The high-energy limit of perturbative QCD: theory and phenomenology

Alessandro Papa

Università della Calabria & INFN - Cosenza

QCD@work - Lecce, June 27, 2022

UNIVERSITÀ DELLA CALABRIA
DIPARTIMENTO DI FISICA

INFN
Istituto Nazionale di Fisica Nucleare
GRUPPO COLLEGATO DI COSENZA

Outline

Introduction and Motivation

- LHC and future colliders (EIC, FCC, ILC, ...) can/will challenge QCD in energy regimes where fixed-order calculations, based on collinear factorization, need to be improved.

- Semihard collision processes, featuring the scale hierarchy

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD} \;, \quad Q \text{ a hard scale} \;,$$

  represent one of these challenges for perturbative QCD,

$$\alpha_s(Q) \log s \sim 1 \implies \text{ all-order resummation needed!}$$

- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach provides a general framework for the large-s / high-energy resummation: it predicts a peculiar behavior of amplitudes at high energies, which should to precede the onset of saturation physics.

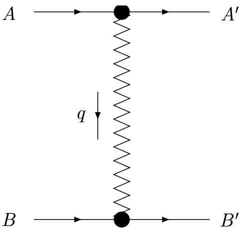
- I will briefly review the theoretical basis of the BFKL approach and present some of its potentialities for phenomenology.

Outline

Gluon Reggeization in perturbative QCD

Elastic scattering process $A + B \longrightarrow A' + B'$

- gluon quantum numbers in the t-channel: octet color representation, negative signature
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s)
- all-order resummation:
  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$



$$\left(\mathcal{A}_8^-\right)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t) \;, \qquad j(0) = 1$$

$\omega(t)$ – Reggeized gluon trajectory

$$\Gamma_{A'A}^c = g \langle A' | T^c | A \rangle \Gamma_{A'A}$$

T^c fundamental (quarks) or adjoint (gluons)

Gluon Reggeization in perturbative QCD

Interlude: Sudakov decomposition

$$p = \beta p_1 + \alpha p_2 + p_\perp \ , \qquad p_\perp^2 = -\vec{p}^{\,2}$$

(p_1, p_2) light-cone basis of the initial particle momenta plane

$$p_A = p_1 + \frac{m_A^2}{s} p_2 \ , \qquad p_B = p_2 + \frac{m_B^2}{s} p_1 \ , \qquad 2\, p_1 \cdot p_2 = s$$

The gluon Reggeization has been first verified in fixed order calculations, then rigorously proved

- in the LLA [Ya.Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A} \ , \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{D/2}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^\epsilon$$

$$D = 4 + 2\epsilon \ , \qquad t = q^2 \simeq q_\perp^2$$

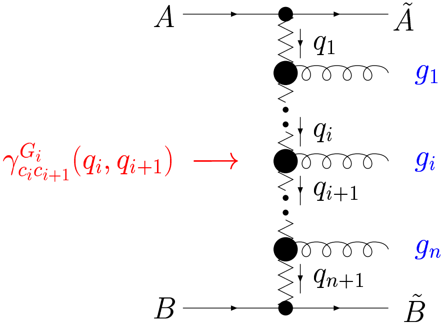
- in the NLA [V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko (2006)]

$$\Gamma^{(1)}_{A'A} = \delta_{\lambda_{A'}\lambda_A} \Gamma^{(+)}_{AA} + \delta_{\lambda_{A'},-\lambda_A} \Gamma^{(-)}_{AA} \ , \qquad \omega^{(2)}(t)$$

Outline

BFKL in leading accuracy

Inelastic scattering process $A + B \rightarrow \tilde{A} + \tilde{B} + n$ in the LLA



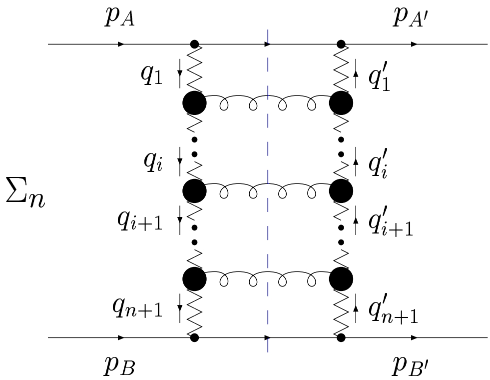
$$\mathrm{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

s_i invariant mass of the $\{g_{i-1}, g_i\}$ system, proportional to s

s_R energy scale, irrelevant in the LLA

BFKL in leading accuracy

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity

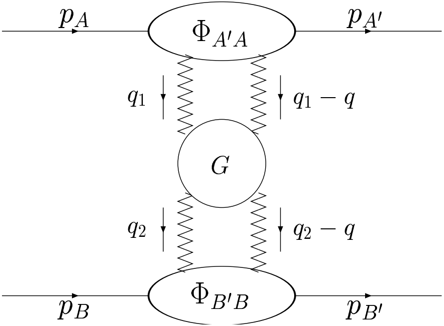


$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \ , \qquad \mathcal{R} = 1 \text{ (singlet)}, 8^- \text{ (octet)}, \ldots$$

The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

BFKL in leading accuracy

Structure of the amplitude:



$$\mathrm{Im}_s(\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1;\vec{q})$$

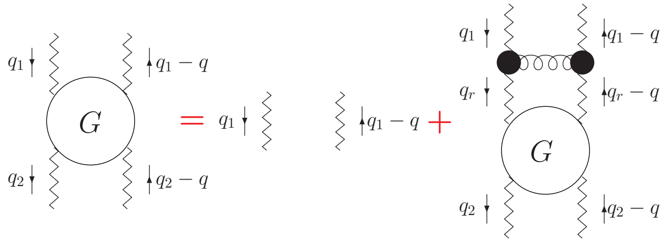
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2,\vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2;-\vec{q})$$

BFKL in leading accuracy

- $G_\omega^{(\mathcal{R})}$ – Mellin transform of the Green's functions for Reggeon-Reggeon scattering

$$\omega G_\omega^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2,\vec{q}) = \vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2\delta^{(D-2)}(\vec{q}_1-\vec{q}_2) + \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}(\vec{q}_r-\vec{q})^2}\mathcal{K}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_r;\vec{q})\,G_\omega^{(\mathcal{R})}(\vec{q}_r,\vec{q}_2;\vec{q})$$

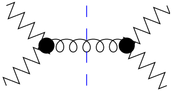
BFKL equation: $t=0$ and singlet color representation
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]



BFKL in leading accuracy

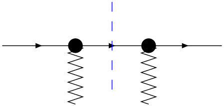
$$\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2; \vec{q}\right) = \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-(\vec{q}_1 - \vec{q})^{\,2}\right)\right] \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2; \vec{q}\right)$$

In the LLA: $\omega(t) = \omega^{(1)}(t), \quad \mathcal{K}_r = \mathcal{K}_{RRG}^{(B)}$



- $\Phi_{A'A}^{(\mathcal{R},\nu)}$ – impact factors in the t-channel color state $(\mathcal{R},\nu)$

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^c_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^*$$



constant in the LLA

BFKL in leading accuracy

Pomeron channel: $t = 0$ and singlet color representation in the t-channel

Redefinition: $G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}$, $\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}$

$$\omega G_\omega\left(\vec{q}_1, \vec{q}_2\right) = \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \int d^{D-2} q_r \mathcal{K}\left(\vec{q}_1, \vec{q}_r\right) G_\omega\left(\vec{q}_r, \vec{q}_2\right)$$

$$\mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) = 2\omega(-\vec{q}_1^{\,2}) \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r\left(\vec{q}_1, \vec{q}_2\right)$$

Infrared divergences cancel in the singlet kernel

$\mathcal{K}\left(\vec{q}_1, \vec{q}_2\right)$ is scale-invariant $\longrightarrow$ its eigenfunctions are powers of $\vec{q}_2^{\,2}$:

$$\int d^{D-2} q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) (\vec{q}_2^{\,2})^{\gamma-1} = \frac{N\alpha_s}{\pi} \chi(\gamma) \, (\vec{q}_1^{\,2})^{\gamma-1}$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \,, \quad \psi(\gamma) = \frac{\Gamma'(\gamma)}{\Gamma(\gamma)}$$

The set of functions $(\vec{q}_2^{\,2})^{\gamma-1}$, with $\gamma = 1/2 + i\nu$, $\nu \in (-\infty, +\infty)$ is complete.

