Neutron star mergers as materials science

Prof. Mark Alford
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Alford, Bovard, Hanauske, Rezzolla, Schwenzer
arXiv:1707.09475

Alford and Harris, arXiv:1907.03795

Alford, Harutyunyan, Sedrakian, arXiv:2006.07975, 2108.07523

Alford and Haber, arXiv:2009.05181

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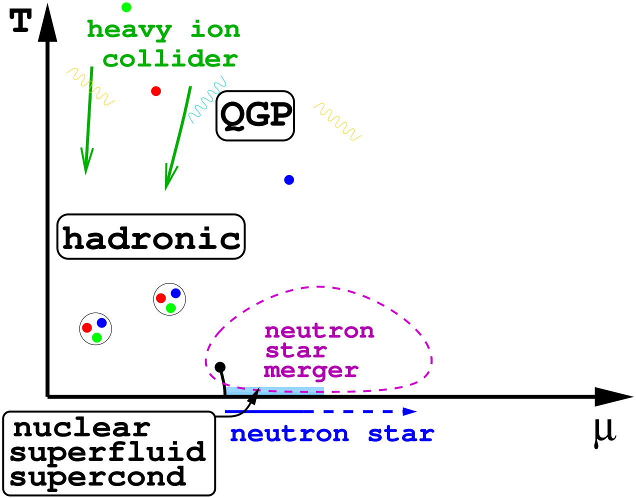
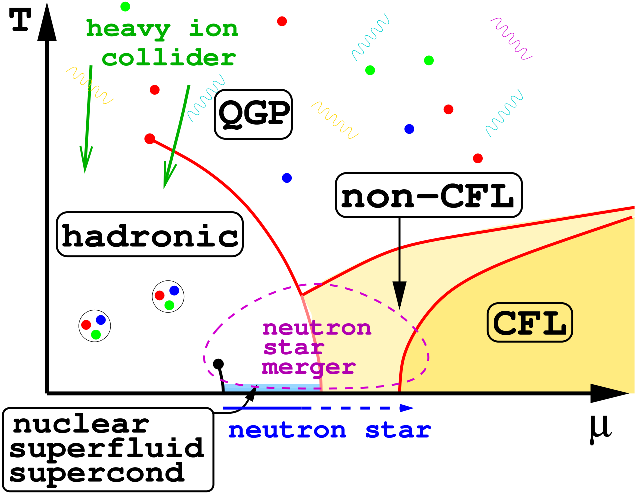
Outline

- Neutron star mergers test the properties of dense matter, e.g. **Equation of State**
- Equilibration:
  Thermal equilibration — thermal conductivity
  Shear flow equilibration — shear viscosity
  Flavor equilibration — bulk viscosity
  Better than the equation of state for probing phase structure!
- Is bulk viscosity important in mergers?
  - How does bulk viscosity arise?
  - Bulk viscosity is a resonance
  - Damping time for density oscillations

QCD Phase diagram



Conjectured QCD Phase diagram



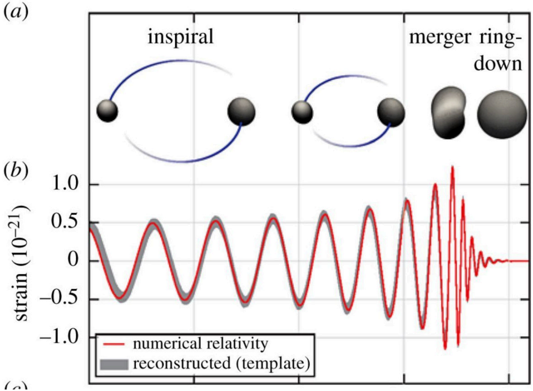
heavy ion collisions: deconfinement crossover and chiral critical point

neutron stars: quark matter core?

neutron star mergers: dynamics of warm and dense matter

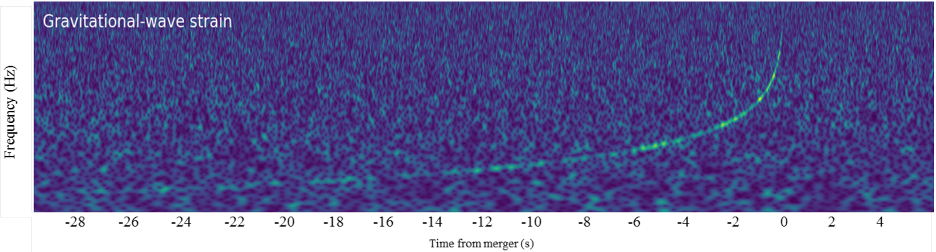
Grav waves from mergers: prediction

Prediction of gravitational waves is done by intense numerical computation



Grav waves from mergers: observation

LIGO Data from the event GW170817

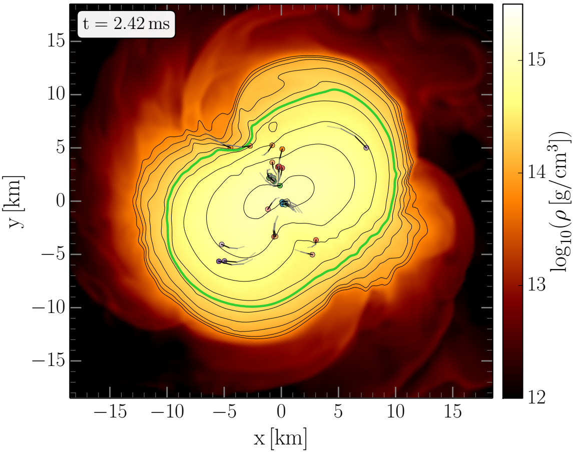


Omega Scan of LIGO's First Neutron Star Merger Detection

With LIGO we only see the inspiral, not the merger itself.

