Naturalness and Hierarchy
An analysis and an Outrageous Proposal

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Renormalization and Renormalization Group

... Under the spell of Wilson RG ...

Steven Weinberg, **Why The Renormalization Group Is A Good Thing**

“I think that this in the end is what the renormalization group is all about ... you may use any degrees of freedom you like to describe a physical system, **but if you use the wrong ones you’ll be sorry**.”

Renormalization and Renormalization Group

RENORMALIZATION AND EFFECTIVE LAGRANGIANS

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There is a strong intuitive understanding of renormalization, due to Wilson, in terms of the scaling of effective lagrangians. We show that this can be made the basis for a proof of perturbative renormalization. We first study renormalizability in the language of renormalization group flows for a toy renormalization group equation. We then derive an exact renormalization group equation for a four-dimensional $\lambda\phi^4$ theory with a momentum cutoff. We organize the cutoff dependence of the effective lagrangian into relevant and irrelevant parts, and derive a linear equation for the irrelevant part. A lengthy but straightforward argument establishes that the piece identified as irrelevant actually is so in perturbation theory. This implies renormalizability. The method extends immediately to any system in which a momentum-space cutoff can be used, but the principle is more general and should apply for any physical cutoff. Neither Weinberg's theorem nor arguments based on the topology of graphs are needed.

Theory contains an ultimate scale $\Lambda_{phys} \Rightarrow \mathcal{L}_{\Lambda_{phys}}$

Above Λ_{phys} : UV completion needed

Below Λ_{phys} : ok $\mathcal{L}_{\Lambda_{phys}}$ Effective Field Theory

. . . But it seems we have some Problems . . .

Problems: Naturalness and Hierarchy

The Higgs mass m_H^2 receives **Unnaturally Large** contributions from the quantum fluctuations: $\Delta m_H^2 \sim \Lambda_{phys}^2$

Quadratic sensitivity to the ultimate scale of the Theory

Physically: left-over from the UV completion of the SM

Large hierarchy between $m_H^2(\mu_F)$ and $m_H^2(\Lambda_{phys})$

(from now on $\Lambda_{phys} \to \Lambda$)

Very useful example: Scalar Theory in d-dimensions

$d =$ integer dimension (no dim reg)

- Wilsonian Effective Action: $S_k[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + V_k(\phi)\right]$

Wilson (Polchinski) RG Equation (LPA)

$$k \frac{\partial}{\partial k} V_k(\phi) = -\frac{k^d}{(4\pi)^{\frac{d}{2}} \Gamma\left(\frac{d}{2}\right)} \ln \left(\frac{k^2 + V_k''(\phi)}{k^2} \right)$$

- UV boundary: $V_\Lambda(\phi) \equiv V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\mu^{4-d}\lambda_0}{4!}\phi^4$

Approximating $V_k(\phi)$ in the rhs as $V_k(\phi) \to V_\Lambda(\phi)$

One-loop effective potential

$$V_{1l}(\phi) = V_0(\phi) + \underbrace{\frac{1}{2}\int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln \left(1 + \frac{m_0^2 + \frac{1}{2}\mu^{4-d}\lambda_0\,\phi^2}{k^2} \right)}_{\delta V(\phi)}$$

Lesson: One-loop Effective Potential Approx. of the Wilsonian Potential

Let us focus on the Radiative Correction $\delta V(\phi)$

$$\delta V(\phi) = \frac{1}{2} \int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{M^2(\phi)}{k^2}\right) \equiv \delta V_1(\phi) + \delta V_2(\phi)$$

where $$M^2(\phi) \equiv m_0^2 + \tfrac{1}{2}\mu^{4-d}\lambda_0\,\phi^2$$

$$\delta V_1(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}} \int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\,(1-t)^{\frac{d}{2}-1}\,t^{-\frac{d}{2}}$$

$$\delta V_2(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \left(\frac{\Lambda}{\mu}\right)^{d} \ln\left(1 + \frac{M^2(\phi)}{\Lambda^2}\right)$$