BFKL in leading accuracy

Total cross section for the process $A + B \to$ all

$$\sigma_{AB}(s) = \frac{\mathcal{I}m_s\left(\mathcal{A}_{AB}^{AB}\right)}{s}$$

$$= \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_A}{\vec{q}_A^{\,2}} \Phi_A(\vec{q}_A) \int \frac{d^{D-2}\vec{q}_B}{\vec{q}_B^{\,2}} \Phi_B(-\vec{q}_B) \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_A, \vec{q}_B)$$

Using the complete set of kernel eigenfunctions, the BFKL equation and $D = 4$

$$\sigma_{AB}(s) = \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \int_{-\infty}^{+\infty} \frac{d\nu}{\omega - \frac{N\alpha_s}{\pi}\chi(1/2 + i\nu)}$$

$$\times \int \frac{d^2\vec{q}_A}{2\pi} \int \frac{d^2\vec{q}_B}{2\pi} \left(\frac{s}{s_0}\right)^{\omega} \Phi_A(\vec{q}_A) \frac{(\vec{q}_A^{\,2})^{-i\nu-3/2}}{\pi\sqrt{2}} \Phi_B(-\vec{q}_B) \frac{(\vec{q}_B^{\,2})^{i\nu-3/2}}{\pi\sqrt{2}}$$

Infrared finiteness guaranteed for colorless colliding particles

[V.S. Fadin, A.D. Martin (1999)]

BFKL in leading accuracy

Contour integration over ω

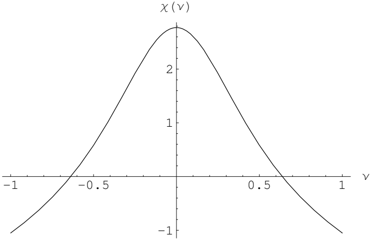
$$\sigma_{AB}(s) = \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi^2} \int \frac{d^2\vec{q}_A}{2\pi} \int \frac{d^2\vec{q}_B}{2\pi} \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s \chi(\nu)} \Phi_A(\vec{q}_A)(\vec{q}_A^{\,2})^{-i\nu-3/2} \Phi_B(-\vec{q}_B)(\vec{q}_B^{\,2})^{i\nu-3/2}$$

$$\bar{\alpha}_s \equiv \frac{N\alpha_s}{\pi}\,, \qquad \chi(\nu) \equiv \chi(1/2+i\nu)$$

Saddle point approximation:
$\chi(\nu) = 4\ln 2 - 14\zeta(3)\nu^2 + O(\nu^4)$

$$\sigma_{AB}(s) \sim \frac{s^{4\bar{\alpha}_s \ln 2}}{\sqrt{\ln s}}$$

$\omega_P = 4\bar{\alpha}_s \ln 2 \simeq 0.40$ for $\alpha_s = 0.15$



- unitarity is violated; BFKL cannot be applied at asymptotically high energies
- the scale of s and the argument of the running coupling constant are not fixed in the LLA $\longrightarrow$ NLA

Outline

BFKL in next-to-leading accuracy

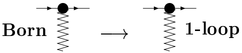
Production amplitudes keep the simple factorized form

$$\mathrm{Re}\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{P_i}(q_i,q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

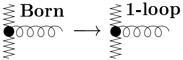
but, with respect to the LLA case, one replacement is allowed among the following:

multi-Regge kinematics

- $\omega^{(1)} \longrightarrow \omega^{(2)}$
- $\Gamma_{P'P}^{c\ (\mathrm{Born})} \longrightarrow \Gamma_{P'P}^{c\ (\mathrm{1\text{-}loop})}$



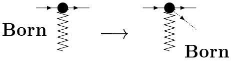
- $\gamma_{c_i c_{i+1}}^{G_i(\mathrm{Born})} \longrightarrow \gamma_{c_i c_{i+1}}^{G_i(\mathrm{1\text{-}loop})}$



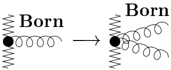
BFKL in next-to-leading accuracy

quasi-multi-Regge kinematics

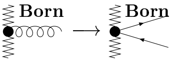
- $\Gamma^{c\,\text{(Born)}}_{P'P} \longrightarrow \Gamma^{c\,\text{(Born)}}_{\{f\}P}$



- $\gamma^{G_i\text{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\overline{Q}\text{(Born)}}_{c_i c_{i+1}}$



- $\gamma^{G_i\text{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{GG\text{(Born)}}_{c_i c_{i+1}}$



This is the program of calculation of radiative corrections to the LLA BFKL

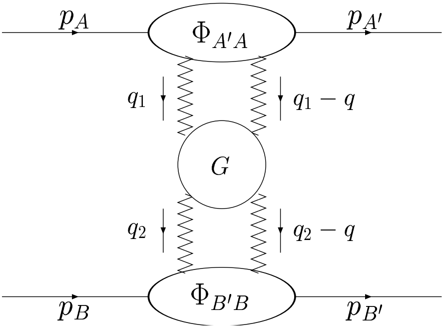
[V.S. Fadin, L.N. Lipatov (1989)]

BFKL in next-to-leading accuracy

- $\omega^{(2)}(t)$ [V.S. Fadin (1995)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)] [V.S. Fadin, R. Fiore, A. Quartarolo (1996)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)] [V.S. Fadin, M.I. Kotsky (1996)]
- $\gamma_{c_i c_{i+1}}^{G_i}$ (1-loop) [V.S. Fadin, L.N. Lipatov (1993)] [V.S. Fadin, R. Fiore, A. Quartarolo (1994)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)] [V.S. Fadin, R. Fiore, A. P. (2001)]
- $\Gamma_{P'P}^{c}$ (1-loop) [V.S. Fadin, R. Fiore (1992)] [V.S. Fadin, L.N. Lipatov (1993)] [V.S. Fadin, R. Fiore, A. Quartarolo (1994)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)]
- $\gamma_{c_i c_{i+1}}^{Q\overline{Q}}$ (Born) [V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)] [S. Catani, M. Ciafaloni, F. Hautmann (1990)] [G. Camici, M. Ciafaloni (1996)]
- $\gamma_{c_i c_{i+1}}^{GG}$(Born) [V.S. Fadin, L.N. Lipatov (1996)] [V.S. Fadin, M.I. Kotsky, L.N. Lipatov (1997)]

BFKL in next-to-leading accuracy

Structure of the amplitude:



$$\mathrm{Im}_s (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2} (\vec{q}_1 - \vec{q})^2} \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2} (\vec{q}_2 - \vec{q})^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1; \vec{q}; s_0)$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2; -\vec{q}; s_0)$$

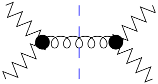
BFKL in next-to-leading accuracy

- $G_\omega^{(\mathcal{R})}$ – Mellin transform of the Green's functions for Reggeon-Reggeon scattering

$$\omega G_\omega^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2,\vec{q}) = \vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^{\,2}\delta^{(D-2)}(\vec{q}_1-\vec{q}_2) + \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}(\vec{q}_r-\vec{q})^{\,2}}\mathcal{K}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_r;\vec{q})\,G_\omega^{(\mathcal{R})}(\vec{q}_r,\vec{q}_2;\vec{q})$$

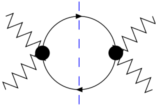
$$\mathcal{K}^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q}) = \left[\omega\left(-\vec{q}_1^{\,2}\right)+\omega\left(-(\vec{q}_1-\vec{q})^{\,2}\right)\right]\delta^{(D-2)}(\vec{q}_1-\vec{q}_2)+\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q})$$

In the NLA: $\omega(t)=\omega^{(1)}(t)+\omega^{(2)}(t)$, $\quad \mathcal{K}_r=\mathcal{K}_{RRG}^{(B)}+\mathcal{K}_{RRG}^{(1)}+\mathcal{K}_{RRQ\overline{Q}}^{(B)}+\mathcal{K}_{RRGG}^{(B)}$



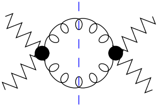
$\mathcal{K}_{RRG}^{(1)}$

$t=0$: [V.S. Fadin, L.N. Lipatov (1993)]
[V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
[V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]
$t\neq 0$: [V.S. Fadin, R. Fiore, A. P. (2001)]



$\mathcal{K}_{RRQ\overline{Q}}^{(B)}$

$t=0$: [V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)]
$t\neq 0$: [V.S. Fadin, R. Fiore, A. P. (1999)]



$\mathcal{K}_{RRGG}^{(B)}$

$t=0$: [V.S. Fadin, L.N. Lipatov, M.I. Kotsky (1997)]
$t\neq 0$: [V.S. Fadin, D.A. Gorbachev (2000)]
[V.S. Fadin, R. Fiore (2005)]

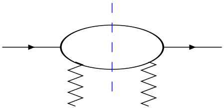
$-$ **counterterm**

BFKL in next-to-leading accuracy

- $\Phi^{(\mathcal{R},\nu)}_{A'A}$ – impact factors in the t-channel color state $(\mathcal{R},\nu)$

