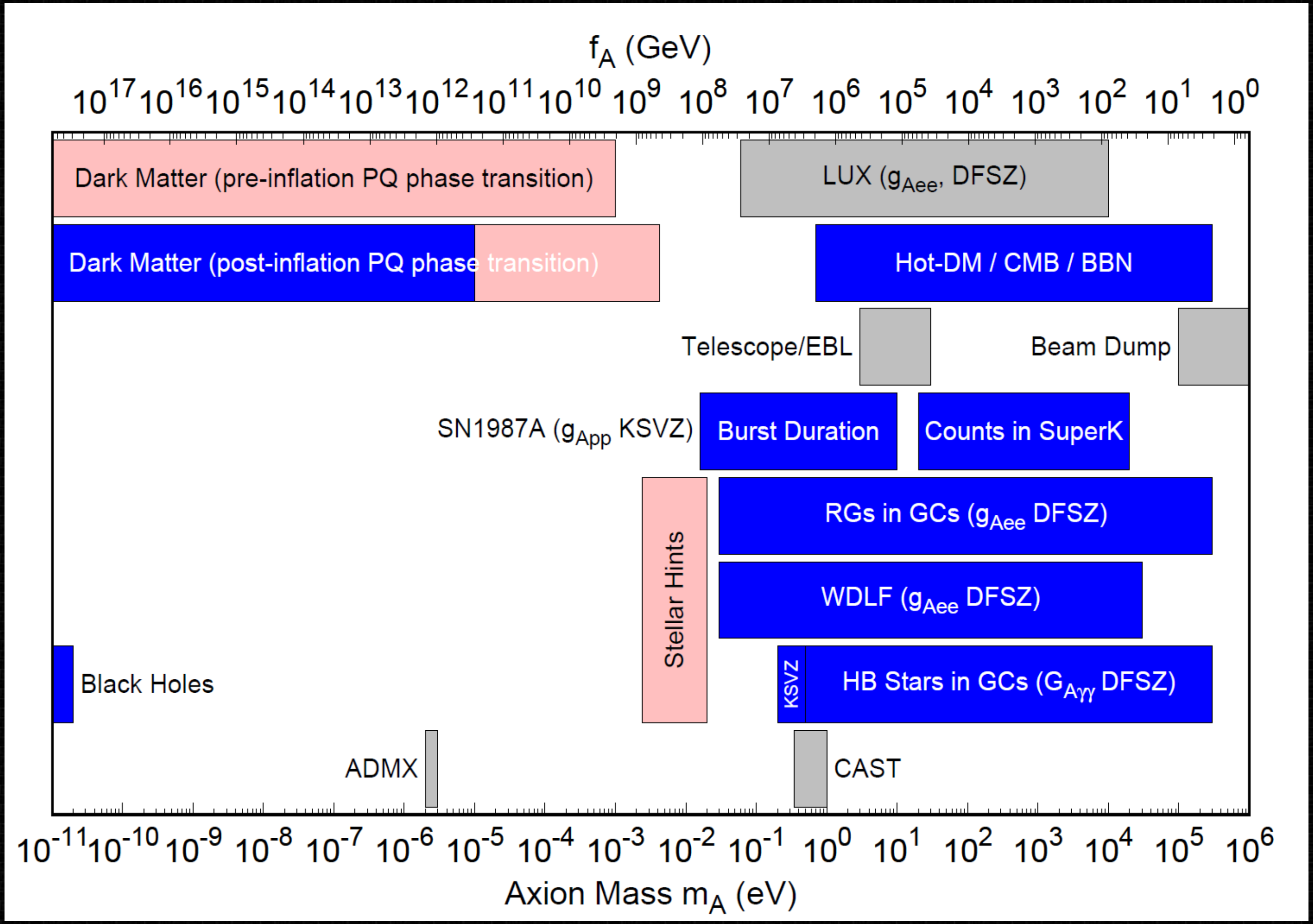
Conclusions and outlook

- *NJL-like models are useful tools to study the axion potential, and the θ-vacua, of hot and dense QCD*
- *Axion potential is very sensitive to the QCD phase transition*
- *Axion self-coupling enhanced near critical endpoint*
- *Axion walls might be abundant in hot&dense quark matter*

- o *Trapped axions and cooling by axions in mergers (see also Alford's talk)*
- o *Coupling to EM field (see also De La Incera and Ferrer's talks)*
- o *Axions in color-superconductive phases.*

