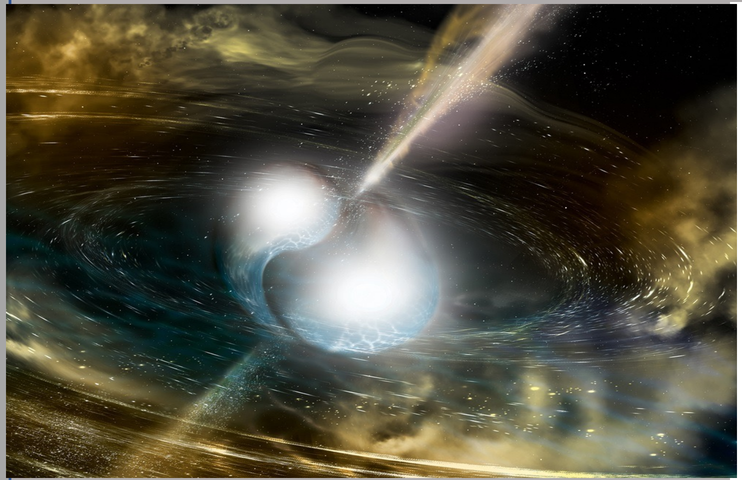
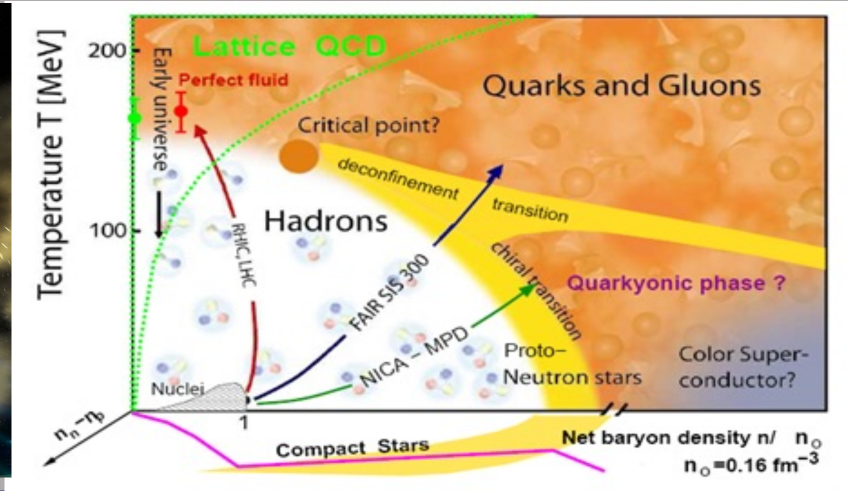
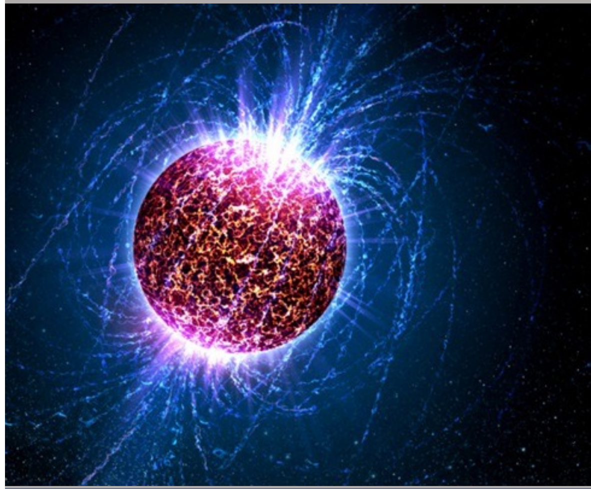
Electromagnetism in Quark Matter at Intermediate Densities







