From PT Quantum Mechanics to PT Holography

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Based on:

- **SciPost Phys. 9 (2020) no.3, 032, [arXiv:1912:06647]**
  with *Daniel Areán, Ignacio Salazar-Landea*

- **e-Print 2203.02524 [hep-th]**
  with *Sergio Morales-Tejera*

QCD@work, Lecce 28. June, 2022

Resources

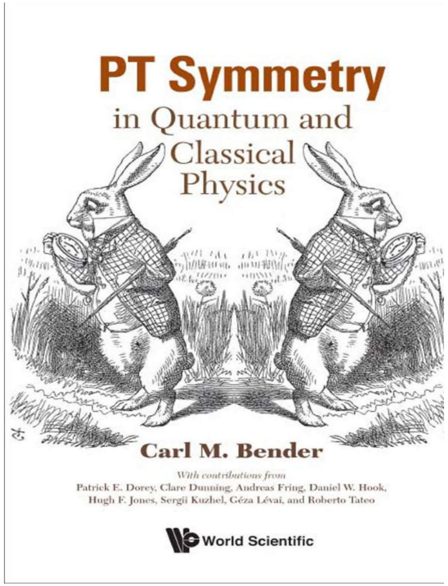
Some references:

- Seminal article:

  Bender, Boettcher: Phys.Rev.Lett. 80 (1998) 5243-5246

- Book:
  Bender: “PT symmetry in quantum and classical physics” (2019)

- Time dependence:
  - **Mostafazadeh:** Entropy 22 (2020) 4, 471 **(Review)**
  - **Fring:** e-Print: 2201.05140 [quant-ph] **(Lecture notes)**

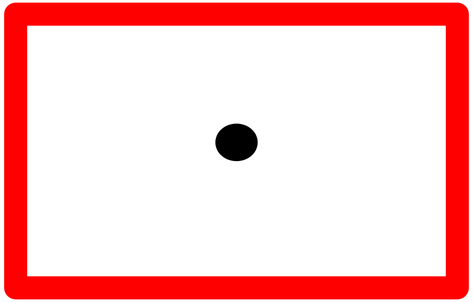


Outline

- PT Quantum Mechanics
- Holographic model
  - Static properties, PT phase transition
  - Dynamics: Hermitian vs. Non-Hermitian
- Conclusions

