

Pseudo and quasi gluon PDF in the BFKL approximation

Giovanni Antonio Chirilli

University of Regensburg

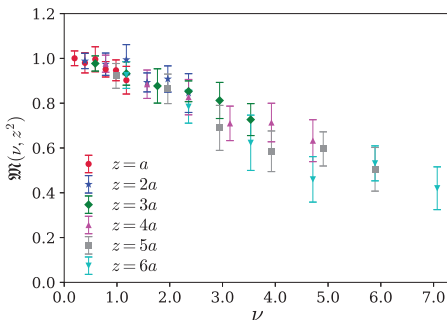
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- Lattice gauge theory is formulated in Euclidean space
 - ▶ direct calculation of the PDFs would be impossible for objects that are defined through the light-cone matrix element of gauge-invariant bi-local operators
- Idea Consider equal-time correlators and perform the Lattice analysis in coordinate space through the loffe-time distributions.
- Fourier transform in momentum space
 - ▶ quasi-PDF X. Ji (2013)
 - ▶ pseudo-PDF A. Radyushkin (2017)
- Lattice calculations provide values of the loffe-time distributions for a limited range of the distance separating the bi-local operators. In order to perform the Fourier transform for the quasi-PDF or the pseudo-PDF, it is then necessary to extrapolate the large-distance behavior.



$z = \{1, 2, 3, 4, 5, 6\} \times a = 0.094 \text{ fm}, 0.188 \text{ fm}, 0.282 \text{ fm}, 0.376 \text{ fm}, 0.470 \text{ fm}, 0.564 \text{ fm}$

loffe-time distribution at high energy

$$z_\mu z_\nu \langle P | G^{ai\mu}(z) [z, 0] G_i^{b\nu}(0) | P \rangle = 2 \varrho^2 \mathcal{M}_{pp}(\varrho, z^2)$$

loffe-time $\varrho \equiv z \cdot P$

z^μ space-like vector

$i = 1, 2$

Tensor decomposition over invariant amplitudes of the gluon matrix element

Tensor structures are build from P^μ , x^μ , and $g^{\mu\nu}$

$$\begin{aligned} M_{\mu\alpha;\lambda\beta} &\equiv \langle P | G_{\mu\alpha}(z) [z, 0] G_{\lambda\beta}(0) | P \rangle \\ &= I_{1\mu\alpha;\lambda\beta} \mathcal{M}_{pp} + I_{2\mu\alpha;\lambda\beta} \mathcal{M}_{zz} + I_{3\mu\alpha;\lambda\beta} \mathcal{M}_{zp} \\ &\quad + I_{4\mu\alpha;\lambda\beta} \mathcal{M}_{pz} + I_{5\mu\alpha;\lambda\beta} \mathcal{M}_{ppzz} + I_{6\mu\alpha;\lambda\beta} \mathcal{M}_{gg} \end{aligned}$$

the amplitudes \mathcal{M} are functions of the invariants z^2 and $z \cdot P = \varrho$ (Ioffe time)

light-cone Gluon distribution is obtained from

$$g_{\perp}^{\alpha\beta} M_{+\alpha;\beta+}(z^+, P) = -2(P^-)^2 \mathcal{M}_{pp}$$

The PDF is determined by the \mathcal{M}_{pp} structure

$$M_{+i;+i} = M_{0i;0i} + M_{3i;3i} + M_{0i;3i} + M_{3i;0i}$$

$$M_{0i;0i} + M_{ji;j} = 2p_0^2 \mathcal{M}_{pp} \xrightarrow{\text{high-energy}} M_{+i;+i}$$

At high energy (Regge limit) the transverse components are suppressed and we do not distinguish between the 0-component and the 3-component

Ioffe-time distribution at high energy

$$z_\mu z_\nu \langle P | G^{ai\mu}(z)[z, 0] G_i^{b\nu}(0) | P \rangle = 2 \varrho^2 \mathcal{M}_{pp}(\varrho, z^2)$$

Ioffe-time $\varrho \equiv z \cdot P$ z^μ space-like vector $i = 1, 2$

Gluon PDF $D_g(x_B)$ is defined as

$$\mathcal{M}_{pp}(z \cdot P, 0) = \frac{1}{2} \int_{-1}^1 dx_B e^{iz \cdot P x_B} x_B D_g(x_B)$$

