Nonperturbative renormalization in large-N QCD and topological strings

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Talk mainly based on:

M. B. An asymptotic solution of large-N QCD for the glueball and meson spectrum and the collinear S matrix HADRON 2015, AIP Conference Proceedings 1735(2016) 1, 030004

M. B. The large-N Yang-Mills S matrix is ultraviolet finite, but the large-N QCD S matrix is only renormalizable, Phys. Rev. D 95 (2017) 054010, arXiv:1701.07833 [hep-th]

M. B. Renormalization in large-N QCD is incompatible with open/closed string duality, Phys. Lett. B 783 (2018) 341, arXiv:1703.10176 [hep-th]

and to appear in arXiv …

Plan of the talk

(1) We work out the nonperturbative renormalization properties of large-N QCD and QCD-like theories

(2) On the basis of the aforementioned renormalization properties, we demonstrate two versions of a no-go theorem for the existence of a canonical string solution - matching the topology of the 't Hoof large-N expansion and conformal invariant on the world sheet - of large-N QCD and n=1 SUSY QCD

(3) We define a new class of noncanonical string theories that may evade the no-go theorem, and we investigate their generic spectral and UV properties

Part I

The first aim of this talk is to answer the following fundamental question that, surprisingly, has not been asked for more than 40 years

We know that YM theory and QCD are not UV finite, but only renormalizable, in perturbation theory

Yet, we may ask which are the renormalization properties of the YM theory and QCD (with massless quarks at first, for simplicity) nonperturbatively in the large-N 't Hooft expansion

We recall that the large-N 't Hooft limit of SU(N) QCD (with N_f massless quarks):

$$Z = \int \delta A \delta\psi \delta\bar{\psi} \exp\left(-\frac{N}{g^2} \int TrF^2 + \sum_{N_f} \bar{\psi}_f \gamma_\alpha D_\alpha \psi_f\right)$$

is a free theory of glueballs and mesons to leading 1/N order, which become weakly coupled to the next order with couplings O(1/N) and O(1/sqrt N) respectively (the leading nontrivial 1/N order is 't Hooft planar theory)

In the glueball sector: (G. 't Hooft 1974)

$$< \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n) >_{conn} \sim N^{2-n}$$

In the meson sector:

$$< \mathcal{M}_1(x_1)\mathcal{M}_2(x_2)\cdots\mathcal{M}_k(x_k) >_{conn} \sim N^{1-\frac{k}{2}}$$

In the meson/glueball sector:

$$< \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)\mathcal{M}_1(x_1)\mathcal{M}_2(x_2)\cdots\mathcal{M}_k(x_k) >_{conn} \sim N^{1-n-\frac{k}{2}}$$

Indeed, to the leading 1/N order,
because of the vanishing of the interaction associated to 3 and multi-point correlators,

the connected two-point correlators, by assuming confinement, are an infinite sum of free propagators satisfying the the Kallen-Lehmann representation (A. Migdal, 1977):

$$\int \langle \mathcal{O}^{(s)}(x)\mathcal{O}^{(s)}(0)\rangle_{conn} e^{-ip\cdot x} d^4x = \sum_{n=1}^{\infty} P^{(s)}\left(\frac{p_\alpha}{m_n^{(s)}}\right) \frac{|<0|\mathcal{O}^{(s)}(0)|p,n,s>'|^2}{p^2+m_n^{(s)2}}$$

$$<0|\mathcal{O}^{(s)}(0)|p,n,s,j> = e_j^{(s)}\left(\frac{p_\alpha}{m}\right) <0|\mathcal{O}^{(s)}(0)|p,n,s>'$$

$$\sum_j e_j^{(s)}\left(\frac{p_\alpha}{m}\right)\overline{e_j^{(s)}\left(\frac{p_\alpha}{m}\right)} = P^{(s)}\left(\frac{p_\alpha}{m}\right)$$

Moreover, the residues of the poles of the propagators

contain the scheme-indepedent information on the anomalous dimensions and the beta function of the theory,

as the following asymptotic theorem shows

M.B. Glueball and meson propagators of any spin in large-N QCD
M. B. Nucl. Phys. B 875 (2013) 621[hep-th/1305.0273]

Asymptotic Theorem:

$$\int \langle \mathcal{O}^{(s)}(x)\mathcal{O}^{(s)}(0)\rangle_{conn}\, e^{-ip\cdot x} d^4x \sim \sum_{n=1}^{\infty} P^{(s)}\big(\frac{p_\alpha}{m_n^{(s)}}\big)\frac{m_n^{(s)2D-4} Z_n^{(s)2} \rho_s^{-1}(m_n^{(s)2})}{p^2+m_n^{(s)2}}$$

$$= P^{(s)}\big(\frac{p_\alpha}{p}\big)\, p^{2D-4}\sum_{n=1}^{\infty}\frac{Z_n^{(s)2}\rho_s^{-1}(m_n^{(s)2})}{p^2+m_n^{(s)2}} + \cdots$$

$$\sim P^{(s)}\big(\frac{p_\alpha}{p}\big)\, p^{2D-4}\int_{m_1^{(s)2}}^{\infty}\frac{Z^{(s)2}(m)}{p^2+m^2}dm^2 + \cdots$$