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QCD@Work – International Workshop on QCD Theory and Experiment
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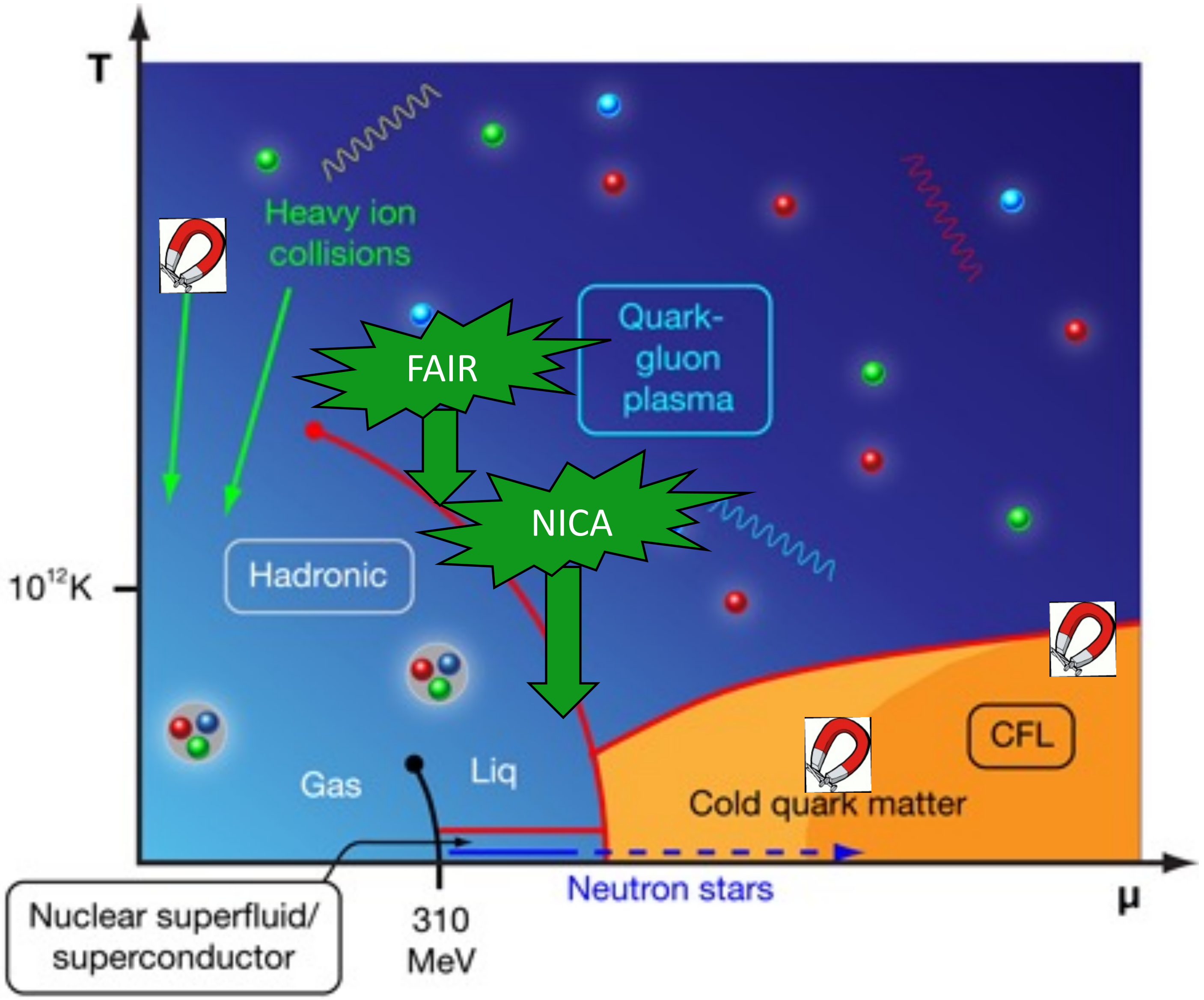
QCD Phase Diagram



Outline

1. Anomalies in the MDCDW Phase
2. Axion Electrodynamics & Anomalous Quantities
3. Photon-Phonon Interaction & Axion Polariton Modes
4. Primakoff Effect & NS Collapse under γ-ray radiation

Magnetic Dual Chiral Density Wave Model

2-flavor NJL model + QED at finite baryon density and with magnetic field B∥ z

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu}) + \gamma_0\mu]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2].$$