Definition of the pseudo and quasi gluon PDF

loffe-time distribution at high energy

$$z_\mu z_\nu \langle P | G^{ai\mu}(z)[z, 0] G_i^{b\nu}(0) | P \rangle = 2 \varrho^2 \mathcal{M}_{pp}(\varrho, z^2)$$

loffe-time $\varrho \equiv z \cdot P$ z^μ space-like vector $i = 1, 2$

Pseudo-PDF: Fourier transform with respect to P keeping its orientation fixed

$$G_p(x_B, z^2) = \int \frac{d\varrho}{2\pi} e^{-i\varrho x_B} \mathcal{M}_{pp}(\varrho, z^2)$$

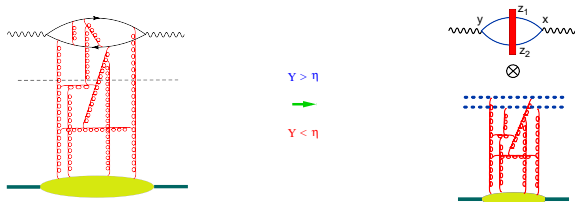
Quasi-PDF: Fourier transform with respect to z keeping its orientation fixed

$$G_q(x_B, P_\xi) = P_\xi \int \frac{d\varsigma}{2\pi} e^{-i\varsigma P_\xi x_B} \mathcal{M}_{pp}(\varsigma P_\xi, \varsigma^2)$$

$$\xi^\mu = \frac{z^\mu}{|z|} \quad P_\xi = P \cdot \xi$$

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2 I_{\mu\nu}^{\text{LO}}(z_1, z_2, x, y) \text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}$$

- Calculate LO Impact factor: $I_{\mu\nu}^{\text{LO}}(z_1, z_2, x, y)$
- Calculate evolution of matrix element $\text{Tr}\{\hat{U}_{z_1}^\eta \hat{U}_{z_2}^{\dagger\eta}\}$: BK/JIMWLK equation
 - ▶ we need only linear terms: BFKL
- Solve the evolution equation with initial condition: GBW/MV model
- Convolute the solution of the evolution equation with the impact factor



$$\langle P | G^{a,i-}(x)[x,0]G^{b,i-}(0) | P \rangle = \int d^2z_2 d^2z_z I_g(z_1, z_2; x) \langle P | \text{Tr}\{U(z_1)U^\dagger(z_2)\} | P \rangle$$

$$\langle P | \bar{\psi}(x)\gamma^- [x,0]\psi(0) | P \rangle = \int d^2z_2 d^2z_z I_q(z_1, z_2; x) \langle P | \text{tr}\{U(z_1)U^\dagger(z_2)\} | P \rangle$$

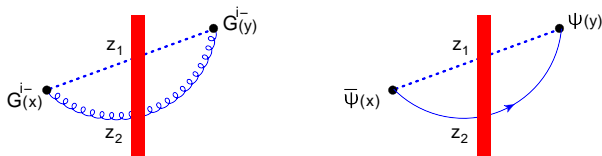
- Calculate coefficient functions (impact factors) I_g and I_q
- Convolute them with the solution of the evolution equation of relative matrix elements

High-energy OPE for loffe-time distribution

$$\langle P | G^{a i^-}(x) [x, 0] G^{b, i^-}(0) | P \rangle = \int d^2 z_2 d^2 z_z I_g(z_1, z_2; x) \langle P | \text{Tr} \{ U(z_1) U^\dagger(z_2) \} | P \rangle$$

$$\langle P | \bar{\psi}(x) \gamma^- [x, 0] \psi(0) | P \rangle = \int d^2 z_2 d^2 z_z I_q(z_1, z_2; x) \langle P | \text{tr} \{ U(z_1) U^\dagger(z_2) \} | P \rangle$$

Diagrams for the gluon impact factor I_g and quark impact factor I_q respectively

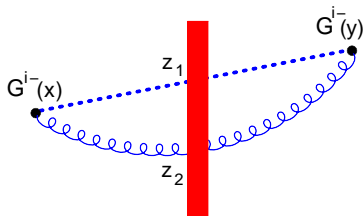


- Gluon: Tr trace in the adjoint representation;
- Quark: tr trace in the fundamental representation.