Neutron star mergers

Mergers probe the properties of nuclear/quark matter at high density (up to $\sim 4n_{\rm sat}$) and temperature (up to $\sim 60\,\mathrm{MeV}$)



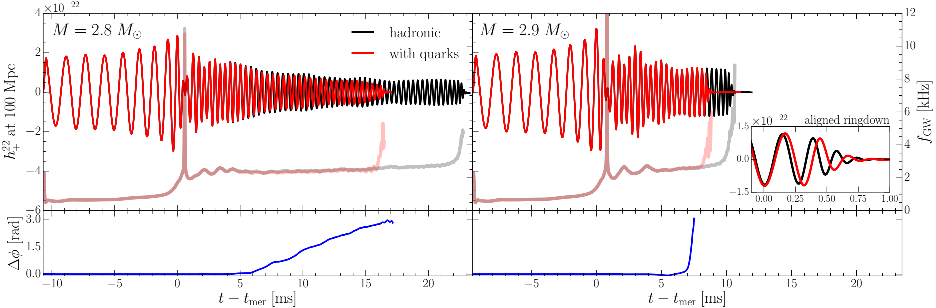
If we want to use mergers to learn about nuclear matter, we need to include all the relevant physics in our simulations.

Rezzolla group, Frankfurt

Video

Using grav waves to probe dense matter

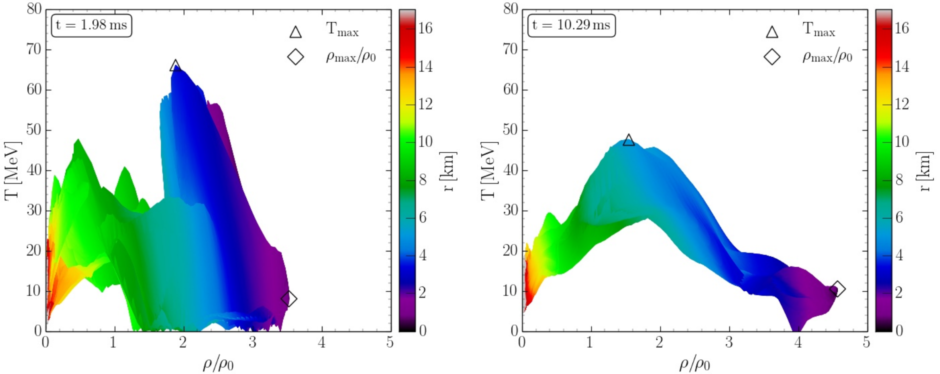
Current simulations try to connect the gravitational wave signal with features of the **Equation of State**, such as a first-order phase transition:



Most et. al., `arXiv:1807.03684`

solid lines: gravitational wave strain
translucent lines: instantaneous frequency

Nuclear material in a neutron star merger



M. Hanauske, Rezzolla group, Frankfurt

Significant spatial/temporal variation in:

- temperature
- fluid flow velocity
- density

so we need to allow for

- thermal conductivity
- shear viscosity
- bulk viscosity

Non-equilibrium physics in mergers

The important dissipation mechanisms are the ones whose equilibration time is $\lesssim 20\,\mathrm{ms}$

- Thermal equilibration: If neutrinos are trapped, and there are short-distance temperature gradients, then thermal transport might be fast enough to play a role.

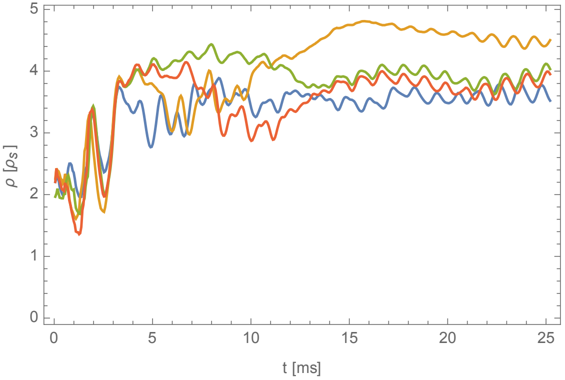
$$\tau_\kappa^{(\nu)} \approx 700\,\mathrm{ms}\left(\frac{z_{\mathrm{typ}}}{1\,\mathrm{km}}\right)^2\left(\frac{T}{10\,\mathrm{MeV}}\right)^2\left(\frac{0.1}{x_p}\right)^{1/3}\left(\frac{m_n^*}{0.8\,m_n}\right)^3\left(\frac{\mu_e}{2\mu_\nu}\right)^2$$

- Shear flow equilibration: similar conclusion.
  (related to shear viscosity)
- Flavor equilibration: could influence the merger's evolution.
  (related to bulk viscosity)

Density oscillations in mergers

Density vs time for tracers in merger

Bulk viscosity neglected



Tracers (co-moving fluid elements) show dramatic density oscillations, especially in the first 5 ms.