$$\sim P^{(s)}\big(\frac{p_\alpha}{p}\big)\, p^{2D-4}\Big[\frac{1}{\beta_0\log(\frac{p^2}{\Lambda^2_{QCD}})}\Big(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{p^2}{\Lambda^2_{QCD}})}{\log(\frac{p^2}{\Lambda^2_{QCD}})}+O(\frac{1}{\log(\frac{p^2}{\Lambda^2_{QCD}})})\Big)\Big]^{\frac{\gamma_0}{\beta_0}-1}$$

$$\sum_{n=1}^{\infty} f(m_n^{(s)2}) \sim \int_1^{\infty} f(m_n^{(s)2})dn = \int_{m_1^{(s)2}}^{\infty} f(m^2)\rho_s(m^2)dm^2$$

$$Z_n^{(s)} \equiv Z^{(s)}(m_n^{(s)}) = \exp\int_{g(\mu)}^{g(m_n^{(s)})}\frac{\gamma_{\mathcal{O}^{(s)}}(g)}{\beta(g)}dg$$

$$Z_n^{(s)2} \sim \Big[\frac{1}{\beta_0\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}}\Big(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}}{\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}}+O(\frac{1}{\log\frac{m_n^{(s)2}}{\Lambda^2_{QCD}}})\Big)\Big]^{\frac{\gamma_0}{\beta_0}}$$

Hence, the large-N nonperturbative solution would replace QCD

viewed as a theory of
gluons and quarks that is strongly coupled in the infrared in perturbation theory,

with a theory of glueballs and mesons
that is weakly coupled at all energy scales

S-matrix renormalization

Nonperturbatively, the S matrix in YM and massless QCD only depends on the RG-invariant scale:

$$\Lambda_{RG} = const\,\Lambda\ \exp\left(-\frac{1}{2\beta_0 g^2}\right)\left(\beta_0 g^2\right)^{-\frac{\beta_1}{2\beta_0^2}}\left(1+\sum_{n=1} c_n g^{2n}\right)$$

because the S matrix cannot depend on the anomalous dimensions of the gauge-invariant operators that create the asymptotic states (indeed, the aforementioned residues of the poles cancel in the LSZ formulas)

Of course, the finiteness of Λ_{RG} as the cutoff diverges is equivalent to the gauge coupling renormalization

Therefore, the UV finiteness of the S matrix is equivalent to the UV finiteness of 1/N expansion of the RG invariant scale in terms of the planar RG invariant

In large-N YM the first-two coefficients of the beta function are only planar without 1/N corrections

$$\beta_0 = \beta_0^P = \frac{1}{(4\pi)^2}\frac{11}{3}$$

$$\beta_1 = \beta_1^P = \frac{1}{(4\pi)^4}\frac{34}{3}$$

Hence, the further 1/N corrections contribute at most only a finite change of renormalization scheme:

$$\Lambda_{YM} \sim const\, \Lambda_{YM}^P \left(1 + \sum_{n=1} c_n\, O(\frac{1}{\log^n(\frac{\Lambda}{\Lambda_{YM}^P})})\right)$$

Thus, all glueball loops are UV finite in the YM S matrix since,

if they were not, they would imply a divergent

renormalization of the planar RG invariant scale,

which is the only parameter in the S matrix,

contrary to what we have just proved

Hence, the $1/N$ expansion of the YM S matrix is UV finite,
and the UV finiteness is a consequence of the RG group and
asymptotic freedom (AF) of the YM planar theory

Instead, in large-N QCD the first-two coefficients of the beta function get corrections to the order of N_f/N

$$\beta_0 = \beta_0^P + \beta_0^{NP} = \frac{1}{(4\pi)^2}\frac{11}{3} - \frac{1}{(4\pi)^2}\frac{2}{3}\frac{N_f}{N}$$

$$\beta_1 = \beta_1^P + \beta_1^{NP} = \frac{1}{(4\pi)^4}\frac{34}{3} - \frac{1}{(4\pi)^4}\big(\frac{13}{3} - \frac{1}{N^2}\big)\frac{N_f}{N}$$

Thus, in large-N QCD the planar RG invariant gets a log-divergent renormalization starting from the order of N_f/N:

$$
\begin{aligned}
\sqrt{T} = \Lambda_{QCD} \sim \ & \Lambda \exp(-\frac{1}{2\beta_0 g^2}) \\
= \ & \Lambda \exp(-\frac{1}{2\beta_0^P(1+\frac{\beta_0^{NP}}{\beta_0^P})g^2}) \\
\sim \ & \Lambda \exp(-\frac{(1-\frac{\beta_0^{NP}}{\beta_0^P})}{2\beta_0^P g^2}) \\
\sim \ & \Lambda \exp(-\frac{1}{2\beta_0^P g^2}) \exp(\frac{\frac{\beta_0^{NP}}{\beta_0^P}}{2\beta_0^P g^2}) \\
\sim \ & \Lambda \exp(-\frac{1}{2\beta_0^P g^2})(1+\frac{\frac{\beta_0^{NP}}{\beta_0^P}}{2\beta_0^P g^2}+\cdots) \\
\sim \ & \Lambda \exp(-\frac{1}{2\beta_0^P g^2})(1+\frac{\beta_0^{NP}}{\beta_0^P}\log(\frac{\Lambda}{\Lambda_{QCD}^P})+\cdots) \\
= \ & \Lambda_{QCD}^P(1+\frac{\beta_0^{NP}}{\beta_0^P}\log(\frac{\Lambda}{\Lambda_{QCD}^P})+\cdots) \\
= \ & \sqrt{T^P}(1+\frac{\beta_0^{NP}}{\beta_0^P}\log(\frac{\Lambda}{\sqrt{T^P}})+\cdots)
\end{aligned}
$$