Calculating $\delta V(\phi)$

For any **integer** *d*:

$$\delta V_1(\phi) = \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}}\int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\, t^{-\frac{d}{2}}(1-t)^{\frac{d}{2}-1} =$$

$$= \lim_{z\to d}\left[A_1(z) - A_2(z)\right]$$

where z is **complex**, and

$$A_1(z) \equiv F(z)\cdot \overline{B}\left(1-\frac{z}{2},\frac{z}{2}\right) \qquad A_2(z) \equiv F(z)\cdot \overline{B}_i\left(1-\frac{z}{2},\frac{z}{2};\frac{M^2(\phi)}{M^2(\phi)+\Lambda^2}\right)$$

$$F(z) \equiv \frac{\mu^z}{z(4\pi)^{\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{z}{2}}$$

$\overline{B}$ and $\overline{B}_i$ are (the analytic extensions of) the Beta functions

Both $\overline{B}$ and $\overline{B}_i$ have poles in $z = 2, 4, 6, \ldots$

$\delta V_1(\phi)$ finite $\Rightarrow$ the poles of A_1 and A_2 have to cancel each other

Example: $\delta V(\phi)$ in $d = 4$ dimensions

$z \equiv 4 - \epsilon$. Expanding in powers of ϵ and M^2/Λ^2

$$A_1(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)}$$

$$A_2(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)} + \cancel{\mathcal{O}\left(\frac{M^2}{\Lambda^2}\right)}$$

$$- \frac{\mu^{-\epsilon}}{64\pi^2}\left[M^2(\phi)\right]^2\left(\frac{\Lambda^2}{M^2(\phi)} - \log\frac{\Lambda^2}{M^2(\phi)}\right)$$

Remember: $\delta V_1(\phi) = \lim_{\epsilon\to 0}\left[A_1(4-\epsilon) - A_2(4-\epsilon)\right]$. Adding $\delta V_2(\phi)$

$$\delta V(\phi) = \delta V_1 + \delta V_2 = \frac{\Lambda^2 M^2(\phi)}{32\pi^2} - \frac{\left[M^2(\phi)\right]^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2(\phi)} + \frac{1}{2}\right) + \cancel{\mathcal{O}\left(\frac{\phi^6}{\Lambda^2}\right)}$$

$$\Rightarrow \quad V_{1l}(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 + \frac{\Lambda^2 M^2}{32\pi^2} - \frac{\left(M^2\right)^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2} + \frac{1}{2}\right)$$

No reference whatsoever to ϵ (of course!)

With $\Omega_0 = \Omega + \delta\Omega_\Lambda$, $m_0^2 = m^2 + \delta m_\Lambda^2$, $\lambda_0 = \lambda + \delta\lambda_\Lambda$

and $\delta\Omega_\Lambda = -\frac{m^2\Lambda^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$; $\delta m_\Lambda^2 = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$

$\delta\lambda_\Lambda = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$

... where $\delta\Omega_\Lambda$ and δm_Λ^2 realize fine-tunings (*) ...

$\Rightarrow$ **Renormalized One-Loop Effective Potential (take $\Omega = 0$)**

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

(*) Physically ... in the parameter space of the theory we go close to the Critical region, or Critical Surface ...

... Let's move now to Dim Reg ...

Radiative correction $\delta V(\phi)$ in Dim. Reg.

- $\delta V(\phi)$ in **Dim Reg.** $\boxed{d \to \text{complex}\,,\; d \equiv 4-\epsilon}$

$$\delta V(\phi) \to \delta V_\epsilon(\phi) \equiv -\frac{\mu^{4-\epsilon}}{2(4\pi)^{2-\frac{\epsilon}{2}}}\left(\frac{M^2(\phi)}{\mu^2}\right)^{2-\frac{\epsilon}{2}} \overline{\Gamma}\left(\frac{\epsilon}{2}-2\right)$$

$$= \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon}+\gamma+\ln\frac{M^2(\phi)}{4\pi\mu^2}-\frac{3}{2}\right)+\mathcal{O}(\epsilon)$$

