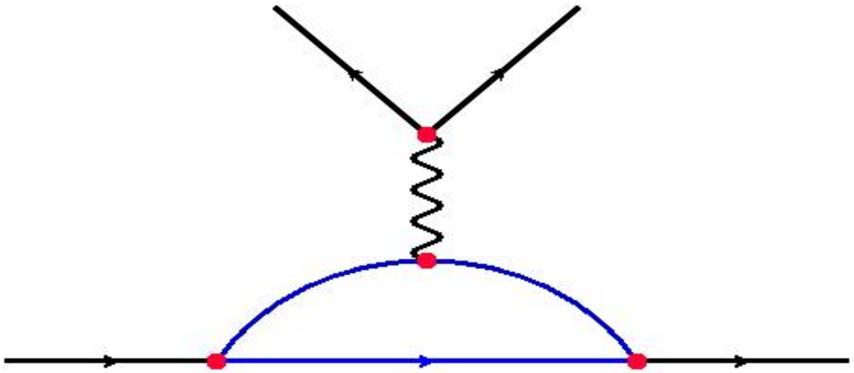
Weak Decays of the Double-Beauty Tetraquark



Birten BARSBAY

27-30 June 2022

QCD@Work-International Workshop on QCD-Theory and Experiment

OUTLINE

Standard Hadrons

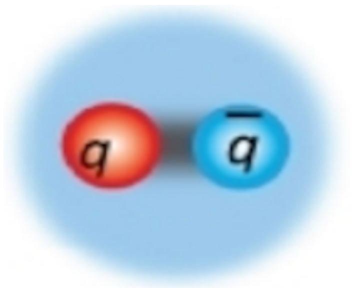
Exotic Hadrons

QCD Sum Rules

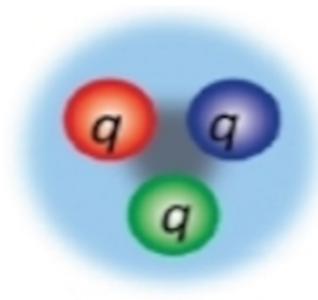
Semileptonic and nonleptonic decays

Concluding Remarks

Standard Hadrons

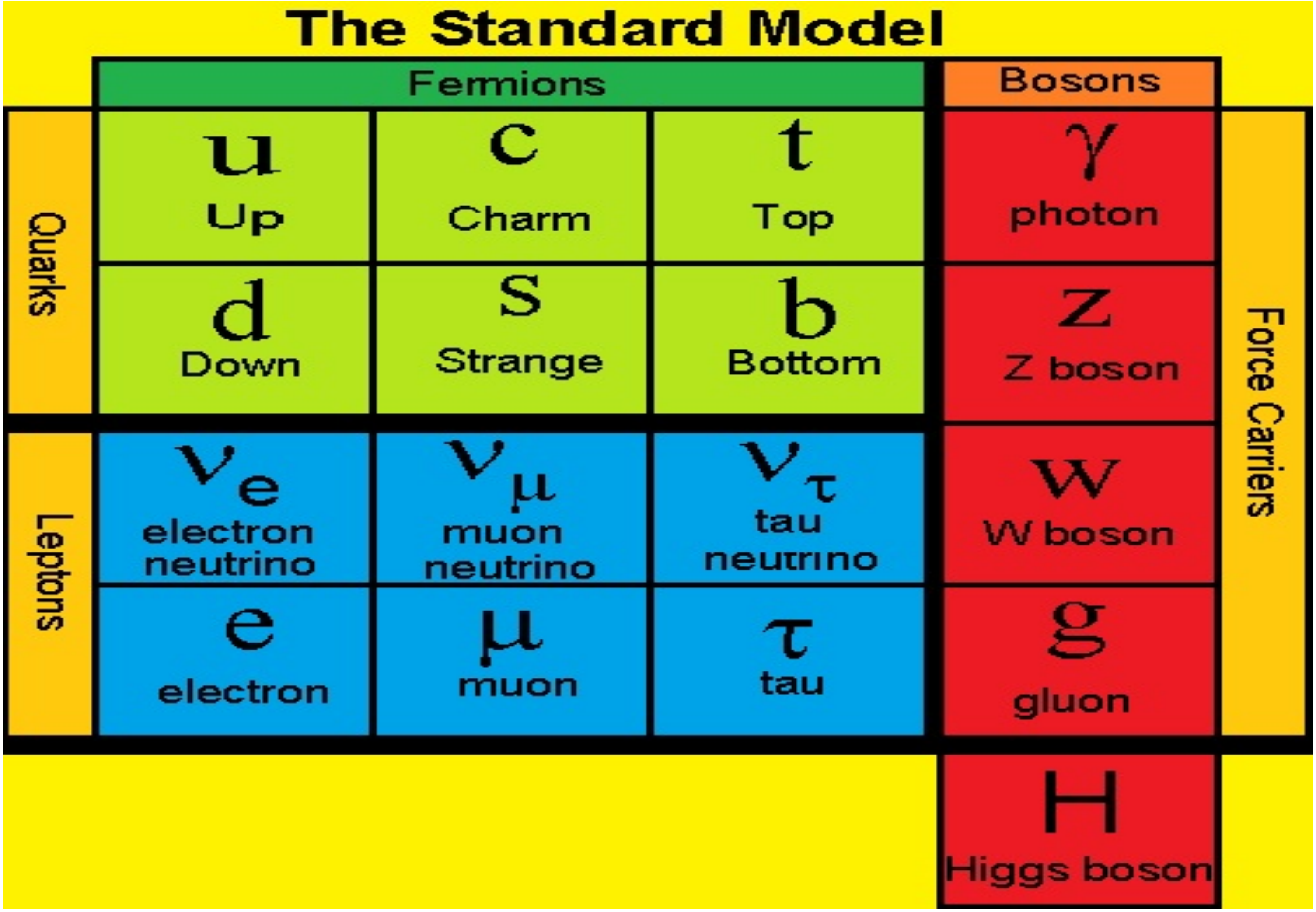


Meson



Baryon

Classification of basic particles: 12 basic fermions, 4 basic bosons and Higgs boson



In addition to standard particles, there might exist hadrons with different quark-gluon structures, which cannot be included into the ordinary $q\bar{q}$ and qqq !

R.L.Jaffe & K.Johnson, Phys. Let. 60 B,201(**1976**)
R.L.Jaffe,Phys. Rev. D15,281(**1977**)

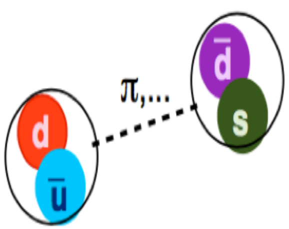
Experiment
X(3872) : 2003 Belle
P_c(4380) ve P_c(4450) : 2015 LHCb

Exotic Hadrons

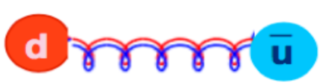
Tetraquark

Pentaquark

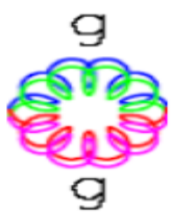
Hegzaquark,H-dibaryon



Hybrid



Glueball



- Tetraquarks observed by various Collaborations:

X(3872) : 2003 Belle

D_{sJ}(2632) : 2004 Fermilab SELEX

Z(4430): 2007 Belle

Y(4140) : 2009 Fermilab, 2012 CMS, 2013 D0,
Belle X

Z_c(3900) : 2013 BESIII, Belle

Z(4430) : 2014 LHCb

X(5568) : Şubat 2016 D0, LHCb X ve CMS X

X(4274), X(4500) ve X(4700) : Haziran 2016 LHCb

X_0(2900), X_1(2900) : 2020 LHCb

X(6900) : 2020 LHCb

Z_{cs}(3985) : 2020 BESIII

X(4630) : 2021 LHCb

Exotic Hadrons

- It will help to understand new physics (extra dimensions, supersymmetry…) models.