$$\Lambda_{QCD} \sim \Lambda_{QCD}^{P} \left[1 + \frac{\beta_0^{NP}}{\beta_0^{P}} \log(\frac{\Lambda}{\Lambda_{QCD}^{P}}) + \frac{\beta_1^{P}}{2{\beta_0^{P}}^2} \log\log(\frac{\Lambda}{\Lambda_{QCD}^{P}}) \left(\frac{\beta_1^{NP}}{\beta_1^{P}} - \frac{\beta_0^{NP}}{\beta_0^{P}} \right) + \ldots \right]$$

Hence, since the glueball and meson masses are proportional to the RG invariant scale, glueball and meson self-energies are log divergent in large-N QCD starting from the order of N_f/N.

The log divergence arises because of

the asymptotic freedom (AF) of the planar theory,
i.e. of the YM theory,

and of the change of the beta function
to the order of N_f/N due to the quark loops

Part 2

The second aim of this talk is to work out the implications of these seemingly innocuous renormalization properties

for the existence of a would-be canonical string solution

of large-N YM and QCD

Indeed,

the large-N limit of SU(N) QCD (with massless quarks):

$$Z=\int \delta A \delta\psi \delta\bar{\psi} \exp(-\frac{N}{g^2}\int TrF^2+\sum_{N_f}\bar{\psi}_f\gamma_\alpha D_\alpha\psi_f)$$

(G. 't Hooft 1974)

is universally believed to be solved by a yet-to-be-found string theory, of closed strings in the glueball sector, and of open strings in the meson sector

The main evidence is the large-N counting of Feynman diagrams

In the glueball sector:

$$<\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)>_{conn}\sim N^{2-n}$$

In the meson sector:

$$<\mathcal{M}_1(x_1)\mathcal{M}_2(x_2)\cdots\mathcal{M}_k(x_k)>_{conn}\sim N^{1-\frac{k}{2}}$$

In the meson/glueball sector:

$$<\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)\mathcal{M}_1(x_1)\mathcal{M}_2(x_2)\cdots\mathcal{M}_k(x_k)>_{conn}\sim N^{1-n-\frac{k}{2}}$$

This is exactly the canonical counting that we would get from a string theory with string coupling: g_s=1/N

of closed strings in the glueball sector:

a sphere with n punctures

of open strings in the meson sector:

a disk with k punctures on the boundary

and of open/closed strings in the meson/glueball sector:

a disk with k punctures on the boundary and n in the interior

This is the 't Hooft planar theory, that describes tree amplitudes

Then, unitarization introduces higher-genus contributions, matching the topology of the 't Hooft expansion as well, that correct the planar theory by string diagrams with a weight that is 1/N to a power equal to minus the Euler characteristic

Physically, this is the standard picture of confinement where

mesons are bound states of quarks linked by a chromo-electric flux tube and

glueballs are rings of chromo-electric flux

with the string world-sheet identified with the flux tube

Yet, this physical interpretation is not necessary in the canonical string framework. You may consider an arbitrary string background (higher dimensions, curvature, D-branes, RR sectors…) provided that it leads to a conformal-invariant string theory on the world sheet.

Now, the UV finiteness of the large-N YM theory, due to AF and RG, which we have just found out on the gauge side, is

compatible

with the universally believed UV finiteness of (consistent) closed-string theories (due to the underlying modular invariance on the closed-string side)

Thus, a canonical string solution of the pure large-N YM theory may exist

But, contrary to the universal belief,
we prove in the present talk a first NO-GO THEOREM that the aforementioned renormalization properties in large-N QCD +

the existence of the glueball mass gap at the lowest 1/N order, i.e. in the planar theory

+

UV finiteness of closed string trees

are incompatible with the open/closed duality of a would-be canonical string solution (canonical means that matches topologically 't Hooft expansion)

As a consequence, the long sought-after canonical string solution of large-N QCD does not actually exist.