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^c_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^*$$

$$\times \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\frac{\omega(-\vec{q}_1^{\,2})}{2}} \left(\frac{s_0}{(\vec{q}_1 - \vec{q})^2}\right)^{\frac{\omega(-(\vec{q}_1-\vec{q})^2)}{2}}$$



$-$ **counterterm**

non-trivial momentum and scale-dependence

BFKL in next-to-leading accuracy

Pomeron channel: $t = 0$ and singlet color representation in the t-channel

$$\left(\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}} \; , \; \gamma = \frac{1}{2} + i\nu \right)$$

$$\int d^{D-2} q_2 \mathcal{K} \left(\vec{q}_1, \vec{q}_2\right) (\vec{q}_2^{\,2})^{\gamma-1} = \frac{N\alpha_s(\vec{q}_1^2)}{\pi} \left(\chi(\gamma) + \frac{N\alpha_s(\vec{q}_1^2)}{\pi} \chi^{(1)}(\gamma) \right) (\vec{q}_1^{\,2})^{\gamma-1}$$

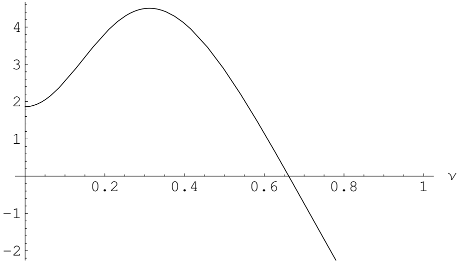
- broken scale invariance
- large corrections: $\quad -\left.\frac{\chi^{(1)}(\gamma)}{\chi(\gamma)}\right|_{\gamma=1/2} \simeq 6.46 + 0.05 \frac{n_f}{N} + 0.96 \frac{n_f}{N^3}$

[V.S. Fadin, L.N. Lipatov (1998)] [G. Camici, M. Ciafaloni (1998)]

BFKL in next-to-leading accuracy

$\chi(\nu) + \bar{\alpha}_s(\vec{q}_1^{\,2})\chi^{(1)}(\nu)$ *vs* ν

$\bar{\alpha}_s(\vec{q}_1^{\,2}) \equiv \frac{\alpha_s(\vec{q}_1^{\,2})N}{\pi} = 0.15$



Double maxima $\longrightarrow$ oscillations in momentum space after ν-integration

Ways out:

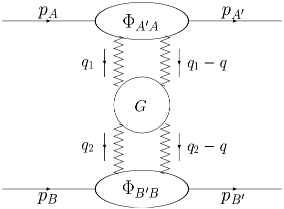
- rapidity veto [C.R. Schmidt (1999)] [J.R. Forshaw, D.A. Ross, A. Sabio Vera (1999)]
- collinear improvement [G. Salam (1998)] [M. Ciafaloni, D. Colferai (1999)]
- renormalization with a physical scheme [S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov (1999)]
- ...

Outline

BFKL factorization

Scattering $A + B \longrightarrow A' + B'$ in the Regge kinematical region $s \to \infty$, t fixed

$\Rightarrow$ BFKL factorization for $\mathrm{Im}_s \mathcal{A}$:
convolution of a Green's function with the impact factors of the colliding particles.



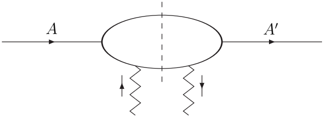
Valid both in
LLA (resummation of all terms $(\alpha_s \ln s)^n$)
NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

$$\mathrm{Im}_s \mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_{AA'}(\vec{q}_1, \vec{q}; s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_{BB'}(-\vec{q}_2, -\vec{q}; s_0)$$
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

The Green's function is process-independent and is determined through the BFKL equation. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)$$

Impact factors are process-dependent;
only very few of them known in the NLA ...



- $A = A'$=quark, $A = A'$=gluon [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)] [M. Ciafaloni and G. Rodrigo (2000)]
- $A = \gamma^*$, $A' = V$, with $V = \rho^0, \omega, \phi$ (forward)
  twist-2 (long. polarization) [D.Yu. Ivanov, M.I. Kotsky, A.P. (2004)]
  twist-3 (transv. polarization) [I.V. Anikin et al. (2009)]
- $A = A' = \gamma^*$ (forward) [J. Bartels, S. Gieseke, C.F. Qiao (2001)] [J. Bartels, S. Gieseke, A. Kyrieleis (2002)] [J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)] [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)] [J. Bartels, A. Kyrieleis (2004)] [I. Balitsky, G.A. Chirilli (2013)] [G.A. Chirilli, Yu.V. Kovchegov (2014)]

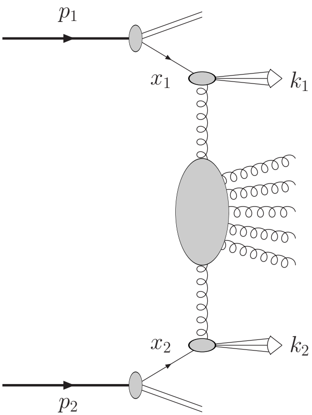
... so that only a very limited number of predictions can be built for exclusive processes or total cross sections (among them the 'gold plated' $\gamma^*\gamma^* \to$ all), even hardly testable in present colliders.

Outline

Inclusive processes

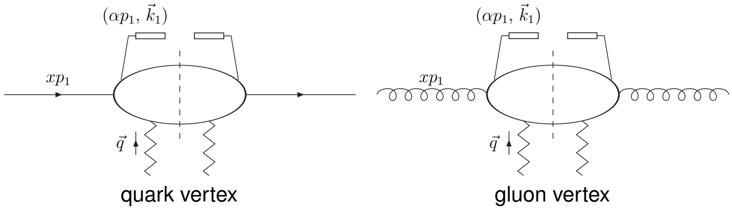
A lot more possibilities open for inclusive processes, with jets or identified particles in the final state, produced in the fragmentation regions...

... and if the fragmentation subprocess is hybridized with collinear factorization.



Identified 'object' (jet, hadron) with momentum k_1 (k_2) in the forward (backward) region; all the rest undetected.

Straightforward adaptation of the BFKL factorization: just restrict the summation over final states entering the definition of impact factors.



- "open" one of the integrations over the phase space of the intermediate state to allow one (or more) parton(s) to generate a jet or one parton to fragment into a given hadron
- use QCD collinear factorization

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes (D_a^h / S_a^J) \quad + \quad f_g \otimes (\text{gluon vertex}) \otimes (D_g^h / S_g^J)$$

$f_{a,g}$: unpolarized collinear PDFs, $D_{a,g}^h$: unpolarized collinear FFs,

$S_{a,g}^J$: jet selection functions

A few more impact factors became available in the hybrid collinear/high-energy factorization in the NLA ...

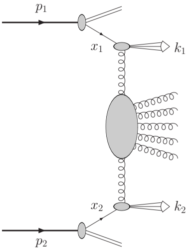
- jet vertex [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P., A. Perri (2011)]
  [D.Yu. Ivanov, A.P. (2012)] (small-cone approximation)
  [D. Colferai, A. Niccoli (2015)]

- hadron vertex [D.Yu. Ivanov, A.P. (2012)]

- Higgs vertex [M. Hentschinski, K. Kutak, A. van Hameren (2020)]
  [F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2022)]

... to be added to several LA ones:

- J/Ψ vertex [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- Drell-Yan pair vertex [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- heavy-quark production vertex
  [A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2019)]



$$\text{proton}(p_1) + \text{proton}(p_2) \longrightarrow \text{object}_1(k_1) + X + \text{object}_2(k_2)$$

Taking also into account the large variety of available FFs, a plethora of predictions were recently produced for the inclusive production at the LHC of a forward and a backward identified 'objects'.