Amplitude: up to 50%
Period: 1–2 ms

What gets driven out of equilibrium?

Density oscillations and beta equilibration

Density oscillations lead to departure from *flavor equilibrium* (proton fraction).

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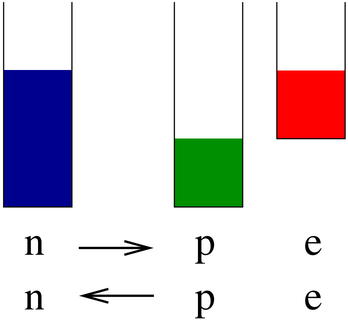
When you compress nuclear matter, the proton fraction wants to change.



Only the weak interaction can change proton fraction;
It operates on a macroscopic time scale, comparable to the merger ($\sim$ ms)

Flavor equilibration and bulk viscosity

- The finite rate of flavor equilibration via the weak interaction could be important for merger dynamics.
- Relaxation to flavor equilibrium will lead to bulk viscosity and damping of the density oscillations.
- How long does it take for bulk viscosity to dissipate a sizeable fraction of the energy of a density oscillation?

What is the damping time τ_ζ for density oscillations?

Density oscillation damping time τ_ζ

Density oscillation of amplitude Δn at angular freq ω:

$$n(t) = \bar{n} + \Delta n \cos(\omega t)$$

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Energy of density oscillation:
($K =$ nuclear incompressibility)

$$E_{\mathrm{comp}} = \frac{K}{18}\bar{n}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

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$$W_{\rm comp} = \zeta\frac{\omega^2}{2}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

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$$\boxed{\text{Damping Time:} \quad \tau_\zeta = \frac{E_{\rm comp}}{W_{\rm comp}} = \frac{K\bar{n}}{9\omega^2\zeta}}$$

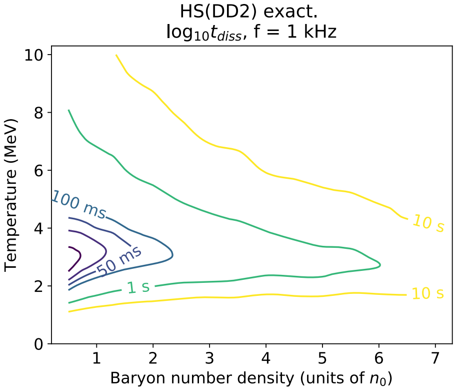
Bulk visc is only important in mergers if $\tau_\zeta \lesssim 20\,\text{ms}$

Damping time calculation (ν-transparent)

Damping time:

$$\boxed{\tau_\zeta = \frac{K\bar{n}}{9\omega^2\zeta}}$$

Damping can be fast enough to affect merger dynamics!



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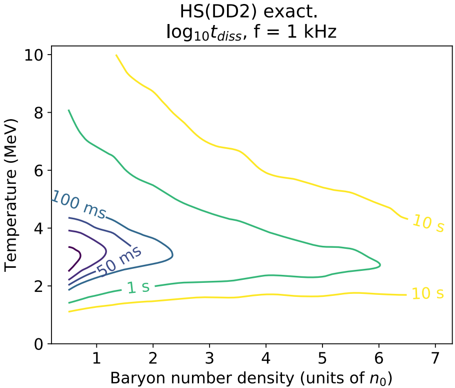

- Damping gets *slower at higher density*.
  Baryon density $\bar{n}$ and incompressibility K are both increasing.
  Oscillations carry more energy $\Rightarrow$ slower to damp

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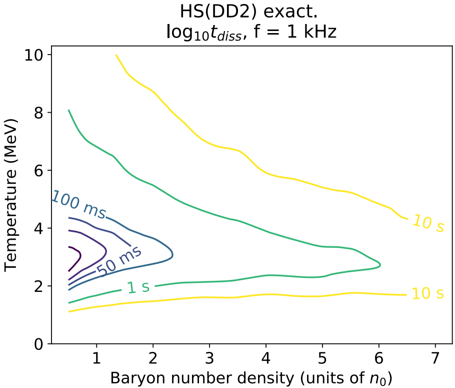

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  Baryon density $\bar{n}$ and incompressibility K are both increasing.
  Oscillations carry more energy $\Rightarrow$ slower to damp
- *Non-monotonic T-dependence:* damping is fastest at $T \sim 3\,\text{MeV}$.
  Damping is slow at very low or very high temperature.
  Non-monotonic dependence of bulk viscosity on temperature

Bulk viscosity: phase lag in system response

Some property of the material (proton fraction) takes time to equilibrate.

