NNLO accurate QCD predictions for the LHC

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QCD@Work 2022, Lecce, June 29, 2022

Outline

Introduction

• NNLO computations

- heavy quark production

- The way to jet processes
- First NLO results
- Summary and Outlook

QCD at hadron colliders



High- p_T interactions are characterised by the presence of a hard scale Q(invariant mass of a lepton pair, high- p_T jet, heavy-quark mass...)



Can be controlled through the factorisation theorem

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij}(p_1, p_2, \alpha_S(\mu_R), Q^2; \mu_F^2, \mu_R^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)$$

Parton distributions: universal but not perturbatively computable Hard partonic cross section: process dependent but computable in perturbation theory T Power-suppressed contributions

The factorisation picture is systematically improvable (until the power-suppressed contributions become quantitative relevant...)

Fully differential predictions



LHC detectors are able to measure leptons, photons and jets only if they have a finite (relatively large) transverse momentum and not too large rapidity

Fully differential predictions needed

At LO everything is finite but at NLO real and virtual contributions are separately divergent and after renormalisation IR poles appear as $D \rightarrow 4$

The need of regularizing the divergences in $D = 4 - 2\epsilon$ dimensions prevents a straightforward implementation of numerical techniques

$$d\sigma = \int_{n+1} r d\Phi_{n+1} + \int_n v d\Phi_n$$

Add and subtract a (local) counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton Catani, Seymour (1995) Frixione, Kunszt, Signer (1996)

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Fully differential predictions



Beam pipe

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Subtraction method

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NLO

- Nowadays tree-level and one-loop amplitudes can be computed automatically Recola, Openloops, Gosam....
- This "next-to-leading order (NLO) revolution" has left us with flexible tools that make possible to carry out relatively precise computations at NLO in QCD and EW theory



But the precision of experimental data now calls for a step forward and on the inclusion of next-to-next-to-leading order (NNLO) QCD corrections

The quest for NNLO



data/theory

NNLO: building blocks



Crucial to keep the calculation fully differential: corrections for fiducial and inclusive rates may be significantly different (H in VBF, WW...)

NNLO methods

Broadly speaking there are two approaches that we can follow:

- Organise the calculation from scratch so as to cancel all the singularities
 - sector decomposition
 - antenna subtraction
 - "colourful" subtraction

- join subtraction and sector decomposition

Binoth, Heinrich (2000,2004) Anastasiou, Melnikov, Petriello (2004)

Gehrmann, Glover (2005)

Somogyi, Trocsanyi, Del Duca (2005, 2007)

Czakon (2010,2011) Boughezal, Melnikov, Petriello (2011) Caola, Melnikov, Rontsch (2017)

- Start from an inclusive NNLO calculation (sometimes obtained through resummation) and combine it with an NLO calculation for n+1 parton process
 - q_T subtraction
 - "N-jettiness" method
 - "Born projection" method for VBF

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NNLO progress



NNLO results lead to much better description of the data

NNLO: deployment of results

NNLO computations are generally rather expensive (may need millions of CPU hours for a production run): most results obtained through private codes

Lack of public code makes deployment of NNLO precision difficult and requires manual intervention and involvement of the authors of the various calculations in concrete applications

Up to few years ago public codes available only for limited processes

Essentially two general purpose public codes

• MATRIX

https://matrix.hepforge.org

• MCFM

https://mcfm.fnal.gov

Both are based on non-local subtraction schemes (q_T and jettiness)

MATRIX v2.1.0

Kallweit, Wiesemann, MG (June 2017) + Buonocore, Devoto, Mazzitelli, Rottoli......