High-energy operator product expansion

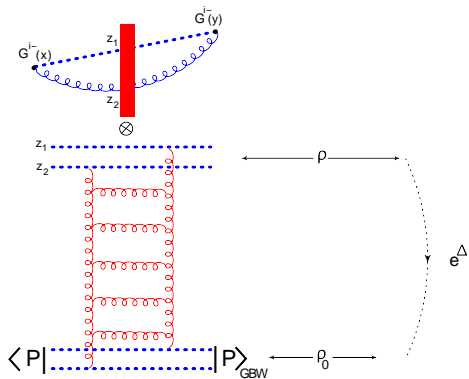
At high-energy we do not distinguish between the 0 and 3 components

$$\langle P | G^{ai-}(z)[z, 0] G_i^{b-}(0) | P \rangle = 2(P^-)^2 \mathcal{M}_{pp}(\varrho, z^2)$$

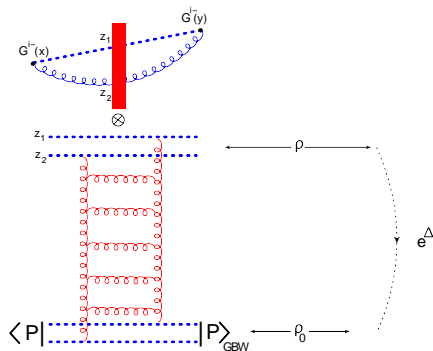


High-energy operator product expansion formalism is formulated in coordinate space \Rightarrow is suitable to reach our goal.

High-energy operator product expansion



High-energy operator product expansion



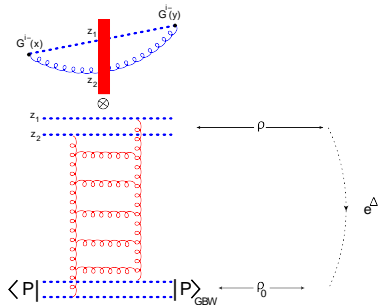
Resum $\alpha_s \ln \varrho$ with BFKL eq.

$$a = -\frac{2x^+y^+}{(x-y)^2 a_0}$$

$$2a \frac{d}{da} \mathcal{V}_a(z_\perp) = \frac{\alpha_s N_c}{\pi^2} \int d^2 z' \left[\frac{\mathcal{V}_a(z'_\perp)}{(z-z')_\perp^2} - \frac{(z, z')_\perp \mathcal{V}_a(z_\perp)}{z_\perp^2 (z-z')_\perp^2} \right]$$

$$\frac{1}{z_\perp^2} \mathcal{U}(z_\perp) \equiv \mathcal{V}(z_\perp) \quad \mathcal{U}(x_\perp, y_\perp) = 1 - \frac{1}{N_c} \text{tr}\{U(x_\perp)U^\dagger(y_\perp)\}$$

High-energy operator product expansion



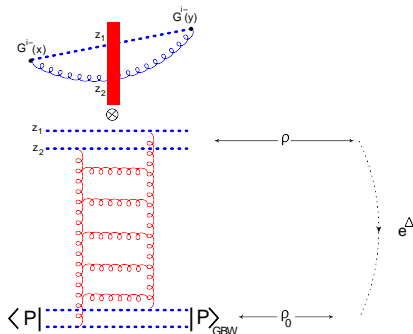
Resum $\alpha_s \ln g$ with BFKL eq.