It favors the formation of an inhomogeneous chiral condensate

$$\langle\bar{\psi}\psi\rangle = m\cos q_{\mu}x^{\mu}, \qquad \langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = m\sin q_{\mu}x^{\mu}, \qquad q^{\mu} = (0,0,0,q)$$

Mean-field Lagrangian

$$\mathcal{L}_{MF} = \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu}) + \gamma_0\mu]\psi - m\bar{\psi}e^{i\tau_3\gamma_5 q_{\mu}x^{\mu}}\psi - \frac{m^2}{4G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Complex mass term

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

Chiral Transformation & Asymmetric Spectrum

Performing the chiral local transformation

$$\psi \to U_A \psi = e^{-i\tau_3\gamma_5 \frac{qz}{2}}\psi \qquad \bar{\psi} \to \bar{\psi}\bar{U}_A = \bar{\psi} e^{-i\tau_3\gamma_5\frac{qz}{2}}$$

The MF Lagrangian acquires a constant mass term plus a $\gamma_3\gamma_5$ term

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu 0} + iQA_\mu - i\tau_3\gamma_5\delta_{\mu 3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

For $A^\mu = (0, 0, Bx, o)$ the corresponding fermion spectrum is

$$E_k^{LLL} = \epsilon\sqrt{\Delta^2 + k_3^2} + q/2, \quad \epsilon = \pm$$

LLL mode is Asymmetric

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1, 2, 3, \ldots$$

Stability of the MDCDW Phase

Feng/EJF/Portillo *PRD* 101 (2020) 056012

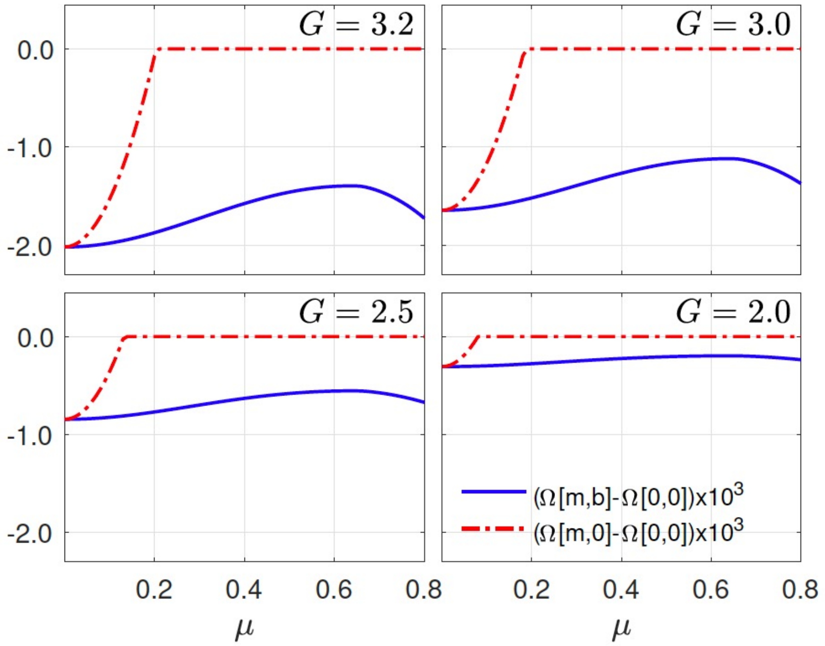
Topology emerges due to the LLL spectral asymmetry & to the axion term.

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_{T}(B,\mu,T) + \frac{m^2}{4G}.$$

$$\Omega^{f}_{anom} = -\frac{N_c|e_f B|}{(2\pi)^2} q\mu$$

$$\rho^{A}_{B} = 3\frac{|e|}{4\pi^2} qB$$

Anomalous baryon number density



The anomaly makes the MDCDW solution energetically favored over the homogeneous condensate

Axion Term

EJF & Incera, PLB' 2017; NPB' 2018

Key observation: the fermion measure is not invariant under U_A

$$D\bar{\psi}D\psi \to (\det U_A)^{-2} D\bar{\psi}D\psi \qquad (\det U_A)_R^{-2} = e^{i\int d^4x \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}}$$