- It will provide information about the perturbative and non-perturbative nature of QCD.

- It will help analyze the experiments that have been done.

- It will shed light on the experiments to be done.

Quantum Chromodynamics (QCD)

NON-PERTURBATIVE THEORIES:

- Lattice QCD
- Bag Model
- Effective Lagrangian Model
- Potential Model
- Phenomenological Quark Model
- QCD Sum Rules

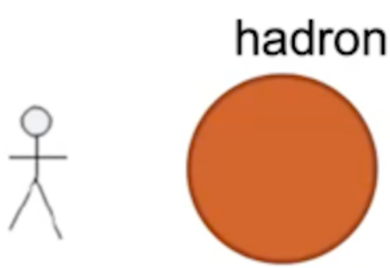
QCD Sum Rules

Correlation Function

Phenomenological
according to hadronic parameters

$\cong$

QCD(Theoretical)
according to quark,gluon and their interactions



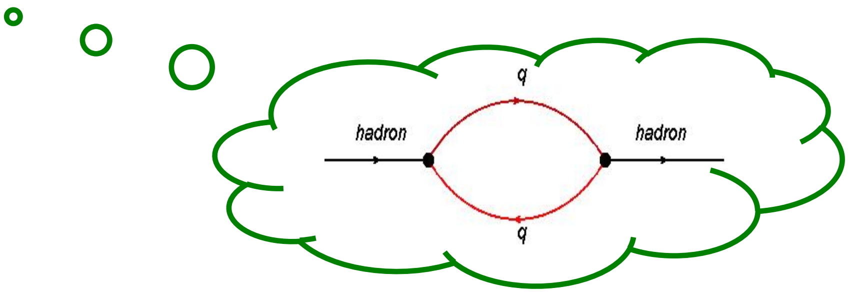
Correlation Function

Aim: To create a correlation function expressed in terms of interpolating current.

! The correlation function injects quarks at the origin and analyze the evolution of quarks to the space-time point x.

- Two-point correlation function:

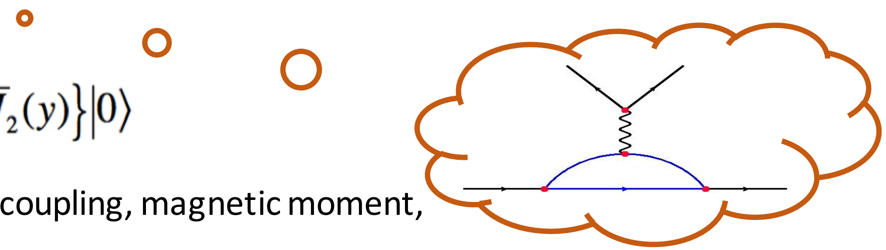
$$\Pi(q^2) = i^2 \int d^4x e^{iq\cdot x} \langle 0 | \mathrm{T}[J(x)\bar{J}(0)] | 0 \rangle$$



⟹ mass , decay constant

- Three- point correlation function:

$$\Pi(p^2, p'^2) = i^2 \int d^4x d^4y e^{-ip.x} e^{-ip'.y} \langle 0 | T\{J_1(x) J_3(0) \bar{J}_2(y)\} | 0 \rangle$$



⟹ transition amplitude, decay rate, form factor, strong coupling, magnetic moment, decay width.

(three-point sum rules or light cone QCD sum rules)

$m = (10250 \pm 270)$ MeV

10692 MeV

10695 MeV

$$T_{bb;\overline{u}\overline{s}} \to B^{*}\overline{B}_s^{0}$$

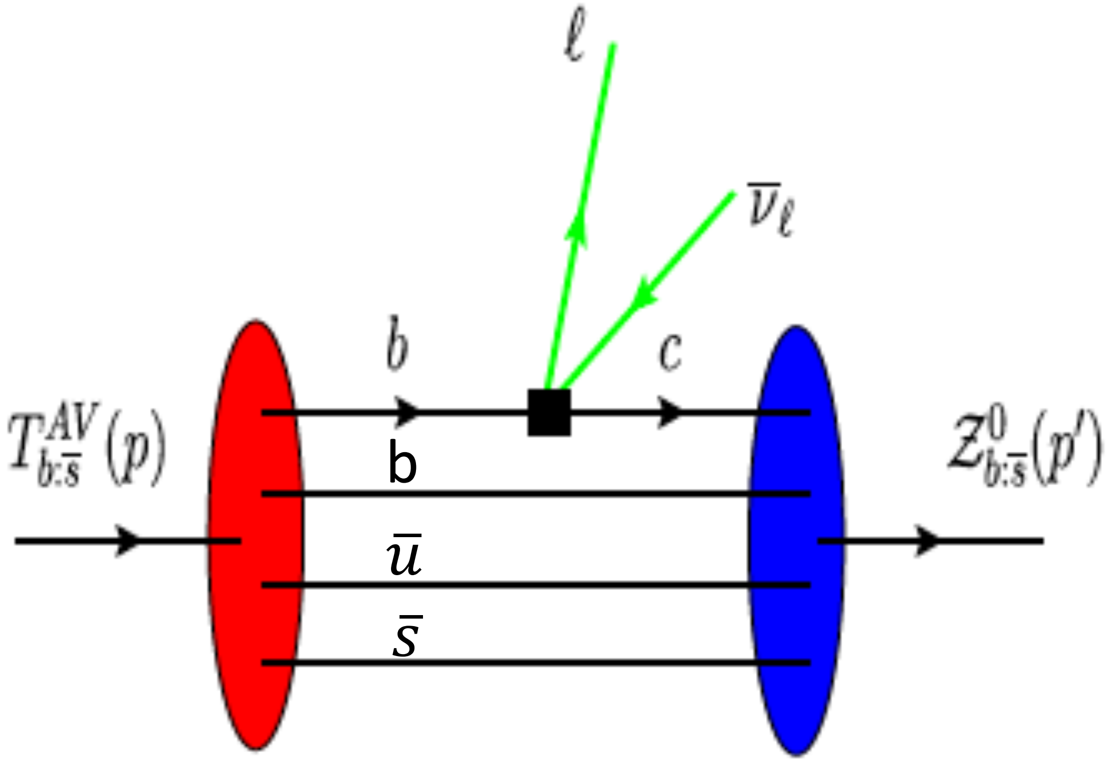
$$T_{bb;\overline{u}\overline{s}} \to B^{-}\overline{B}_s^{*}$$

Strong decays

$$T^{-}_{bb:\overline{u}\overline{s}} \to Z^{0}_{bc;\overline{u}\overline{s}}\, l\overline{\nu}_l$$

Semileptonic decay

$T^{AV}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} l \bar{\nu}_l$ Semileptonic Decay



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Why are semileptonic decays important?

- The experiment results obtained for weak decay channels do not match with the standard model prediction.
- There are predictions that this inconsistency may have an impact on new physics (extra dimensions, supersymmetry…).
- Studies on semileptonic channels of exotic particles will be compared with experimental results and this uncertainty in the literature will be tried to be explained.
- It will help to understand new physics models.