process_id	<u>III</u>	process	11	description
pph21	>>	рр> Н	>>	on-shell Higgs production (NNLO)
ppz01	>>	p p> Z	>>	on-shell Z production (NNLO,NLO EW)
ppw01	>>	p p> W^-	>>	on-shell W- production with CKM (NNLO)
ppwx01	>>	p p> W^+	>>	on-shell W+ production with CKM (NNLO)
ppeex02	>>	p p> e^- e^+	>>	Z production with decay (NNLO,NLO EW)
ppnenex02	>>	p p> v_e^- v_e^+	>>	Z production with decay (NNLO,NLO EW)
ppenex02	>>	p p> e^- v_e^+	>>	W- production with decay and CKM (NNLO,NLO EW)
ppexne02	>>	p p> e^+ v e^-	>>	W+ production with
ppaa02	>>	p p> gamma gamma	>>	gamma gamma product NNLO QCD + NLO EW for
ppeexa03	>>	$p p> e^- e^+ gamma$	>>	Z gamma production all the single and massive
ppnenexa03	>>	$p p> v e^- v e^+ gamma$	>>	Z gamma production
ppenexa03	>>	$p p> e^- v e^+ gamma$	>>	W- gamma production diboson processes
ppexnea03	>>	$p p> e^+ v e^-$ gamma	>>	W+ gamma production with decay (MALO)
ppzz02	>>	p p> Z Z	>>	on-shell ZZ production (NNLO)
ppwxw02	>>	p p> W^+ W^-	>>	on-shell WW production (NNLO)
ppemexmx04	>>	$p p = -> e^{-} mu^{-} e^{+} mu^{+}$	>>	ZZ production with deca
ppeeexex04	>>	p p> e^- e^- e^+ e^+	>>	ZZ production with deca NLO QCD for loop
ppeexnmnmx04	>>	p p> e^- e^+ v_mu^- v_mu^+	>>	ZZ production with deca inclused an contribution
ppemxnmnex04	>>	$p p = -> e^{-} mu^{+} v mu^{-} v e^{+}$	>>	WW production with deca made age contribution
ppeexnenex04	>>	p p> e^- e^+ v_e^- v_e^+	>>	ZZ/WW production with d for WW and ZZ
ppemexnmx04	>>	$p p = -> e^{-} mu^{-} e^{+} v_{mu}^{+}$	>>	W-Z production with deca, there have been
ppeeexnex04	>>	p p> e^- e^- e^+ v_e^+	>>	W-Z production with decay (NNLO,NLO EW)
ppeexmxnm04	>>	p p> e^- e^+ mu^+ v mu^-	>>	W+Z production with decay (NNLO,NLO EW)
ppeexexne04	>>	p p> e^- e^+ e^+ v e^-	>>	W+Z production with decay (NNLO,NLO EW)
ppttx20	>>	p p> top anti-top	>>	on-shell top-pair production (NNLO)
ppaaa03	>>	p p> gamma gamma gamma	>>	gamma gamma gamma production (NNLO)
======================================	>>			

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ppemexnmx0	Section 4		>>	W-Z production with decay (marchine any
ppeeexnex0 1	and	www.now.available_v_e^+	>>	W-Z production with decay (NNLO, NLO EW)
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The method

Catani, MG (2007)

Consider the hard-scattering process $pp \rightarrow F + X$ (*F* colourless system, heavy quark pair....)

Use a dimensionless resolution variable $r > r_{cut}$ (e.g. $r = q_T/Q$)

Real contribution with one additional resolved jet, divergent as $r_{cut} \rightarrow 0$

Subtraction counterterm that cancels the $r_{cut} \rightarrow 0$ singularity

$$d\sigma_{NNLO}^{F+X} = \mathcal{H}_{NNLO}^{F} \otimes d\sigma_{LO}^{F} + \left[d\sigma_{NLO}^{F+\text{jets}} - d\sigma_{NNLO}^{CT,F} \right] + \mathcal{O}(r_{\text{cut}}^{p})$$

Virtual contribution after subtraction of IR singularities + collinear and large-angle soft radiation (beam, jet and soft function)

Power suppressed contribution whose size determines the efficiency of the computation

Catani, Devoto, Kallweit, Mazzitelli, Sargsyan, MG (2019)

Extension of qT subtraction to heavy-quark production now completed

$$d\sigma_{(N)NLO}^{t\bar{t}} = \mathcal{H}_{(N)NLO}^{t\bar{t}} \otimes d\sigma_{LO}^{t\bar{t}} + \left[d\sigma_{(N)LO}^{t\bar{t}+\text{jets}} - d\sigma_{(N)LO}^{CT} \right]$$

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Modified subtraction counterterm fully known

Additional perturbative ingredient: soft anomalous dimension Γ_t known at NNLO

Mitov, Sterman, Sung (2009) Neubert et al (2009)

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 \checkmark Additional soft contributions needed to evaluate $\mathcal{H}_{NNLO}^{t\bar{t}}$