The effective MF Lagrangian acquires an axion term:

$$\mathcal{L}_{eff} = \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu - i\tau_3\gamma_5\partial_\mu\theta) + \gamma_0\mu - m]\psi - \frac{m^2}{4G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu},$$

$\theta = mqz$

$\kappa = 2\alpha/\pi m$

Integrating out the fermions, we find the electromagnetic effective action in the MDCDW model

$$\Gamma(A) = V\Omega + \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}\right] - \int d^4x A^\mu(x)J_\mu(x) + \cdots,$$

QED in MDCDW is Axion QED

EJF & Incera, Phys.Lett. B769 (2017) 208; Nucl.Phys. B931 (2018) 192

$$\nabla \cdot \mathbf{E} = J^0 + \frac{e^2}{4\pi^2} qB,$$

$$\nabla \times \mathbf{B} - \partial \mathbf{E}/\partial t = \mathbf{J} - \frac{e^2}{4\pi^2} \mathbf{q} \times \mathbf{E},$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \partial \mathbf{B}/\partial t = 0,$$

Anomalous charge

Anomalous Hall current ⊥ to both B and E

Anomalous Hall conductivity

$$\sigma_{xy}^{anom} = e^2 q / 4\pi^2$$

Magnetoelectricity

$$\nabla \cdot \mathbf{D} = J_0 \quad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_V$$

$$\nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\mathbf{H} = \mathbf{B} - \kappa\theta\mathbf{E} \qquad \mathbf{D} = \mathbf{E} + \kappa\theta\mathbf{B}$$

Anomalous magnetization

Anomalous polarization

MDCDW Symmetry-Breaking Pattern

Explicit Symmetry Breaking by the Magnetic field

$$SU_V(2)\times SU_A(2)\times SO(3)\times R^3 \rightarrow U_V(1)\times U_A(1)\times SO(2)\times R^3$$

MDCDW Single-Modulated Density Wave Ansatz

$$M(z) = me^{iqz}$$

Spontaneous Symmetry Breaking by the Inhomogeneous Condensate

$$U_V(1)\times U_A(1)\times SO(2)\times R^3 \rightarrow U_V(1)\times SO(2)\times R^2$$

Thus, the most relevant fluctuations of the condensate at low energy come from the two Goldstone Bosons: A pion and a phonon.

GL Expansion

EJF & Incera, PRD`2020

Low Energy GL Expansion of the MDCDW Free Energy

$$\mathcal{F} = a_{2,0}|M|^2 - i\frac{b_{3,1}}{2}[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] + a_{4,0}|M|^4 + a_{4,2}^{(0)}|\nabla M|^2$$
$$+ a_{4,2}^{(1)}(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) - i\frac{b_{5,1}}{2}|M|^2[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M]$$
$$+ \frac{ib_{5,3}}{2}[(\nabla^2 M^*)\hat{B}\cdot\nabla M - \hat{B}\cdot\nabla M^*(\nabla^2 M)] + a_{6,0}|M|^6 + a_{6,2}^{(0)}|M|^2|\nabla M|^2$$
$$+ a_{6,2}^{(1)}|M|^2(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) + a_{6,4}|\nabla^2 M|^2 + \cdots.$$

MDCDW ansatz

$$M(z) = me^{iqz}$$

→

$$\mathcal{F} = a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4$$
$$+ b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2,$$

The b coefficients are a consequence of the asymmetry of the LLL spectrum

The $a_{x,y}^{(1)}$ coefficients are a consequence of having an external vector

Phonon Low Energy Theory

Isospin and translation transformations are locked

$$M(z) \to e^{i\tau} M\big(z+u(x)\big) = e^{i(\tau + qu(x))} M(z)$$

Phonon Fluctuation Field $u(x)$

$$M(x) = M\big(z+u(x)\big) \approx M_0(z) + M_0'(z)u(x) + \frac{1}{2} M_0''(x)u^2(x)$$