$T^{AV}_{b:\bar{s}} \rightarrow Z^{0}_{b:\bar{s}} l\bar{\nu}_l$ Semileptonic Decay

- Spectroscopic parameters of the axial-vector $T^{AV}_{b:\bar{s}}$ and scalar $Z^{0}_{b:\bar{s}}$tetraquarks:

$$\Pi_{\mu\nu}(p) = \mathrm{i}\int \mathrm{d}^4 x e^{\mathrm{i}px}\langle 0|\mathcal{T}\{J_\mu(x)J_\nu^\dagger(0)\}|0\rangle,$$

$$\Pi(p) = \mathrm{i}\int \mathrm{d}^4 x e^{\mathrm{i}px}\langle 0|\mathcal{T}\{J_Z(x)J_Z^\dagger(0)\}|0\rangle.$$

Two-point correlation function

The phenomenological side:

$$\Pi^{\mathrm{Phys}}_{\mu\nu}(p) = \frac{\langle 0|J_\mu|T^{\mathrm{AV}}_{b:\bar{s}}(p)\rangle\langle T^{\mathrm{AV}}_{b:\bar{s}}(p)|J_\nu^\dagger|0\rangle}{m^2_{\mathrm{AV}} - p^2} + \cdots$$

$$\langle 0|J_\mu|T^{\mathrm{AV}}_{b:\bar{s}}(p)\rangle = m_{\mathrm{AV}} f_{\mathrm{AV}} \epsilon_\mu,$$

$$\Pi^{\mathrm{Phys}}_{\mu\nu}(p) = \frac{m^2_{\mathrm{AV}} f^2_{\mathrm{AV}}}{m^2_{\mathrm{AV}} - p^2}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2_{\mathrm{AV}}}\right) + \cdots$$

$$\Pi^{\mathrm{Phys}}(p) = \frac{\langle 0|J^Z|Z(p)\rangle\left\langle Z(p)\middle|J^{Z^+}\middle|0\right\rangle}{m_Z^2 - f_Z^2}$$

$$\langle 0|J^Z|Z(p)\rangle = f_Z m_Z$$

$$\Pi^{\mathrm{Phys}}(p) = \frac{m_Z^2 f_Z^2}{m_Z^2 - p^2} + \ldots$$

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$T_{b:\overline{s}}^{AV} \rightarrow Z_{b:\overline{s}}^{0} l\overline{\nu}_l$ Semileptonic Decay

The QCD side:

$$J_\mu(x) = \left[b_a^T(x)C\gamma_\mu b_b(x)\right]\left[\overline{u}_a(x)\gamma_5 C\overline{s}_b^T(x)\right]$$

$$\Pi_{\mu\nu}(p) = i\int d^4x e^{ipx}\langle 0|\mathcal{T}\{J_\mu(x)J_\nu^\dagger(0)\}|0\rangle,$$

$$\Pi_{\mu\nu}^{\mathrm{OPE}}(p) = i\int d^4x e^{ipx}\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{b'b}(-x)\gamma_5 S_u^{a'a}(-x)\right] \times\left\{\mathrm{Tr}\left[\gamma_\nu\widetilde{S}_b^{ba'}(x)\gamma_\mu S_b^{ab'}(x)\right] -\mathrm{Tr}\left[\gamma_\nu\widetilde{S}_b^{aa'}(x)\gamma_\mu S_b^{bb'}(x)\right]\right\},$$

$$J_Z(x) = [b_a^T(x)C\gamma_5 c_b(x)][\overline{u}_a(x)\gamma_5 C\overline{s}_b^T(x) - \overline{u}_b(x)\gamma_5 C\overline{s}_a^T(x)].$$

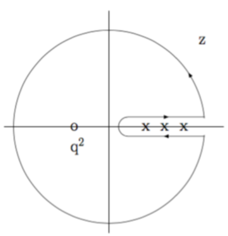
$$\Pi(p) = i\int d^4x e^{ipx}\langle 0|\mathcal{T}\{J_Z(x)J_Z^\dagger(0)\}|0\rangle.$$

$$\Pi^{\mathrm{OPE}}(p) = i\int d^4x e^{ipx}\mathrm{Tr}\left[\gamma_5\widetilde{S}_b^{aa'}(x)\gamma_5 S_c^{bb'}(x)\right] \times\left\{\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{b'b}(-x)\gamma_5 S_u^{a'a}(-x)\right]-\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{a'b}(-x)\times\gamma_5 S_u^{b'a}(-x)\right]-\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{b'a}(-x)\gamma_5 S_u^{a'b}(-x)\right] +\mathrm{Tr}\left[\gamma_5\widetilde{S}_d^{a'a}(-x)\gamma_5 S_u^{b'b}(-x)\right]\right\}.$$

QCD Sum Rules

QCD Side

Hadronic Side



$$\mathcal{B}_{M^2} f(q^2) = lim_{\substack{Q^2, n \to \infty \\ \frac{Q^2}{n} = M^2}} \frac{(-q^2)^n}{(n-1)!} \left(\frac{d}{dq^2}\right)^{n-1} f(q^2),$$

$$\int_{s_0^h}^{\infty} ds \rho^h(s) e^{-s/M^2} = \int_{s_0}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2}$$

$T_{b:\bar{s}}^{AV} \to Z_{b:\bar{s}}^{0} l\bar{v}_l$ Semileptonic Decay

$$m^2 = \frac{\boxed{\Pi'(M^2,s_0)}}{\Pi(M^2,s_0)}$$

$$\Pi'(M^2,s_0) = \frac{d}{d(-1/M^2)}\Pi(M^2,s_0)$$

$$f^2 = \frac{e^{m^2/M^2}}{m^2}\boxed{\Pi(M^2,s_0)}$$

$$\Pi(M^2,s_0) = \int_{\mathcal{M}^2}^{s_0} ds \boxed{\rho^{\mathrm{OPE}}(s)} e^{-s/M^2} + \boxed{\Pi(M^2)}$$

$$\rho^{\mathrm{OPE}}(s) = \rho^{\mathrm{pert.}}(s) + \sum_{N=3}^{8} \rho^{\mathrm{DimN}}(s), \quad \Pi(M^2) = \sum_{N=6}^{10} \Pi^{\mathrm{DimN}}(M^2)$$

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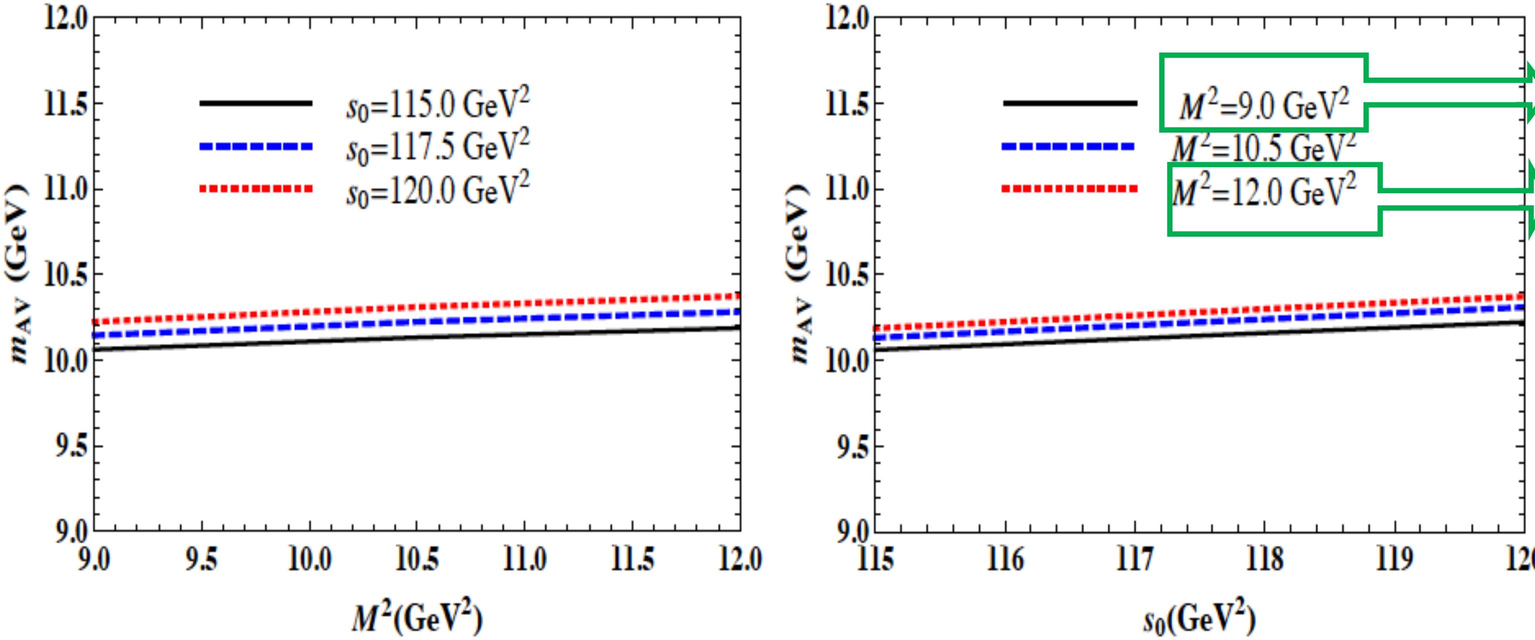
$T^{AV}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} l \bar{\nu}_l$ Semileptonic Decay