$\overline{\Gamma}(-d/2)$ defined for any complex $d \neq 2, 4, 6, \ldots$

- Counterterms in $\overline{MS}$ scheme $\left(\overline{\epsilon} \equiv \epsilon\left(1+\frac{\epsilon}{2}\ln\frac{e^\gamma}{4\pi}\right)\right)$:

$$\delta\Omega_\epsilon = \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon} \quad , \quad \delta m^2_\epsilon = \frac{\lambda m^2}{16\pi^2\overline{\epsilon}} \quad , \quad \delta\lambda_\epsilon = \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

- **Renormalized One-loop Effective Potential (take $\Omega = 0$) as before**

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2+\frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2+\frac{\lambda}{2}\phi^2}{\mu^2}\right)-\frac{3}{2}\right]$$

Before going on with our analysis ... Let's hear "news" from the Literature

“Dim Reg” versus “Wilson” (= “successive elimination of modes”)

Views on “Dim Reg” and “Wilson”

1) Typical textbook statement . . . “Dimensional Regularization has no direct physical interpretation” (J. Zinn-Justin - **Quantum field theory of critical phenomena**)

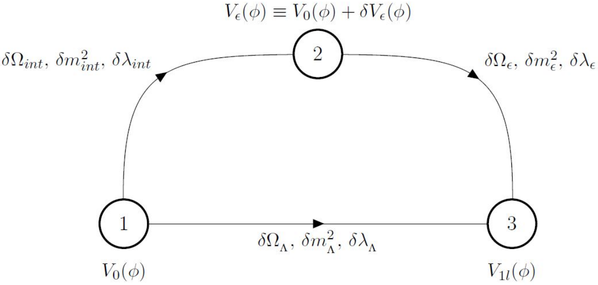
2) Recent ideas (gaining lot of followers)

“Maybe power divergences vanish because the ultimate unknown physical cut-off behaves like dimensional regularization” (M. Farina, D. Pappadopulo and A. Strumia, JHEP 08 (2013) 022)

“Wilsonian computation techniques attribute physical meaning to momentum shells of loop integrals” . . . “The naturalness problem can be more generically formulated as a problem of the **Effective Theory Ideology**” (A. Salvio and A. Strumia, JHEP 06 (2014) 080)

Accordingly **DR** should have special physical properties that make it the **correct way** to calculate the quantum fluctuations ... while **Wilson** ... **incorrect** ...

Dim Reg.: Physical Meaning? ... Special Physical Properties?



$$V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 = (\Omega + \delta\Omega_\Lambda) + \frac{1}{2}(m^2 + \delta m^2_\Lambda)\,\phi^2 + \frac{1}{4!}(\lambda + \delta\lambda_\Lambda)\phi^4$$

$$= (\Omega + \delta\Omega_{int} + \delta\Omega_\epsilon) + \frac{1}{2}(m^2 + \delta m^2_{int} + \delta m^2_\epsilon)\phi^2 + \frac{1}{4!}(\lambda + \delta\lambda_{int} + \delta\lambda_\epsilon)\phi^4$$

$\Rightarrow$ Ren.Pot. : $$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

Dim Reg.: Physical Meaning? ... Special Physical Properties?

DR secretly realizes the fine-tuning:

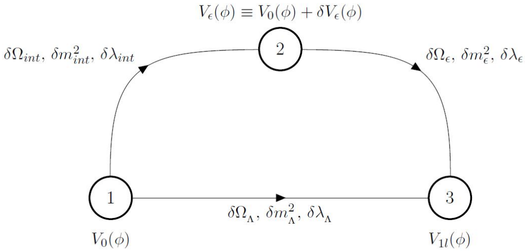
$$\delta\Omega_{int} = -\frac{m^2\Lambda^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon}$$

$$\delta m^2_{int} = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{\lambda m^2}{16\pi^2\overline{\epsilon}}$$

$$\delta\lambda_{int} = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

DR has a Physical Meaning but No Special Physical Properties. It implements the Wilsonian iterative elimination of modes for including the quantum fluctuations in the Effective Theory, and **secretly** realizes the fine-tuning

Summary on DR



DR setting, "Bubble (2)", obtained by introducing an intermediate step, (1) → (2), in the process of obtaining the Renormalized Potential, "Bubble (3)".