Appendix

The axion's exclusion region



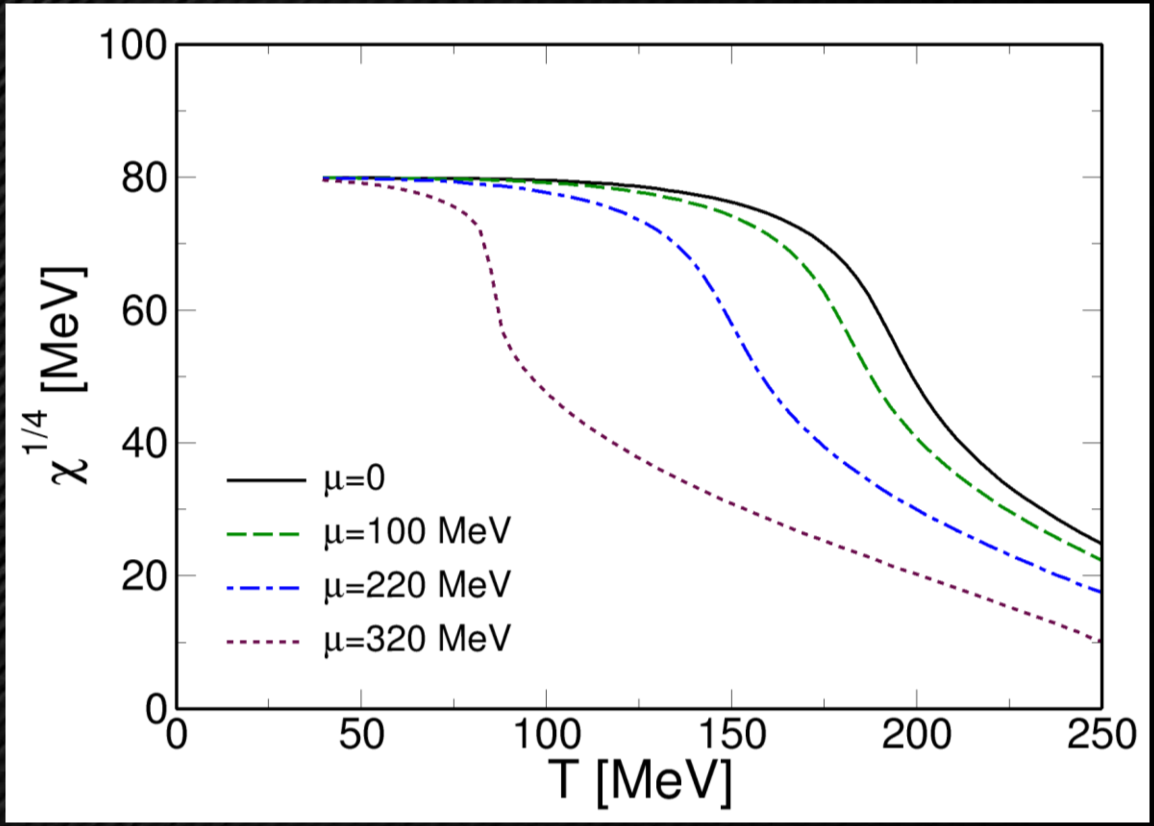
Topological susceptibility

Put

$$\theta = a/f_a$$

then

$$\chi = \left.\frac{\partial^2\Omega}{\partial\theta^2}\right|_{\theta=0}$$



Qualitative understanding

$$\chi = |\langle \bar{q}q \rangle| \frac{m_u m_d}{m_u + m_d}$$

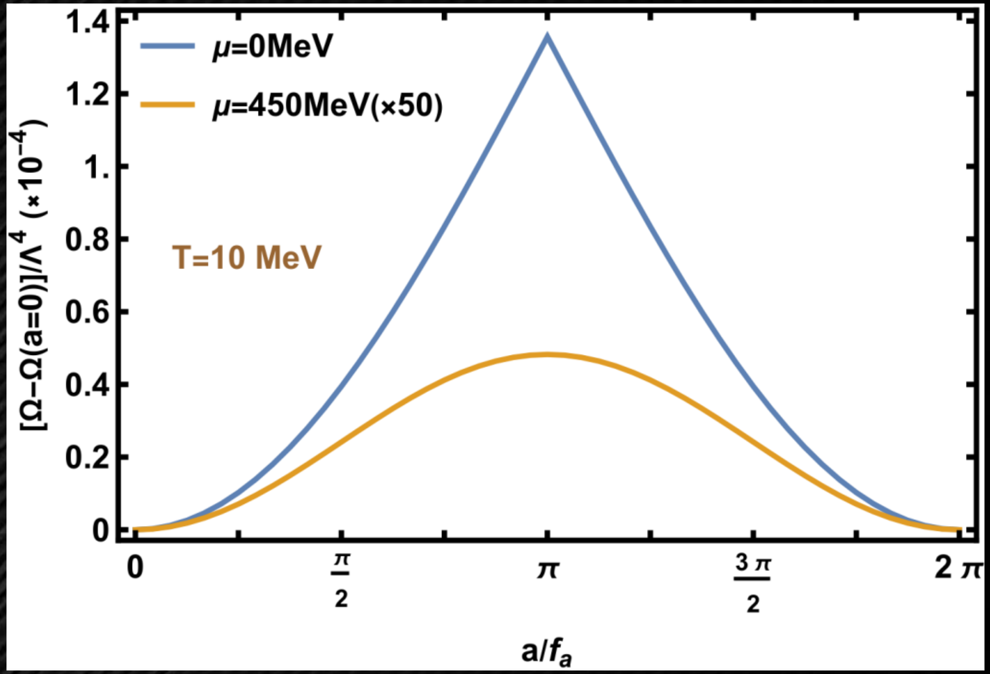
For vacuum-QCD:
Veneziano (1979), Di Vecchia-Veneziano (1980)
It works faily well for thermal/dense QCD within NJL model
M.R. and Gatto (2011)

Measures fluctuations of the topological charge

The axion walls

$$\mathcal{L} = \frac{1}{2}\partial^\mu a \partial_\mu a - V(a/f_a)$$

$$V(\theta) = \Omega(\theta) - \Omega(0)$$



In general

$$(m_a f_a)\int_\pi^{\theta(m_a x)} \frac{d\theta}{\sqrt{V(\theta)}} = \pm\sqrt{2} m_a x$$