$$a = -\frac{2x^+y^+}{(x-y)^2 a_0}$$

$$2a \frac{d}{da} \mathcal{V}_a(z_\perp) = \frac{\alpha_s N_c}{\pi^2} \int d^2 z' \left[\frac{\mathcal{V}_a(z'_\perp)}{(z-z')_\perp^2} - \frac{(z, z')_\perp \mathcal{V}_a(z_\perp)}{z_\perp'^2 (z-z')_\perp^2} \right]$$

solution
$$\mathcal{V}^a(z_{12}) = \int \frac{d\nu}{2\pi^2} (z_{12}^2)^{-\frac{1}{2}+i\nu} \left(\frac{a}{a_0} \right)^{\frac{N(\nu)}{2}} \int d^2 \omega (\omega_\perp^2)^{-\frac{1}{2}-i\nu} \mathcal{V}^{a_0}(\omega_\perp)$$

loffe-time distribution in the saddle-point approximation



Saddle point approximation

$$\mathcal{M}_{pp}(\varrho, z^2) = \frac{3N_c^2 Q_s \sigma_0}{128 \varrho |z|} \left(\frac{2\varrho^2}{z^2 M_N^2} + i\epsilon \right)^{\bar{\alpha}_s 2 \ln 2} \frac{e^{-\frac{\ln^2 \frac{Q_s |z|}{\varrho}}{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2\varrho^2}{z^2 M_N^2} + i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2\varrho^2}{z^2 M_N^2} + i\epsilon\right)}}$$

saturation scale Q_s , $\sigma_0 = 29.1 \text{ mb}$, M_N mass of the nucleon

Pseudo-PDF: Fourier transform with respect to $z \cdot P$

$$G_p(x_B, z^2) = \int \frac{d\varrho}{2\pi} e^{-i\varrho x_B} \mathcal{M}_{pp}(\varrho, z^2)$$

$$G_p(x_B, z^2) = -i \frac{3N_c^2 Q_s \sigma_0}{4\pi^4 |z|} \int d\nu \left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon \right)^{\frac{\aleph(\gamma)}{2}} \left(\frac{Q_s |z|}{2} \right)^{2i\nu} \\ \times \frac{\gamma \Gamma^2(1-\gamma) \Gamma^3(1+\gamma) \Gamma(\aleph(\gamma))}{\Gamma(2+2\gamma)} \sin\left(\frac{\pi}{2} \aleph(\gamma)\right) \text{sign}(x_B)$$

Let's take saddle point approximation

Pseudo-PDF: Fourier transform with respect to $z \cdot P$

$$G_p(x_B, z^2) = \int \frac{d\varrho}{2\pi} e^{-i\varrho x_B} \mathcal{M}_{pp}(\varrho, z^2)$$

Saddle point approximation

$$G_p(x_B, z^2) \simeq -i \frac{3N_c^2}{256} \frac{Q_s \sigma_0}{|z|} \frac{\text{sign}(x_B) e^{\frac{-\ln^2 \frac{Q_s |z|}{2}}{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon\right)}} \left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon \right)^{\bar{\alpha}_s 2 \ln 2}$$

Quasi-PDF: Fourier transform with respect to z^μ keeping its orientation fixed

$$G_q(x_B, P_\xi) = P_\xi \int \frac{d\zeta}{2\pi} e^{-i\zeta P_\xi x_B} \mathcal{M}_{pp}(\zeta P_\xi, \zeta^2)$$

$$\xi^\mu = \frac{z^\mu}{|z|} \quad P_\xi = P \cdot \xi$$

$$G_q(x_B, P_\xi) = i \frac{3N_c^2}{4\pi^4} Q_s \sigma_0 P_\xi |x_B| \int d\nu \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)^{\frac{\kappa(\gamma)}{2}} \left(\frac{Q_s^2}{4P_\xi^2 x_B^2} \right)^{i\nu} \\ \times \frac{\gamma \Gamma^2(1-\gamma) \Gamma^3(1+\gamma) \Gamma(2\gamma-2)}{\Gamma(2+2\gamma)} \sinh(\pi\nu)$$

and the saddle point approximation is

Quasi-PDF: Fourier transform with respect to z^μ keeping its orientation fixed

$$G_q(x_B, P_\xi) = P_\xi \int \frac{d\zeta}{2\pi} e^{-i\zeta P_\xi x_B} \mathcal{M}_{pp}(\zeta P_\xi, \zeta^2)$$

$$\xi^\mu = \frac{z^\mu}{|z|} \quad P_\xi = P \cdot \xi$$

Saddle point approximation

$$G_q(x_B, P_\xi) \simeq -\frac{3N_c^2}{256} Q_s \sigma_0 P_\xi |x_B| \frac{e^{-\frac{\ln^2 \frac{Q_s}{2P_\xi |x_B|}}{7\bar{\alpha}_s \zeta(3) \ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s \ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}} \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)^{\bar{\alpha}_s 2 \ln 2}$$