Full-NLA analyses:

- jet + jet (Mueller-Navelet)
  [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]
  [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P. (2014)]
  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2015,2016)]
- light hadron + light hadron
  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2016,2017)]
- light hadron + jet
  [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2018)]
- $\Lambda + \Lambda$, Λ + jet [F.G. Celiberto, D.Yu. Ivanov, A.P. (2020)]
- $\Lambda_c + \Lambda_c$, Λ_c + jet [F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2020)]
- *b*-hadron + *b*-hadron, *b*-hadron + jet
  [F.G. Celiberto, M. Fucilla, M.M.A. Mohammed, A.P. (2022)]
- $B_c(B_c^*)$ + *b*-hadron, $B_c(B_c^*)$ + jet [F.G. Celiberto (2022)]

Partial-NLA analyses:

- J/Ψ + jet [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- Drell-Yan pair + jet [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- Higgs + jet [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2021)]
- light jet + heavy jet
  [A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2021)]
- Higgs + c-hadron [F.G. Celiberto, M. Fucilla, M.M.A. Mohammed, A.P. (2022)]

Observables

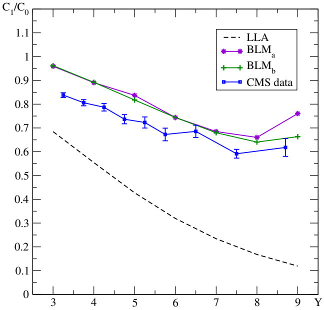
BFKL-factorized form, with "differential" impact factors:

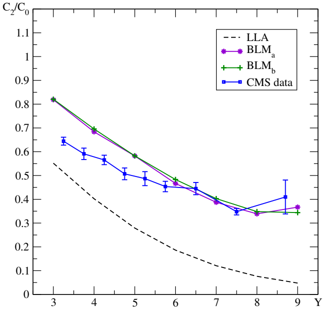
$$\frac{d\sigma}{dy_1 dy_2\, d|\vec{k}_1|\, d|\vec{k}_2| d\phi_1 d\phi_2} = \frac{d\Phi_1}{dy_1\, d|\vec{k}_1| d\phi_1} \otimes G \otimes \frac{d\Phi_2}{dy_2\, d|\vec{k}_2| d\phi_2}$$

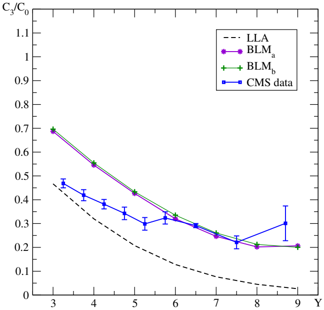
$$= \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\, \mathcal{C}_n \right] , \quad \phi = \phi_1 - \phi_2 - \pi$$

- azimuthal correlations, $\langle \cos(n\phi) \rangle = \mathcal{C}_n / \mathcal{C}_0$, and ratios between them
  [A. Sabio Vera, F. Schwennsen (2007)]
- cross sections differential in $Y \equiv y_1 - y_2$
- ...

Mueller-Navelet jets







$C_n/C_0 = \langle \cos(n\phi) \rangle$ *vs* $Y = y_{J_1} - y_{J_2}$

$|\vec{k}_{1,2}| \geq 35$ GeV, $|y_{1,2}| < 4.7$

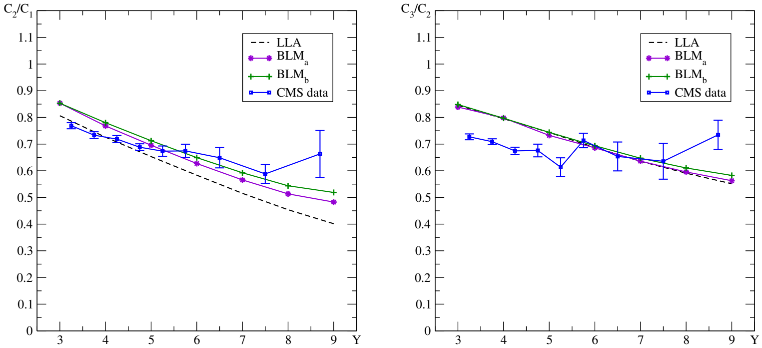
□ CMS (7 TeV)

small-cone approximation,

BLM scale setting: choose μ_R to make β_0-dependent terms in the amplitude vanish
[S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P. (2014)]

Mueller-Navelet jets



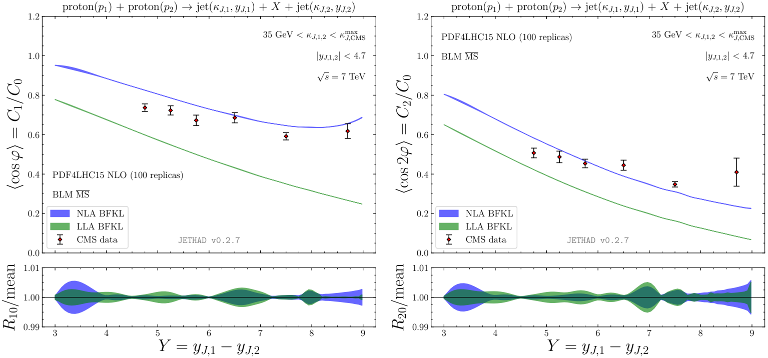
$$C_n/C_m = \frac{\langle \cos(n\phi)\rangle}{\langle \cos(m\phi)\rangle} \; vs \; Y = y_{J_1} - y_{J_2}$$

$|\vec{k}_{1,2}| \geq 35$ GeV, $|y_{1,2}| < 4.7$, □ CMS (7 TeV)

small-cone approximation, BLM scale setting

Similar results obtained with the exact jet vertices
[B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]

Mueller-Navelet jets



$$C_n/C_0 = \langle \cos(n\phi) \rangle \; vs \; Y = y_{J_1} - y_{J_2}$$

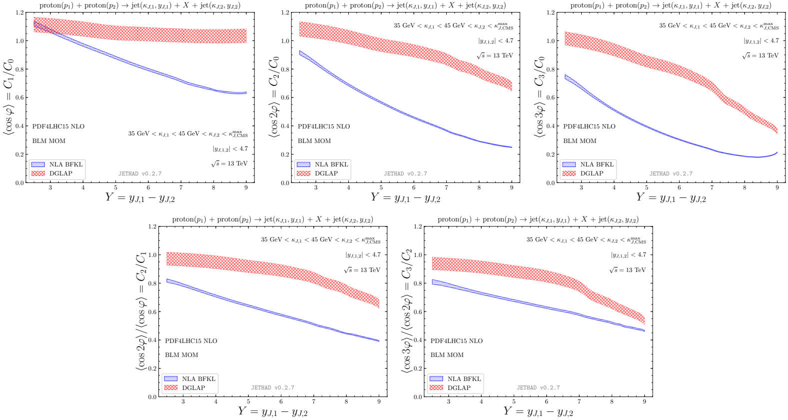
$$|\vec{k}_{1,2}| \geq 35 \text{ GeV}, \quad |y_{1,2}| < 4.7$$

◊ CMS (7 TeV)

small-cone approximation, BLM scale setting (exact implementation)

Taken from F.G. Celiberto, Eur. Phys. J. C 81 (2021) 8, 691 [arXiv:2008.07378] with author's permission.

Mueller-Navelet jets: BFKL *vs* high-energy DGLAP



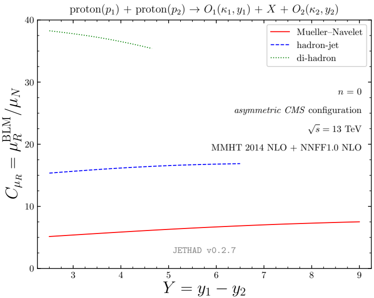
C_n/C_0 and C_n/C_m *vs* $Y = y_{J_1} - y_{J_2}$ at $\sqrt{s}$=13 TeV

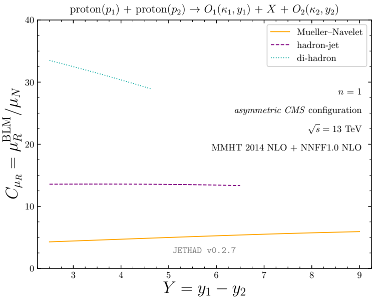
$35\ \mathrm{GeV} < |\vec{k}_1| < 45\ \mathrm{GeV}$, $\quad |\vec{k}_2| > 45\ \mathrm{GeV}$ (asymm. kinematics), $\quad |y_{1,2}| < 4.7$

small-cone approximation, BLM scale setting (exact implementation)

Taken from F.G. Celiberto, Eur. Phys. J. C 81 (2021) 8, 691 [arXiv:2008.07378] with author's permission.

Mueller-Navelet jet (and other processes)





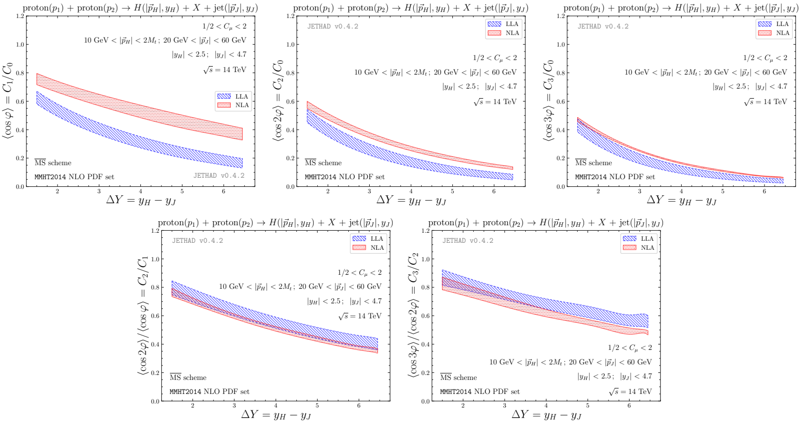
Values of the optimal BLM choice for μ_R of C_0 and C_1 at $\sqrt{s}$=13 TeV

asymmetric kinematics for transverse momenta

$\mu_N \equiv \sqrt{|\vec{k}_1||\vec{k}_2|}$ should be the "natural" scale of the process

Taken from F.G. Celiberto, Eur. Phys. J. C 81 (2021) 8, 691 [arXiv:2008.07378] with author's permission.