Baryon density n and hence fluid element volume V gets out of phase with applied pressure P:

$$\text{Dissipation} = -\int P\, dV = -\int P\, \frac{dV}{dt} dt$$

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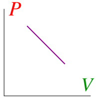
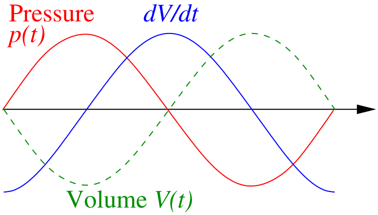
Dissipation = 0





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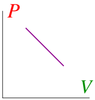
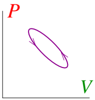
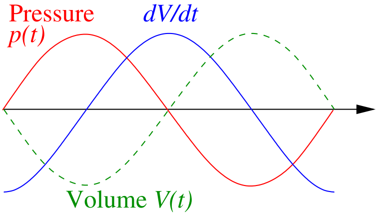
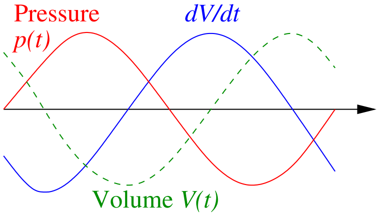


Some phase lag.
Dissipation > 0





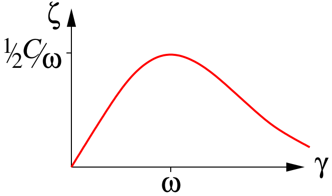


Bulk viscosity: a resonant phenomenon

Bulk viscosity is **maximum** when

$$\underset{\gamma}{\text{(internal equilibration rate)}} = \underset{\omega}{\text{(freq of density oscillation)}}$$

$$\zeta = C \frac{\gamma}{\gamma^2 + \omega^2}$$



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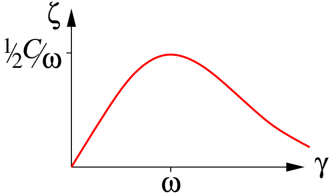

- Fast equilibration: $\gamma \to \infty \ \Rightarrow \ \zeta \to 0$
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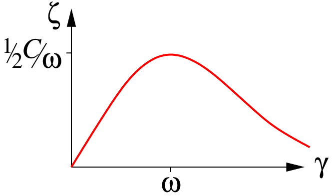

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- Slow equilibration: $\gamma \to 0 \;\Rightarrow\; \zeta \to 0$.
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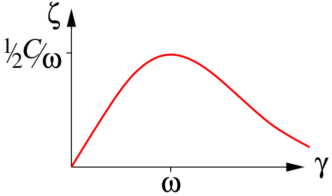

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- Slow equilibration: $\gamma \to 0 \;\Rightarrow\; \zeta \to 0$.
  System does not try to equilibrate: proton number and neutron number are both conserved. Proton fraction fixed.
- Maximum phase lag when $\omega = \gamma$.

Resonant peak in bulk viscosity

We now see why bulk visc is a <u>non-monotonic</u> fn of temperature.

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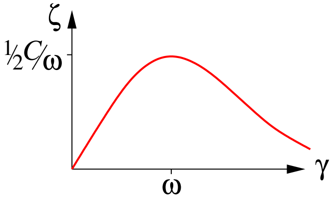
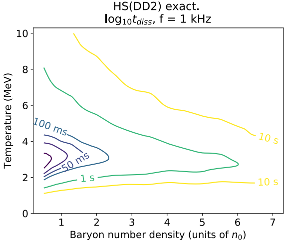
$$\zeta(T) = C\,\frac{\gamma(T)}{\gamma(T)^2 + \omega^2}$$

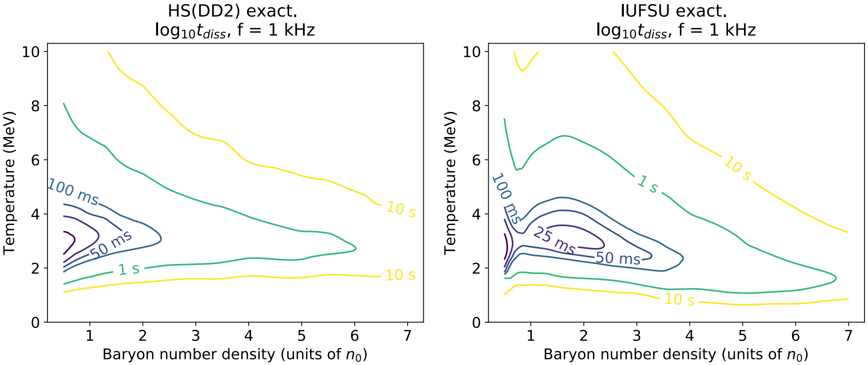




Beta equilibration rate γ rises with temperature (phase space at Fermi surface)

Maximum bulk viscosity in a neutron star merger will be when equilibration rate matches typical compression frequency $f \approx 1\,\text{kHz}$. I.e. when $\gamma \sim 2\pi \times 1\,\text{kHz}$

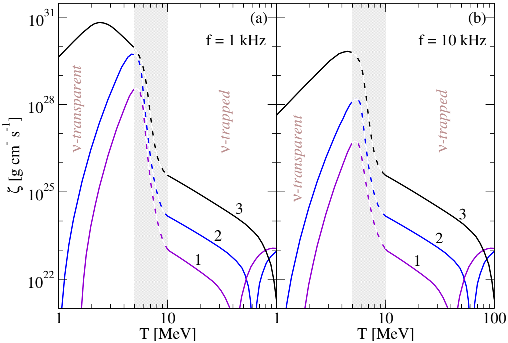
Two different EoSes



The damping time for density oscillations is shortest around $T \sim 3\,\mathrm{MeV}$, independent of the EoS.