Open/closed string duality in a nutshell

annulus= one-loop in the open-string sector

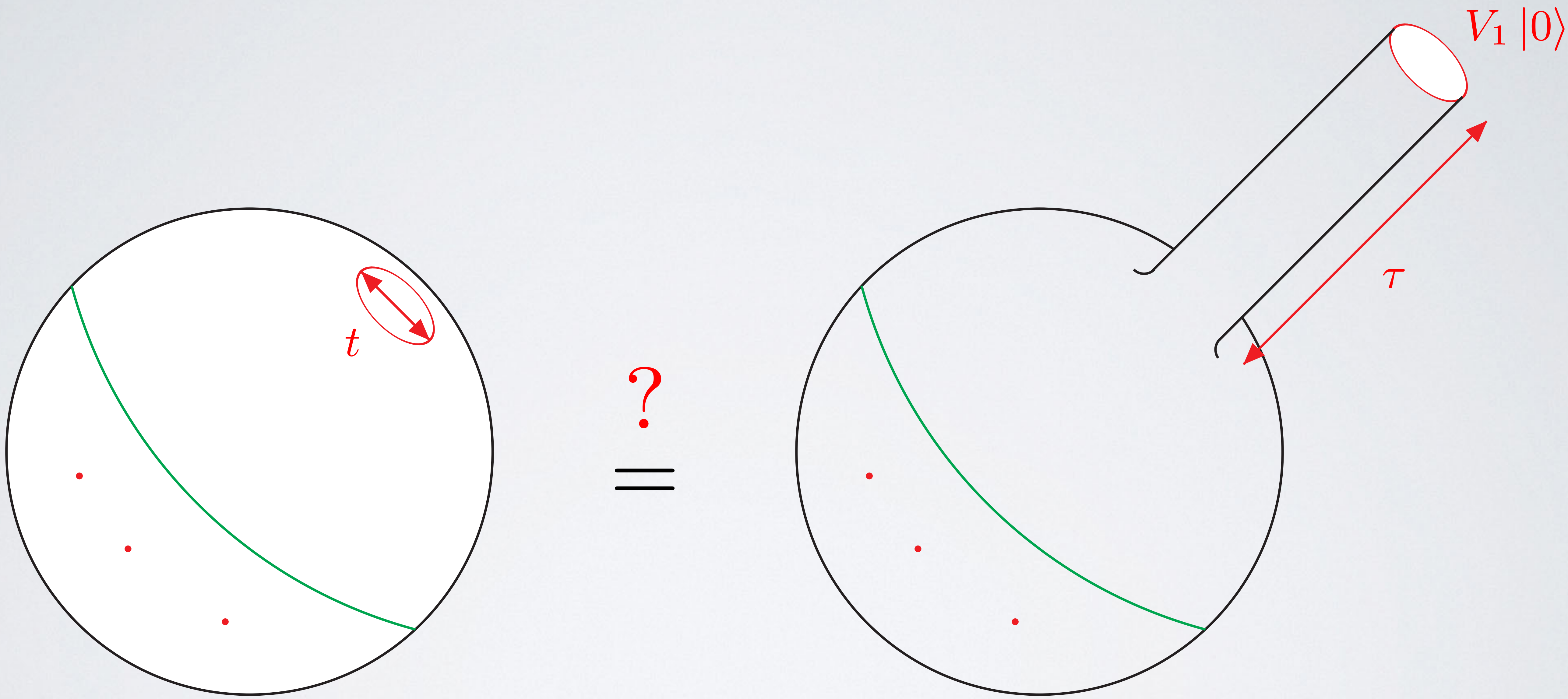
is topologically the same as the

cylinder = tree amplitude in the closed-string sector

Moreover, there is a conformal map that exchanges the world-sheet UV with the IR, under which the annulus, i.e. a disk with a small hole, is mapped into a long cylinder and vice versa.

Thus, if conformal symmetry on the world-sheet is not anomalous, i.e. the string theory really exists, the annulus and the cylinder are identical.

An example of open/closed string duality



Thus, open/closed string duality implies in large-N QCD:

$$< TrF^2 ... TrF^2 >_{conn}^{1-OpenStringLoop} = < TrF^2 ... TrF^2 \, V_1 >_{conn}^{TreeClosedString}$$

and the stronger equation, which is the equality of the open- and closed-string integrands before integrating on the world-sheet moduli:

$$\langle D_k(m_i)| Annulus(t) \rangle dm_1 \wedge \cdots \wedge dm_i \wedge \frac{dt}{t}$$
$$= \langle D_k(m_i)| \exp(-\tau H_{Closed}) V_1 |0\rangle dm_1 \wedge \cdots \wedge dm_i \wedge d\tau$$

In the 4d QCD S matrix there is no ambiguity in classifying UV and IR divergences.

Therefore, when we say that a corresponding string diagram is UV or IR divergent we mean,

as the canonical string is supposed to solve for the QCD S matrix,

that it is so the corresponding S matrix amplitude in QCD as a field theory

Now, we have just proved that in the large-N 4d QCD S matrix the one-loop graph on the left-hand side, which lives in the mixed glueball-meson sector, must be UV log divergent as the hole shrinks to a point

But, by open/closed duality, it may diverge only if the conformally equivalent tree glueball diagram in the 4d S matrix on the right-hand side, which naively cannot be UV divergent because it is both a closed and a tree string diagram, has an infrared divergence

corresponding to a scalar massless glueball propagating in the infinitely long cylinder on the right-hand side

But such a massless scalar glueball does not exist in the glueball sector of planar large-N QCD. Hence, open/closed duality cannot hold, and the canonical string solution does not exist !

