Nonfactorizable charming loops in exclusive rare FCNC B decays

Dmitri Melikhov

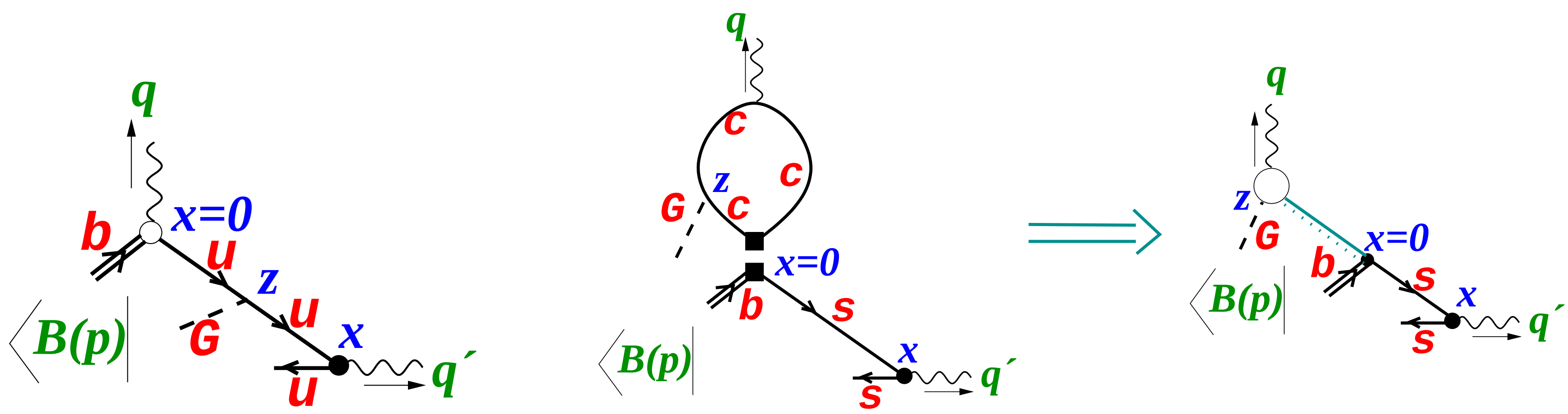
I compare nonfactorizable correction induced by charm-quark loops in exclusive FCNC B-decays with a specific correction to SL B-decay form factor (ff); both are given via three-particle distribution amplitude (3DA) of the B-meson $\langle 0|\bar{q}(x)G_{\mu\nu}(z)b(0)|B(p)\rangle$:

- **The ff correction is dominated by the *collinear* LC configuration:**
$x^2 = 0$, $z^2 = 0$, and $z_\mu = ux_\mu$, $0 < u < 1$.

- **In contrast, the FCNC amplitude is dominated by a different LC configuration with *non-collinear* arguments:**
$x^2 = 0$, $z^2 = 0$, but $(x - z)^2 \neq 0$.