$$R(M^2) = \frac{\Pi^{\mathrm{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)} \leq 0.01$$

$$\mathrm{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)} > 0.2$$

$$m_{\mathrm{AV}} = (10215 \pm 250)\ \mathrm{MeV},$$
$$f_{\mathrm{AV}} = (2.26 \pm 0.57) \times 10^{-2}\ \mathrm{GeV}^4$$

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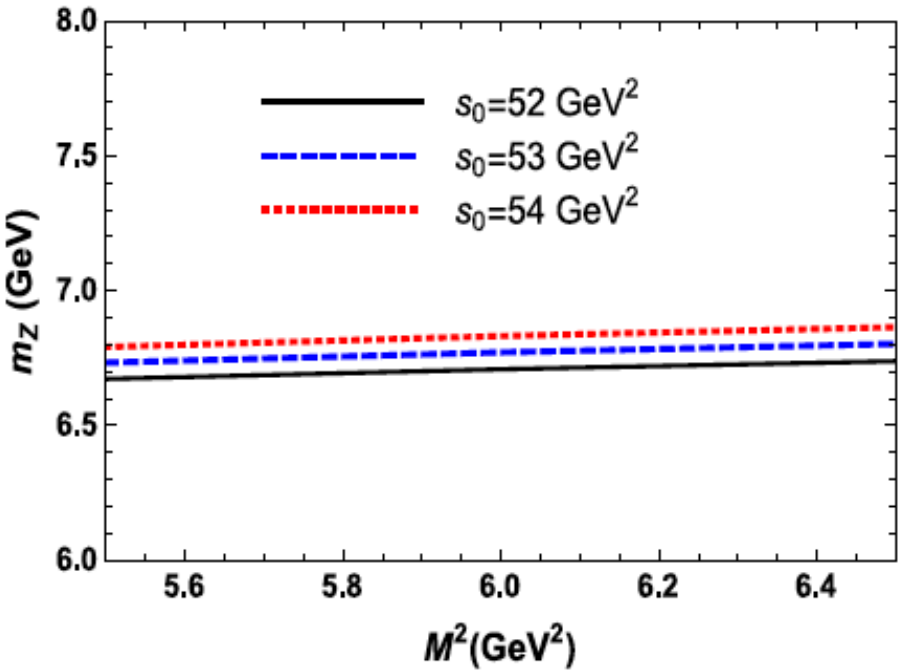
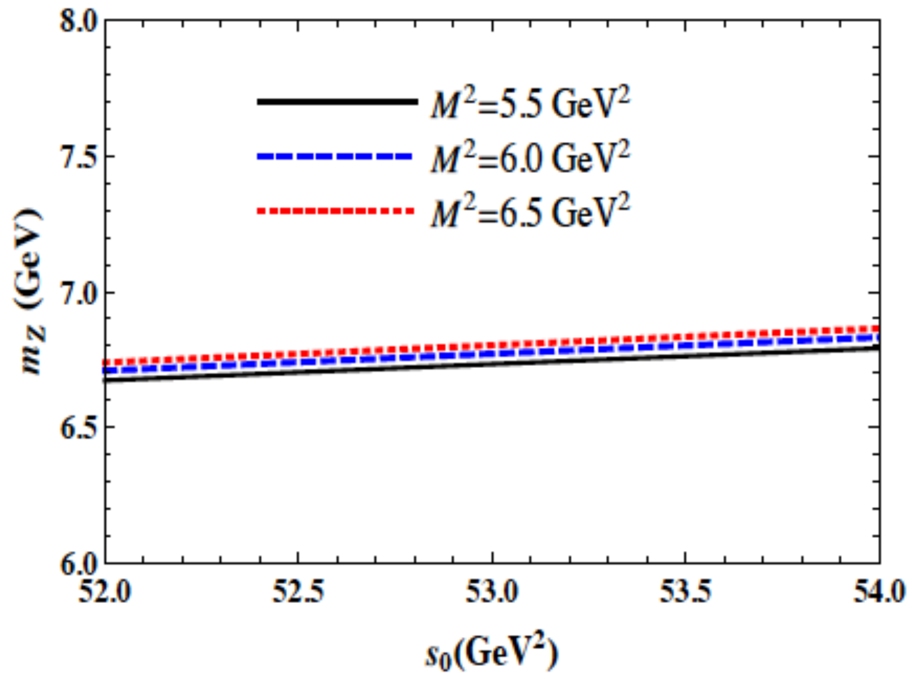
$T_{b:\overline{s}}^{AV} \rightarrow Z_{b:\overline{s}}^{0} l\overline{\nu}_l$ Semileptonic Decay





$$
\begin{aligned}
m_Z &= (6770 \pm 150)\ \mathrm{MeV}, \\
f_Z &= (6,3 \pm 1.3) \times 10^{-3}\ \mathrm{GeV}^4
\end{aligned}
$$

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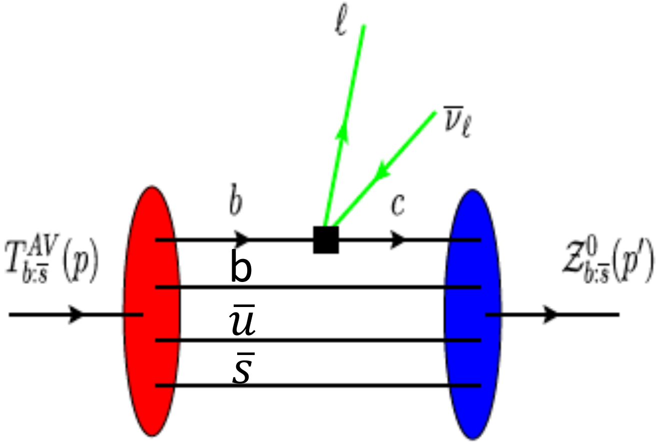
$T_{b:\bar{s}}^{AV} \to Z_{b:\bar{s}}^{0} l\bar{\nu}_l$ Semileptonic Decay

- The effective Hamiltonian to describe the subprocess $b \to cl\bar{\nu}$ at the tree-level is given by the expression.

$$\mathcal{H}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} \bar{c}\gamma_\mu (1-\gamma_5) b \bar{l} \gamma^\mu (1-\gamma_5) \nu_l$$



G_F is the Fermi coupling contant

V_{bc} is the corresponding element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

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$T^{AV}_{b:\overline{s}} \rightarrow Z^{0}_{b:\overline{s}} l\overline{\nu}_l$ Semileptonic Decay

- Sandwiching the H^{eff} between the initial and final tetraquarks we get the matrix element for the weak transition current

$$J_\mu^{\mathrm{tr}} = \bar{c}\gamma_\mu(1-\gamma_5)b$$

$$\begin{aligned}\langle \mathcal{Z}^0_{b:\bar{s}}(p')|J^{\mathrm{tr}}_\mu|T^{\mathrm{AV}}_{b:\bar{s}}(p)\rangle =& \widetilde{G}_0(q^2)\epsilon_\mu + \widetilde{G}_1(q^2)(\epsilon p')P_\mu \\ &+ \widetilde{G}_2(q^2)(\epsilon p')q_\mu \\ &+ \mathrm{i}\widetilde{G}_3(q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu p^\alpha p'^\beta .\end{aligned}$$