Higgs + jet (partial NLA)

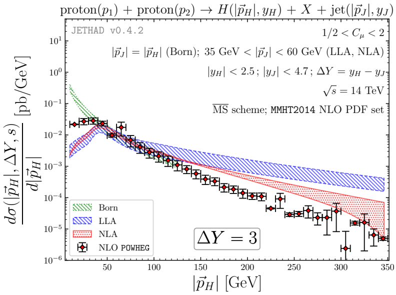


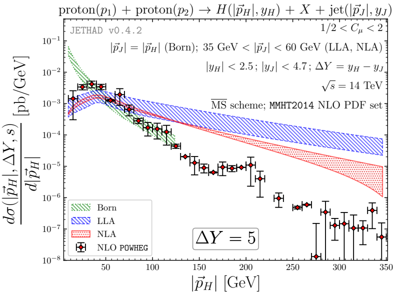
C_n/C_0 and C_n/C_m *vs* $\Delta Y = y_{J_H} - y_{J_J}$ at $\sqrt{s}$ = 14 TeV

10 GeV $< |\vec{p}_H| < 2M_t$, $|y_H| < 2.5$, 20 GeV $< |\vec{p}_J| <$ 60 GeV, $|y_J| < 4.7$,

small-cone jet approximation, $\overline{\text{MS}}$ scale setting (!)

Higgs + jet (partial NLA)





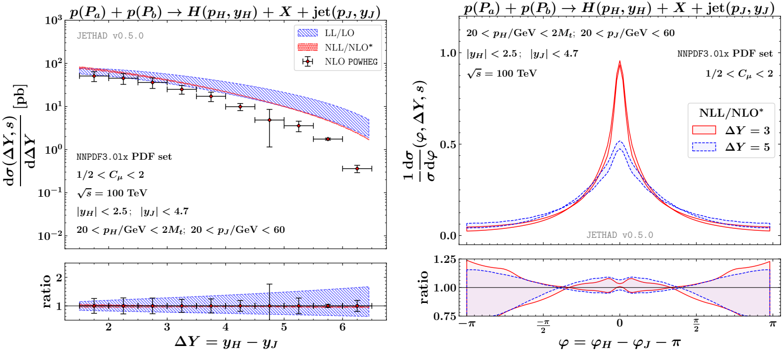
$d\sigma/d|\vec{p}_H|$ *vs* $|\vec{p}_H|$ at $\sqrt{s}$ = 14 TeV for $\Delta Y = 3$ and 5

$|y_H| < 2.5$, 35 GeV $< |\vec{k}_J| <$ 60 GeV, $|y_J| < 4.7$,

small-cone jet approximation, $\overline{\rm MS}$ scale setting (!)

[F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2021)]

Higgs + jet (partial NLA) @FCC



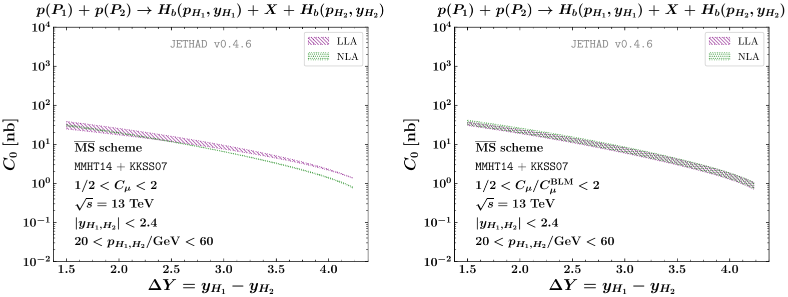
$d\sigma/d\Delta Y$ and $1/\sigma d\sigma/d\phi$ at $\sqrt{s} = 100$ TeV

$20 \text{ GeV} < |\vec{k}_H| < 2M_t$, $|y_H| < 2.5$, $\quad 20 \text{ GeV} < |\vec{k}_J| < 60 \text{ GeV}$, $|y_J| < 4.7$,

small-cone jet approximation, $\overline{\text{MS}}$ scale setting (!)

[F.G. Celiberto, A.P., FCC week 2022]

b-hadron + b-hadron



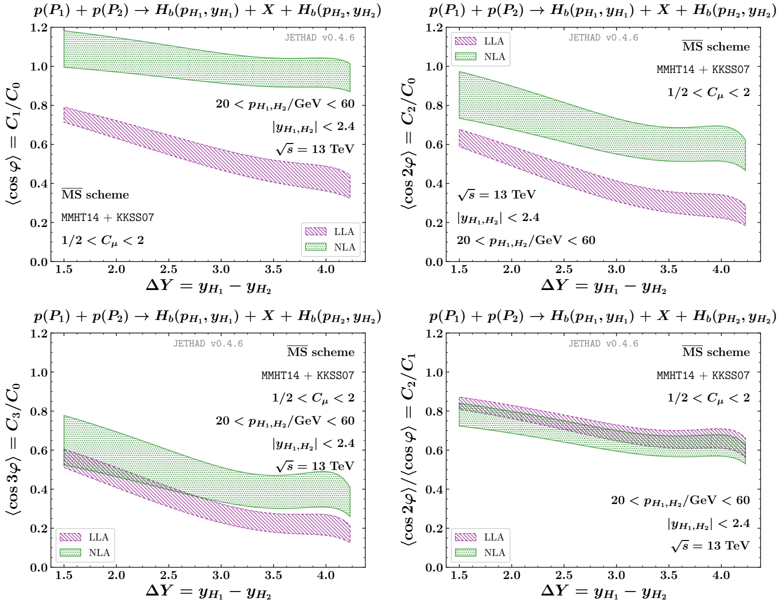
C_0 *vs* $\Delta Y = y_{H_1} - y_{H_2}$ at $\sqrt{s}$=13 TeV

$20 \text{ GeV} < |\vec{k}_{1,2}| < 60 \text{ GeV} \quad |y_{H_{1,2}}| < 2.4$

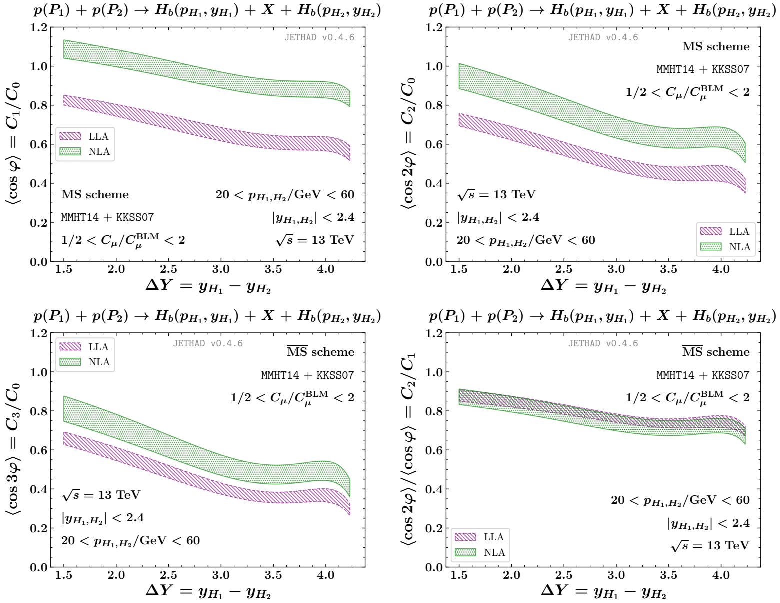
“natural” scales (left) BLM scale setting (right)

[F.G. Celiberto, M. Fucilla, M.M.A. Mohammed, A.P. (2022)]