The basic object is

$$\int dx \exp(iqx)\langle 0|T(j_1(x), j_2(0)|B(p)\rangle$$



- **Ffs**

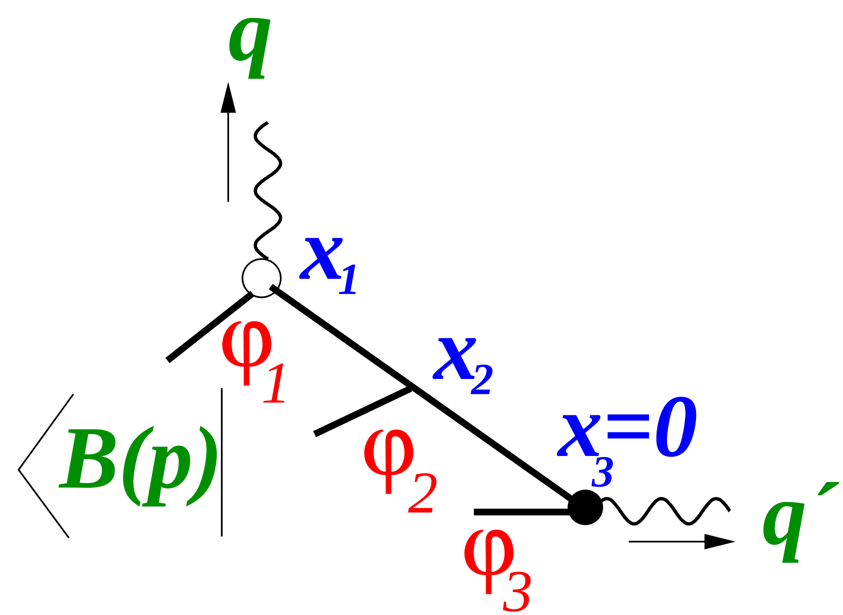
$$\int dx dz d\kappa_b \frac{1}{m_s^2-(q+\kappa_b)^2} \exp(i\kappa_b z) \frac{dk \exp(-ikx+iq'x)}{m_s^2-k^2} \langle 0|\bar{s}(x)G(z)b(0)|B_s(p)\rangle .$$

- **FCNC**

$$\int dx dz d\kappa_b \Gamma_{cc}(\kappa_b, q) \exp(i\kappa_b z) \frac{dk \exp(-ikx+iq'x)}{m_s^2-k^2} \langle 0|\bar{s}(x)G(z)b(0)|B_s(p)\rangle .$$

where

$$\Gamma_{cc}(\kappa, q) = \frac{1}{8\pi^2} \int_0^1 du \int_0^1 dv \frac{\theta(u+v<1)}{m_c^2+2uv\kappa q-u(1-u)\kappa^2-v(1-v)q^2}.$$



$$\int_0^1 dv \int \frac{d\tilde{k}dz'dxd\kappa}{\left[m^2(1-v)+\mu^2 v-\tilde{k}^2-v(1-v)\kappa^2\right]^2} e^{i(\tilde{k}+q)x+i\kappa z'}\langle 0|\varphi_1(x)\varphi_2((1-v)x+z')\varphi_3(0)|B(p)\rangle,$$

where κ is the momentum carried by φ_2 and transmitted in the "central" vertex.

One can, e.g., expand $\langle 0|\varphi_1(x)\varphi_2((1-v)x+z')\varphi_3(0)|B(p)\rangle$ in powers of z and obtain a tower of operators. Or, equivalently, one can expand in powers of κ. The expansion is meaningful if κ is soft (as quark propagators have virtualities $\Lambda_{\rm QCD} m_b$).

The amplitude is dominated by a collinear LC configuration of the 3DA if the momentum tranferred in the central vertex is soft

Amplitude of the FCNC B-decay

B-meson 3DA with non-collinear arguments

$$\langle 0|s^\dagger(x)G(z)b(0)|B_s(p)\rangle = \int d\lambda e^{-i\lambda xp} \int d\omega e^{-i\omega zp}\Phi(\omega,\lambda)\left[1+O\left(\Lambda_{\rm QCD}^2 x^2, \Lambda_{\rm QCD}^2 z^2, \Lambda_{\rm QCD}^2 (x-z)^2\right)\right].$$

• LC contribution of 3DA

The contribution of the LC term in 3DA, $\Phi(\omega,\lambda)$, to $A(q,p)$ is easy to calculate.

$$A(q,p) = \int_0^\infty d\lambda \int_0^\infty d\omega\, \Phi(\lambda,\omega)\Gamma_{cc}\left(-\omega p, q\right) \frac{1}{m_s^2-(\lambda p - q')^2}.$$

• The triangle charming loop is easily calculable

$$\Gamma_{cc}(\kappa,q) = \frac{1}{8\pi^2}\int_0^1 du \int_0^1 dv \frac{\theta(u+v<1)}{m_c^2 + 2uv\kappa q - u(1-u)\kappa^2 - v(1-v)q^2}.$$

The ω-integral is peaked at $\omega \sim \Lambda_{\rm QCD}/m_b$ so the gluon is soft:
$\kappa = -\omega p$ and $\kappa^2 \sim O(\Lambda_{\rm QCD}^2) \ll m_c^2$.