Pseudo-PDF vs. Quasi-PDF in the Saddle point approximation

$$G_p(x_B, z^2) \simeq -i \frac{3N_c^2}{256} \frac{Q_s \sigma_0}{|z|} \frac{\text{sign}(x_B) e^{\frac{-\ln^2 \frac{Q_s |z|}{2}}{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon\right)}} \left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon \right)^{\bar{\alpha}_s 2 \ln 2}$$

$$G_q(x_B, P_\xi) \simeq -\frac{3N_c^2}{256} Q_s \sigma_0 P_\xi |x_B| \frac{e^{\frac{\ln^2 \frac{Q_s}{2P_\xi |x_B|}}{7\bar{\alpha}_s \zeta(3) \ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s \ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}} \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)^{\bar{\alpha}_s 2 \ln 2}$$

n -th moment of the structure function

The Q^2 behavior of DIS structure function is obtained from the anomalous dimension of twist-two operators

$$\mu \frac{d}{d\mu} F_{\xi_+}^a \nabla_+^{n-2} F_+^{a \xi} = \gamma(\alpha_s, n) F_{\xi_+}^a \nabla_+^{n-2} F_+^{a \xi}$$

Dipole DIS cross-section can be written as

$$\sigma^{\gamma^* p}(x_B, Q^2) = \int d\nu F(\nu) x_B^{-\aleph(\nu)-1} \left(\frac{Q^2}{P^2} \right)^{\frac{1}{2}+i\nu}$$

$-q^2 = Q^2 \gg P^2$, and $s = (P+q)^2 \gg Q^2$

$\aleph(\gamma)$ BFKL pomeron intercept.

The n -th moment of the structure function is

$$\int_0^1 dx_B x_B^{n-1} \sigma^{\gamma^* p}(x_B, Q^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma \frac{F(\gamma)}{n-1-\aleph(\gamma)} \left(\frac{Q^2}{P^2} \right)^\gamma$$

Integrating over γ -parameter we get the anomalous dimensions of the leading and higher twist operators at the *unphysical point* $n = 1$.

$$\int_0^1 dx_B x_B^{n-1} \sigma^{\gamma^* P}(x_B, Q^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma \frac{F(\gamma)}{\omega - \aleph(\gamma)} \left(\frac{Q^2}{P^2} \right)^\gamma$$

Analytic continuation: $n - 1 \rightarrow \omega$ complex continuous variable

\Rightarrow Residues $\omega = \aleph(\gamma)$; expand $\aleph(\gamma)$ for small γ and solve for γ

$$\gamma(\alpha_s, \omega) = \frac{\alpha_s N_c}{\pi \omega} + \mathcal{O}(\alpha_s^2), \quad F(\omega, Q^2) \sim \left(\frac{Q^2}{P^2} \right)^{\frac{\alpha_s N_c}{\pi \omega}}$$

Thus, we get the analytic continuation of anomalous dimension at the *unphysical point* $j \rightarrow 1$ of twist-2 gluon operator $F_{\xi_+}^a \nabla^{-1} F_{\xi_+}^a$

Analytic continuation of light-ray operators at $j = 1$

$$F_{\xi+}^a(x) \nabla_+^{j-2} F_+^{a\xi}(x) \Big|_{x=0} = \frac{\Gamma(2-j)}{2\pi i} \int_0^{+\infty} du u^{1-j} F_{\xi+}^a(0) [0, un]^{ab} F_+^{b\xi}(un)$$

OPE in light-ray operators in QCD (Balitsky, Braun (1989))

2-point function in BFKL limit (Balitsky; Balitsky, Kazakov, Sobkov (2013-2018))

2-point function in triple Regge limit (Balitsky 2018)

A lot of activity on light-ray operators in CFT (e.g. Kravchuk, Simmons-Duffin (2018))