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	Inclusive cross section							
$\sigma_{\rm NNLO} \ [{\rm pb}]$		Matrix	TOP++					
8	TeV	$238.5(2)^{+3.9\%}_{-6.3\%}$	$238.6^{+4.0\%}_{-6.3\%}$					
$13 { m TeV}$		$794.0(8)^{+3.5\%}_{-5.7\%}$	$794.0^{+3.5\%}_{-5.7\%}$					
$100 { m TeV}$		$35215(74)^{+2.8\%}_{-4.7\%}$	$35216^{+2.9\%}_{-4.8\%}$					

Tree and loop amplitudes from Openloops 2 (cross checked with Recola)

Two-loop amplitudes from Czakon et al. (0.1% effect at 13 TeV)

statistical+systematic

scale uncertainties



Extension to bottom production

Catani, Devoto, Kallweit, Mazzitelli, MG (2020)



The case of jet processes

Buonocore, Haag, Savoini, Rottoli, MG (2022)

Transverse momentum is a viable resolution variable to describe arbitrary processes in which heavy quarks and colourless particles are produced at Born level

Besides $t\bar{t}$ and $b\bar{b}$ production further possible applications are:

 $t\bar{t}H, Wt\bar{t}, Zt\bar{t}, Wb\bar{b}, WWb\bar{b}....$

However q_T cannot regularise final state collinear singularities

$$q_T = |p_V + p_j|_T \neq 0$$

N-jettiness

N-jettiness is a global shape variable smoothly describing the $N + 1 \rightarrow N$ jet transition

Stewart, Tackmann, Waalewijn (2010)

$$\tau_N = \frac{2}{Q^2} \sum_k \min\{q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \dots, q_N \cdot p_k\}$$

It is considered the natural extension of q_T to jet processes

0-jettiness successfully used to compute NNLO corrections to several coloursinglet processes

Boughezal et al (2016) Campbell, Ellis, Li, Williams (2016) Heinrich, Jahn, Jones, Kerner, Pires (2017)

1-jettiness applied to NNLO computations of V+jet and H+jet

Boughezal et al (2015, 2016)

Also used as evolution variable in matching NNLO computations to parton shower simulations

Never applied to more complicated processes

The quantitative impact of power suppressed contributions can limit the performance of non-local subtraction methods

$$d\sigma_{NNLO}^{F+X} = \mathscr{H}_{NNLO}^{F} \otimes d\sigma_{LO}^{F} + \left[d\sigma_{NLO}^{F+\text{jets}} - d\sigma_{NNLO}^{CT,F} \right] + \mathcal{O}(r_{\text{cut}}^{p})$$

The larger the power corrections, the smaller values of r_{cut} must be chosen

The computation of leading power suppressed terms may help to obtain better quantitative predictions Ebert, Moult, Stewart, Tackmann, Vita (2018)

Jert, Mount, Stewart, Tackinanni, Vita (2010)

Boughezal, Isgro', Petriello (2018,2019)

Cieri, Oleari, Rocco (2019)

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When fixed-order predictions are supplemented with all-order resummation and compared with data hadronisation and multi-partonic interactions (MPI) can be large and substantially diluite the power of the considered observable







Exploring jet resolution variables

We look for a resolution variable with some specific good properties

- It reduces to q_T when jets are not present at Born level
- Linear (in the worse case) power corrections
- No non-global logs
- Can be extended to an arbitrary number of jets

We consider the inclusive *N*-jet production process

$$\begin{split} h_1(P_1) + h_2(P_2) \to j(p_1) + j(p_2) + \dots j(p_N) + F(p_F) + X \\ & \bigstar \\ & \texttt{Possible colourless final-state} \end{split}$$

Our proposal: k_T^{ness}

We introduce a global dimensionful variable able to capture the $N + 1 \rightarrow N$ jet transition

The variable represents an effective transverse momentum controlling the singularities in the $N + 1 \rightarrow N$ jet transition

When the unresolved radiation is close to the beam k_T^{ness} coincides with the transverse momentum of the final state system

When the unresolved radiation is close to one of the final state jets k_T^{ness} represents the relative transverse momentum with respect to the jet direction

The variable takes its name from the k_T clustering algorithm

Catani, Dokshitzer, Seymour, Webber (1993) Ellis, Soper (1993)