PT Quantum Mechanics

Particle in a Box



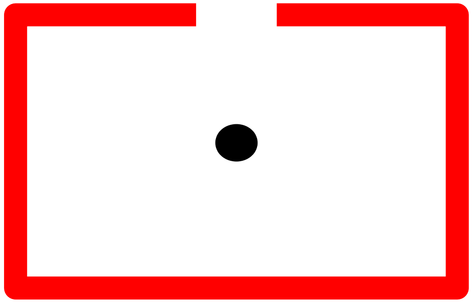
$$\Psi(t) = e^{-iEt}$$

$$H = E$$

$$p = |\Psi|^2 = 1$$

PT Quantum Mechanics

Particle in a leaky Box



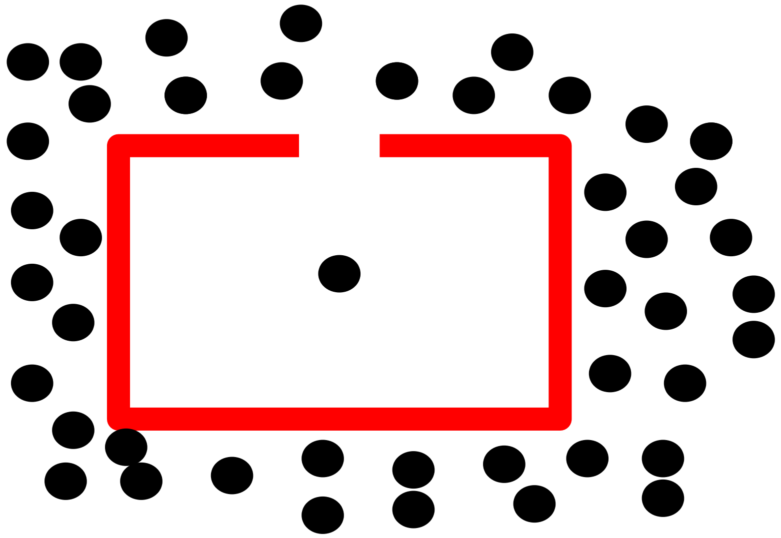
$$p(t) = e^{-2\Gamma t}$$

$$``\Psi(t)`` = e^{-i(E-i\Gamma)t}$$

$$``H`` = E - i\Gamma$$

PT Quantum Mechanics

Particle in a leaky Box in a reservoir



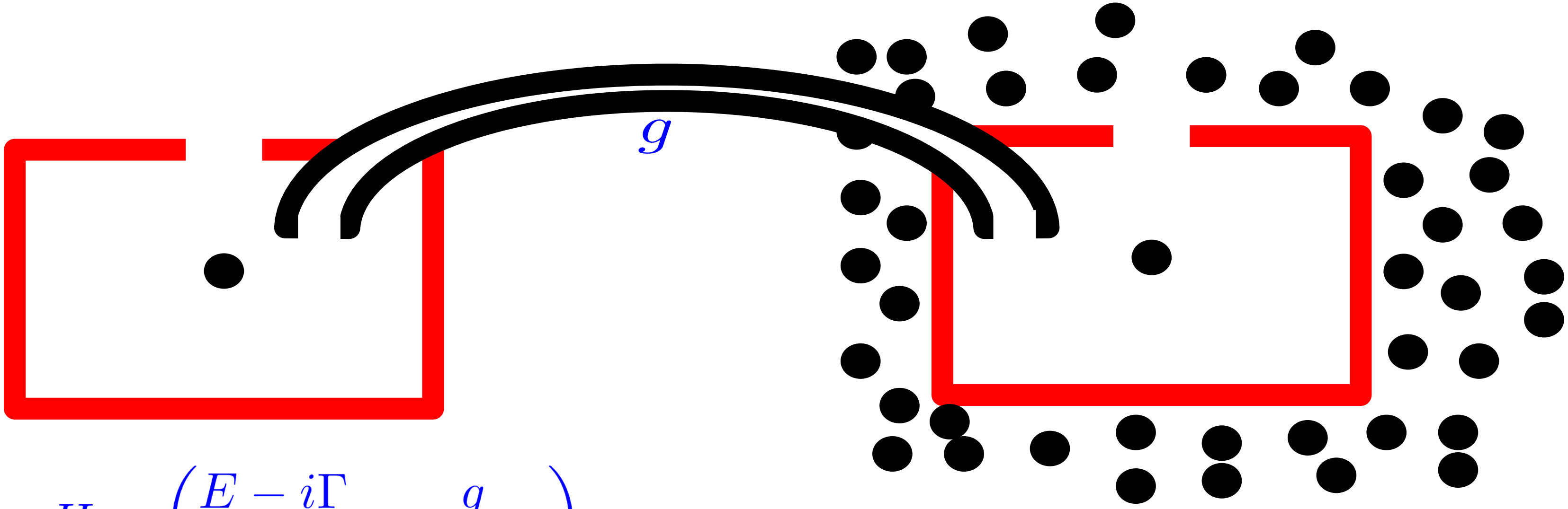
$$n(t) = e^{2\Gamma t}$$

$$``\Psi(t)`` = e^{-i(E+i\Gamma)t}$$

$$``H`` = E + i\Gamma$$

PT Quantum Mechanics

Particle in a leaky Box coupled to a particle in a Box immersed in a reservoir



$$H = \begin{pmatrix} E - i\Gamma & g \\ g & E + i\Gamma \end{pmatrix} \qquad H^\dagger \neq H$$

$$\epsilon_\pm = E \pm \sqrt{g^2 - \Gamma^2}$$ Real if $|g| \geq |\Gamma|$

PT Hamiltonian = Open system with exact balance between source and sink

PT Quantum Mechanics

Time reversal: $\mathcal{T}(H) = H^*$ $\quad i \to -i$

Parity: exchange the two boxes $\mathcal{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$[\mathcal{PT}, H] = 0$$

$$(\mathcal{PT})^2 = 1$$

$$(\mathcal{PT})^2|n\rangle = \lambda_n^* \lambda_n |n\rangle \;\Rightarrow\; \lambda_n = e^{i\phi}$$

$$\epsilon_n^* = \epsilon_n$$

PT Symmetric phase $|g| \geq |\Gamma|$

Or:

$$\mathcal{PT}|n,+\rangle = |n,-\rangle$$

$$\epsilon_{n,+} = \epsilon_{n,-}^*$$

PT broken phase $|g| < |\Gamma|$

PT Quantum Mechanics

Symmetry perspective: $h = E\mathbf{1}_2 + \vec{g}.\vec{\sigma}$ Hermitian 2x2

Any two theories are related by SU(2) $D = e^{i\vec{\alpha}.\frac{\vec{\sigma}}{2}}$

$$\vec{g} = \begin{pmatrix} g' \\ 0 \\ 0 \end{pmatrix} \qquad D^\dagger h D = \begin{pmatrix} E - g'\sin(\alpha) & g'\cos(\alpha) \\ g'\cos(\alpha) & E + g'\sin(\alpha) \end{pmatrix}$$