$\rightarrow$ $P_\mu = \acute{p}_\mu + p_\mu$

$\rightarrow$ $q_\mu = p_\mu - \acute{p}_\mu$

$\rightarrow$ $m_l^2 \le q^2 \le (m_{AV} - m_Z)^2$

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$T^{AV}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} l\bar{\nu}_l$ Semileptonic Decay

- The sum rules for the form factors $G_i(q^2)$ can be obtained by analyzing the three-point correlation function

$$\Pi_{\mu\nu}(p,p') = i^2 \int d^4x d^4y e^{i(p'y-px)} \times \langle 0|\mathcal{T}\{J_Z(y) J^{\mathrm{tr}}_{\nu}(0) J^{\dagger}_{\mu}(x)\}|0\rangle$$

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$T^{AV}_{b:\overline{s}} \rightarrow Z^{0}_{b:\overline{s}} l \overline{\nu}_l$ Semileptonic Decay

- The phenomenological side:

$$\Pi^{\mathrm{Phys}}_{\mu\nu}(p,p') = \frac{\langle 0|J_{\mathcal{Z}}|\mathcal{Z}^0_{b:\overline{s}}(p')\rangle\langle\mathcal{Z}^0_{b:\overline{s}}(p')|J^{\mathrm{tr}}_{\nu}|T^{\mathrm{AV}}_{b:\overline{s}}(p,\epsilon)\rangle}{(p^2-m^2_{\mathrm{AV}})(p'^2-m^2_{\mathcal{Z}})} \times\langle T^{\mathrm{AV}}_{b:\overline{s}}(p,\epsilon)|J^{\dagger}_{\mu}|0\rangle + \cdots ,$$

$$\langle 0|J_{\mathcal{Z}}|\mathcal{Z}^0_{b:\overline{s}}(p')\rangle = f_{\mathcal{Z}} m_{\mathcal{Z}}$$

$$\langle\mathcal{Z}^0_{b:\overline{s}}(p')|J^{\mathrm{tr}}_{\mu}|T^{\mathrm{AV}}_{b:\overline{s}}(p)\rangle = \widetilde{G}_0(q^2)\epsilon_{\mu} + \widetilde{G}_1(q^2)(\epsilon p')P_{\mu} + \widetilde{G}_2(q^2)(\epsilon p')q_{\mu} + i\widetilde{G}_3(q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\nu}p^{\alpha}p'^{\beta}$$

$$\langle 0|J_{\mu}|T^{\mathrm{AV}}_{b:\overline{s}}(p)\rangle = m_{\mathrm{AV}} f_{\mathrm{AV}} \epsilon_{\mu}$$

$$\Pi^{\mathrm{Phys}}_{\mu\nu}(p,p') = \frac{f_{\mathrm{AV}} m_{\mathrm{AV}} f_{\mathcal{Z}} m_{\mathcal{Z}}}{(p^2-m^2_{\mathrm{AV}})(p'^2-m^2_{\mathcal{Z}})} \times\left\{\widetilde{G}_0(q^2)\left(-g_{\mu\nu}+\frac{p_{\mu}p_{\nu}}{m^2_{\mathrm{AV}}}\right)+\left[\widetilde{G}_1(q^2)P_{\mu} + \widetilde{G}_2(q^2)q_{\mu}\right]\left(-p'_{\nu}+\frac{m^2_{\mathrm{AV}}+m^2_{\mathcal{Z}}-q^2}{2m^2_{\mathrm{AV}}}p_{\nu}\right) - i\widetilde{G}_3(q^2)\varepsilon_{\mu\nu\alpha\beta}p^{\alpha}p'^{\beta}\right\} + \cdots .$$

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$T_{b:\bar{s}}^{AV} \to Z_{b:\bar{s}}^{0} l \bar{\nu}_l$ Semileptonic Decay

The QCD side:

$$
\begin{aligned}
\Pi_{\mu\nu}^{\mathrm{OPE}}(p,p') = \int \mathrm{d}^4x\mathrm{d}^4y \mathrm{e}^{\mathrm{i}(p'y-px)} \Big\{ & \mathrm{Tr}\left[\gamma_5 \widetilde{S}_s^{ba'}(x-y) \times \gamma_5 S_u^{a'b}(x-y)\right]\Big(\mathrm{Tr}\left[\gamma_\mu \widetilde{S}_b^{aa'}(y-x)\gamma_5 S_c^{bi}(y)\gamma_\nu(1-\gamma_5) \times S_b^{ib'}(-x)\right] \\
& + \mathrm{Tr}\left[\gamma_\mu \widetilde{S}_b^{ia'}(-x)(1-\gamma_5)\gamma_\nu \widetilde{S}_c^{bi}(y)\gamma_5 \times S_b^{ab'}(y-x)\right]\Big) - \mathrm{Tr}\left[\gamma_5 \widetilde{S}_s^{b'a}(x-y)\gamma_5 S_u^{a'b}(x-y)\right] \\
& \times \Big(\mathrm{Tr}\left[\gamma_\mu \widetilde{S}_b^{aa'}(y-x)\gamma_5 S_c^{bi}(y)\gamma_\nu(1-\gamma_5) S_b^{ib'}(-x)\right] + \mathrm{Tr}\left[\gamma_\mu \widetilde{S}_b^{ia'}(-x)(1-\gamma_5)\gamma_\nu \widetilde{S}_c^{bi}(y)\gamma_5 S_b^{ab'}(y-x)\right]\Big)\Big\}.
\end{aligned}
$$

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$T_{b:\bar{s}}^{AV} \rightarrow Z_{b:\bar{s}}^{0} l\bar{\nu}_l$ Semileptonic Decay

$$\Pi_{\mu}^{\mathrm{Phys}}(p,p') = \Pi_{\mu}^{\mathrm{OPE}}(p,p')$$

$$\hat{B}_{M_1^2}\hat{B}_{M_2^2}\frac{1}{(p^2-m_1^2)^a}\frac{1}{(p'^2-m_2^2)^b} \rightarrow (-1)^{a+b}\frac{1}{\Gamma(a)}\frac{1}{\Gamma(b)}e^{-m_1^2/M_1^2}e^{-m_2^2/M_2^2}\frac{1}{(M_1^2)^{a-1}}\frac{1}{(M_2^2)^{b-1}}$$

$$\widetilde{G}_i(\mathbf{M}^2,\mathbf{s}_0,\mathbf{q}^2) = \frac{1}{f_{\mathrm{AV}}m_{\mathrm{AV}}f_Z m_Z}\int_{\mathcal{M}^2}^{s_0}\mathrm{d}s\,e^{(m_{\mathrm{AV}}^2-s)/M_1^2} \times \int_{\widetilde{\mathcal{M}}^2}^{s'_0}\mathrm{d}s'\rho_i(s,s')\mathrm{e}^{(m_Z^2-s')/M_2^2},$$

$\boldsymbol{M}^2=(M_1^2,\,M_2^2)$

$\boldsymbol{s}_0=(s_0,\,s'_0)$

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$T_{b:\bar{s}}^{AV} \to Z_{b:\bar{s}}^{0} l\bar{\nu}_l$ Semileptonic Decay

- There are numerous analytical expressions for the fit functions. In the present paper we use