It is defined through a recursive procedure

Our proposal: k_T^{ness}

Define the distances

$$d_{ij} = \min(p_{Ti}, p_{Tj})\Delta R_{ij}/D$$
 (pseudo)-particle distance
 $d_{iB} = p_{Ti}$ particle-beam distance

where *D* is a parameter of order unity and $\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$ is the standard separation in rapidity and azimuth

Our variable is defined via a recursive procedure through which close-by particles are combined with each other or with the beam until *N* + 1 jets remain

When particles are recombined with the beam we keep track of their transverse momentum through $p_{\rm rec} \to p_{\rm rec} + p_i$

When N + 1 protojets are left we still evaluate all the d_{iB} and d_{ij}

- If the minimum is a d_{ij} we set $k_T^{\text{ness}} = \min(d_{ij})$
- If the minimum is a d_{iB} add the recoil to p_i and set $k_T^{\text{ness}} = p_{Ti}$

Sample NLO results: H+jet

As a first application we consider H+jet

Compared to jettiness the power suppressed contributions are mild and scale linearly (no logarithmic enhancement)



Sample NLO results: H+jet

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Hadronisation and MPI effects

We have generated an LO sample for Z+jet with POWHEG and showered them with PYTHIA8

We compare 1-jettiness with $1-k_T^{ness}$

As expected the hadronisation effects on jettiness are relatively large while MPI completely distorts the shape of the distribution

The k_T^{ness} distribution has a peak at $k_T^{\text{ness}} \sim 15 \text{ GeV}$ and features much smaller hadronisation and MPI effects



Further NLO results: 2 and 3 jets

We have recently implemented k_T^{ness} subtraction in MATRIX

Flexible implementation and consistent comparisons with CS subtraction

Nice linear convergence to the result obtained with CS subtraction



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Summary & Outlook

The current and expected precision of LHC data requires NNLO accurate QCD predictions for the most relevant processes

NNLO results now available for essentially all the relevant $2 \rightarrow 1$ and $2 \rightarrow 2$ processes and lead to an improved description of the data for many benchmark processes

NNLO computations are challenging both from a technical view point but also as far as computing resources are concerned

The only general purpose publicly available codes able to compute NNLO corrections in QCD are based on non-local subtraction schemes

For the hadronic production of colourless systems and heavy quarks q_T has proven to be an extremely efficient resolution variable

Summary & Outlook

For processes involving jets N-jettiness has been the only player so far but
 leads to large power suppressed contributions and, more generally to large non-perturbative and MPI effects

- We introduced a new variable, that we dub k_T^{ness} : this variable represents an effective transverse momentum controlling the $N + 1 \rightarrow N$ transition
- We have presented NLO results for processes involving one or more jets and successfully compared them to results obtained with dipole subtraction
- Our results show that power corrections for k_T^{ness} scale linearly and are relatively small, providing a promising candidate for NNLO applications
- The new variable appears to be quite stable with respect to hadronisation and MPI and might prove useful also for Parton Showers

Backup

Catani, Devoto, Kallweit, Mazzitelli, MG (2019)

Fully differential results

LO, NLO and NNLO predictions obtained using NNPDF3.1 PDFs with $\alpha_S(m_Z)=0.118$ at the corresponding order

CMS data of CMS-TOP-17-002 in the lepton+jets channel

Extrapolation to parton level in the inclusive phase space



Our calculation is carried out without cuts

To compare with data we multiply our absolute predictions by 0.438 (semileptonic BR of the t \overline{t} pair) times 2/3 (only electrons and muons)



Good description of the data except in the first bin

Issues in extrapolation ? Smaller mt ?

A smaller m_t (just by about 2 GeV) leads to a higher theoretical prediction in this bin and to small changes at higher m_{tt}

CMS-TOP-18-004: leptonic channel: a fit with the same PDFs leads to $m_t=170.81 \pm 0.68$ GeV



The first m_{tt} interval now extends up to 450 GeV \rightarrow better agreement with the data

Single-differential distributions



As noted in various previous analyses the measured p_T distribution is slightly softer than the NNLO prediction

Perturbative prediction relatively stable when going from NLO to NNLO

Data and theory are consistent within uncertainties

NNLOPS

Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi (2020)