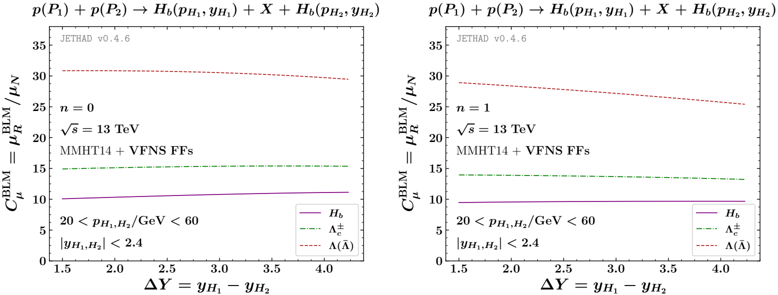
b-hadron + b-hadron: “natural” scales



b-hadron + b-hadron: BLM scale



b-hadron + b-hadron

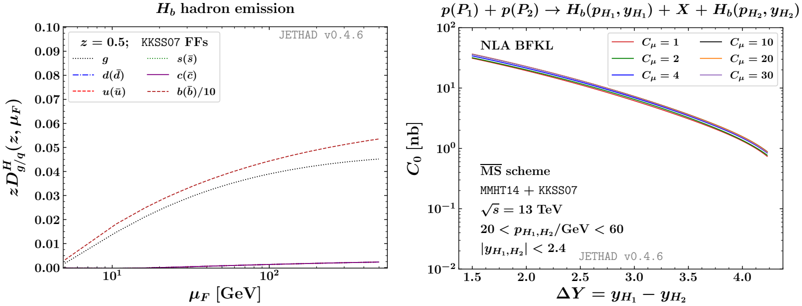


Values of the optimal BLM choice for μ_R of C_0 and C_1 at $\sqrt{s}$=13 TeV

$\mu_N \equiv \sqrt{|\vec{k}_1||\vec{k}_2|}$ should be the “natural” scale of the process

The heavier the hadron, the closer the BLM scale to the natural one.

b-hadron + b-hadron: scale stability

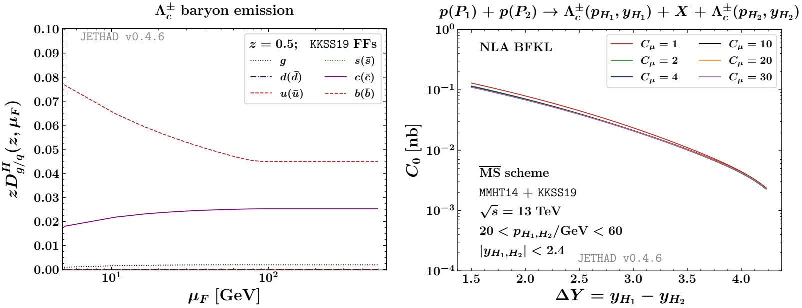


b-flavored hadrons

left: energy-scale dependence of FFs for b-flavored hadrons (reduced by a factor 10) at $z = 0.5$

right: C_0 *vs* ΔY in the double hadron production channel

$\Lambda_c^{\pm}$-hadron + $\Lambda_c^{\pm}$-hadron: scale stability

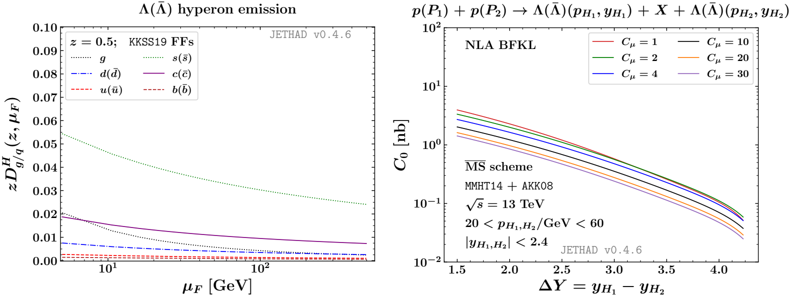


$\Lambda_c^{\pm}$ hadrons

left: energy-scale dependence of FFs for $\Lambda_c^{\pm}$ at $z = 0.5$

right: C_0 *vs* ΔY in the double hadron production channel

$\Lambda(\bar{\Lambda})$-hadron + $\Lambda(\bar{\Lambda})$-hadron: scale stability



$\Lambda(\bar{\Lambda})$ hadrons

left: energy-scale dependence of FFs for $\Lambda(\bar{\Lambda})$ at $z = 0.5$

right: C_0 *vs* ΔY in the double hadron production channel

Conclusions

- The BFKL approach is based on a remarkable property of perturbative QCD, the gluon Reggeization, now proved both in the leading and in the next-to-leading logarithmic approximation, and valid for both elastic and inelastic amplitudes.
- This approach provides a common basis for the description semi-hard processes ($s \gg Q^2 \gg \Lambda^2_{\rm QCD}$) and leads to a factorized form of physical amplitudes, in terms of a universal Green's function and the impact factors of the colliding particles.
- The consideration of BFKL dynamics has improved the analyses of several processes at HERA and LHC; an even clearer evidence of its manifestation could emerge in future high-energy colliders.
- BFKL and DGLAP are complementary approaches, undergoing an intense cross-fertilization. The combination with other resummation methods (soft/threshold, transverse momentum, etc.) can lead us to a more and more accurate description of strong interactions.

Conclusions

- The BFKL approach is based on a remarkable property of perturbative QCD, the gluon Reggeization, now proved both in the leading and in the next-to-leading logarithmic approximation, and valid for both elastic and inelastic amplitudes.

- This approach provides a common basis for the description semi-hard processes ($s \gg Q^2 \gg \Lambda^2_{\rm QCD}$) and leads to a factorized form of physical amplitudes, in terms of a universal Green's function and the impact factors of the colliding particles.

- The consideration of BFKL dynamics has improved the analyses of several processes at HERA and LHC; an even clearer evidence of its manifestation could emerge in future high-energy colliders.

- BFKL and DGLAP are complementary approaches, undergoing an intense cross-fertilization. The combination with other resummation methods (soft/threshold, transverse momentum, etc.) can lead us to a more and more accurate description of strong interactions.

Conclusions

- The BFKL approach is based on a remarkable property of perturbative QCD, the gluon Reggeization, now proved both in the leading and in the next-to-leading logarithmic approximation, and valid for both elastic and inelastic amplitudes.
- This approach provides a common basis for the description semi-hard processes ($s \gg Q^2 \gg \Lambda^2_{\rm QCD}$) and leads to a factorized form of physical amplitudes, in terms of a universal Green's function and the impact factors of the colliding particles.
- The consideration of BFKL dynamics has improved the analyses of several processes at HERA and LHC; an even clearer evidence of its manifestation could emerge in future high-energy colliders.
- BFKL and DGLAP are complementary approaches, undergoing an intense cross-fertilization. The combination with other resummation methods (soft/threshold, transverse momentum, etc.) can lead us to a more and more accurate description of strong interactions.

Conclusions

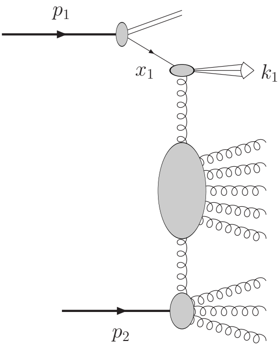
- The BFKL approach is based on a remarkable property of perturbative QCD, the gluon Reggeization, now proved both in the leading and in the next-to-leading logarithmic approximation, and valid for both elastic and inelastic amplitudes.
- This approach provides a common basis for the description semi-hard processes ($s \gg Q^2 \gg \Lambda^2_{\rm QCD}$) and leads to a factorized form of physical amplitudes, in terms of a universal Green's function and the impact factors of the colliding particles.
- The consideration of BFKL dynamics has improved the analyses of several processes at HERA and LHC; an even clearer evidence of its manifestation could emerge in future high-energy colliders.
- BFKL and DGLAP are complementary approaches, undergoing an intense cross-fertilization. The combination with other resummation methods (soft/threshold, transverse momentum, etc.) can lead us to a more and more accurate description of strong interactions.


