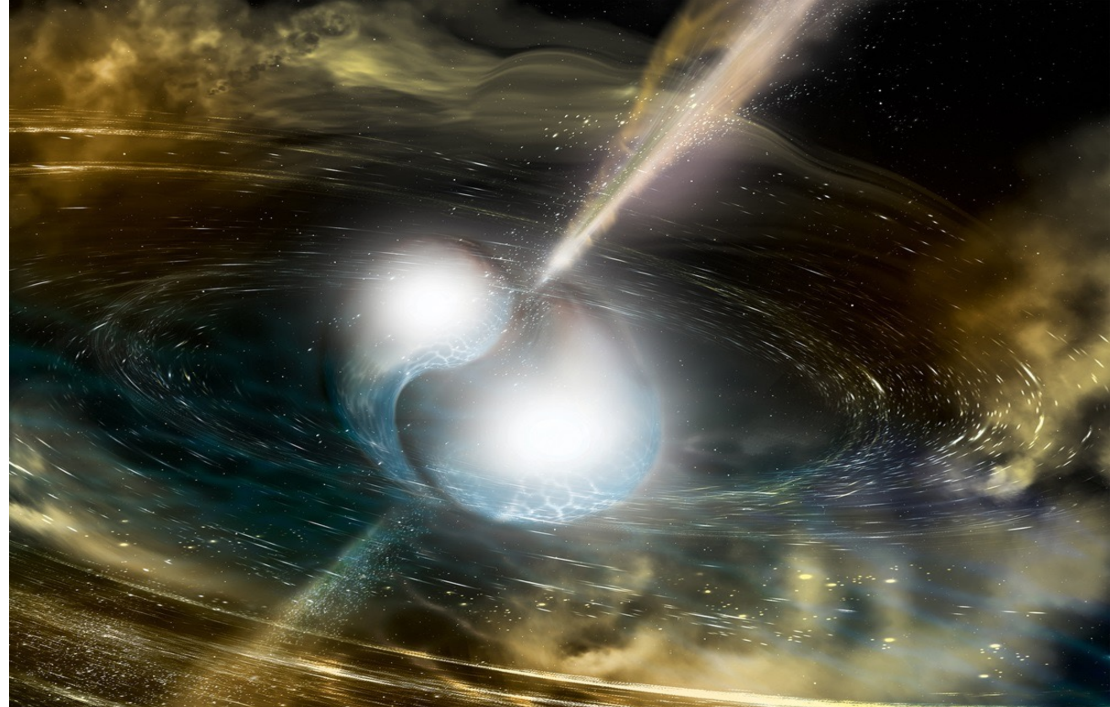
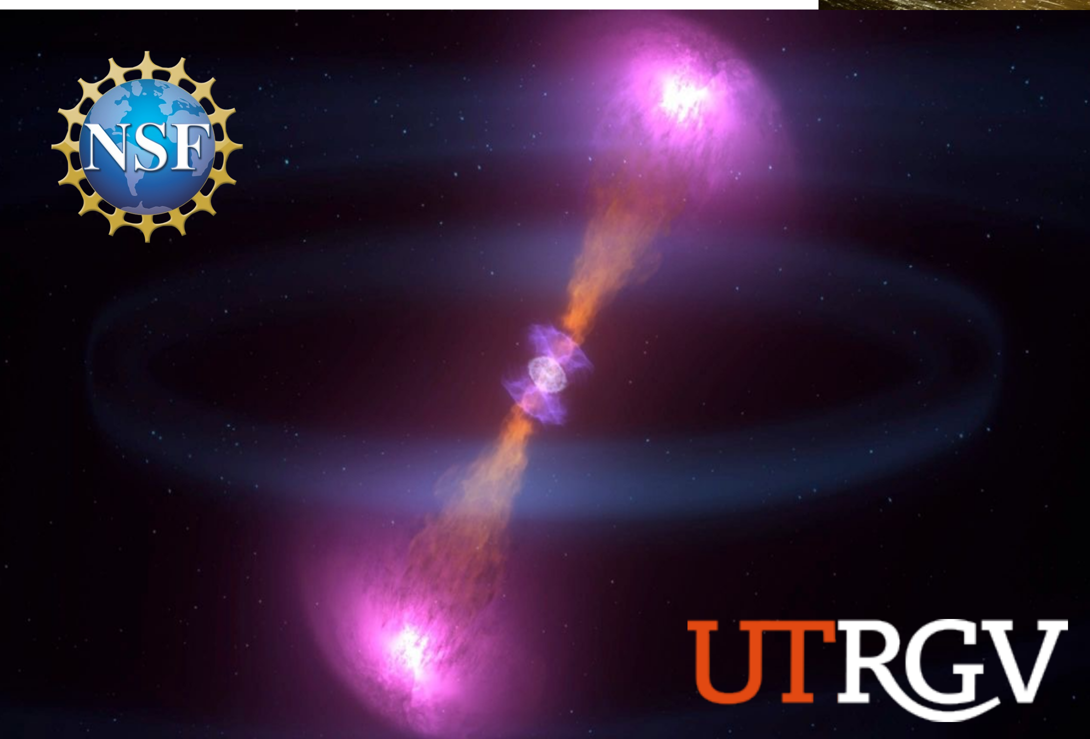
Topology's Role in the Feasibility of a Neutron Star's Matter Phase