DR provides a shortcut: "Bubble (3)" is reached starting from "Bubble (2)". The fine-tuning step "Bubble (1)" → "Bubble (2)" is skipped (secretly realized)

Lesson: DR is a way to implement the Wilson's strategy in the perturbative regime, where the *fine-tuning* (in the Wilsonian language: tuning toward the critical regime, critical surface) is secretly performed.

Naturalness and Dimensional Regularization

What should we then say on those attempts to solve the Naturalness/Hierarchy problem with DR?

- **Classically Scale Invariant BSM**. The theory does not possess mass or length scales ⇒ only dimension four operators
- Dimensional Regularization used ⇒ Scale Invariance only softly broken ⇒ apparently **no fine-tuning needed** . . . seems good . . .
- . . . But . . . we have just shown . . . DR **secretly realizes the fine-tuning**

⇒ **No way to solve the Naturalness/Hierarchy problem with DR**

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Consider now attempts to solve the NH problem in a RG framework

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- S. Mooij and M. Shaposhnikov, arXiv:2110.15925.

"Wilson" versus "Perturbatively-Renormalized" RG Equations

Scalar Theory : $\mathcal{L}_\Lambda = \frac{1}{2}(\partial_\mu \phi_\Lambda)^2 + \frac{1}{2} m_\Lambda^2 \phi_\Lambda^2 + \frac{\lambda_\Lambda}{4!}\phi_\Lambda^4$

Wilson-Polchinski RG Equations

$$\mu \frac{d\Omega}{d\mu} = -\frac{m^2\mu^2}{16\pi^2} + \frac{m^4}{32\pi^2} \quad ; \quad \mu \frac{dm^2}{d\mu} = -\frac{\lambda\mu^2}{16\pi^2} + \frac{\lambda m^2}{16\pi^2} \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2}$$

$\mu \in [0, \Lambda]$ is the running scale. Λ is the UV boundary (physical cut-off)

Define:

$$m_{\rm cr}^2(\mu) \equiv \frac{\lambda(\mu)}{16\pi^2}\mu\,\delta\mu \quad \text{and} \quad \widetilde{m}^2(\mu-\delta\mu) \equiv m^2(\mu-\delta\mu) - m_{\rm cr}^2(\mu)$$

$$\Omega_{\rm cr}(\mu) \equiv \frac{\widetilde{m}^2(\mu)}{16\pi^2}\mu\,\delta\mu \quad \text{and} \quad \widetilde{\Omega}(\mu-\delta\mu) \equiv \Omega(\mu-\delta\mu) - \Omega_{\rm cr}(\mu)$$

Perturbatively-Renormalized RG Equations $(\delta\mu \to 0)$

$$\mu \frac{d\widetilde{\Omega}}{d\mu} = \frac{\widetilde{m}^4}{32\pi^2} = \beta_\Omega \quad ; \quad \mu \frac{d\widetilde{m}^2}{d\mu} = \frac{\lambda \widetilde{m}^2}{16\pi^2} = \widetilde{m}^2\gamma_m \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2} = \beta_\lambda$$

The Perturbatively-Renormalized RG Equations **contain the fine-tuning**

Physically: **Tuning towards the Critical Surface**

Perturbatively-Renormalized RG equations in the Standard Model

Well-known Standard Model perturbative RG equations (*)

$$\mu \frac{d}{d\mu} \lambda_i = \beta_{\lambda_i} \qquad \mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