• The s-quark propagator takes the form

$$m_s^2 - (\lambda p - q')^2 = m_s^2 - \lambda q^2 - (1-\lambda)q'^2 + (1-\lambda)\lambda M_B^2.$$

In the bulk of λ-integration the virtuality of the s-quark propagator is $O(M_B)$.

• off-LC contribution of 3DA

The difficulty of the problem is the existence of two heavy quark scales, one of which is much heavier than the other:

$$\boxed{\Lambda_{\rm QCD} \ll m_c \ll m_b, \textbf{ and } \Lambda_{\rm QCD} m_b/m_c^2 \simeq 1}$$

We need to sum all powers of the parameter $\lambda_{\rm QCD} m_b/m_c^2$.

Contributions of other terms to the amplitude $A(q,p)$ relative to the $\Phi(\omega,\lambda)$ term:

$$\Lambda_{\rm QCD}^2 y^2 \to \frac{\Lambda_{\rm QCD}}{m_b}, \qquad \Lambda_{\rm QCD}^2 z^2 \to \frac{\Lambda_{\rm QCD}^3 m_b}{m_c^4}, \qquad \Lambda_{\rm QCD}^2 xz \to \frac{\Lambda_{\rm QCD} m_b}{m_c^2}.$$

The knowledge of its functional dependence on $(x-z)^2$ is essential for a proper resummation of large $\Lambda_{\rm QCD} m_b/m_c^2$ corrections.

CONCLUSIONS

• **3DA contributions to SL B form factors are to charming loops in FCNC B-decays both are given via three-particle distribution amplitude (3DA) of the B-meson $\langle 0|\bar q(x)G_{\mu\nu}(z)b(0)|B(p)\rangle$ but have different properites:**

The ff correction is dominated by the *collinear* LC configuration:
$x^2=0$**,** $z^2=0$**, and** $z_\mu = u x_\mu$**,** $0<u<1$**.**

In contrast, the FCNC amplitude is dominated by a different LC configuration with *non-collinear* arguments:
$x^2=0$**,** $z^2=0$**, but** $(x-z)^2\neq 0$**.**

• **In QCD, B-meson 3DAs with non-collinear arguments involve new Lorentz structures compared to LC 3DAs. Respectively, new invariant amplitudes arise.**

$$\langle 0|\bar s(x)G_{\alpha\beta}(ux)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda xv}\int d\omega e^{-i\omega uxv}\left[\frac{x_\alpha v_\beta}{xv}-\frac{x_\beta v_\alpha}{xv}\right]\Phi(\lambda,\omega).$$

For non-collinear case, new structures and new amplitudes arise:

$$\langle 0|\bar s(x)G_{\alpha\beta}(z)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda xv}\int d\omega e^{-i\omega zv}$$
$$\times\frac{1}{2}\left[\left(\frac{x_\alpha v_\beta}{xv}-\frac{x_\beta v_\alpha}{xv}+\frac{z_\alpha v_\beta}{zv}-\frac{z_\beta v_\alpha}{zv}\right)\Phi_S(\lambda,\omega)+\left(\frac{x_\alpha v_\beta}{xv}-\frac{x_\beta v_\alpha}{xv}-\frac{z_\alpha v_\beta}{zv}+\frac{z_\beta v_\alpha}{zv}\right)\Phi_A(\lambda,\omega)\right].$$

$\Phi_S=\Phi$**, whereas** Φ_A **is new. If the contributions induced by** Φ_A **are not suppressed, a consistent calculation of the decay amplitude** A **needs further inputs.**