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University of Texas Rio Grande Valley

QCD @ Work
Lecce, Italy, June 27-30, 2022

The Case for MDCDW in NS: Role of Topology

- 2203.14209 - Will Gyory and VI
- Universe 7, 458, 2021 - E.J. Ferrer and VI
- PRD 103, 123013, 2021 - E.J. Ferrer, VI, and P. Sanson
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✓ Compatible with NS's densities and temperatures

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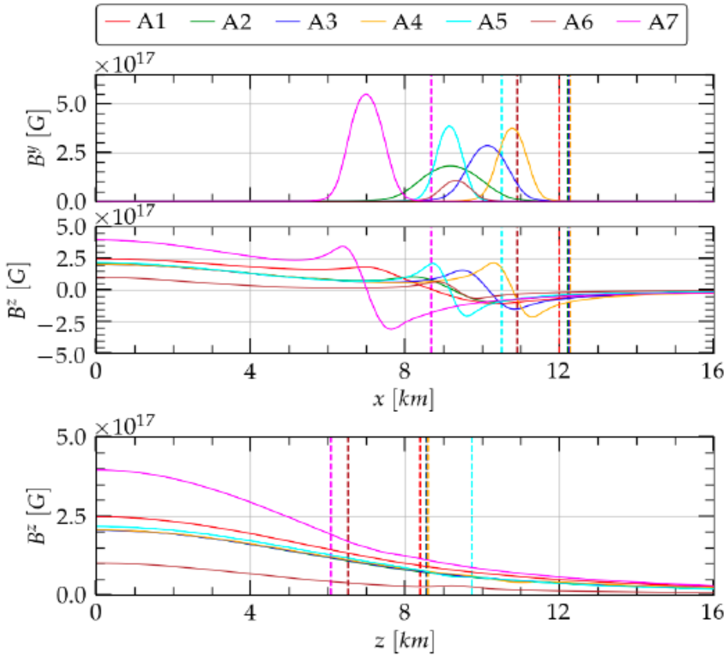
✓ Consistent with observed NS heat capacity limit

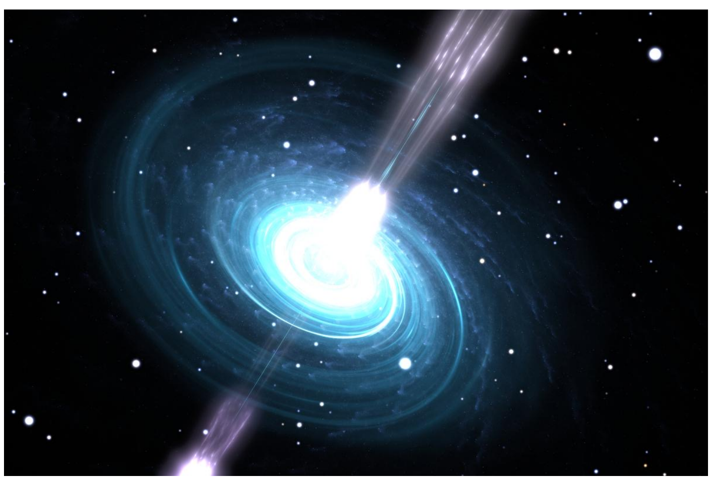
✓ Compatible with 2 $M_\odot$

✓ A candidate for transient NS in BNS Mergers?