Forward matrix elements

$$\begin{aligned}\mathcal{S}_1^j &= \mathcal{F}^j + \frac{j-1}{4}\Lambda^j - j(j-1)\frac{1}{2}\Phi^j, \\ \mathcal{S}_2^j &= \mathcal{F}^j - \frac{1}{4}\Lambda^j + \frac{j(j+1)}{6}\Phi^j, \\ \mathcal{S}_3^j &= \mathcal{F}^j - \frac{j+2}{2}\Lambda^j - \frac{(j+1)(j+2)}{2}\Phi^j.\end{aligned}$$

Correlation function in CFT at high-energy, $j \rightarrow 1$

$$\langle \mathcal{F}^j(x_\perp) \mathcal{F}^{j'}(y_\perp) \rangle = \langle \mathcal{S}_1^j(x_\perp) \mathcal{S}_1^{j'}(y_\perp) \rangle \stackrel{\text{CFT}}{=} \delta(j-j') \frac{C(\Delta, j) s^{j-1}}{[(x-y)_\perp^2]^{\Delta-1}} \mu^{-2\gamma_a}$$

Δ : canonical dimension d plus anomalous dim. γ_a

μ : normalization point.

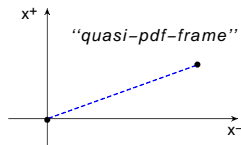
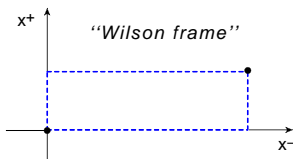
$C(\Delta, j)$: unknown structure constant. Calculate it in the BFKL limit.

Wilson frame vs quasi-pdf frame

In the BFKL limit the two-point correlation function is UV divergent.

Regularization: point splitting \Rightarrow

- **Wilson frame** Balitsky (2013, 2019), Balitsky, Kazhakov, Sobko (20013-2018)
 - ▶ Motivation: Give an example of actual calculation of correlation function; **goal**: understanding full dynamics of $\mathcal{N}=4$ SYM.
- **quasi-pdf frame** G.A.C. *Quark and Gluon quasi-pdf at low-x*
 - ▶ Motivation: check of the calculation comparing with expected CFT general result; **goal**: calculate the behavior of the quasi-pdf at small- x_B .



Leading and next-to-leading twist

Analytic continuation of local-operator \Rightarrow light-ray operators

$$F_{p_1\xi}^a(x)\nabla^{j-2}F_{p_1\xi}^a(x)\Big|_{x=0} \stackrel{\text{forw.}}{=} \frac{1}{\Gamma(2-j)} \int_0^\infty dv v^{1-j} F_{p_1\xi}^a(0)[0, vp_1]^{ab} F_{p_1\xi}^b(vp_1)$$

$\omega = j - 1 \rightarrow 0 \Leftrightarrow x_B \rightarrow 0$ at $\frac{\alpha_s}{\omega} \sim 1 \Rightarrow$ resummation: BFKL eq.

To get the leading and next-to-leading residues we need to approach the DGLAP limit $\alpha_s \ll \omega \ll 1$

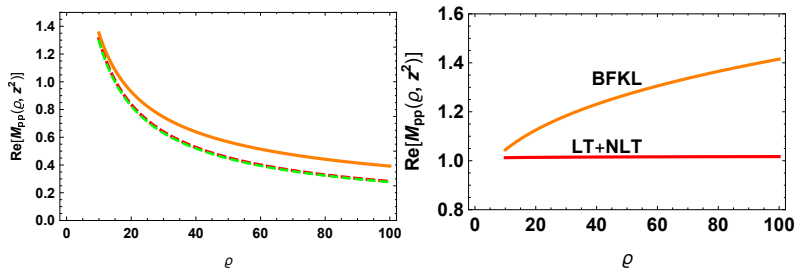
Leading and next-to-leading twist for the Ioffe-time-distribution

$$\mathcal{M}_{pp}(\varrho, z^2) = \frac{N_c^2}{8\pi^2\bar{\alpha}_s} \frac{Q_s^2\sigma_0}{\varrho} \left(\frac{4\bar{\alpha}_s \left| \ln \frac{Q_s|z|}{2} \right|}{\ln\left(\frac{2\varrho^2}{z^2 M_N^2} + i\epsilon\right)} \right)^{\frac{1}{2}} I_1(\tilde{t}) \left(1 + \frac{Q_s^2|z|^2}{5} \right) + \mathcal{O}\left(\frac{Q_s^4|z|^4}{16}\right)$$