λ_i $(i = 1, \ldots, 5)$ are the SM couplings

$(*)$ similarly for SM extensions

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 1 : Quantum Gravity "**miracle**" G.F. Giudice, PoS EPS-HEP2013, 163 (2013)

$$m_H^2(\Lambda) \ll \Lambda^2$$

With the SM perturbative γ_m $(\gamma_m \ll 1)$ $\Rightarrow$

Apparently no Hierarchy Problem : $m_H^2(\Lambda) \sim m_H^2(\mu_F)$

. . . But . . . remember . . . in the above RG Equation m_H^2 is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the RG Equation above

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 2 : "**Self-organized criticality**"

J. M. Pawlowski, M. Reichert, C. Wetterich and M. Yamada, Phys. Rev. D **99**, 086010 (2019)

Assumes Quantum Gravity might give a non-perturbative $\gamma_m \sim 2 \;\Rightarrow$

Hierarchy can be tolerated : $m_H^2(\Lambda) \gg m_H^2(\mu_F)$

. . . But . . . remember . . . m_H^2 is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the above RG Equation

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 3 : $m_H^2(\mu)$ from $\lambda(\mu)$ and $v(\mu)$ ...

P. H. Chankowski, A. Lewandowski, K. A. Meissner and H. Nicolai, Mod. Phys. Lett. A **30**, 1550006 (2015)

M. Holthausen, K. S. Lim and M. Lindner, JHEP **02**, 037 (2012)

Apparently no large corrections : $m_H^2(\mu_F) \sim 125\,\mathrm{GeV}$

... However ... same problem as before ... Tuning encoded in the RG equation for the vev $v(\mu)$ (equivalent to the above RG equation for $m_H^2(\mu)$)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbative-Renormalized RG equations in the Standard Model

Attempt 4 : "Finite formulation" of QFT using RG equations *à la* Callan-Symanzik for the Green's functions . . .

S. Mooij and M. Shaposhnikov, arXiv:2110.05175

S. Mooij and M. Shaposhnikov, arXiv:2110.15925

Apparently no quadratic corrections for the mass m^2 of scalar particles

However . . . Tuning encoded in taking derivatives with respect to m^2 of the Green's functions, until they become finite

Callan has shown that this is just a way of implement the subtraction of Λ^2 and $\log \Lambda$ terms

C. G. Callan, Jr., Conf. Proc. C **7507281**, 41-77 (1975)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

... Let us re-think to the whole problem ...

... What do we know, after all? ...

The inclusion of quantum fluctuations in the Effective Theory through the Wilson successive elimination of modes is **PHYSICALLY MANDATORY**

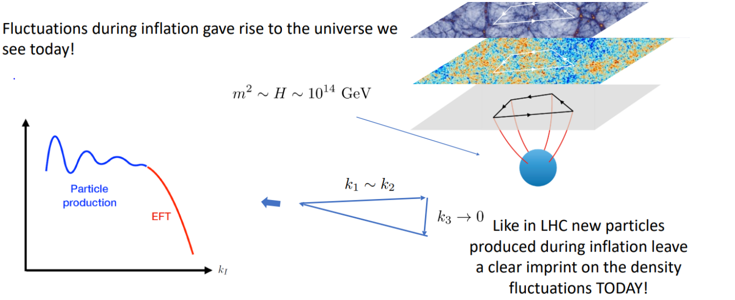
... Under the spell of Wilson RG ...

We have to take seriously the consequences of this physical and unavoidable starting point

... Starting point ?

... Yes, Starting Point ...

… Initial condition for our EFT (Effective Field Theory) …



Picture from Massimo Taronna talk at this conference

We need the Boundary Condition for our Effective Field Theory (EFT), that might be the Standard Model itself, or an extension of it … or …

… But once we have the EFT we have to take seriously the consequences of that ….