Generate a non-Hermitian Matrix by setting $\alpha \to i\hat{\alpha}$

Equivalently: <u>transform couplings:</u> $\vec{g} \to O(\alpha).\vec{g}$

PT Quantum Mechanics

Symmetry perspective: $h = E\mathbf{1}_2 + \vec{g}.\vec{\sigma}$

Complexify transformation: $\eta = e^{\hat{\alpha}.\frac{\sigma_2}{2}}$ Hermitian

$$\vec{g} = \begin{pmatrix} g' \\ 0 \\ 0 \end{pmatrix}$$

$$H = \eta^{-1} h \eta = \begin{pmatrix} E - ig' \sinh(\hat{\alpha}) & g' \cosh(\hat{\alpha}) \\ g' \cosh(\hat{\alpha}) & E + g' i \sinh(\hat{\alpha}) \end{pmatrix}$$

$$g = g' \cosh(\hat{\alpha}) \quad , \quad \Gamma = g' \sinh(\hat{\alpha})$$

$$\epsilon_{\pm} = E \pm \sqrt{g'^2 (\cosh(\hat{\alpha})^2 - \sinh^2(\hat{\alpha})^2)} = E \pm g'$$

PT Quantum Mechanics

Pseudo-Hermitian Hamiltonians $$H = \eta^{-1} h \eta \,, \quad h^\dagger = h \,, \quad \eta^\dagger = \eta$$

Schrödinger Equation: $i\partial_t|\psi\rangle = H|\psi\rangle$ Metric: η^2

Unitary time evolution: $$\frac{d}{dt}\langle\Phi|\eta^2|\Psi\rangle = i\langle\Phi|H^\dagger\eta^2|\Psi\rangle - i\langle\Phi|\eta^2 H|\Psi\rangle = 0$$

PT symmetric phase = pseudo-Hermitian Hamiltonian

PT Quantum Field Theory

Dirac mass term $\mathcal{O}_1 = \bar{\psi}\psi$

Axial mass term $\mathcal{O}_2 = i\bar{\psi}\gamma_5\psi$

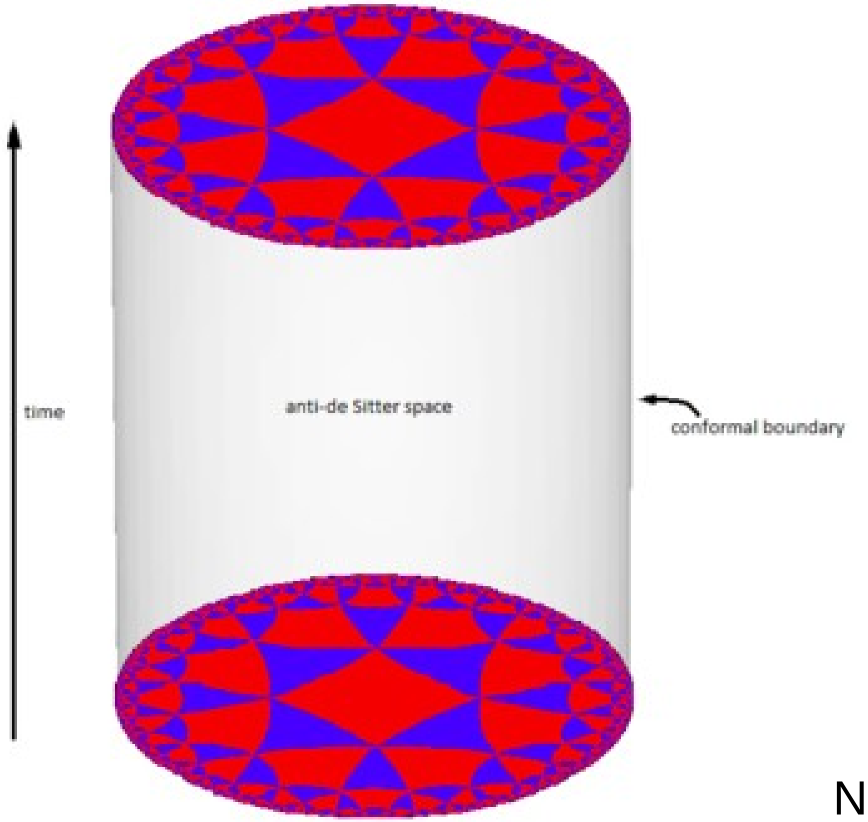
$\mathcal{O} = \mathcal{O}_1 + i\mathcal{O}_2$ Has charge 2 under axial U(1) $e^{i\alpha\gamma_5}$

Non-hermitian mass term $M_0\bar{\psi}\psi + M_5\bar{\psi}\gamma_5\psi$ ($Me^{2\hat{\alpha}}\bar{\mathcal{O}} + Me^{-2\hat{\alpha}}\mathcal{O}$)

Spectrum $\omega = \sqrt{\vec{k}^2 + M_0^2 - M_5^2}$

Real for $|M_0| > |M_5|$

Holography (Gauge/Gravity Duality)



Gravity in asymptotically AdS = QFT

Holographic Dictionary	
Gravity	Quantum Field Theory
Metric	*Energy Momentum Tensor*
Gauge field	*Conserved current*
Scalar field	*Scalar operator*