In fact, a second stronger version of the NO-GO THEOREM holds (M. B. Phys. Lett. B 783 (2018) 341)

that does assume

neither the existence of the glueball mass gap

nor the UV finiteness of the tree closed-string diagram on the right-hand side,

but it only follows
from the renormalization properties of large-N QCD

and from a low-energy theorem of NSVZ type
proved in M.B. Phys. Rev. D 95 054010

An NSVZ low-energy theorem in QCD-like theories

$$< O_1 ... O_i >= Z^{-1} \int O_1 ... O_i e^{-\frac{N}{2g^2} \int TrF^2(x) d^4 x + ...}$$

$$\frac{\partial < O_1 ... O_i >}{\partial \log g}$$
$$= \frac{N}{g^2} \int < O_1 ... O_i TrF^2(x) > - < O_1 ... O_i >< TrF^2(x) > d^4 x$$

Nonperturbative version of the low-energy theorem: trade g for Lambda_QCD in AF QCD-like theories

$$\frac{\partial < O_1 ... O_i >}{\partial \log \Lambda_{QCD}}$$
$$= -\frac{N\beta(g)}{g^3} \int < O_1 ... O_i Tr F^2(x) > - < O_1 ... O_i >< Tr F^2(x) > d^4x$$

and employ lowest-order large-N renormalization:

$$\left(< Tr F^2 ... Tr F^2 >^{NP} \right)_{div} = \frac{\partial < Tr F^2 ... Tr F^2 >^P}{\partial \Lambda_{QCD}} \Lambda^{NP}_{QCD}$$

with:

$$-\frac{\beta_0^P N \Lambda_{QCD}^{NP}}{\Lambda_{QCD}^P} = -N\beta_0^{NP} \left(\log\left(\frac{\Lambda}{\Lambda_{QCD}^P}\right) + \frac{1}{2\beta_0^P}\left(\beta_1^{NP} - \beta_0^{NP}\frac{\beta_1^P}{\beta_0^P}\right) \log\log\left(\frac{\Lambda}{\Lambda_{QCD}^P}\right)\right)$$
$$= \frac{1}{(4\pi)^2}\frac{2}{3}N_f \log\left(\frac{\Lambda}{\Lambda_{QCD}^P}\right) + \cdots$$

The new low-energy theorem follows:

$$
\begin{aligned}
&(< TrF^2 ... TrF^2 >^{NP})_{div} \\
&= \frac{N\beta^P(g)\Lambda^{NP}_{QCD}}{g^3\Lambda^P_{QCD}} \int < TrF^2 ... TrF^2 >^P < TrF^2(x) >^P - < TrF^2 ... TrF^2 TrF^2(x) >^P d^4x \\
&\equiv < TrF^2 ... TrF^2 (V_1^P)_{div} >^P - < TrF^2 ... TrF^2 >^P < (V_1^P)_{div} >^P
\end{aligned}
$$

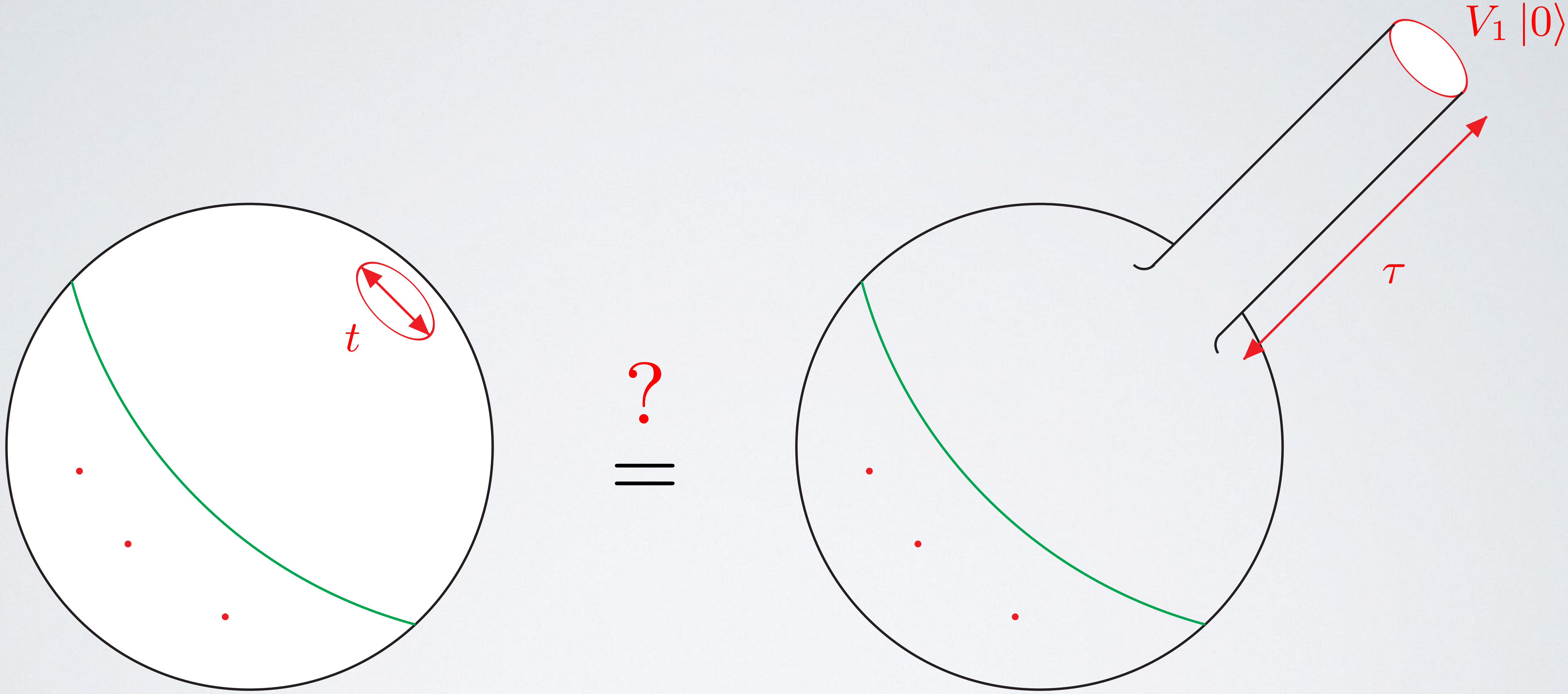
with:

$$
(V_1^P)_{div} = -\frac{N\Lambda^{NP}_{QCD}}{\Lambda^P_{QCD}} \frac{\beta^P(g)}{g^3} \int TrF^2 d^4x = N(\beta_0^{NP} \log(\frac{\Lambda}{\Lambda^P_{QCD}}) + \cdots) \int TrF^2 d^4x
$$

As an independent check, by direct QCD computation:

$$
\begin{aligned}
V_1 &= N_f Tr \log \frac{\gamma_\alpha D_\alpha(A) + Z_m^P m^P}{\gamma_\alpha D_\alpha(0) + Z_m^P m^P} \\
&= N\beta_0^{NP} \log(\frac{\Lambda}{\Lambda^P_{QCD}}) \int TrF^2 d^4x + nonlocal - UV - finite - terms
\end{aligned}
$$

Again, both sides may admit a string interpretation:



t
?
=
$V_1 |0\rangle$
τ

But now, as opposed to perturbation theory,

the new low-energy theorem in large-N QCD is incompatible with open/closed string duality

Proof

To say it in a nutshell, in the string interpretation the left-hand side is log UV divergent because of the integration on the modular parameter, t, of the annulus

By open/closed duality, the right-hand side must be log divergent because of the integration on the dual modular parameter, tau, of the cylinder

On the contrary, the new low-energy theorem in large-N QCD implies that the tau integration is, in fact, log UV finite, because of the extra 1/log^2 factor in the planar OPE with respect to perturbation theory:

$$\beta_0 F^2(z)\beta_0 F^2(0) \sim (1-\frac{1}{N^2})\frac{1}{z^8}\frac{48\beta_0^2}{\pi^4}(\frac{1}{\beta_0\log(\frac{1}{z^2\Lambda^2_{QCD}})})^2+\frac{1}{z^4}\frac{4\beta_0^2}{\pi^2}(\frac{1}{\beta_0\log(\frac{1}{z^2\Lambda^2_{QCD}})})^2\frac{\beta_0}{N}F^2(0)$$

while the boundary state V_110>

is log UV divergent before integrating on tau because it is a log UV-divergent counterterm due to the quark loops

Part 3

Is there a - necessarily noncanonical - way-out
to evade the no-go theorem ?

Yes, by giving up conformal symmetry and/or the matching
with the topology of the 't Hooft expansion !

This sounds exotic, since without conformal symmetry there
seems to be no consistent string theory ...

Not quite !

The topological A-model can be given a meaning without
conformal symmetry, i.e. without the Calabi-Yau constraint.

Its effective action is a Chern-Simons gauge theory
(Witten 1992)

Therefore,

we propose new versions of the open-string topological A model (on noncommutative twistor space) coupled to D branes. The effective action is 4d complex Chern-Simons coupled to D branes. Conjecturally for YM theory, it has the structure:

$$S(B) = i\frac{k\sqrt{T}}{8\pi}Tr(BdB + \frac{2}{3}B^3) - \log Det(\frac{d}{d\lambda} + B_\lambda) + h.c.$$

The interesting part is the log of the functional determinant that arises from (glueball) branes, which should be related to the generating functional of the glueball one-loop effective action in a certain sector. This theory is noncanonical in many ways: Glueball are open strings (and by consistency also closed strings), and the A-model does not need to be conformal invariant on the world sheet !

Mesons are coupled to glueballs in the open-string sector by means of another set of D branes (meson branes)