GRMHD simulations of magnetars' field evolution leads to several times 10^{17} G for B_z at the core

Tsokaros, Ruiz, Shapiro, & Uryu, PRL 128, 2022



Magnetars:

Core: $B < 8 \times 10^{18}$ G

Cardall, Prakash, and Lattimer, ApJ 554, 2001

Then, for magnetars

surface: $B \sim 10^{15}$G

core $B \sim 10^{17}$- 10^{18} G

$$l_m = 7 \times 10^9 (n_e/n_s)/B^{*2} fm, \qquad B^* = B/B_e^c, \qquad R \gg l_m$$

B can be considered uniform and constant as far as effects on the EOS

Broderick, Prakash, and Lattimer, ApJ 537, 2000

Magnetic Dual Chiral Density Wave Model

2-flavor NJL model at finite baryon density and with magnetic field B∥ z

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu) + \gamma_0\mu]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2].$$

It favors the formation of an inhomogeneous chiral condensate

$$\langle\bar{\psi}\psi\rangle = \Delta\cos q_\mu x^\mu, \qquad \langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = \Delta\sin q_\mu x^\mu \qquad q^\mu = (0,0,0,q)$$

Mean-field Lagrangian

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu 0} + iQA_\mu - i\tau_3\gamma_5\delta_{\mu 3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

$E_0 = \epsilon\sqrt{m^2 + k_3^2} + b, \quad \epsilon = \pm,$ $b=q/2$, **LLL mode is Asymmetric!**

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1, 2, 3, \ldots$$

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

Nontrivial Topology of the MDCDW Phase

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_T(B,\mu,T) + \frac{m^2}{4G}.$$

$$\Omega^f_{vac} = \frac{1}{4\sqrt{\pi}} \frac{N_c\,|e_f B|}{(2\pi)^2} \int_{-\infty}^{+\infty} dk \sum_{\ell\xi\epsilon} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^{3/2}} e^{-s(E_\ell)^2}$$

$$\Omega^f_{anom} = -\frac{N_c\,|e_f B|}{(2\pi)^2} 2b\mu$$

Anomalous term due to the LLL spectral asymmetry. It leads to anomalous baryon number

$$\Omega^f_{\mu} = -\frac{N_c\,|e_f B|}{(2\pi)^2} \int_{-\infty}^{+\infty} dk \sum_{\xi,\ell>0} [(\mu - E_\ell)\theta(\mu - E_\ell)]\Big|_{\epsilon=+} + \Omega^{f,LLL}_{\mu}$$

$$\Omega^f_{T} = -\frac{N_c\,|e_f B|}{(2\pi)^2} \frac{1}{\beta} \int_{-\infty}^{+\infty} dk \sum_{\ell\xi\epsilon} \ln\left(1 + e^{-\beta|E_\ell - \mu|}\right)$$

$$\Omega^{f,LLL}_{\mu} = -\frac{1}{2} \frac{N_c\,|e_f B|}{(2\pi)^2} \int_{-\infty}^{+\infty} dk \sum_{\epsilon} (|E_0 - \mu| - |E_0|)_{reg}\,,$$

Improved GL Expansion

$$\Omega = \alpha_{2,0}m^2 + \beta_{3,1}bm^2 + \alpha_{4,0}m^4 + \alpha_{4,2}b^2m^2 + \beta_{5,1}bm^4 + \beta_{5,3}b^3m^2 + \alpha_{6,0}m^6 + \alpha_{6,2}b^2m^4 + \alpha_{6,4}b^4m^2.$$

$$\alpha_{n_b+2,n_b} \sim \frac{\delta_{0,n_b}}{4G} + \sum_{j=0,2,4,\ldots} |eB|^j \frac{B_j}{j!} \cdot \frac{1+2^j}{2\pi^2 3^{j-1}} \cdot \frac{1}{(n_b-1)!!} I_{n_b+2j-2}(\mu,T)$$