It is short enough to be relevant for mergers, especially at low density.

The “hot” (neutrino-trapped) regime



Beta equilibration now includes neutrinos in the initial state too:

$\nu_e + n \leftrightarrow p + e^-$

Bulk viscosity is lower in hot matter ($T \gtrsim 5\,\mathrm{MeV}$).

- β equilibration is too fast, above resonant temperature, because there so much phase space at the Fermi surfaces
- The relevant susceptibilities are smaller, so the peak bulk visc is smaller

Summary

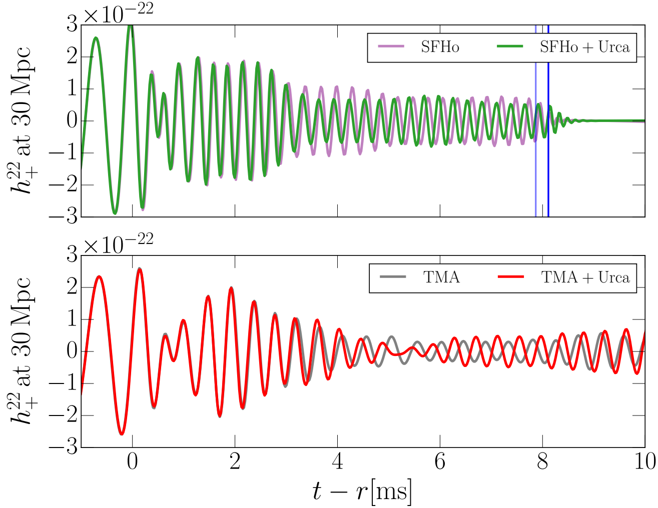
- Neutron star mergers probe the dynamical response of high-density matter, including dissipation properties.
- Thermal conductivity and shear viscosity may become significant in the neutrino-trapped regime ($T \gtrsim 5\,\mathrm{MeV}$) if there are fine-scale gradients ($z \lesssim 100\,\mathrm{m}$).
- In neutrino-transparent nuclear matter (at low density and $T \sim 3\,\mathrm{MeV}$) beta equilibration occurs on the timescale of the merger. So, for example, bulk viscosity may damp density oscillations.
- Under these conditions, the Fermi Surface approximation and detailed balance are not valid.
  Rate calculations must include the whole phase space.

Next steps

- Include beta equilibration / bulk viscosity in merger simulations.
- Do better calculations of beta equilibration rates in warm ($T \sim$ MeV) nuclear matter
- Calculate beta equilibration rates for other forms of matter: hyperonic, pion condensed, nuclear pasta, quark matter, etc
- Other manifestations? (Heating, neutrino emission,...)
- Beyond Standard Model physics?

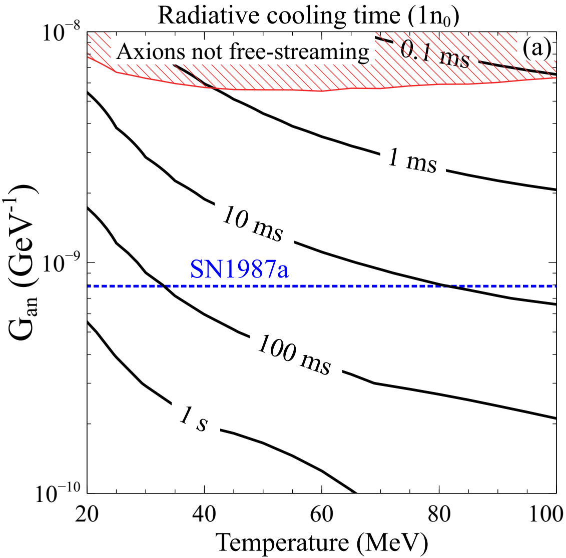
Influence of beta equilibration on gravitational wave signal

Most, Haber, Harris, Zhang, Alford, Noronha, `in progress`



Cooling by axion emission

Time for a hot region to cool to half its original temperature:



Harris, Fortin, Sinha, Alford
`arXiv:2003.09768`

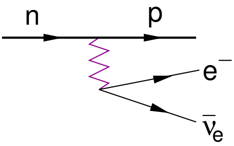
Extra slides

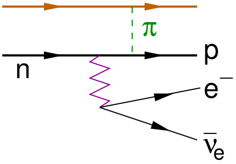
Urca processes

We will focus on the neutrino-transparent regime, $T \lesssim 5\,\mathrm{MeV}$

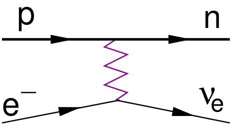
Direct Urca | Modified Urca

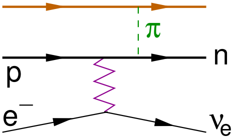
n decay





e^- capture

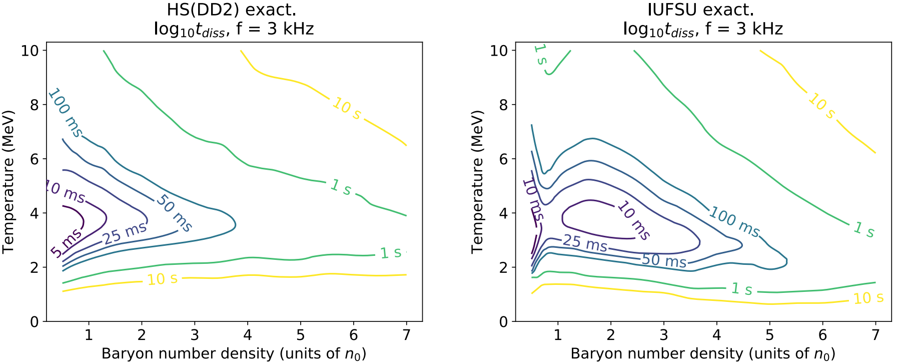




direct Urca only occurs above
direct Urca threshold density

Higher frequency oscillations

If 3 kHz oscillations occur then they would be damped even faster.

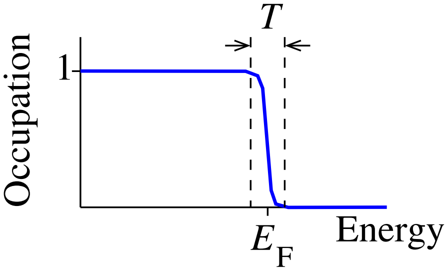


Note that max damping occurs at a slightly higher temperature, to get the beta equilibration rate to match the higher oscillation frequency.

Fermi Surface approximation

If the temperature is low enough, we can analyse beta equilibration processes in a simple way using the *Fermi Surface* (FS) approximation.

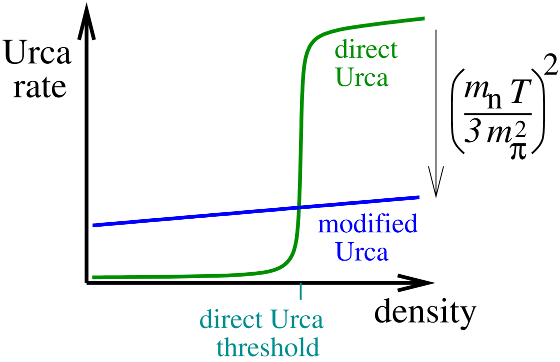
$$T \ll (E_F - m)$$
$$\text{or} \quad (\mu - m)$$



In the FS approximation, all the particles participating in beta equilibration processes are close to their Fermi surfaces.

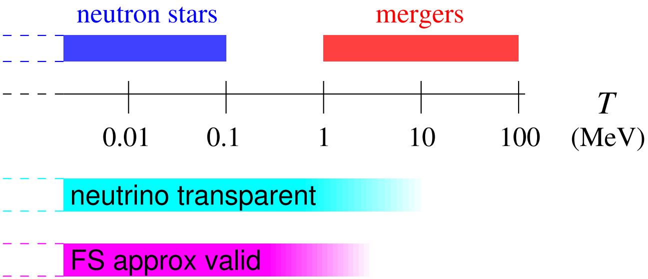
Urca in the cold regime

So in the cold regime, $T \ll 1\,\mathrm{MeV}$, the picture is



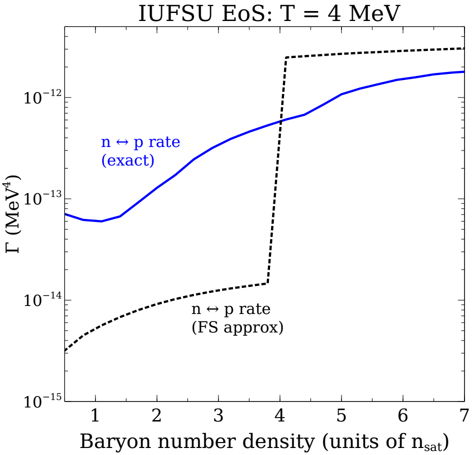
Is this picture still valid at merger temperatures: $T = 1$ to $100\,\mathrm{MeV}$?

Temperature regimes for neutron stars



- Cold ($T \ll 1\,\mathrm{MeV}$): Fermi Surface approx is valid, and neutrinos escape.
- Warm ($1\,\mathrm{MeV} \lesssim T \lesssim 5\,\mathrm{MeV}$): Fermi Surface approx is not accurate, but neutrinos still escape
- Hot ($T \gtrsim 5\,\mathrm{MeV}$): Fermi Surface approx is not accurate, neutrinos are trapped.

The "warm" regime: Beyond the Fermi Surface approx



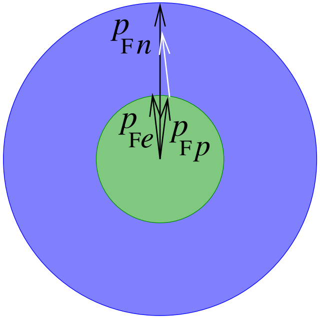
At $T \gtrsim 1\,\mathrm{MeV}$ the proton Fermi surface is sufficiently thermally blurred to smooth out the switch-on of direct Urca.