Low-Energy Theory:

$$\mathcal{L}_1 = \frac{1}{2}[(\partial_0\theta)^2 - v_z^2(\partial_z\theta)^2 - v_\perp^2(\partial_\perp\theta)^2],$$

$$\theta = qmu(x)$$

$$v_z^2 = a_{4.2} + m^2 a_{6.2} + 6q^2 a_{6.4} + 3qb_{5,3},$$

$$v_\perp^2 = a_{4.2} + m^2 a_{6.2} + 2q^2 a_{6.4} + qb_{5,3} - a_{4.2}^{(1)} - m^2 a_{6.2}^{(1)}.$$

Photon-Phonon Axion Electrodynamic at $B \neq 0$

Taking now into account the contribution of the anomalous photon-phonon interaction $\frac{\kappa}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$, the axion-electrodynamic/phonon equations are:

$$\nabla \cdot \mathbf{E} = J^0 + \frac{\kappa}{2}\nabla\theta_0 \cdot \mathbf{B} + \frac{\kappa}{2}\nabla\theta \cdot \mathbf{B},$$

$$\nabla \times \mathbf{B} - \partial\mathbf{E}/\partial t = \mathbf{J} - \frac{\kappa}{2}(\frac{\partial\theta}{\partial t}\mathbf{B} + \nabla\theta \times \mathbf{E}),$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \partial\mathbf{B}/\partial t = 0$$

$$\partial_0^2\theta - v_z^2\partial_z^2\theta - v_\perp^2\partial_\perp^2\theta + \frac{\kappa}{2}\mathbf{B} \cdot \mathbf{E} = 0$$

Here we assume that a linearly polarized electromagnetic wave with electric field parallel to the background magnetic field B_0 propagates in the MDCDW medium

Linearized Field Equations

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \nabla^2 \mathbf{E} + \frac{\kappa}{2}\frac{\partial^2 \theta}{\partial t^2}\mathbf{B}_0$$

$$\frac{\partial^2 \theta}{\partial t^2} - v_z^2 \frac{\partial^2 \theta}{\partial z^2} - v_\perp^2 \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + \frac{\kappa}{2}\mathbf{B}_0 \cdot \mathbf{E} = 0.$$

In momentum space these field equations can be written as

$$\left(\omega^2 - p^2\right) E - \left(\frac{\kappa}{2}\omega^2 B_0\right)\theta = 0$$

$$-\left(\frac{\kappa}{2}B_0\right) E + \left(\omega^2 - P^2\right)\theta = 0$$

Hybridized Propagating Modes

The dispersion relations of the hybrid modes are

$$\omega_0^2 = \omega_1 - \omega_2, \quad \omega_\delta^2 = \omega_1 + \omega_2$$

with

$$\omega_1 = \frac{1}{2}[p^2 + P^2 + (\frac{\kappa}{2}B_0)^2],$$

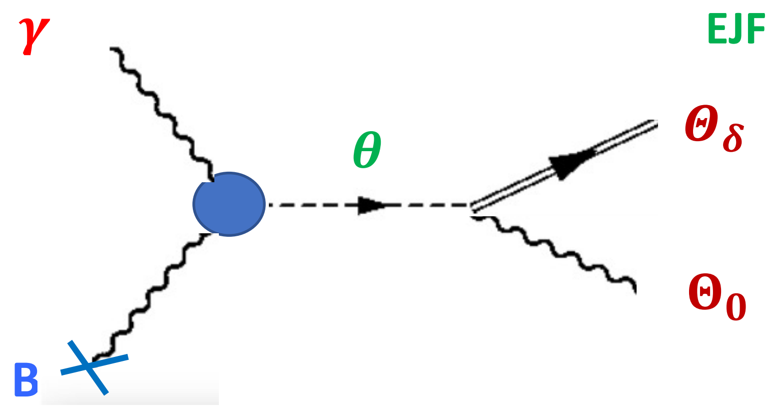
$$\omega_2 = \frac{1}{2}\sqrt{[p^2 + P^2 + (\frac{\kappa}{2}B_0)^2]^2 - 4p^2P^2}.$$