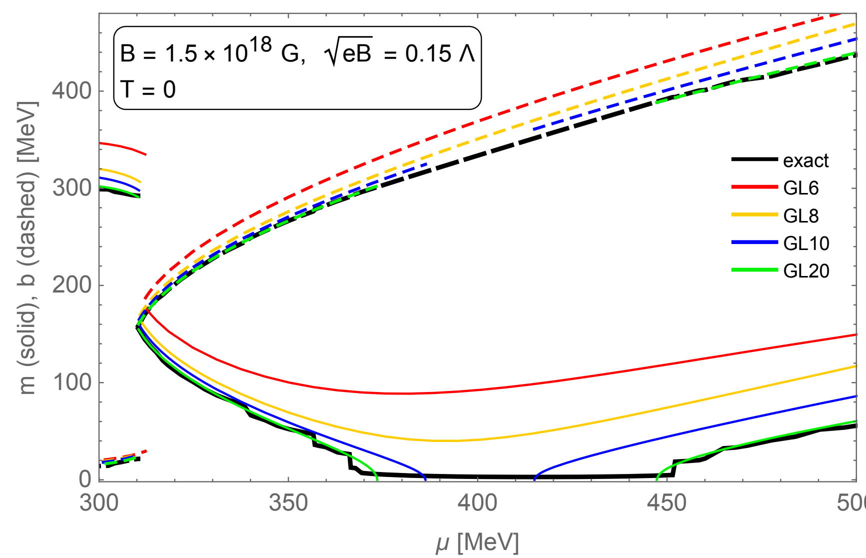
$$\beta_{n_b+2,n_b} = \frac{3|eB|}{(2\pi)^2} \cdot \begin{cases} \frac{1}{n_b!}\frac{1}{(2\pi T)^{n_b}} \mathrm{Re}\left[(-i)^{n_b}\psi^{(n_b)}\left(\frac{1}{2}+i\frac{\mu}{2\pi T}\right)\right] & T>0, \\ -\frac{1}{n_b \mu^{n_b}} & T=0, \end{cases}$$

$$I_{-2}(\mu,T) = -\frac{1}{4}\Lambda^2 + \frac{1}{2}\mu^2 + \frac{\pi^2}{3}T^2$$

$$I_0(\mu,T) = -\frac{\gamma}{2} - \left\{\ln\left(\frac{4\pi T}{\Lambda}\right) + \mathrm{Re}\left[\psi\left(\frac{1}{2}+i\frac{\mu}{2\pi T}\right)\right]\right\}$$

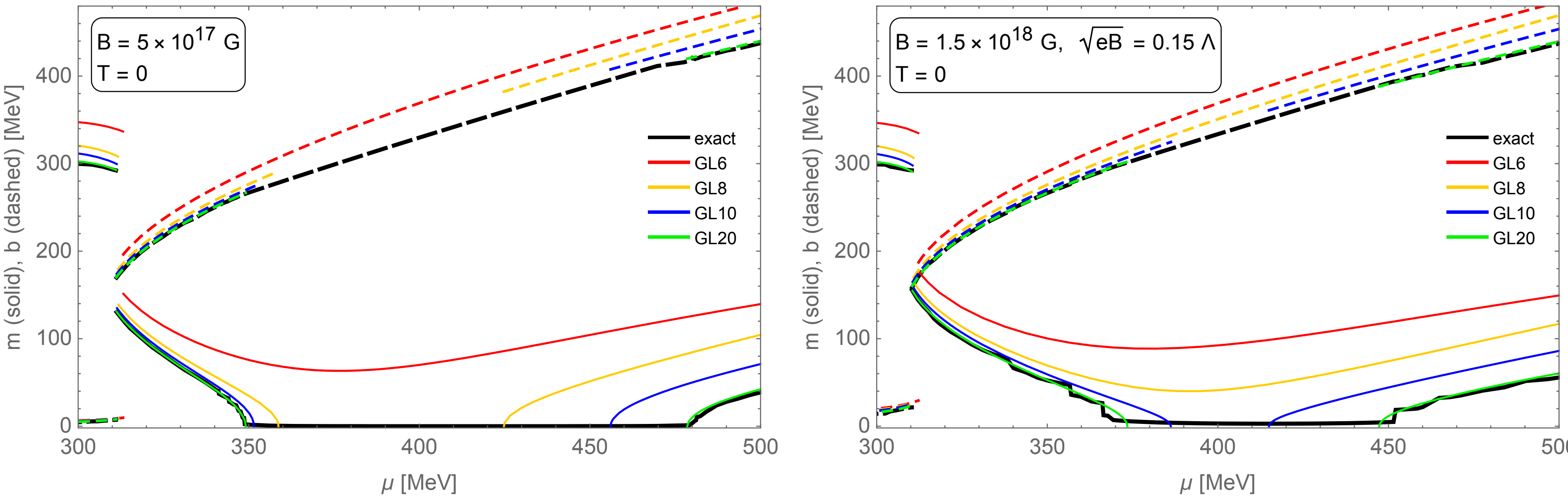
$$I_{p>0}(\mu,T) = -\frac{1}{p}\left(\frac{i\sqrt{2}}{\Lambda}\right)^p - \frac{1}{p!!}\left\{\frac{1}{(2\pi T)^p}\mathrm{Re}\left[(-i)^p\psi^{(p)}\left(\frac{1}{2}+i\frac{\mu}{2\pi T}\right)\right]\right\}$$