Notice: Symmetry in QFT = gauge principle in AdS

PT-Holography

$$S = \int d^4x\sqrt{-g}\left[R - 2\Lambda - |D\phi|^2 - m^2|\phi|^2 - \frac{v}{2}|\phi|^4 - \frac{1}{4}F^2\right]$$

PT = Non-Hermitian boundary conditions at $r = \infty$

$$\phi \approx \frac{\phi_0}{r} + \frac{\langle\bar{\mathcal{O}}\rangle}{r^2} + \dots$$

$$\bar{\phi} \approx \frac{\bar{\phi}_0}{r} + \frac{\langle\mathcal{O}\rangle}{r^2} + \dots$$

Hermitian

$$\phi_0 = M'$$

$$\bar{\phi}_0 = M'$$

$\longrightarrow$

Non-Hermitian

$$\phi_0 = M'e^{\hat{\alpha}} = (1-\xi)M$$

$$\bar{\phi}_0 = M'e^{-\hat{\alpha}} = (1+\xi)M$$

PT-Holography

$$S = \int d^4x\sqrt{-g}\left[R - 2\Lambda - |D\phi|^2 - m^2|\phi|^2 - \frac{v}{2}|\phi|^4 - \frac{1}{4}F^2\right]$$

How does PT work here: $\mathcal{T} : A \to -A \quad , \quad i \to -i \quad , \quad ds^2 \to ds^2$

$$\phi_R \to \phi_R \quad , \quad \phi_I \to -\phi_I$$

$$\mathcal{P} : A \to -A \quad , \quad ds^2 \to ds^2$$

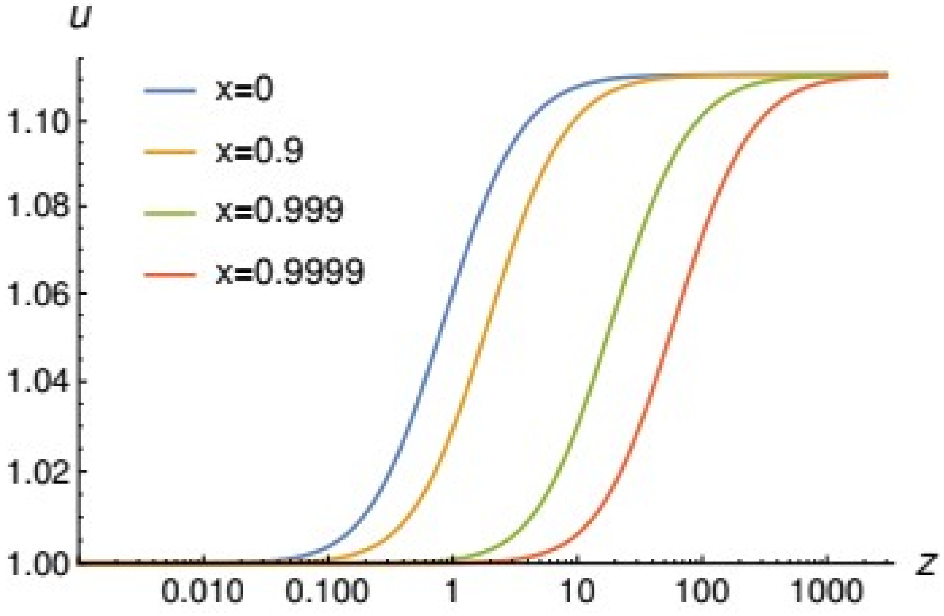
$$\phi_R \to \phi_R \quad , \quad \phi_I \to -\phi_I$$

PT leaves boundary conditions invariant: PT Hamiltonian

PT-Holography

T=0:

IR Solution: $$\phi_{IR} = \sqrt{\frac{-m^2}{v}\frac{1+\xi}{1-\xi}}$$



- We find usual domain wall solutions
- Domain wall moves towards IR with ξ approaching 1
- For |ξ|>1 no real solution!
  PT phase transition
- At ξ=1 exceptional line: formally pure AdS
- Background is equivalent to hermitian solution with bcs