with

$$\tilde{t} = \left[4\bar{\alpha}_s \left| \ln \frac{Q_s|z|}{2} \right| \ln\left(\frac{2\varrho^2}{z^2 M_N^2} + i\epsilon\right) \right]^{\frac{1}{2}}$$

loffe-time distribution at large-longitudinal distances



- Left panel
 - ▶ Orange curve is the BFKL resummation
 - ▶ Green-dash and red-dash are the LT and LT+NLT respectively.
- Right panel: BFKL resummation (Orange) and LT+NLT (red) both normalized to the LT.

BFKL resummation

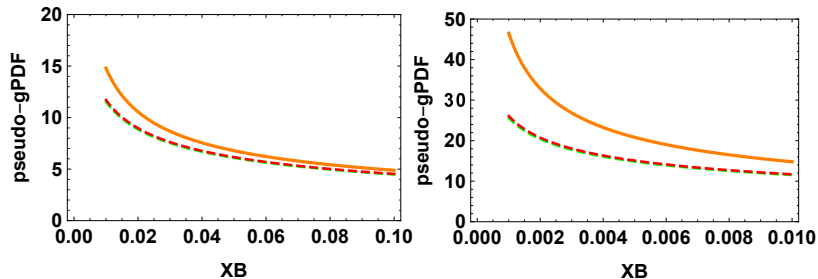
$$G_p(x_B, z^2) = -i \frac{3N_c^2 Q_s \sigma_0}{256 |z|} \frac{\text{sign}(x_B) e^{\frac{-\ln^2 \frac{Q_s |z|}{2}}{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s \ln\left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon\right)}} \left(\frac{2}{x_B^2 z^2 M_N^2} + i\epsilon \right)^{\bar{\alpha}_s 2 \ln 2}$$

Leading and next-to-leading twist

$$G_p(x_B, z^2) = \frac{N_c^2 Q_s^2 \sigma_0}{16\pi^3 \bar{\alpha}_s} \left(1 + \frac{Q_s^2 |z|^2}{5} \right) I_0(h) + O\left(\frac{Q_s^4 |z|^4}{16}\right)$$

$$h = \left[2\bar{\alpha}_s \left| \ln \frac{4}{|z|^2 Q_s^2} \right| \ln \frac{2}{x_B^2 |z|^2 M_N^2} \right]^{\frac{1}{2}}$$

Pseudo gluon PDF



Pseudo-PDF have typical behavior of gluon distribution at low- x_B .

BFKL resummation

$$\aleph(\gamma) \equiv \frac{\alpha_s N_c}{\pi} \left(2\psi(1) - \psi(\gamma) - \psi(1 - \gamma) \right) \quad \gamma = \frac{1}{2} + i\nu$$

$$G_q(x_B, P_\xi) \simeq -\frac{3N_c^2}{256} Q_s \sigma_0 P_\xi |x_B| \frac{e^{-\frac{\ln^2 \frac{Q_s}{2P_\xi |x_B|}}{7\bar{\alpha}_s \zeta(3) \ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)}}}{\sqrt{7\zeta(3) \bar{\alpha}_s \ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)}} \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)^{\bar{\alpha}_s 2 \ln 2}$$

Leading + next-to-leading twist

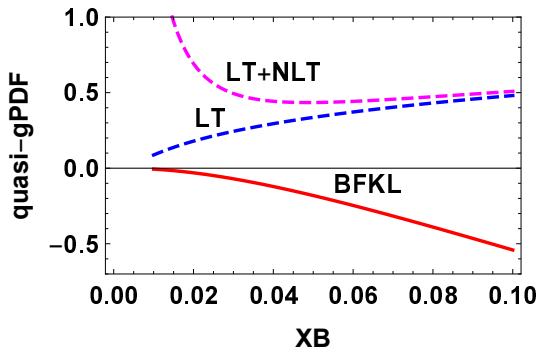
$$G_q(x_B, P_\xi) \simeq -\frac{N_c^2 Q_s^2 \sigma_0}{16\bar{\alpha}_s^2 \pi^3} \frac{1}{2\pi i} \int_{1-i\infty}^{1+i\infty} d\omega \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)^{\frac{\omega}{2}} \left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon \right)^{\frac{\bar{\alpha}_s}{\omega}} \left(\omega + \frac{2\bar{\alpha}_s Q_s^2}{5} \frac{1}{P_\xi^2 x_B^2} \right)$$

Usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B , is missing.