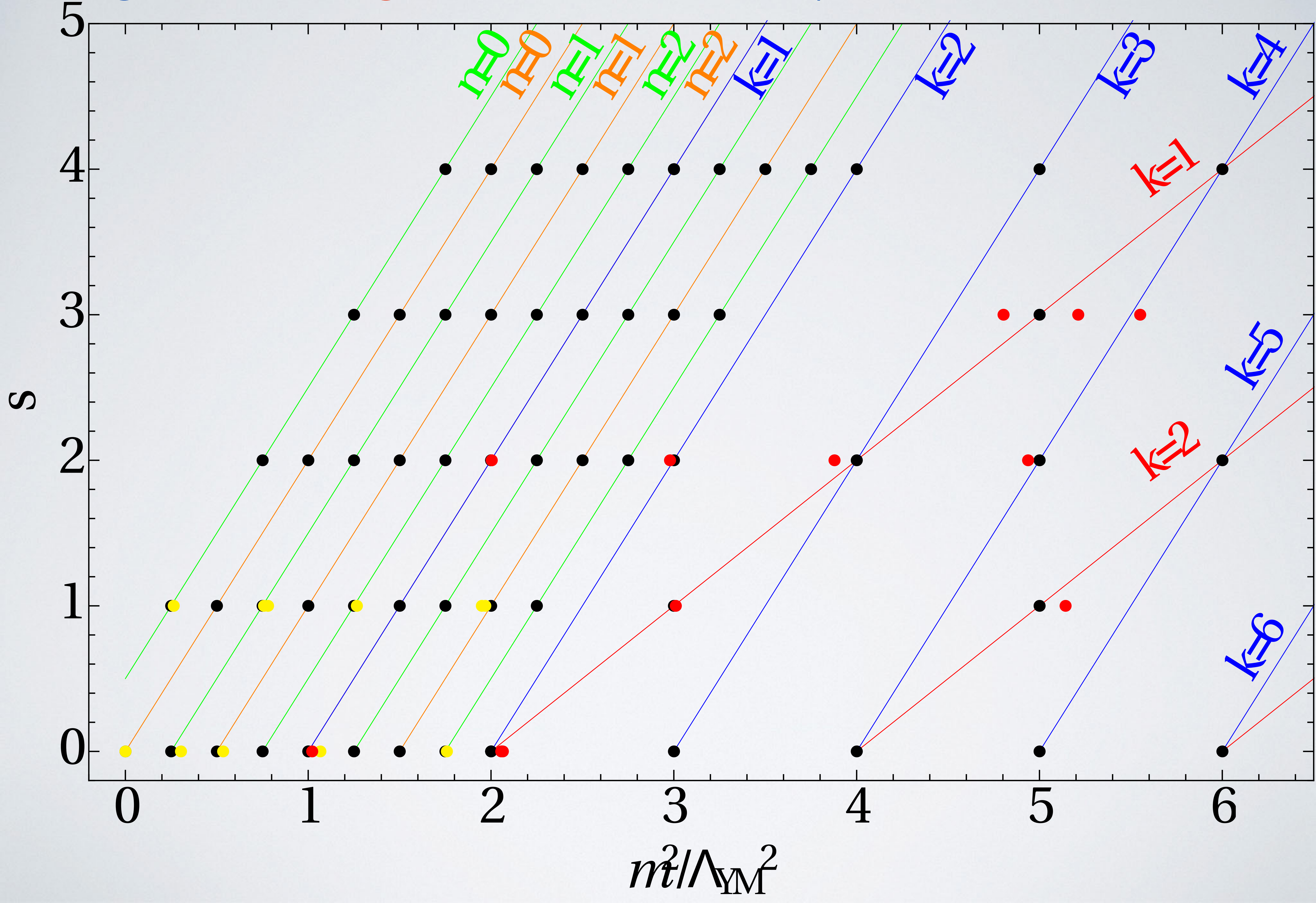
$$\log Det_R(\frac{d}{d\lambda} + B_\lambda) + h.c.$$

Generic prediction of the new models: Glueball and meson Regge trajectories are linear in the spin as a function of the mass squared. The glueball Regge trajectories have two slopes, differing by a factor of 2, arising from the open and closed sectors respectively

Moreover, the glueball Regge trajectories in the open string sector have the same slope as the meson Regge trajectories, because both belong to the open sector !

This looks exotic, but here it is the comparison with the glueball and meson spectrum computed by lattice gauge theories at large-N