In the chiral restored phase

$$V(\theta) = V_0(1-\cos\theta) = m_a^2 f_a^2 (1-\cos\theta)$$

$$\theta_\pm(x) = 4\arctan\exp\left(\pm m_a x\right)$$

Sine-Gordon soliton

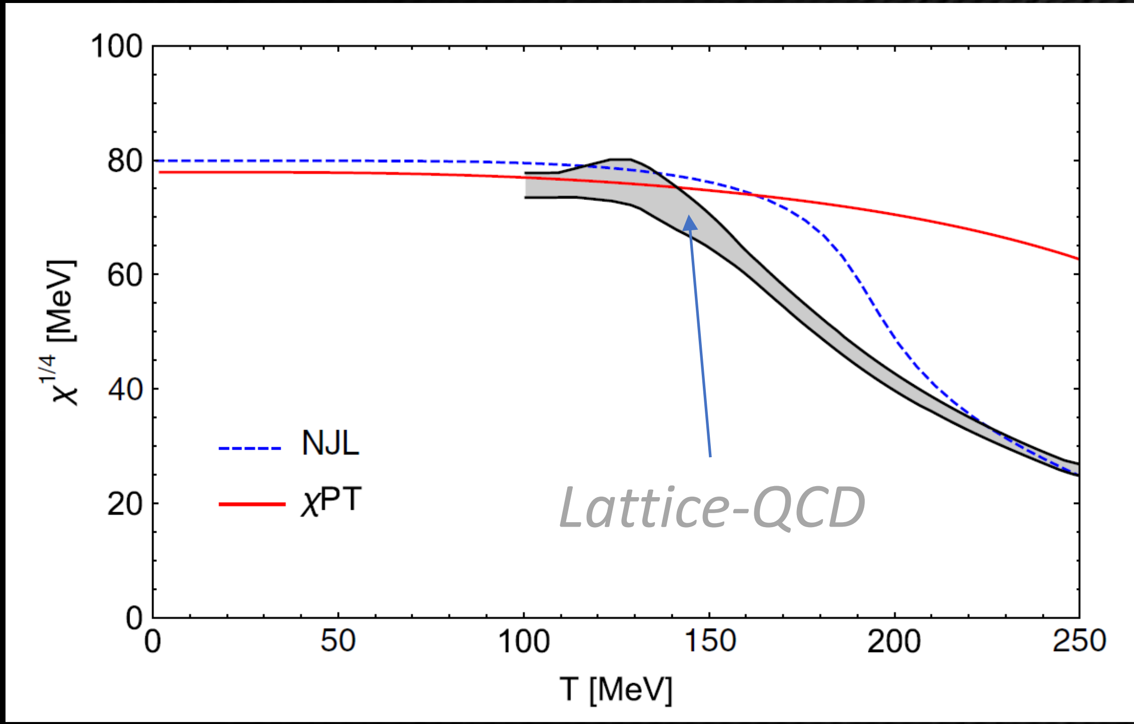
Plan of the talk

- *The QCD axion (a quick, informal introduction)*
- *Modeling the hot&dense QCD medium*
- *The axion potential*
  - *The axion mass*
  - *Axion self-coupling near the quark-hadron phase transition*