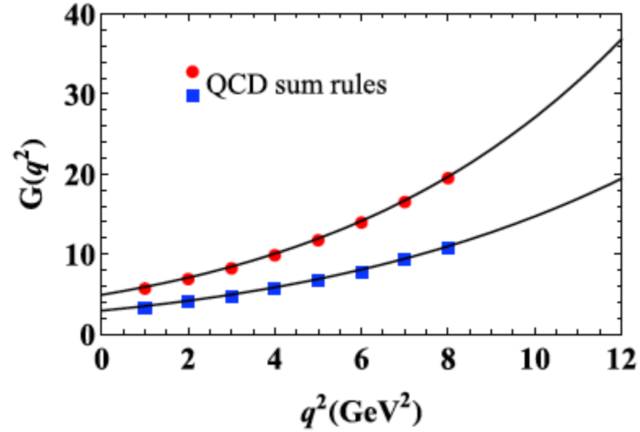

Fig. (color online) Sum rule results for the form factors $G_0(q^2)$ (red circles) and $G_1(q^2)$ (blue squares). The solid curves are fit functions $G_0(q^2)$ and $G_1(q^2)$.

$$G_i(q^2) = G_0^i \exp\left[g_1^i \frac{q^2}{m_{\mathrm{AV}}^2} + g_2^i \left(\frac{q^2}{m_{\mathrm{AV}}^2}\right)^2\right]$$

$$m_l^2 \leqslant q^2 \leqslant (m_{\mathrm{AV}} - m_Z)^2$$

Table 1. Parameters of the extrapolating functions $G_i(q^2)$.

$G_i(q^2)$	G_0^i	g_1^i	g_2^i
$G_0(q^2)$	4.91	19.29	−15.34
$G_1(q^2)$	2.94	18.73	−20.09
$G_2(q^2)$	−22.67	20.50	−22.95
$G_3(q^2)$	−21.14	20.77	−23.62

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$T^{AV}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} l\overline{\nu}_l$ Semileptonic Decay

- As a result, for the full decay width of the processes $T^{AV}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} l\overline{\nu}_l$, l=e, μ and τ we find

$$\Gamma(T^{\mathrm{AV}}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} e^{-}\overline{\nu}_e) = (5.34 \pm 1.43) \times 10^{-8}\ \mathrm{MeV},$$
$$\Gamma(T^{\mathrm{AV}}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} \mu^{-}\overline{\nu}_\mu) = (5.32 \pm 1.41) \times 10^{-8}\ \mathrm{MeV},$$
$$\Gamma(T^{\mathrm{AV}}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}} \tau^{-}\overline{\nu}_\tau) = (2.15 \pm 0.54) \times 10^{-8}\ \mathrm{MeV}.$$

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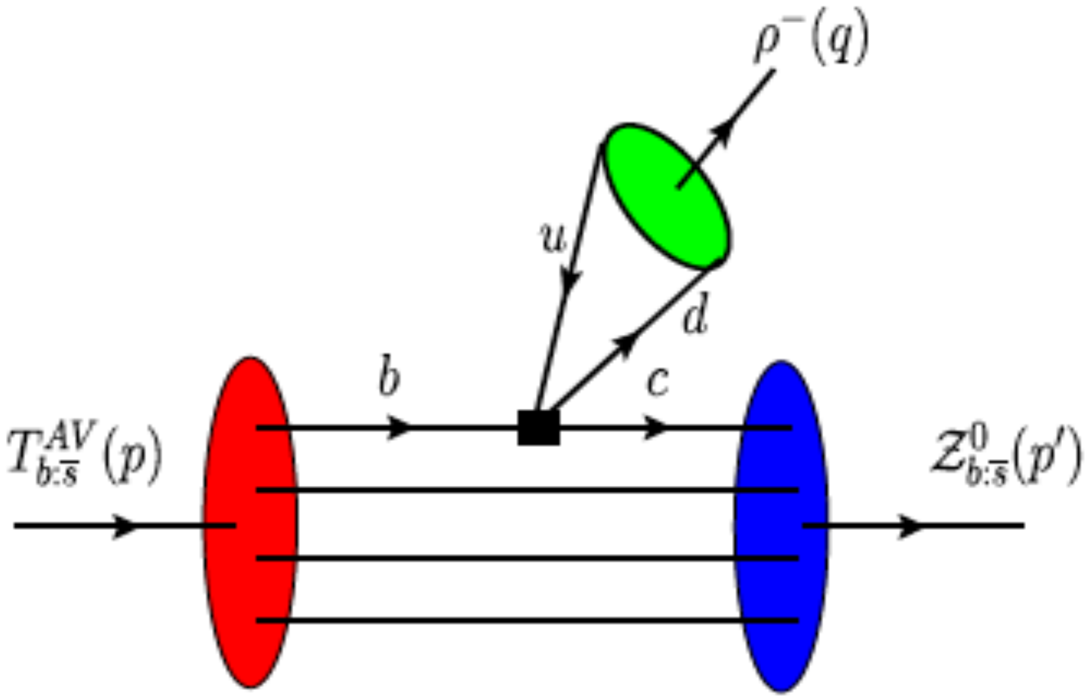
$T^{AV}_{b:\bar{s}} \to Z^{0}_{b:\bar{s}}\rho^{-}(K^{*}(892), D^{*}(2010)^{-}, D_s^{*-})$ Nonleptonic Decays



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- We study the nonleptonic weak decays $T^{AV}_{b:\bar{s}} \to Z^0_{b:\bar{s}}\rho^-(K^*(892), D^*(2010)^-, D_s^{*-})$ of the tetraquark $T^{AV}_{b:\bar{s}}$ in the framework of the QCD factorization method.
- At the quark level, the effective Hamiltonian for this decay is given by the expression

$$\boxed{\mathcal{H}^{\text{eff}}_{\text{n.-lep}} = \frac{G_F}{\sqrt{2}} V_{bc} V^*_{ud} [c_1(\mu) Q_1 + c_2(\mu) Q_2]}$$

where

$$Q_1 = \left(\bar{d}_i u_i\right)_{\text{V-A}} \left(\bar{c}_j b_j\right)_{\text{V-A}},$$
$$Q_2 = \left(\bar{d}_i u_j\right)_{\text{V-A}} \left(\bar{c}_j b_i\right)_{\text{V-A}},$$

i and j are the color indices, and $(\overline{q_1} q_2)_{V-A}$ means

$$(\bar{q}_1 q_2)_{\text{V-A}} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2.$$

The short-distance Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$ are given on the factorization scale μ.

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In the factorization method, the amplitude of the decay $T^{AV}_{b:\bar{s}} \to Z^0_{b:\bar{s}}\rho^-$ has the form

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{bc} V^*_{ud} a(\mu) \langle \rho^-(q) | \left(\overline{d}_i u_i\right)_{\mathrm{V-A}} |0\rangle \times \langle \mathcal{Z}^0_{b:\bar{s}}(p') | \left(\overline{c}_j b_j\right)_{\mathrm{V-A}} | T^{\mathrm{AV}}_{b:\bar{s}}(p)\rangle,$$

where

$$a(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu),$$

with N_c=3 being the number of quark colors. The only unknown matrix element $<\rho^-(q)|(\bar{d}_i u_i)_{V-A}|0>$ in A can be defined in the following form

$$\begin{aligned} \langle \mathcal{Z}^0_{b:\bar{s}}(p') | J^{\mathrm{tr}}_\mu | T^{\mathrm{AV}}_{b:\bar{s}}(p)\rangle = {} & \widetilde{G}_0(q^2)\epsilon_\mu + \widetilde{G}_1(q^2)(\epsilon p')P_\mu \\ & + \widetilde{G}_2(q^2)(\epsilon p')q_\mu \\ & + i\widetilde{G}_3(q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu p^\alpha p'^\beta \end{aligned}$$

$$\langle \rho^-(q) | \left(\overline{d}_i u_i\right)_{\mathrm{V-A}} |0\rangle = f_\rho m_\rho \epsilon^*_\mu(q).$$