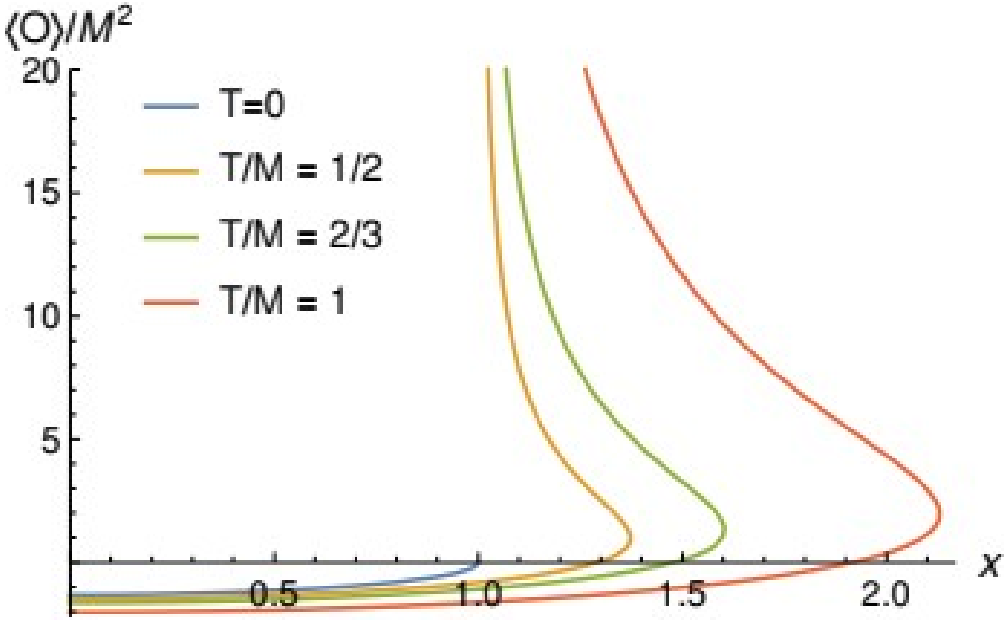
$$\phi_0 = \sqrt{1-\xi^2}M' \quad , \quad \bar{\phi}_0 = \sqrt{1-\xi^2}M'$$

PT-Holography

T>0:

Finite Temperature = black hole $u = u_0(r - r_H) + \ldots$

Surprisingly we find
real solutions for |ξ|>1 !



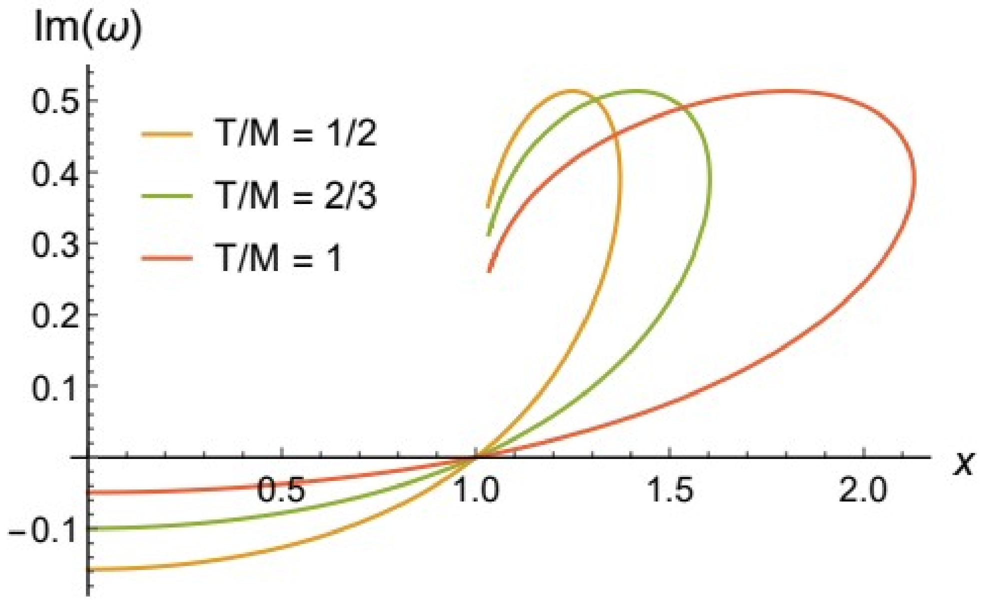
PT-Holography

What is going on? Does finite temperature stabilize the system beyond the PT “quantum” phase transition even for |ξ|>1 ?

NO!

Quasi-normal mode spectrum shows instability precisely at x=1 (“pseudo”-diffusion mode),
No stable solution found

$$\omega = -i\Gamma(x)$$



PT-Holography

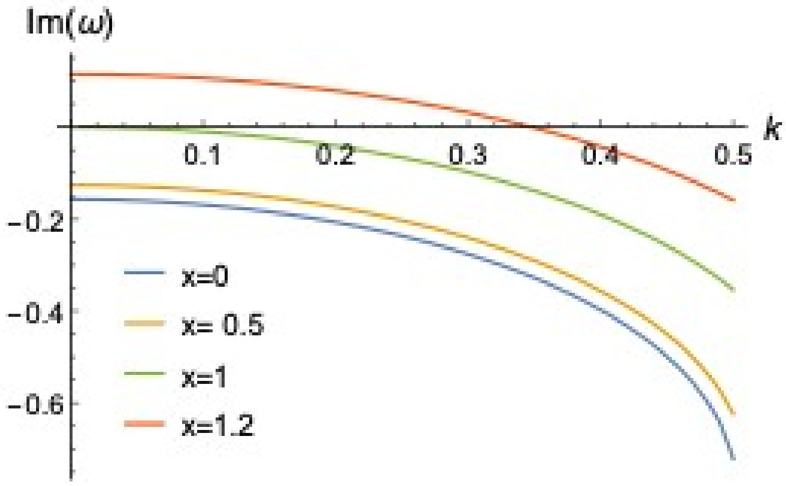
QFT homework:

$$E = \sqrt{\vec{k}^2 + M^2 - M_5^2 + \delta M_T^2}$$

Thermal mass

Instability in (axial) density fluctuations

$$\langle \rho_5 \rho_5 \rangle \approx \frac{i\sigma k^2}{\omega + iDk^2 + i\Gamma}$$



PT-Holography

Time dependent non-hermitian coupling: $$\frac{\partial}{\partial t}\langle\Psi|\Phi\rangle_\eta = 2\langle\Psi|\eta^{-1}\partial_t\eta|\Phi\rangle$$

[Mostafazadeh]
[Fring]

Non-unitary time evolution, what to do?

2 Options:

- Just live with it
- Repair unitarity in some way

How to repair? With a “gauge” field !

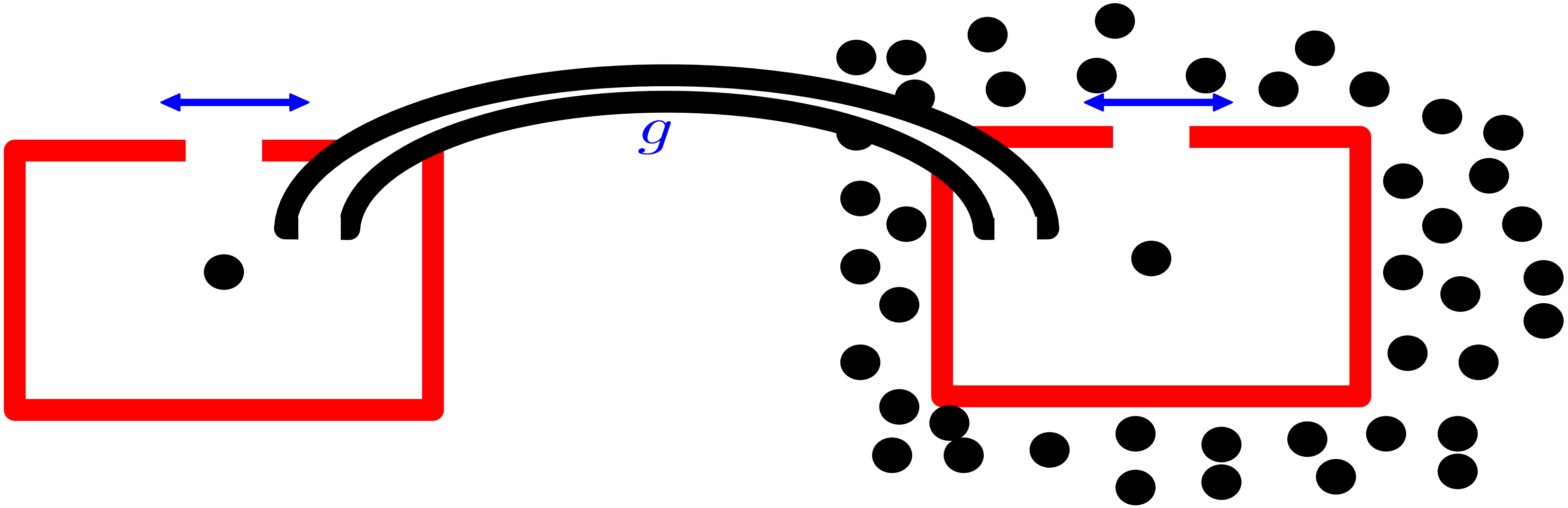
$$i(\partial_t - iA_t)|\Psi\rangle = H|\Psi\rangle$$

$$H = \eta^{-1}(t)h\eta(t) \quad , \quad A_t = i\eta^{-1}\partial_t\eta$$

[Chernodub, Millington]

PT Quenches

PT Hamiltonian = Open system with exact balance between source and sink



- Non-unitary: change openings simultaneously and in the same way
- Unitary: change openings in the same way but also do something more, corresponding to the gauge field (forward vs backward hopping ...)

$$h = g\sigma_x \quad , \quad \eta = e^{\hat{\alpha}\sigma_y} \quad , \quad A_t = i\partial_t\hat{\alpha}\sigma_y$$

PT Quenches

In Holography this can be done explicitly, time evolution can be calculated in both cases!