Diffraction and Low-x
Sep 24 – 30, 2022
Corigliano Calabro (Italy)
Diffraction in e-p and e-ion collisions
Soft and low-mass diffraction
Photon-photon physics and hard diffraction
Spin Physics
Recent theoretical results on QCD and saturation
Low x, PDFs and hadronic final states
ORGANIZING COMMITTEE
Marcella Capua (Cosenza)
Luigi Delle Rose (Cosenza)
Salvatore Fazio (Cosenza)
Evelin Meoni (Cosenza)
Andrea Nava (Cosenza)
Alessandro Papa (Cosenza, co-chair)
Christophe Royon (Lawrence, co-chair)
Marco Rossi (Cosenza)
Enrico Tassi (Cosenza)
UNIVERSITÀ DELLA CALABRIA
KU THE UNIVERSITY OF KANSAS
EMMI
Consiglio regionale della Calabria
INFN Istituto Nazionale di Fisica Nucleare
https://indico.cern.ch/e/difflow2022

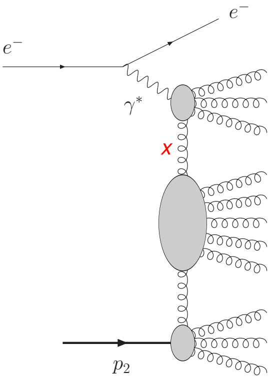
Backup

Inclusive processes: single forward

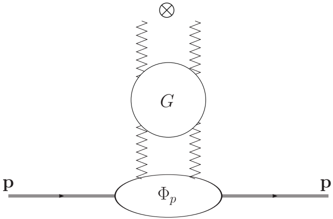
Another important class of inclusive processes arises when there is just one identified object (jet, hadron, Higgs boson, ...), produced in the fragmentation region of one of the colliding particles.



Single-forward inclusive production of an identified object with momentum k_1



Small-x
deep inelastic scattering



$$\underbrace{G \otimes \Phi_p}_{\text{UGD}}$$

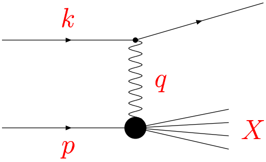
Straightforward emergence of the "unintegrated gluon distribution (UGD)":

- valid both in the LLA and in the NLA
- not unambiguous: normalization, s_0-scale setting, etc., follow from the definition of the impact factor to be convoluted with
- non-perturbative ...
  - the proton impact factor Φ_p is intrinsically non-perturbative
  - unavoidable diffusion in the infrared of the transverse momentum integration
- ... but ($\ln x$)-resummation automatically encoded.

BFKL-inspired UGD models adopted for deep inelastic scattering at HERA
[G. Chachamis, M. Déak, M. Hentschinski, G. Rodrigo, A. Sabio Vera (2013)]
and several (exclusive) electroproduction processes
[G. Chachamis, M. Déak, M. Hentschinski, G. Rodrigo, A. Sabio Vera (2015)]
[I. Bautista, A. Fernandez Tellez, M. Hentschinski (2016)]
[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A.P. (2018)]
[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A.P., W. Schäfer (2021)]

BFKL and deep inelastic scattering

Deep inelastic electron-proton scattering: $e + p \to e + X$



$$Q^2 = -q^2 > 0$$
$$s = (p+k)^2$$
$$W^2 = (p+q)^2$$

$$x = \frac{Q^2}{2p \cdot q} \simeq \frac{Q^2}{Q^2 + W^2} \,, \qquad W^2 \simeq \frac{Q^2(1-x)}{x}$$

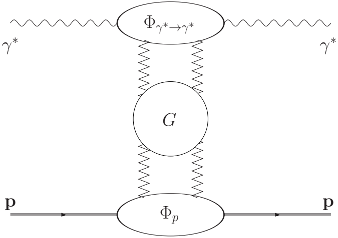
$$y = \frac{p \cdot q}{p \cdot k} \simeq \frac{Q^2}{xs}$$

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left\{ [1 + (1-y)^2] F_2(x, Q^2) - y^2 F_L(x, Q^2) \right\}$$

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha} [\sigma_T(x, Q^2) + \sigma_L(x, Q^2)]$$

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha} \sigma_L(x, Q^2)$$

Low-x (or large W^2) regime: $W^2 \gg Q^2 \gg M_P^2 \quad \longrightarrow \quad x \ll 1$



$$\sigma_{T,L} = \Phi_{T,L} \otimes \underbrace{G \otimes \Phi_p}_{\text{UGD}}$$

Straightforward emergence of the "unintegrated gluon distribution (UGD)":

- valid both in the LLA and in the NLA
- not unambiguous: normalization, s_0-scale setting, etc., follow from the definition of the (photon) impact factor
- non-perturbative ...

  - the proton impact factor Φ_p is intrinsically non-perturbative

  - unavoidable diffusion in the infrared of the transverse momentum integration
- ... but ($\ln x$)-resummation automatically encoded.

Let's get a closer look (only in the LLA)...

$$\sigma_{T,L}(x,Q^2) = \frac{1}{(2\pi)^2}\int \frac{d^2\vec{k}}{\vec{k}^2}\int \frac{d^2\vec{k}'}{\vec{k}'^2}\Phi_{T,L}(Q^2,\vec{k})\,G(x,\vec{k},\vec{k}')\,\Phi_p(-\vec{k}')$$

$$= 2\pi\int \frac{d^2\vec{k}}{(\vec{k}^2)^2}\underbrace{\Phi_{T,L}(Q^2,\vec{k})}_{\text{known at LO}}\underbrace{\frac{\vec{k}^2}{(2\pi)^3}\int \frac{d^2\vec{k}'}{\vec{k}'^2}G(x,\vec{k},\vec{k}')\,\Phi_p(-\vec{k}')}_{\mathcal{F}(x,\vec{k})}$$

In the limit $Q^2 \to \infty$ **(double leading-log approximation)**:

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \sum_{q=1}^{n_f} e_q^2\frac{\bar{\alpha}_s}{9}G(x,Q^2)\,, \qquad G(x,Q^2) \equiv \int \frac{d^2\vec{k}}{\pi\vec{k}^2}\Theta(Q^2-\vec{k}^2)\mathcal{F}(x,\vec{k})$$

Using a specific model for the proton impact factor, $\Phi_p(\vec{k}) \sim \left(\frac{\vec{k}^2}{\vec{k}^2+\mu^2}\right)^\delta$,

$$\frac{\mathcal{F}(x,\vec{k})}{\vec{k}^2} \propto \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{d\nu}{\sqrt{\mu^2\vec{k}^2}}\left(\frac{\vec{k}^2}{\mu^2}\right)^{i\nu} e^{\bar{\alpha}_s\chi(\nu)\ln 1/x}\,\frac{\Gamma(\delta-1/2-i\nu)\Gamma(1/2+i\nu)}{\Gamma(\delta)}$$

$$F_2(x,Q^2) \approx \mathcal{N}(\delta)\,\bar{\alpha}_s\sum e_q^2\left(\frac{Q^2}{\mu^2}\right)^{1/2} e^{\omega_P\ln 1/x}\,\exp\left(-\frac{\ln^2(Q^2/\mu^2)}{56\bar{\alpha}_s\zeta(3)\ln 1/x}\right)$$

by the saddle point method at $\nu = 0$ (recall, $\omega_P = 4\bar{\alpha}_s\ln 2$)

Mellin transforms

$$\mathcal{F}_N(\vec{k}) = \int_0^1 dx\, x^{N-1}\, \mathcal{F}(x,\vec{k})\ , \quad N \text{ stands for } \omega$$

$$\tilde{\mathcal{F}}_N(\gamma) = \int_1^\infty d\left(\frac{\vec{k}^2}{\mu^2}\right) \left(\frac{\vec{k}^2}{\mu^2}\right)^{-\gamma-1} \mathcal{F}_N(\vec{k})$$

Apply to

$$F_2(x,Q^2) \ = \ \frac{Q^2}{4\pi^2\alpha}\,[\sigma_T(x,Q^2)+\sigma_L(x,Q^2)]$$

$$\sigma_{T,L}(x,Q^2) = 2\pi \int \frac{d^2\vec{k}}{(\vec{k}^2)^2}\Phi_{T,L}(Q^2,\vec{k})\,\mathcal{F}(x,\vec{k})$$

$$\longrightarrow F_{2,N}(Q^2) = \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi i}\frac{h_{2,N}(\gamma)}{\gamma^2} \underbrace{\frac{1}{(2\pi)^3}\frac{\Phi_p(\gamma,\mu)}{N}}_{\tilde{\mathcal{F}}_N^0(\gamma)} \frac{N}{N-\bar{\alpha}_s\chi(\gamma)} \left(\frac{Q^2}{\mu^2}\right)^\gamma$$

with $\gamma = 1/2 + i\nu$ and $\chi(\gamma)$ stands for the previous $\chi(\nu)$

Contour integral in the γ-plane (left half plane):
(a) simple (!) pole at $\gamma = 0$ (contributes with Q^2-independent term)
(b) pole at $\bar{\gamma}$ such that $N = \bar{\alpha}_s\chi(\bar{\gamma})$
(c) other poles (from $h_{2,N}$) contribute with terms suppressed in Q^2