Arbitrarily high-order coefficients can be quickly calculated allowing much higher precision results than previous works



Gyory and VI, arXiv: 2203.14209

Topology Yields Extraordinary Robustness



$$\Omega = \alpha_{2,0} m^2 + \beta_{3,1} b m^2 + \alpha_{4,0} m^4 + \alpha_{4,2} b^2 m^2 + \beta_{5,1} b m^4 + \beta_{5,3} b^3 m^2 + \alpha_{6,0} m^6 + \alpha_{6,2} b^2 m^4 + \alpha_{6,4} b^4 m^2 + \ldots$$

$$\beta_{n_b+2,n_b} = \frac{3|eB|}{(2\pi)^2} \cdot \begin{cases} \frac{1}{n_b!}\frac{1}{(2\pi T)^{n_b}} \mathrm{Re}\left[(-i)^{n_b} \psi^{(n_b)}\left(\frac{1}{2} + i\frac{\mu}{2\pi T}\right)\right] & T > 0, \\ -\frac{1}{n_b \mu^{n_b}} & T = 0, \end{cases}$$

Odd in b=q/2 terms! Come from LLL asymmetry, so origin is topological

All β are <0, thus LLL topology always favors the inhomogeneity

The inhomogenous condensate never disappears at intermediate densities!

Gyory and VI, arXiv: 2203.14209

Topology is Important For the Remnant Mass

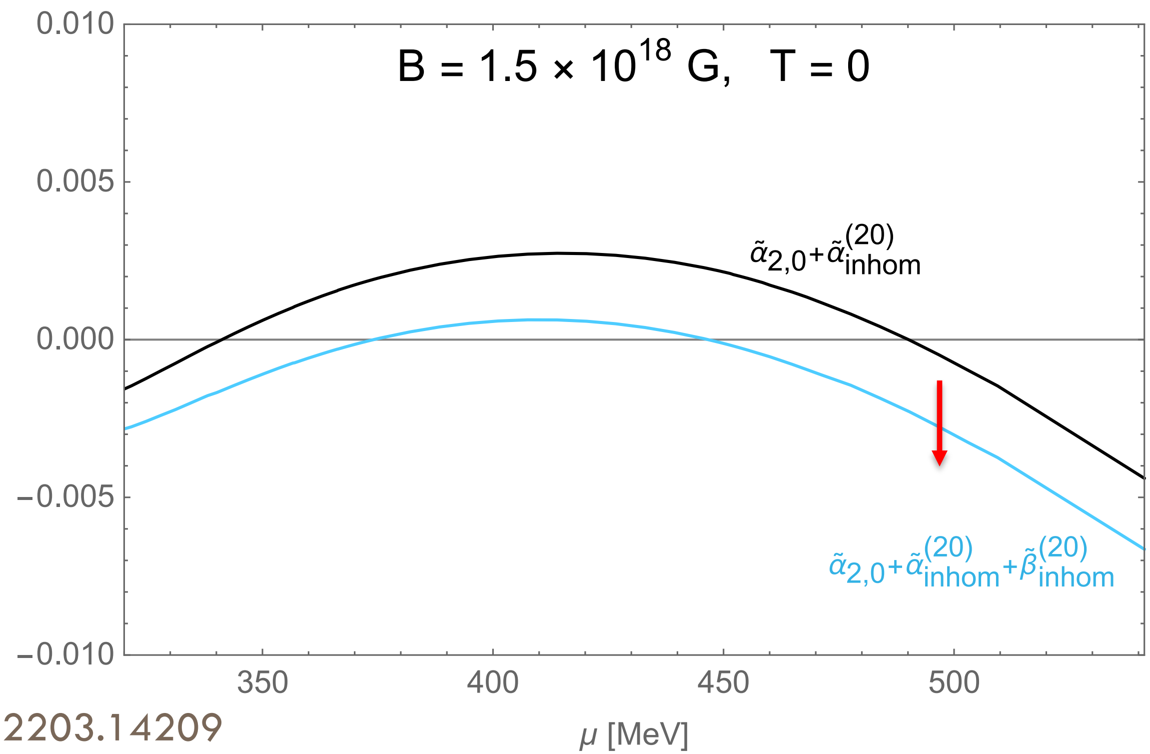
$$\left.\frac{\partial \Omega_{GL,20}}{\partial (m^2)}\right|_{m=0} = \alpha_{2,0} + \alpha_{\text{inhom}}^{(20)} + \beta_{\text{inhom}}^{(20)}$$