Time dependent boundary conditions:

$$\phi = [1 - \xi(t)]M$$
$$\bar{\phi} = [1 + \xi(t)]M$$

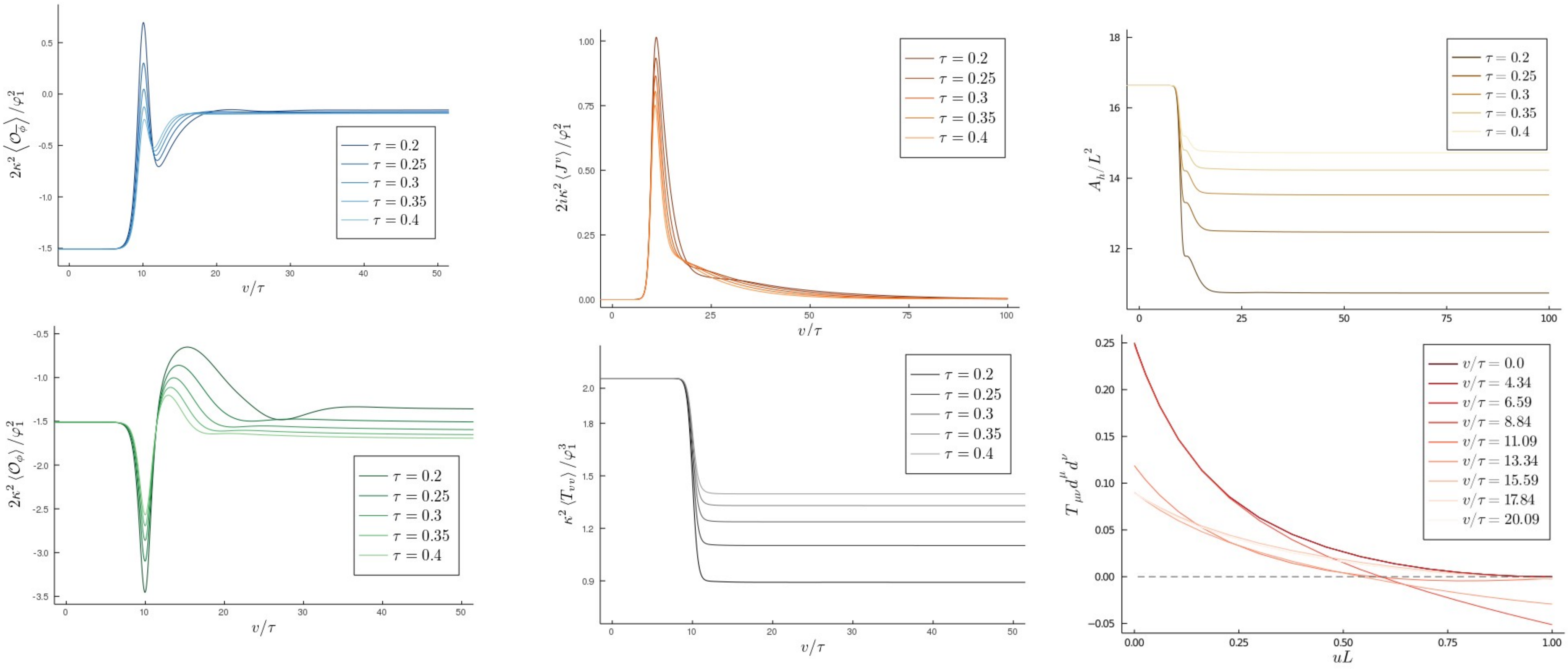
For unitary evolution introduce gauge field:

$$A_t = i\frac{\partial_t \xi}{1 - \xi^2}$$

Note: in the unitary time evolution formalism we can not reach the exceptional point or go into the PT broken regime! Singularity in the gauge field.

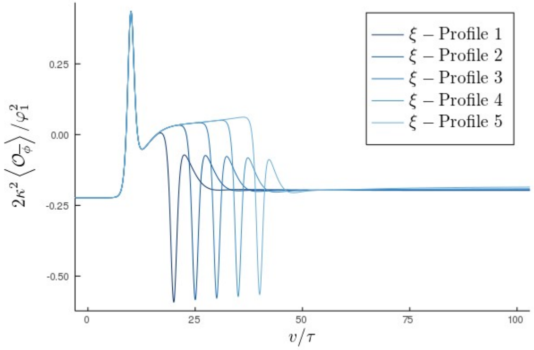
PT Holographic quench

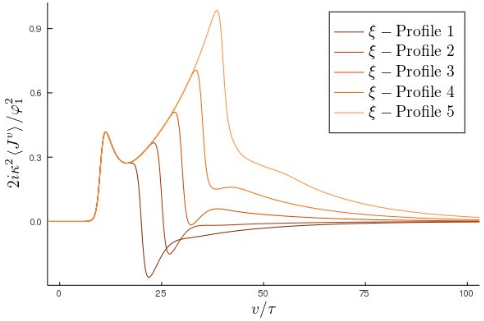
<u>Non-unitary evolution I:</u> $\xi = 0 \rightarrow \xi = 0.8$