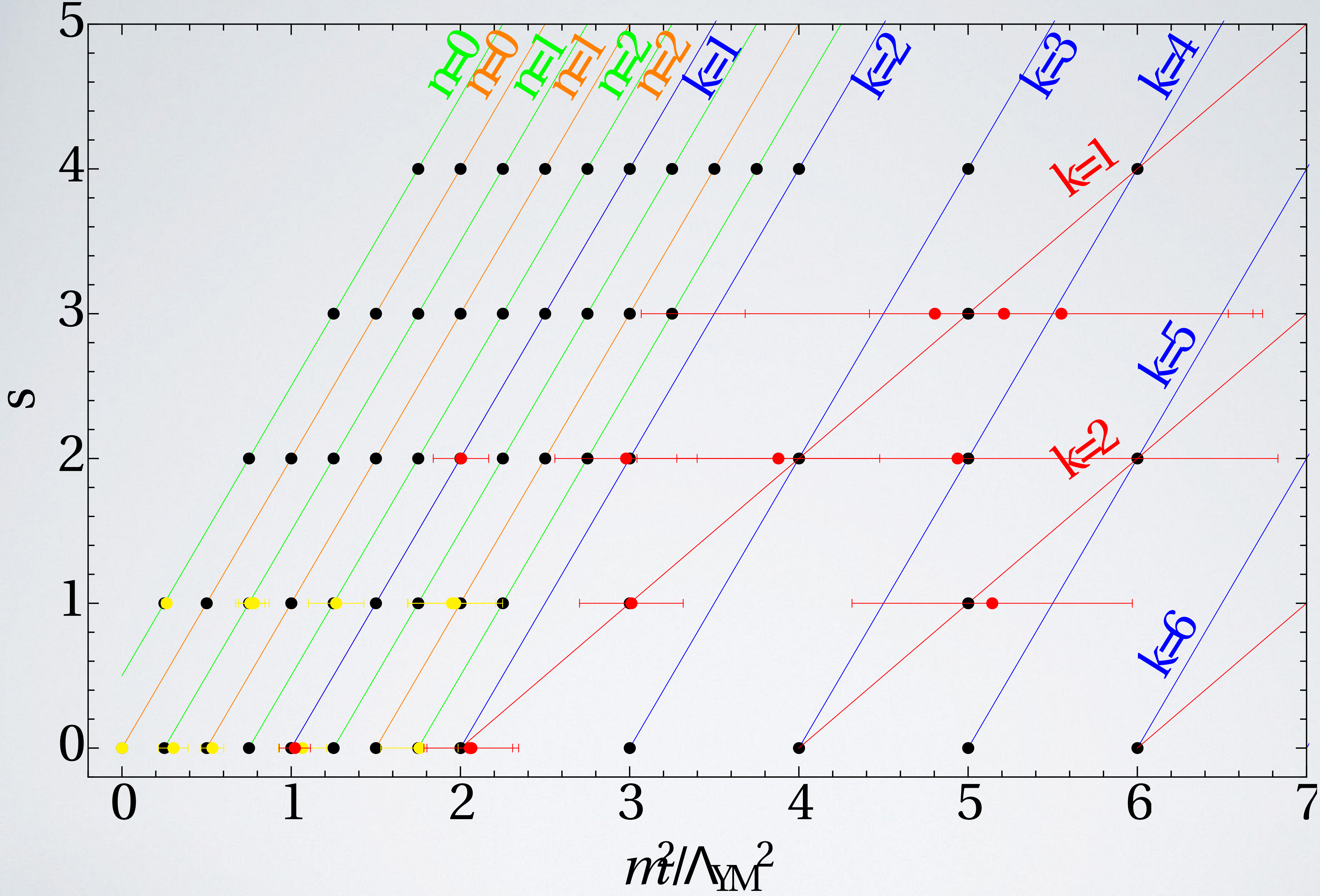
Large-N lattice glueball and meson spectrum versus TTST



The spectrum of the twistorial topological string theory (TTST) in the plot is:

$$
\begin{aligned}
m_k^{(s)2} &= (k+\frac{s}{2})\Lambda^2_{QCD}\ ; s\ even;\ \ k=1,2,\cdots for\, glueballs \\
m_k^{(s)2} &= 2(k+\frac{s}{2})\Lambda^2_{QCD}\ ; s\ odd;\ \ k=1,2,\cdots for\, glueballs \\
m_k^{(s)2} - m^2_{PGB} &= \frac{1}{2}(k+s)\Lambda^2_{QCD}\ ; s=0,1,\cdots ; k=0,1,\cdots for\, mesons \\
m_k^{(s)2} - m^2_{PGB} &= \frac{1}{2}(k+s-\frac{1}{2})\Lambda^2_{QCD}\ ; s=1,\cdots ; k=0,1,\cdots for\, mesons
\end{aligned}
$$

with $m_{PGB} = 0$



n=0
n=0
n=1
n=1
n=2
n=2
k=1
k=2
k=3
k=4
k=1
k=5
k=2
k=6
s
m^2/Λ_{YM}^2
0
1
2
3
4
5
6
7

Outlook: a problem for the future

Can we verify in principle whether a conjectured YM glueball 1PI effective action, for a clever choice of the interpolating fields, actually matches the generating functional of the correlators of twist-2 operators with maximal spin in the YM theory ?

Yes, in principle !

Given the candidate S-matrix, since we know the spectrum from the S-matrix amplitudes,

by working out the LSZ formulae the other way around,

we can attach the propagators, reinsert the square root of the residues given by the asymptotic theorem, and analytically continue back to Euclidean space-time

in order to reconstruct the Euclidean asymptotic correlators

Remarkably, we have already realized one half of this program, since we have computed the UV asymptotics of the generating functional of leading-order nonplanar correlators of twist-2 operators with maximal-spin projection in SU(N) YM theory

M.B., M. Papinutto, F. Scardino, n-point correlators of twist-2 operators in SU(N) YM theory to the lowest order, JHEP08(2021)142

and to appear

and we have found that it has, indeed, the structure of the log of a functional determinant ! :

$$\Gamma^E_{Torus\,asym}[j_{\mathbb{O}'^E},\lambda] = \log\det\left(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y) + \frac{1}{N}\frac{Z^{univ}_{\mathbb{O}'_{s_2}}(\lambda)}{\lambda^{2+s_1-k_1+k_2}}\mathscr{D}^{-1}_{E\ \ s_1k_1,s_2k_2}(x-y)\,j_{\mathbb{O}'^E_{s_2k_2}}(y)\right)$$

with

$$\mathscr{D}^{-1}_{E\ \ s_1k_1,s_2k_2}(x-y) = -\frac{(-i)^{-k_1+k_2}}{8\pi^2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}\partial_z^{s_1-k_1+k_2}\frac{1}{(x-y)^2}$$

The other half of the program consists in finding a one-glueball effective action in the class of string models considered in the present talk that is exactly asymptotic in the UV to the generating functional above

It would be a candidate for a partial string solution, limited to the sector above, of large-N YM, and, eventually, of QCD

Of course, knowing already the asymptotic answer is a powerful guide …