This is why the direct Urca threshold is not clearly visible in the contour plots of the dissipation time.

When can direct Urca happen?

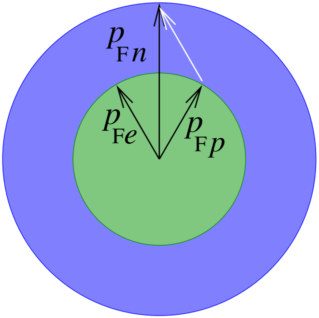
$$n \to p\ e^-\ \bar{\nu}_e, \quad p\ e^- \to n\ \nu_e$$

Low density
Low proton fraction
Direct Urca closed



$\vec{p}_n = \vec{p}_p + \vec{p}_e$ is impossible
because $p_{Fn} > p_{Fp} + p_{Fe}$

High density
High proton fraction
Direct Urca open



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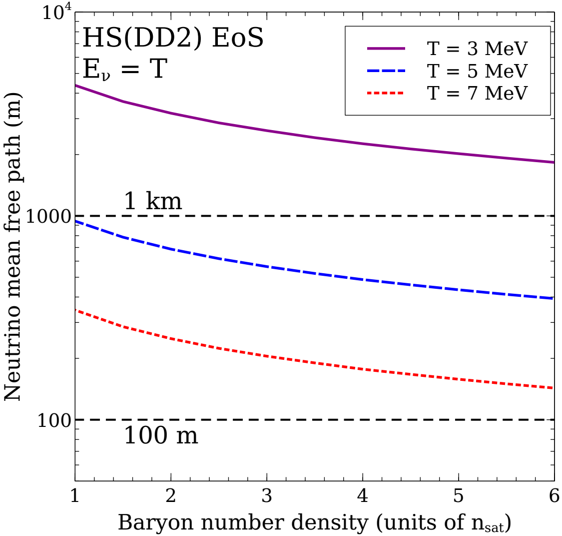
Neutrino mean free path



When does neutrino trapping begin?

mfp $\sim 3\,\mathrm{km}$: $T = 2$-$3\,\mathrm{MeV}$

mfp $\sim 1\,\mathrm{km}$: $T = 4$-$5\,\mathrm{MeV}$

mfp $\sim 0.3\,\mathrm{km}$: $T = 6$-$7\,\mathrm{MeV}$

Why is resonance with $1\,\mathrm{kHz}$ at $T \sim \mathrm{MeV}$?

Let's estimate $\gamma(T)$ and see when it is $2\pi \times 1\,\mathrm{kHz}$.

$$\begin{aligned}\frac{dn_a}{dt} &= -\gamma\,(n_a - n_{a,\mathrm{equil}})\\ \Gamma_{n\to p} - \Gamma_{p\to n} &\sim -\gamma\,\frac{\partial n_a}{\partial \mu_a}\,\mu_a\end{aligned}$$

In FS approx, at β-equilibrium,

$$\Gamma_{n\to p} = \Gamma_{p\to n} \sim G_F^2 \times (p_{Fn}^2 T) \times (p_{Fp} T) \times T^3$$

If we push it away from β equilibrium by adding μ_a, the leading correction is to replace one power of T with μ_a

$$\Gamma_{n\to p} - \Gamma_{p\to n} \sim G_F^2 (p_{Fn}^2 T) \times (p_{Fp} T) \times T^2 \mu_a$$

So

$$\gamma \sim \frac{\partial \mu_a}{\partial n_a} G_F^2\, p_{Fn}^2\, p_{Fp}\, T^4 \sim \frac{1}{(30\,\mathrm{MeV})^2} \frac{(350\,\mathrm{MeV})^2 (150\,\mathrm{MeV})}{(290\,\mathrm{GeV})^4}\, T^4$$

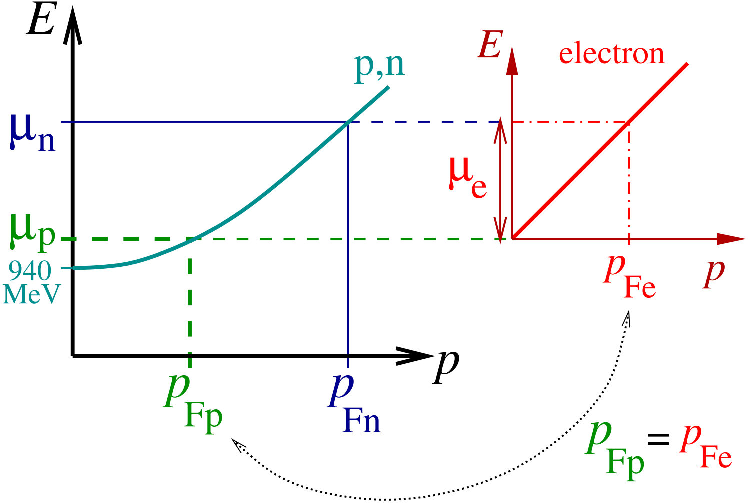
Solve for when $\gamma = 2\pi \times 1\,\mathrm{kHz} = 4 \times 10^{-18}\,\mathrm{MeV}$:

$$T \sim 1\,\mathrm{MeV}$$

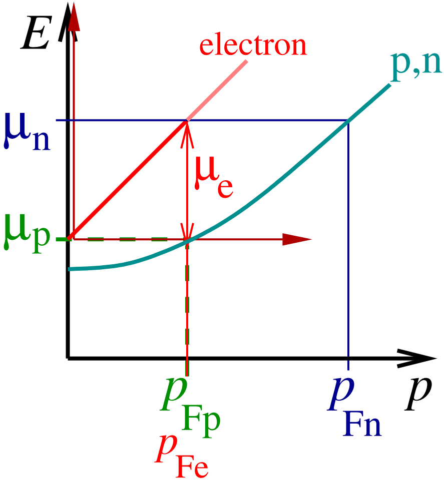
“Cold” beta equilibrium

In the cold regime, beta equilibrium means $\mu_n = \mu_p + \mu_e$

Electrical neutrality means $n_p = n_e \Rightarrow p_{Fp} = p_{Fe}$



"Cold" beta equilibrium



- Choose proton density p_{Fp}. This fixes μ_p and $p_{Fe} = p_{Fp}$
- Superimpose electron dispersion relation with energy zero at μ_p: this automatically adds μ_e to μ_p to give the beta-equilibrated value of μ_n.
- From μ_n we get p_{Fn} which fixes the neutron density.

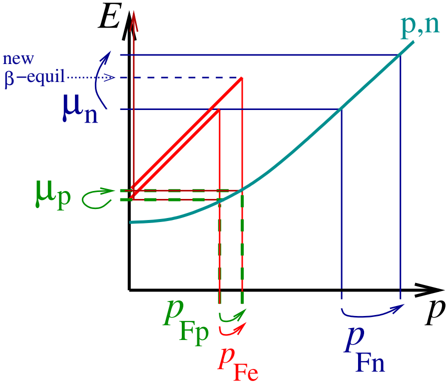
Now, what happens if we compress this β-equilibrated nuclear matter? Does the proton fraction need to change?

Compressing nuclear matter

Suppose we compress β-equilibrated nuclear matter by a factor of 2.

Compressing nuclear matter

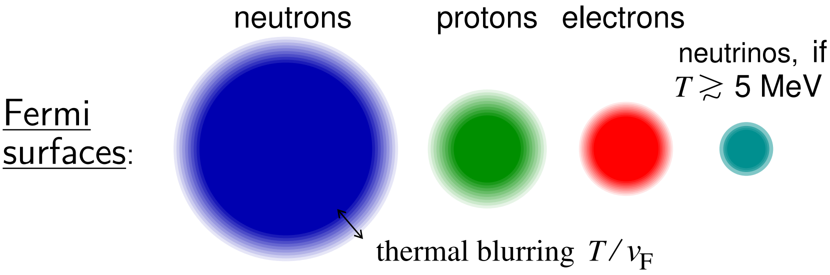
Suppose we compress β-equilibrated nuclear matter by a factor of 2.
All Fermi momenta rise by 26% $\quad (2^{(1/3)} = 1.26)$.
Is the matter still in β equilibrium?



- p_{Fp} rises by 26%. μ_p hardly changes.
- p_{Fe} rises by 26%. Superimpose electron dispersion relation with energy zero at new μ_p. Read off new β-equilibrated value of $\mu_n = \mu_p + \mu_e$
- But actually p_{Fn} rose by 26%, giving actual new μ_n: larger than the β-equilibrated value.

After compression the system is out of β equilibrium; $\mu_n - \mu_p - \mu_e > 0$
There are too many neutrons: proton fraction x needs to rise.

Nuclear material constituents

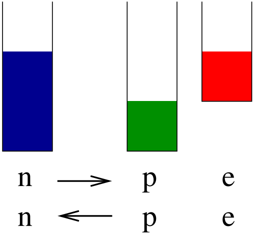


neutrons:	$\sim 90\%$ of baryons	$p_{Fn} \sim 350\,\text{MeV}$
protons:	$\sim 10\%$ of baryons	$p_{Fp} \sim 150\,\text{MeV}$
electrons:	same density as protons	$p_{Fe} = p_{Fp}$
neutrinos:	only present if mfp $\ll 10\,\text{km}$	i.e. when $T \gtrsim 5\,\text{MeV}$

Bulk viscosity and beta equilibration

When you compress nuclear matter, the proton fraction wants to change.

Only weak interactions can change proton fraction



	neutrino-transparent ($T \lesssim 5\,\mathrm{MeV}$)*	neutrino-trapped ($T \gtrsim 5\,\mathrm{MeV}$)*
neutron decay	$n \to p + e^- + \bar{\nu}_e$	$\nu_e + n \to p + e^-$
electron capture	$p + e^- \to n + \nu_e$	$p + e^- \to n + \nu_e$
	forward $\neq$ backward	$A + B \leftrightarrow C + D$

* Neutrino transparency is a finite volume effect, which occurs when the neutrino mean free path is greater than the size of the system. Our system is a neutron star, $R \sim 10\,\mathrm{km}$