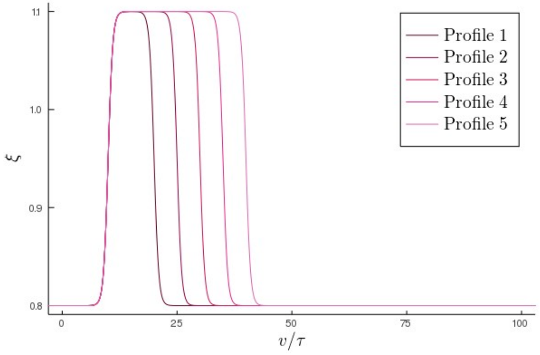


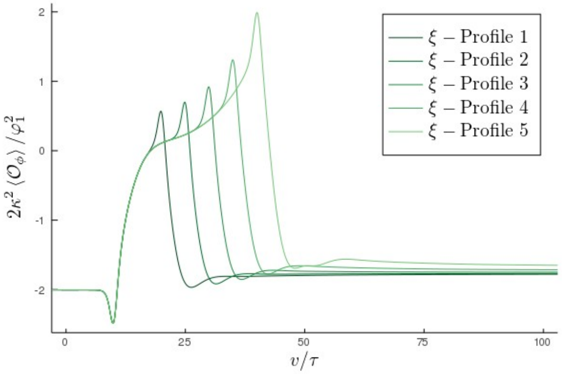
PT Holographic quench

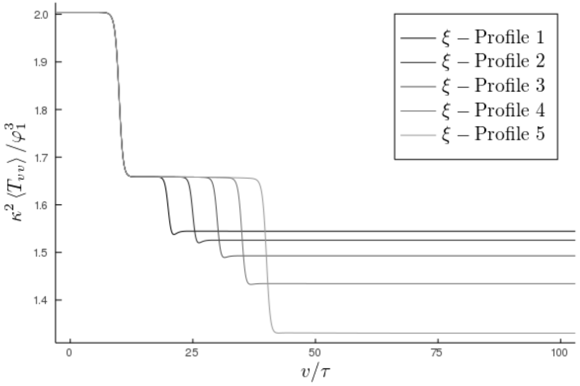
Non-unitary evolution III: $\xi = 0.8 \to \xi = 0.8$ With excursion into PT broken regime!







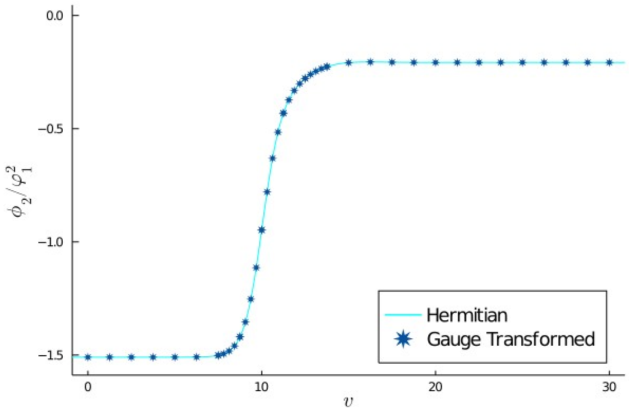


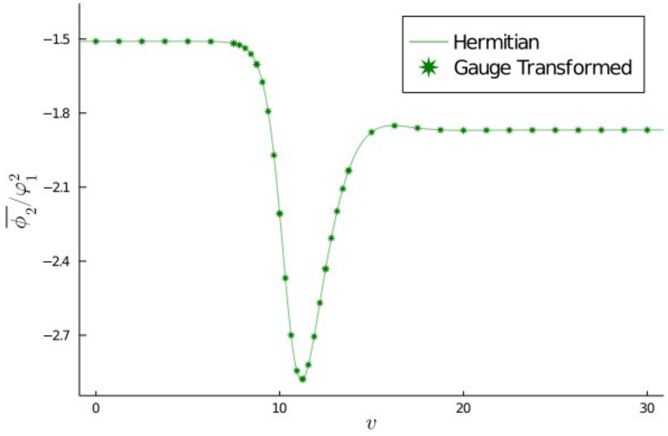


PT Holographic quench

Unitary evolution: scalars as before but also gauge field $A_t(r=\infty) = i\frac{\partial_t \xi}{1-\xi^2}$

Compare to Hermitian description: $\phi = \sqrt{1-\xi(t)^2}\phi(0)$





Summary

- We have constructed the first example of PT holographic model
- PT phase transition exists at $\xi=1$ just as in PT QM
- Predicitons for PT quantum field theory
- PT-quantum quenches show very interesting phenomenology
- Unitary with gauge field equivalent to Hermitian theory
- If $\exists$ holographic material $\rightarrow$ violate energy conditions in Laboratory?

THANK YOU!