Then, it is evident that

$$\begin{aligned} \mathcal{A} = i\frac{G_F}{\sqrt{2}} f_\rho V_{bc} V^*_{ud} a(\mu) \Big[& \widetilde{G}_0(q^2)\epsilon_\mu(p)\epsilon^{*\mu}(q) \\ & + 2\widetilde{G}_1(q^2)(p'\epsilon(p))(p'\epsilon^*(q)) \\ & + i\widetilde{G}_3(q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\mu}(q)\epsilon^\nu(p)p^\alpha p'^\beta \Big]. \end{aligned}$$

$T_{b:\bar{s}}^{AV} \to Z_{b:\bar{s}}^{0}\rho^{-}(K^{*}(892), D^{*}(2010)^{-}, D_{s}^{*-})$ Nonleptonic Decays

- The width of the nonleptonic decay $T_{b:\bar{s}}^{AV} \to Z_{b:\bar{s}}^{0}\rho^{-}$ can be evaluated using the expression

$$\Gamma = \frac{|\mathcal{A}|^2}{24\pi m_{\mathrm{AV}}^2}\lambda\left(m_{\mathrm{AV}}, m_Z, m_\rho\right),$$

$$\lambda(a,b,c) = \frac{1}{2a}\left[a^4 + b^4 + c^4 - 2\left(a^2b^2 + a^2c^2 + b^2c^2\right)\right]^{1/2}$$

$$|\mathcal{A}|^2 = \sum_{j=0,1,2} H_j\widetilde{G}_j^2 + H_3\widetilde{G}_0\widetilde{G}_1,$$

where H_j are given by the expressions

$$H_0 = \frac{m_\rho^4 + (m_{\mathrm{AV}}^2 - m_Z^2)^2 + 2m_\rho^2\left(5m_{\mathrm{AV}}^2 - m_Z^2\right)}{4m_\rho^2 m_{\mathrm{AV}}^2},$$

$$H_1 = \frac{\left[m_\rho^4 + (m_{\mathrm{AV}}^2 - m_Z^2)^2 - 2m_\rho^2\left(m_{\mathrm{AV}}^2 + m_Z^2\right)\right]^2}{4m_\rho^2 m_{\mathrm{AV}}^2},$$

$$H_2 = \frac{1}{2}\left[m_\rho^4 + (m_{\mathrm{AV}}^2 - m_Z^2)^2 - 2m_\rho^2\left(m_{\mathrm{AV}}^2 + m_Z^2\right)\right],$$

$$H_3 = -\frac{1}{2m_\rho^2 m_{\mathrm{AV}}^2}\left[m_\rho^6 + (m_{\mathrm{AV}}^2 - m_Z^2)^3 - m_\rho^4\left(m_{\mathrm{AV}}^2 + 3m_Z^2\right) - m_\rho^2\left(m_{\mathrm{AV}}^4 + 2m_Z^2 m_{\mathrm{AV}}^2 - 3m_Z^2\right)\right].$$

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$T_{b:\bar{s}}^{AV} \to Z_{b:\bar{s}}^{0}\rho^{-}(K^{*}(892), D^{*}(2010)^{-}, D_{s}^{*-})$ Nonleptonic Decays

$$\Gamma(T_{b:\bar{s}}^{\rm AV} \to \mathcal{Z}_{b:\bar{s}}^{0}\rho^{-}) = (3.47 \pm 0.92) \times 10^{-10}\ \mathrm{MeV}$$

$$\Gamma(T_{b:\bar{s}}^{\rm AV} \to \mathcal{Z}_{b:\bar{s}}^{0}K^{*}(892)) = (1.47 \pm 0.37) \times 10^{-11}\ \mathrm{MeV},$$

$$\Gamma(T_{b:\bar{s}}^{\rm AV} \to \mathcal{Z}_{b:\bar{s}}^{0}D^{*}(2010)^{-}) = (1.54 \pm 0.39) \times 10^{-11}\ \mathrm{MeV},$$

$$\Gamma(T_{b:\bar{s}}^{\rm AV} \to \mathcal{Z}_{b:\bar{s}}^{0}D_{s}^{*-}) = (4.97 \pm 1.32) \times 10^{-10}\ \mathrm{MeV}.$$

Quantity	Value
m_{ρ}	(775.26 ± 0.25) MeV
$m_{K^{\star}}$	(891.66 ± 0.26) MeV
$m_{D^{\star}}$	(2010.26 ± 0.05) MeV
$m_{D_s^{\star}}$	(2112.2 ± 0.4) MeV
f_{ρ}	(210 ± 4) MeV
$f_{K^{\star}}$	(204 ± 7) MeV
$f_{D^{\star}}$	(223.5 ± 8.4) MeV
$f_{D_s^{\star}}$	(268.8 ± 6.6) MeV
$\lvert V_{ud}\rvert$	0.97420 ± 0.00021
$\lvert V_{us}\rvert$	0.2243 ± 0.0005
$\lvert V_{cd}\rvert$	0.218 ± 0.004
$\lvert V_{cs}\rvert$	0.997 ± 0.017

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The full width Γ_{full} and the mean lifetime τ

$$\Gamma(T^{AV}_{b:\bar{s}} \to \mathcal{Z}^0_{b:\bar{s}} e^- \bar{\nu}_e) = (5.34 \pm 1.43) \times 10^{-8}\ \text{MeV},$$
$$\Gamma(T^{AV}_{b:\bar{s}} \to \mathcal{Z}^0_{b:\bar{s}} \mu^- \bar{\nu}_\mu) = (5.32 \pm 1.41) \times 10^{-8}\ \text{MeV},$$
$$\Gamma(T^{AV}_{b:\bar{s}} \to \mathcal{Z}^0_{b:\bar{s}} \tau^- \bar{\nu}_\tau) = (2.15 \pm 0.54) \times 10^{-8}\ \text{MeV},$$
$$\Gamma(T^{AV}_{b:\bar{s}} \to \mathcal{Z}^0_{b:\bar{s}} \rho^-) = (3.47 \pm 0.92) \times 10^{-10}\ \text{MeV},$$
$$\Gamma(T^{AV}_{b:\bar{s}} \to \mathcal{Z}^0_{b:\bar{s}} K^*(892)) = (1.47 \pm 0.37) \times 10^{-11}\ \text{MeV},$$
$$\Gamma(T^{AV}_{b:\bar{s}} \to \mathcal{Z}^0_{b:\bar{s}} D^*(2010)^-) = (1.54 \pm 0.39) \times 10^{-11}\ \text{MeV},$$
$$\Gamma(T^{AV}_{b:\bar{s}} \to \mathcal{Z}^0_{b:\bar{s}} D^{*-}_s) = (4.97 \pm 1.32) \times 10^{-10}\ \text{MeV}.$$

$$\Gamma_{\text{full}} = (12.9 \pm 2.1) \times 10^{-8}\ \text{MeV},$$
$$\tau = 5.1^{+0.99}_{-0.71} \times 10^{-15}\ \text{s},$$

$$\hbar = 6{,}58212 \times 10^{-22}\ \text{MeV.s}$$
$$\tau = \hbar / \Gamma_{\text{full}}$$

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Concluding Remarks

- Numerical values of the spectroscopic parameters obtained for the scalar and axial-vector tetraquarks composed of the heavy bb or bc diquarks and light antidiquarks