Up to a Q^2-independent additive term,

$$\frac{\partial F_{2,N}(Q^2)}{\partial \ln Q^2} = h_{2,N}(\bar\gamma) R_N \tilde{\mathcal{F}}^0_N(\bar\gamma) \left(\frac{Q^2}{\mu^2}\right)^{\bar\gamma} , \qquad R_N = \frac{1}{-\bar\alpha_s \bar\gamma \chi'(\bar\gamma)/N}$$

i) Expanding $\chi(\bar\gamma)$ around $\bar\gamma = 1/2$ (i.e. $\nu = 0$) and solving $N = \bar\alpha_s \chi(\bar\gamma)$, one gets

$$\bar\gamma \simeq \frac{1}{2} - \sqrt{\frac{N - \omega_P}{14 \bar\alpha_s \zeta(3)}} , \qquad R_N \simeq -\frac{\omega_P}{\sqrt{14 \bar\alpha_s \zeta(3)(N - \omega_P)}}$$

and the Mellin anti-transform gives back

$$F_2(x, Q^2) \approx \mathcal{N}(\delta)\, \bar\alpha_s \sum_q e_q^2 \left(\frac{Q^2}{\mu^2}\right)^{1/2} e^{\omega_P \ln 1/x} \exp\left(-\frac{\ln^2(Q^2/\mu^2)}{56 \bar\alpha_s \zeta(3) \ln 1/x}\right)$$

ii) Scaling violations even at asymptotically large Q^2:

$$N = \bar\alpha_s \chi(\bar\gamma) , \qquad \chi(\gamma) = \frac{1}{\gamma} + 2 \sum_{r=1}^{\infty} \zeta(2r+1)\, \gamma^{2r} , \quad |\gamma| < 1$$

$$\longrightarrow \bar\gamma = \frac{\bar\alpha_s}{N} + 2\zeta(3) \left(\frac{\bar\alpha_s}{N}\right)^4 + 2\zeta(5) \left(\frac{\bar\alpha_s}{N}\right)^6 + \mathcal{O}\left(\frac{\bar\alpha_s}{N}\right)^7$$

BFKL anomalous dimension

Contact with the DGLAP formalism

$$G(x,Q^2) \equiv xg(x,Q^2)\ , \quad \Sigma(x,Q^2) \equiv \sum_{i=1}^{n_f}[xq_i(x,Q^2) + x\bar{q}_i(x,Q^2)]$$

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} \Sigma_N(Q^2) \\ G_N(Q^2) \end{pmatrix} = \begin{pmatrix} \gamma_{qq}^N & 2n_f\gamma_{qg}^N \\ \gamma_{gq}^N & \gamma_{gg}^N \end{pmatrix}\begin{pmatrix} \Sigma_N(Q^2) \\ G_N(Q^2) \end{pmatrix}$$

$$F_{2,N}(Q^2) = \sum_i e_i^2 C_{i,N}(Q^2/\mu_F^2, \alpha_s(\mu_F^2))Q_{i,N}(\mu_F^2) + C_{g,N}(Q^2/\mu_F^2, \alpha_s(\mu_F^2))G_N(\mu_F^2)$$

(i) Leading order, (ii) $\mu_F = Q$, (iii) neglect quark densities at low x, (iv) neglect derivatives wrt $\ln Q^2$ of coefficient functions (subleading):

$$\frac{\partial F_{2,N}(Q^2)}{\partial \ln Q^2} = (\langle e_q^2\rangle 2n_f\gamma_{qg}^N + C_g^N(1,\alpha_s)\gamma_{gg}^N)G_N(Q^2)$$

$$G_N(Q^2) = G_N(\mu^2)\left(\frac{Q^2}{\mu^2}\right)^{\gamma_{gg}^N}$$

Compare with

$$\frac{\partial F_{2,N}(Q^2)}{\partial \ln Q^2} = h_{2,N}(\bar{\gamma})R_N\tilde{\mathcal{F}}_N^0(\bar{\gamma})\left(\frac{Q^2}{\mu^2}\right)^{\bar{\gamma}}$$

$$\longrightarrow \gamma_{gg}^N = \bar{\gamma} = \frac{\bar{\alpha}_s}{N}$$

Note also that $\gamma_{qg}^N|_{N\to 0} = \frac{\bar{\alpha}_s}{18}$ and $h_{2,N}(\bar{\gamma}) = \bar{\alpha}_s\langle e_q^2\rangle\frac{n_f}{9}$.

BFKL predicts the most singular part of γ_{gg}^N to all orders in α_s.

Collinear improvement of the BFKL kernel

Let's start from the BFKL Green's function in the LLA

$$G(\vec{q}_1, \vec{q}_2, s) = \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega(\gamma)}$$

$$\omega(\gamma) \equiv \bar{\alpha}_s \chi(\gamma) = \alpha_s (2\psi(1) - \psi(\gamma) - \psi(1-\gamma))$$

$$\begin{aligned} G(\vec{q}_1, \vec{q}_2, s) \overset{s_0 = q_1 q_2}{=} & \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q_1 q_2}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega(\gamma)} \\ = & \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q_1^2}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma - \frac{1}{2} + \frac{\omega}{2}} \frac{1}{\omega - \omega(\gamma)} \\ = & \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q_1^2}\right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega(\gamma - \frac{\omega}{2})} \end{aligned}$$

New pole $\bar{\gamma}$: $\omega = \omega(\gamma - \omega/2) = \bar{\alpha}_s \chi(\gamma - \omega/2)$

$$\omega(\gamma - \omega/2) \simeq \frac{\bar{\alpha}_s}{\gamma - \omega/2} \quad \longrightarrow \quad \omega \simeq \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s^2}{2\gamma^3} + \sum_{n=2}^{\infty} \frac{(2n)!}{2^n n! (n+1)!} \frac{(\bar{\alpha}_s)^{n+1}}{\gamma^{2n+1}}$$

- The $\frac{\bar{\alpha}_s^2}{2\gamma^3}$ cancels when the NLA BFKL corrections are taken into account
- The other subleading (but numerically large) terms are not compatible with collinear factorization
- The problem would be solved by the ω-shift: $\omega(\gamma) \to \omega(\gamma + \omega/2)$
  [G. Salam (1998)][M. Ciafaloni, D. Colferai (1999)]

$$G(\vec{q}_1, \vec{q}_2, s) = \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q_1 q_2} \right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega(\gamma + \frac{\omega}{2})}$$

$$= \frac{1}{\pi q_1 q_2} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q_1^2} \right)^{\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega(\gamma)}$$

- ω-shift in the NLA

$$\omega = \bar{\alpha}_s (1 + a\,\bar{\alpha}_s) \big(2\psi(1) - \psi(\gamma + \frac{\omega}{2} - b\,\bar{\alpha}_s) - \psi(1 - \gamma + \frac{\omega}{2} - b\,\bar{\alpha}_s) \big)$$

  with a and b chosen to match the NLA terms from BFKL
- Bottom-line: extra, $\mathcal{O}(\bar{\alpha}_s^3)$, contribution to the NLA BFKL kernel
- When DGLAP- or (RG-) improved kernels are adopted, the optimal values of energy scales (μ_R, s_0) get closer to the "natural" values
  [F. Caporale, A.P., A. Sabio Vera (2007)]

Azimuthal coefficients for di-hadron production

$$\mathcal{C}_n \equiv \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \, \cos[n(\phi_1 - \phi_2 - \pi)] \, \frac{d\sigma}{dy_1 dy_2 \, d|\vec{k}_1| \, d|\vec{k}_2| d\phi_1 d\phi_2}$$

$$= \frac{e^Y}{s} \int_{-\infty}^{+\infty} d\nu \, \left(\frac{\alpha_1 \alpha_2 s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\left[\chi(n,\nu)+\bar{\alpha}_s(\mu_R)\left(\bar{\chi}(n,\nu)+\frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu)+\frac{10}{3}+\ln\frac{\mu_R^4}{\vec{k}_1^2\vec{k}_2^2}\right)\right)\right]}$$

$$\times \alpha_s^2(\mu_R) c_1(n,\nu,|\vec{k}_1|,\alpha_1) c_2(n,\nu,|\vec{k}_2|,\alpha_2)$$

$$\times \left[1 + \alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu,|\vec{k}_1|,\alpha_1)}{c_1(n,\nu,|\vec{k}_1|,\alpha_1)} + \frac{c_2^{(1)}(n,\nu,|\vec{k}_2|,\alpha_2)}{c_2(n,\nu,|\vec{k}_2|,\alpha_2)}\right)\right.$$

$$\left. + \bar{\alpha}_s^2(\mu_R) \ln\frac{\alpha_1 \alpha_2 s}{s_0} \frac{\beta_0}{8N_c} \chi(n,\nu) \left(2 \ln \vec{k}_1^2 \vec{k}_2^2 + i \frac{d \ln \frac{c_1(n,\nu)}{c_2(n,\nu)}}{d\nu}\right)\right] .$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$$

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right)$$