Wilson RG Eqs. for our Effective Theory: Scalar theory

Wilsonian action $\quad S_k[\phi] = \int d^4x \left(\frac{1}{2}\, \partial_\mu \phi\, \partial_\mu \phi + U_k(\phi) \right)$

Truncating the potential $\quad U_k(\phi) = \frac{1}{2} m_k^2 \phi^2 + \frac{1}{4!} \lambda_k \phi^4$

RG equations for m_k^2 and λ_k

$$k\frac{dm_k^2}{dk} = -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2 + m_k^2}$$

$$k\frac{d\lambda_k}{dk} = \frac{k^4}{16\pi^2}\frac{3\lambda_k^2}{(k^2 + m_k^2)^2}$$

When $m_k^2 \ll k^2$

$$k\frac{dm_k^2}{dk} = -\frac{\lambda_k}{16\pi^2}k^2 + \frac{\lambda_k}{16\pi^2}m_k^2$$

$$k\frac{d\lambda_k}{dk} = \frac{3\lambda_k^2}{16\pi^2}$$

Now solve ...

Wilsonian RG Eqs., Scalar theory, solutions

$$\lambda(\mu) = \frac{\lambda_\Lambda}{1 - \frac{3}{16\pi^2}\lambda_\Lambda \log\left(\frac{\mu}{\Lambda}\right)}$$

In the perturbative regime (far from the Landau pole) $\lambda(\mu)$ has only a soft logarithmic running. Replace λ_k with the constant value λ_Λ

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda_\Lambda}{16\pi^2}} \left(m^2_\Lambda + \frac{\lambda_\Lambda \Lambda^2}{32\pi^2 - \lambda_\Lambda}\right) - \frac{\lambda_\Lambda \mu^2}{32\pi^2 - \lambda_\Lambda}$$

Above you see: RG fine-tuning

Outrageous Proposal

Who is afraid of fine-tuning?

The UV completion of our Effective theory forces on us the UV boundary values for the parameters of the Effective Field Theory ... RG Swampland ...

Wilsonian RG Equations for our Effective theory Standard Model

Wilsonian RG equation for the Higgs mass

$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2$$

$\gamma(\mu)$ mass anomalous dimension, $\alpha(\mu)$ combination of SM running couplings (gauge, Yukawa and scalar). For instance at one-loop

$$\alpha(\mu) = 12 y_t^2 - 12\lambda - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2$$

UV Boundary for our Effective Theory: Standard Model

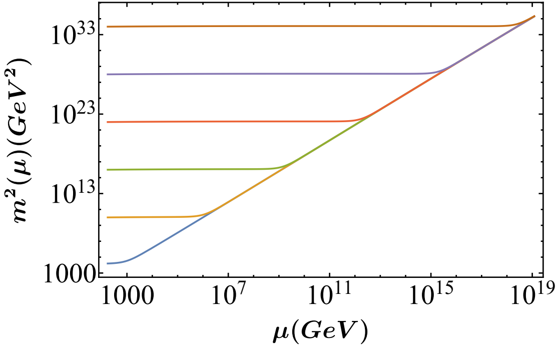


Figure: This figure shows how the RG flow changes when the cancellation between the $m^2(\Lambda)$ and Λ^2 terms is performed on different digits. All the trajectories freeze in the IR, and the degree of cancellation at the UV scale Λ determines where this will happen

RG Swampland

... Take home messages ...

- No way to solve the Naturalness/Hierarchy Problem with DR
- No way to solve the Naturalness/Hierarchy Problem using the SM perturbative RG equations
- If you take derivatives of your loop integrals to get finite result, you can't pretend to solve the NH problem
- The inclusion of quantum fluctuations in the Effective Theory through the Wilson successive elimination of modes is **PHYSICALLY MANDATORY** ... and ... The consequences of this unavoidable physical starting point have to be taken seriously ...
- ... Outrageous (???) proposal: The UV boundary values of the parameters of the Effective Theory (Standard Model) are given by its UV completion. Being the SM an Effective Theory, we can do nothing else than run the Wilson RG equations ... so the UV boundary values of the parameters **NEED** to be the right ones ! ...