Tetraquark (J^P)	Mass (MeV)	Coupling Constant (GeV^4)
$T^-_{bb;\overline{u}\overline{s}}(0^+)$	10250 ± 270	$(2,69 \pm 0,58)\times 10^{-2}$
$Z^0_{bc;\overline{u}\overline{s}}(0^+)$	6830 ± 160	$(7,1 \pm 1,8)\times 10^{-3}$
$T^-_{bb;\overline{u}\overline{d}}(0^+)$	10135 ± 240	$(2,26 \pm 0,57)\times 10^{-2}$
$\tilde{Z}^0_{bc;\overline{u}\overline{d}}(0^+)$	6730 ± 150	$(6,2 \pm 1,4)\times 10^{-3}$
$T^-_{bb;\overline{u}\overline{d}}(1^+)$	10035 ± 260	$(1,38 \pm 0,27)\times 10^{-2}$
$Z^0_{bc;\overline{u}\overline{d}}(0^+)$	6660 ± 150	$(0,51 \pm 0,16)\times 10^{-2}$
$T^-_{bb;\overline{u}\overline{s}}(1^+)$	10215 ± 250	$(2,26 \pm 0,57)\times 10^{-2}$
$Z^0_{bc;\overline{u}\overline{s}}(0^+)$	6770 ± 150	$(6,3 \pm 1,3)\times 10^{-3}$
$T^0_{bc;\overline{u}\overline{d}}(1^+)$	7050 ± 125	$(8,3 \pm 1,3)\times 10^{-3}$

Concluding Remarks

- Numerical values of decay width of the semileptonic and nonleptonic decays for the scalar and axial-vector tetraquarks

	Channels	Decay Width (Γ)
Scalar → Scalar	$T_{b:\bar{s}}^{-} \to Z_{bc}^{0} e^{-} \overline{\nu}_{e}$	$(6,16 \pm 1,74)\times 10^{-10}$ MeV
	$T_{b:\bar{s}}^{-} \to Z_{bc}^{0} \mu^{-} \overline{\nu}_{\mu}$	$(6,15 \pm 1,74)\times 10^{-10}$ MeV
	$T_{b:\bar{s}}^{-} \to Z_{bc}^{0} \tau^{-} \overline{\nu}_{\tau}$	$(2,85 \pm 0,81)\times 10^{-10}$ MeV
	$T_{b:\bar{s}}^{-} \to Z_{bc}^{0} \pi^{-}$	$(6,67 \pm 1,99)\times 10^{-13}$ MeV
	$T_{b:\bar{s}}^{-} \to Z_{bc}^{0} K^{-}$	$(5,33 \pm 1,47)\times 10^{-14}$ MeV
	$T_{b:\bar{s}}^{-} \to Z_{bc}^{0} D^{-}$	$(1,13 \pm 0,31)\times 10^{-13}$ MeV
	$T_{b:\bar{s}}^{-} \to Z_{bc}^{0} D_{s}^{-}$	$(3,88 \pm 1,01)\times 10^{-12}$ MeV
	$T_{b:\bar{d}}^{-} \to \tilde{Z}_{bc}^{0} e^{-} \overline{\nu}_{e}$	$(4,45 \pm 1,28)\times 10^{-10}$ MeV
	$T_{b:\bar{d}}^{-} \to \tilde{Z}_{bc}^{0} \mu^{-} \overline{\nu}_{\mu}$	$(4,44 \pm 1,26)\times 10^{-10}$ MeV
	$T_{b:\bar{d}}^{-} \to \tilde{Z}_{bc}^{0} \tau^{-} \overline{\nu}_{\tau}$	$(1,99 \pm 0,56)\times 10^{-10}$ MeV
	$T_{b:\bar{d}}^{-} \to \tilde{Z}_{bc}^{0} \pi^{-}$	$(5,13 \pm 1,42)\times 10^{-13}$ MeV
	$T_{b:\bar{d}}^{-} \to \tilde{Z}_{bc}^{0} K^{-}$	$(3,93 \pm 1,12)\times 10^{-14}$ MeV
	$T_{b:\bar{d}}^{-} \to \tilde{Z}_{bc}^{0} D^{-}$	$(8,49 \pm 2,41)\times 10^{-14}$ MeV
	$T_{b:\bar{d}}^{-} \to \tilde{Z}_{bc}^{0} D_{s}^{-}$	$(2,92 \pm 0,82)\times 10^{-12}$ MeV
AV → Scalar	$T_{b:\bar{d}}^{AV} \to Z_{bc}^{0} e^{-} \overline{\nu}_{e}$	$(2,65 \pm 0,78)\times 10^{-8}$ MeV
	$T_{b:\bar{d}}^{AV} \to Z_{bc}^{0} \mu^{-} \overline{\nu}_{\mu}$	$(2,64 \pm 0,78)\times 10^{-8}$ MeV
	$T_{b:\bar{d}}^{AV} \to Z_{bc}^{0} \tau^{-} \overline{\nu}_{\tau}$	$(1,88 \pm 0,55)\times 10^{-8}$ MeV
	$T_{b:\bar{s}}^{AV} \to Z_{bc}^{0} e^{-} \overline{\nu}_{e}$	$(5,34 \pm 1,43)\times 10^{-8}$ MeV
	$T_{b:\bar{s}}^{AV} \to Z_{bc}^{0} \mu^{-} \overline{\nu}_{\mu}$	$(5,32 \pm 1,41)\times 10^{-8}$ MeV
	$T_{b:\bar{s}}^{AV} \to Z_{bc}^{0} \tau^{-} \overline{\nu}_{\tau}$	$(2,15 \pm 0,54)\times 10^{-8}$ MeV
	$T_{b:\bar{s}}^{AV} \to Z_{bc}^{0} \rho^{-}$	$(3,47 \pm 0,92)\times 10^{-10}$ MeV
	$T_{b:\bar{s}}^{AV} \to Z_{bc}^{0} K^{*}(892)$	$(1,47 \pm 0,37)\times 10^{-11}$ MeV
	$T_{b:\bar{s}}^{AV} \to Z_{bc}^{0} D^{*}(2010)^{-}$	$(1,54 \pm 0,39)\times 10^{-11}$ MeV
	$T_{b:\bar{s}}^{AV} \to Z_{bc}^{0} D_{s}^{*-}$	$(4,97 \pm 1,32)\times 10^{-10}$ MeV
AV → AV	$T_{b:\bar{d}}^{AV} \to \tilde{T}_{bc}^{AV} e^{-} \overline{\nu}_{e}$	$(2,02 \pm 0,39)\times 10^{-9}$ MeV
	$T_{b:\bar{d}}^{AV} \to \tilde{T}_{bc}^{AV} \mu^{-} \overline{\nu}_{\mu}$	$(1,96 \pm 0,37)\times 10^{-9}$ MeV
	$T_{b:\bar{d}}^{AV} \to \tilde{T}_{bc}^{AV} \tau^{-} \overline{\nu}_{\tau}$	$(1,03 \pm 0,19)\times 10^{-10}$ MeV
	$T_{b:\bar{d}}^{AV} \to \tilde{T}_{bc}^{AV} \pi^{-}$	$(5,84 \pm 1,11)\times 10^{-10}$ MeV
	$T_{b:\bar{d}}^{AV} \to \tilde{T}_{bc}^{AV} K^{-}$	$(6,43 \pm 1,32)\times 10^{-11}$ MeV
	$T_{b:\bar{d}}^{AV} \to \tilde{T}_{bc}^{AV} D^{-}$	$(3,01 \pm 0,64)\times 10^{-11}$ MeV
	$T_{b:\bar{d}}^{AV} \to \tilde{T}_{bc}^{AV} D_{s}^{-}$	$(7,80 \pm 1,54)\times 10^{-10}$ MeV

Concluding Remarks

- These studies are very important in terms of contributing to the theoretical literature as well as shedding light on future experiments.
- In addition, the experimental results obtained recently for the weak decay channels of hadrons and the standard model predictions do not overlap, and there are predictions that this inconsistency may have a new physics effect.
- Therefore, our results for the weak decays of exotic hadrons will also guide the search for new physics.
- In addition, studies include information about the perturbative and non-perturbative aspects of QCD, which is one of the four-basic interactions of the universe.
- Therefore, when the obtained results are combined with the experimental predictions, it will also help to increase our knowledge about the universe.

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“The most incomprehensible thing about the Universe is that it is comprehensible.”

ALBERT EINSTEIN

Thank you

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