NNLO calculation recently deployed into the first NNLO calculation matched
to parton shower for this processMonni, Nason, Re, Wiesemann,
Zanderighi (2019)
Monni, Re, Wiesemann (2020)All-order radiative contributions implemented through
the shower using the MiNNLOPS methodMonni, Re, Wiesemann,
Zanderighi (2019)
Monni, Re, Wiesemann (2020)













Example: V+jet $p_{\rm rec}$

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 $p_{\rm rec}$

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NLO ingredients

We have computed the singular behavior of the cross section for the production of a colourless system accompanied by an arbitrary number of jets as $k_T^{\text{ness}} \rightarrow 0$

The computation starts by organizing the terms relevant in each singular region and removing the double counting

The results are used to construct a subtraction formula

$$d\sigma_{NLO}^{F+N\,\text{jets}+X} = \mathscr{H}_{NLO}^{F+N\,\text{jets}} \otimes d\sigma_{LO}^{F+N\,\text{jets}} + \left[d\sigma_{LO}^{F+(N+1)\,\text{jets}} - d\sigma_{NLO}^{\text{CT},F+N\,\text{jets}}\right]$$

The counterterm is particularly simple

initial- and final-state partons

$$d\hat{\sigma}_{\text{NLO}\,ab}^{\text{CT,F+Njets}} = \frac{\alpha_{\text{S}}}{\pi} \frac{dk_T^{\text{ness}}}{k_T^{\text{ness}}} \left\{ \left[\ln \frac{Q^2}{(k_T^{\text{ness}})^2} \sum_{\alpha} C_{\alpha} - \sum_{\alpha} \gamma_{\alpha} - \sum_{i} C_i \ln (D^2) - \sum_{\alpha \neq \beta} \langle \mathbf{T}_{\alpha} \cdot \mathbf{T}_{\beta} \rangle \ln \left(\frac{2p_{\alpha} \cdot p_{\beta}}{Q^2} \right) \right] \times \right. \\ \left. \times \delta_{ac} \delta_{bd} \delta(1 - z_1) \delta(1 - z_2) + 2\delta(1 - z_2) \delta_{bd} P_{ca}^{(1)}(z_1) + 2\delta(1 - z_1) \delta_{ac} P_{db}^{(1)}(z_2) \right\} \otimes d\hat{\sigma}_{\text{LO}\,cd}^{\text{F+N jets}}$$
initial-state collinear contributions

NLO ingredients

The hard-virtual term reads

collinear (beam) functions

$$\mathcal{H}_{cd;ab}^{\text{F+N jets}} = (\text{HS})_{cd}C_{ca}C_{db}\prod_{i=1,...,N}J_i$$
Soft function

$$(\text{HS})_{cd} = \frac{\langle \mathcal{M}_{cd} | \mathbf{S} | \mathcal{M}_{cd} \rangle}{|\mathcal{M}_{cd}^{(0)}|^2}$$

$$\mathbf{S} = 1 + \frac{\alpha_{\text{S}}(\mu_R)}{\pi}\mathbf{S}^{(1)} + \mathcal{O}(\alpha_{\text{S}}^2)$$

The soft contribution is expressed in terms of one- and two-fold integrals that are evaluated numerically

Sample NLO results: Z+2jets

Next we consider Z+2jets: we require a dilepton pair with 66 GeV $\leq m_{ll} \leq 116$ GeV The leptons have $p_{T,l} > 20$ GeV and $|\eta_l| < 2.5$ with $\Delta R_{ll} > 0.2$ and $\Delta R_{lj} > 0.5$

We require at least two jets with $p_T > 30 \text{ GeV}$ and $|\eta| < 2.5$

Our results for the fiducial cross section nicely converge to the benchmark result in all the partonic channels

Sample NLO results: Z+2jets

Next we consider Z+2jets: we require a dilepton pair with 66 GeV $\leq m_{ll} \leq 116$ GeV The leptons have $p_{T,l} > 20 \text{ GeV}$ and $|\eta_l| < 2.5 \text{ with } \Delta R_{ll} > 0.2 \text{ and } \Delta R_{li} > 0.5$

We require at least two jets with $p_T > 30 \text{ GeV}$ and $|\eta| < 2.5$

Excellent agreement observed also at the level of the distributions

 $pp \rightarrow \ell^+ \ell^- + 2j + X$