For low values of x_B and fixed values of P these corrections are enhanced rather than suppressed at this regime.

quasi gluon PDF

Here $P_\xi = 4$ GeV.



Behavior of curves will not change even for values of $P_\xi = 100$ GeV.

Quasi-PDF have rather unusual behavior at low- x_B .

The usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B , is missing.

- Large-distance behavior of the gluon loffe-time distribution is computed
 - ▶ loffe-time ϱ acts as rapidity parameter.
 - ★ $\alpha_s \ln \varrho$ resummed by BFKL eq.
 - ▶ loffe-time distribution is a very slowly varying function at large values of ϱ .
- Pseudo-PDF and quasi-PDF have a very different behavior at low- x_B .
 - ▶ pseudo-PDF have typical behavior of gluon distribution at low- x_B .
 - ▶ quasi-PDF have rather unusual behavior at low- x_B .
 - ★ usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B , is missing.
- The power corrections in the quasi-PDF do not come in as inverse powers of P but as inverse powers of $x_B P$
 - ▶ for low values of x_B and fixed values of P these corrections are enhanced rather than suppressed at this regime.

- The physical origin of the difference between the two distributions lay in the two different Fourier transforms under which they are defined.
- pseudo-PDF case
 - ▶ the scale is the resolution that is, the square of the length of the gauge link separating the bi-local operator.
- quasi-PDF case
 - ▶ the scale is the energy that is, the momentum of the hadronic target (the nucleon) projected along the direction of the gauge link.

Outlook

- Pseudo- and quasi-quark PDF in the BFKL approximation

LT and NLT quasi-PDF: analytic expression

$$\ln \frac{4P_\xi^2 x_B^2}{Q_s^2} < 0$$

$$G_q(x_B, P_\xi) \simeq \frac{N_c^2 Q_s^2 \sigma_0}{16\bar{\alpha}_s \pi^3} \left[\frac{\ln \left(-\frac{Q_s^2}{4P_\xi^2 x_B^2} - i\epsilon \right)}{\ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)} \left(J_0(m) - J_2(m) - \frac{2}{m} J_1(m) \right) + \frac{2Q_s^2}{5P_\xi^2 x_B^2} \left(\frac{2\bar{\alpha}_s \ln \left(-\frac{Q_s^2}{4P_\xi^2 x_B^2} - i\epsilon \right)}{\ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)} \right)^{\frac{1}{2}} J_1(m) \right]$$

$$m \equiv \left[2\bar{\alpha}_s \ln \left(-\frac{Q_s^2}{4P_\xi^2 x_B^2} - i\epsilon \right) \ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right) \right]^{\frac{1}{2}}$$

LT and NLT quasi-PDF: analytic expression

$$\ln \frac{4P_\xi^2 x_B^2}{Q_s^2} < 0$$

$$G_q(x_B, P_\xi) \simeq -\frac{N_c^2 Q_s^2 \sigma_0}{16\bar{\alpha}_s \pi^3} \left[\frac{\ln \left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon \right)}{\ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)} \left(I_0(\tilde{m}) + I_2(\tilde{m}) - \frac{2}{\tilde{m}} I_1(\tilde{m}) \right) + \frac{2Q_s^2}{5P_\xi^2 x_B^2} \left(\frac{2\bar{\alpha}_s \ln \left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon \right)}{\ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right)} \right)^{\frac{1}{2}} I_1(\tilde{m}) \right]$$

$$\tilde{m} \equiv \left[2\bar{\alpha}_s \ln \left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon \right) \ln \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon \right) \right]^{\frac{1}{2}}$$