The gap of ω_δ is field-dependent and given by

$$\omega_\delta(\vec{p} \to 0) = \delta = \alpha B_0/\pi m$$

Primakoff Effect as a Mechanism to Increase the Star Mass

H. Primakoff, Phys. Rev. 81 (1951) 899;
EJF & Incera, arXiv: 2010.02314 [hep-ph]



$$\begin{bmatrix} \Theta_0 \\ \Theta_\delta \end{bmatrix} = \begin{bmatrix} \cos\beta & i\sin\beta \\ i\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \theta \\ A_3 \end{bmatrix}$$

$$\cos\theta = \frac{1}{\sqrt{2}}\left[1 + \frac{\sqrt{(X_1)^2 - (X_2)^2}}{X_1}\right]^{1/2} \qquad \sin\theta = \frac{1}{\sqrt{2}}\left[1 - \frac{\sqrt{(X_1)^2 - (X_2)^2}}{X_1}\right]^{1/2}$$

$$X_1 = (v_z^2 - 1)p_z^2 + (v_\perp^2 - 1)p_\perp^2 \qquad X_2 = 2\kappa B_0 p_4$$

The Missing Pulsar Problem

- Theoretical analysis predicts at least 10^3 active radio pulsars in a distance of 10 pc of the Galaxy center

- However, these numbers have not been observed.

- This paradox has been magnified by the observation of magnetar SGR J1745-29 by the NuSTAR and Swift satellites.

- These observations revealed that the failures to detect ordinary pulsars at low frequencies could not be simply due to strong interstellar scattering but to an intrinsic deficit produced by other causes.

- On the other hand, the Milky Way galactic center is a very active astrophysical environment with numerous γ-ray emitting point sources.

Missing Pulsar Problem in Galactic Center & Axion Polaritons

EJF & Incera, arXiv: 2010.02314 [hep-ph]

Chandrasekhar limit

$$N_{AP}^{Ch} = \left(\frac{M_{pl}}{\delta}\right)^2 = 1.5 \times 10^{44} \left(\frac{MeV}{c^2\delta}\right)^2$$

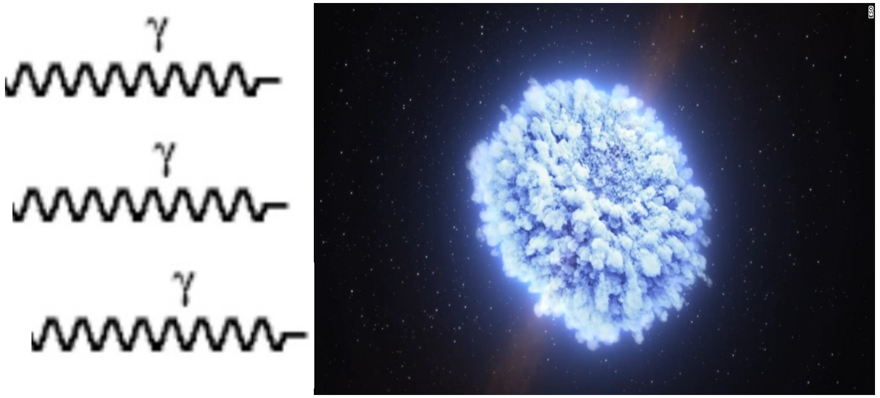
For the range of $\delta' s \sim 0.4\, MeV$ considered

We find that $N_{AP}^{Ch} \sim 10^{45} - 10^{46}$

Each GRB energy output: $10^{56} \sim 10^{59}$ MeV

Photons' energy range: 0.1 – 1 MeV

Photons produced in each event: $10^{55} - 10^{58}$



This means that if just 10^{-10} % of the photons reaching the star interior have energy $0.4 - 1$ MeV, they will generate enough number of axion polaritons to produce the NS collapse into a black hole.

Summary:

- Due to anomalous effects, the electromagnetism in the MDCDW phase of quark matter at intermediate densities is modified giving rise to hybridized modes called axion polaritons.
- The anomalous two-photon/phonon interaction present in the MDCDW phase, produces a new mechanism to increase the strange star mass.

Outlook:

- Needed more measurable NS observables that can discriminate between intermediate density candidates: MDCDW, Quarkyonic, CS Phases.