  - *The axion walls in a hot/dense QCD medium*
- *Conclusions and Outlook*

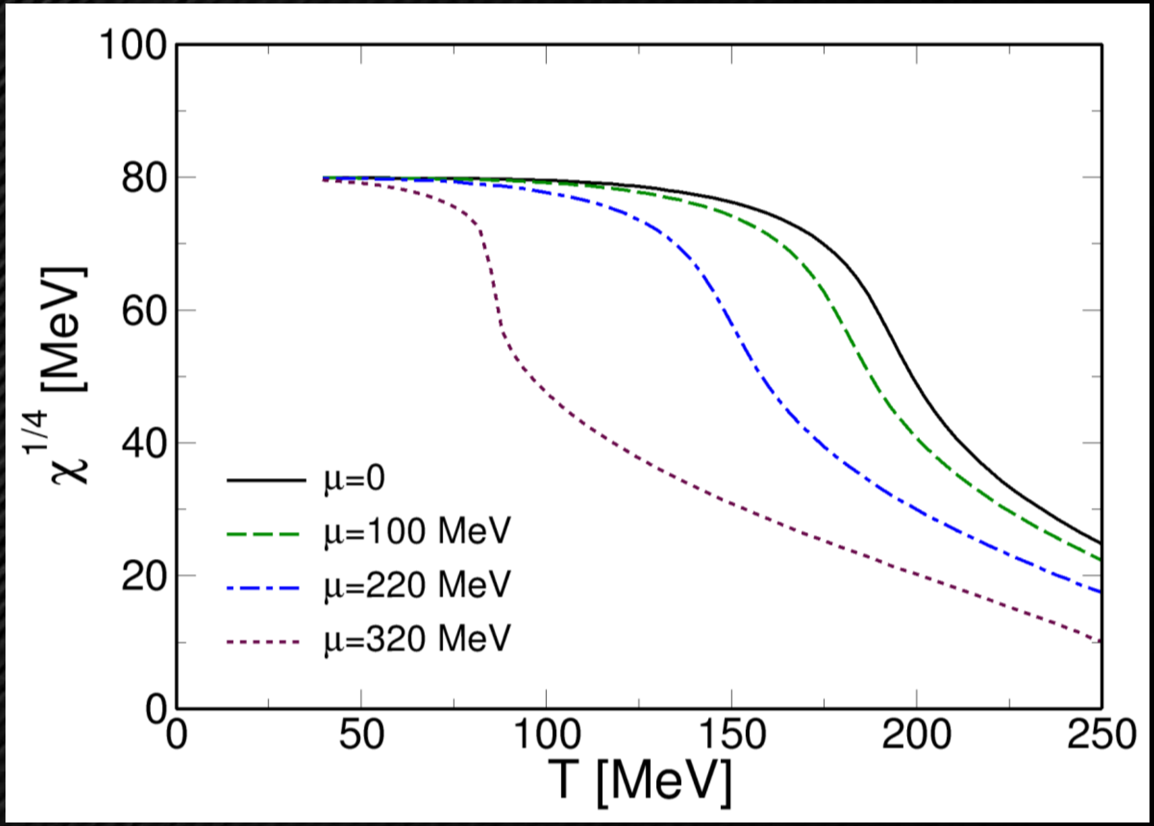
$\theta = a/f_a$

Topological susceptibility

$$\chi = \left.\frac{\partial^2 \Omega}{\partial \theta^2}\right|_{\theta=0}$$



Lu *et al.* (2019)
Lattice data from Borsanyi *et al.* (2016)
χPT result from Grilli di Cortona *et al.* (2016)



Measures fluctuations of the topological charge