RG Swampland

Backup Slides

Wilsonian - Polchinski RG equations

- Flow of the theory parameters:

$$\Lambda \frac{d}{d\Lambda}\Omega_0 = -\frac{m_0^2\Lambda^2}{16\pi^2} + \frac{m_0^4}{32\pi^2} \qquad \Lambda \frac{d}{d\Lambda} m_0^2 = -\frac{\lambda_0\Lambda^2}{16\pi^2} + \frac{\lambda_0 m_0^2}{16\pi^2} \qquad \Lambda \frac{d}{d\Lambda}\lambda_0 = \frac{3\lambda_0^2}{16\pi^2}$$

- From the Wegner-Houghton equation for $d = 4$, inserting the expansion $U_k(\phi) = \Omega_k + \frac{1}{2} m_k^2 \phi^2 + \frac{1}{4!}\lambda_k \phi^4 + \frac{1}{6!}\lambda_k^{(6)}\phi^6 + \dots$ we have the flow equations:

$$k\frac{\partial \Omega_k}{\partial k} = -\frac{k^4}{16\pi^2}\log\left(\frac{k^2+m_k^2}{k^2}\right)$$

$$k\frac{\partial m_k^2}{\partial k} = -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2+m_k^2}$$

$$k\frac{\partial \lambda_k}{\partial k} = \frac{k^4}{16\pi^2}\frac{3\lambda_k^2}{(k^2+m_k^2)^2}$$

- Under the condition $k^2 \gg m_k^2$, i.e. in the UV regime, they reduce to the bare parameters flow equations.

Critical term

- Finite difference RG equation for the mass:

$$m_0^2\left(\Lambda-\delta\Lambda\right)=m_0^2\left(\Lambda\right)+\frac{\delta\Lambda}{\Lambda}\,\frac{\lambda_0\left(\Lambda\right)}{16\pi^2}\,\Lambda^2-\frac{\delta\Lambda}{\Lambda}\,\frac{\lambda_0\left(\Lambda\right)m_0^2\left(\Lambda\right)}{16\pi^2}+\mathcal{O}\left(\frac{\delta\Lambda^2}{\Lambda^2}\right)$$

- Subtracted mass parameter at the scale $\Lambda-\delta\Lambda$

$$\widetilde{m}^2(\Lambda-\delta\Lambda)\equiv m_0^2(\Lambda-\delta\Lambda)-m_{\rm cr}^2(\Lambda)$$

where the *critical mass* $m_{\rm cr}^2$, and the boundary at Λ are given by

$$m_{\rm cr}^2(\Lambda)\equiv\frac{\lambda_0\left(\Lambda\right)}{16\pi^2}\,\Lambda\,\delta\Lambda\qquad\widetilde{m}^2(\Lambda)=m_0^2(\Lambda)$$

- In the limit $\delta\Lambda\to 0$ we recover the perturbative RG equations:

$$\beta_\Omega=\mu\frac{d\Omega}{d\mu}=\frac{m^4}{32\pi^2}\qquad\gamma_m=\frac{1}{m^2}\left(\mu\frac{dm^2}{d\mu}\right)=\frac{\lambda}{16\pi^2}\qquad\beta_\lambda=\mu\frac{d\lambda}{d\mu}=\frac{3\lambda^2}{16\pi^2}$$

- The renormalized RG equations **contain the fine-tuning**: physically, this corresponds to a *tuning towards the critical surface*.

Perturbative-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 5 : hierarchy between M_P and μ_F generated by an instanton configuration contributing to the vev of the Higgs field ...

M. Shaposhnikov and A. Shkerin, Phys. Lett. B **783**, 253 (2018)

M. Shaposhnikov and A. Shkerin, JHEP **10**, 024 (2018)

Apparently Hierarchy explained

however ... quantum corrections calculated with DR, and flow of the parameters studied with the perturbative RG flows ... same problems as before

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Attempts to a gauge invariant Wilsonian RG

- V. Branchina, K. Meissner and G. Veneziano, The Price of an exact, gauge invariant RG flow equation, Phys. Lett. B **574**, 319-324 (2003)
- S.P. de Alwis, Exact RG Flow Equations and Quantum Gravity, JHEP **03**, 118 (2018)