When negative, m is nonzero

$$\alpha_{\text{inhom}}^{(20)} = \alpha_{4,2} b^2 + \cdots + \alpha_{20,18} b^{18}$$

$$\beta_{\text{inhom}}^{(20)} = \beta_{3,1} b + \cdots + \beta_{19,17} b^{17}$$

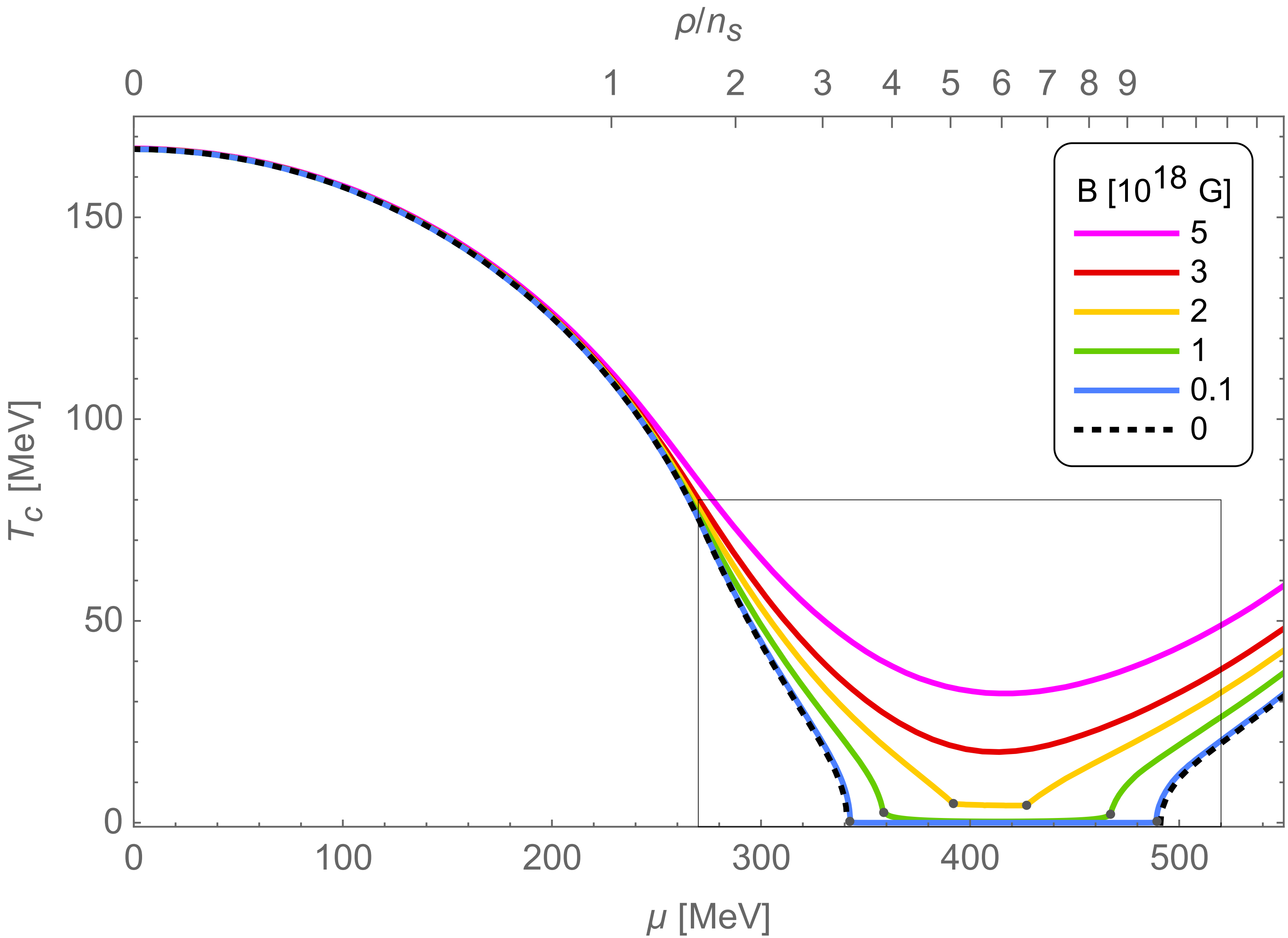
Purely anomalous contribution



Beta terms push it down

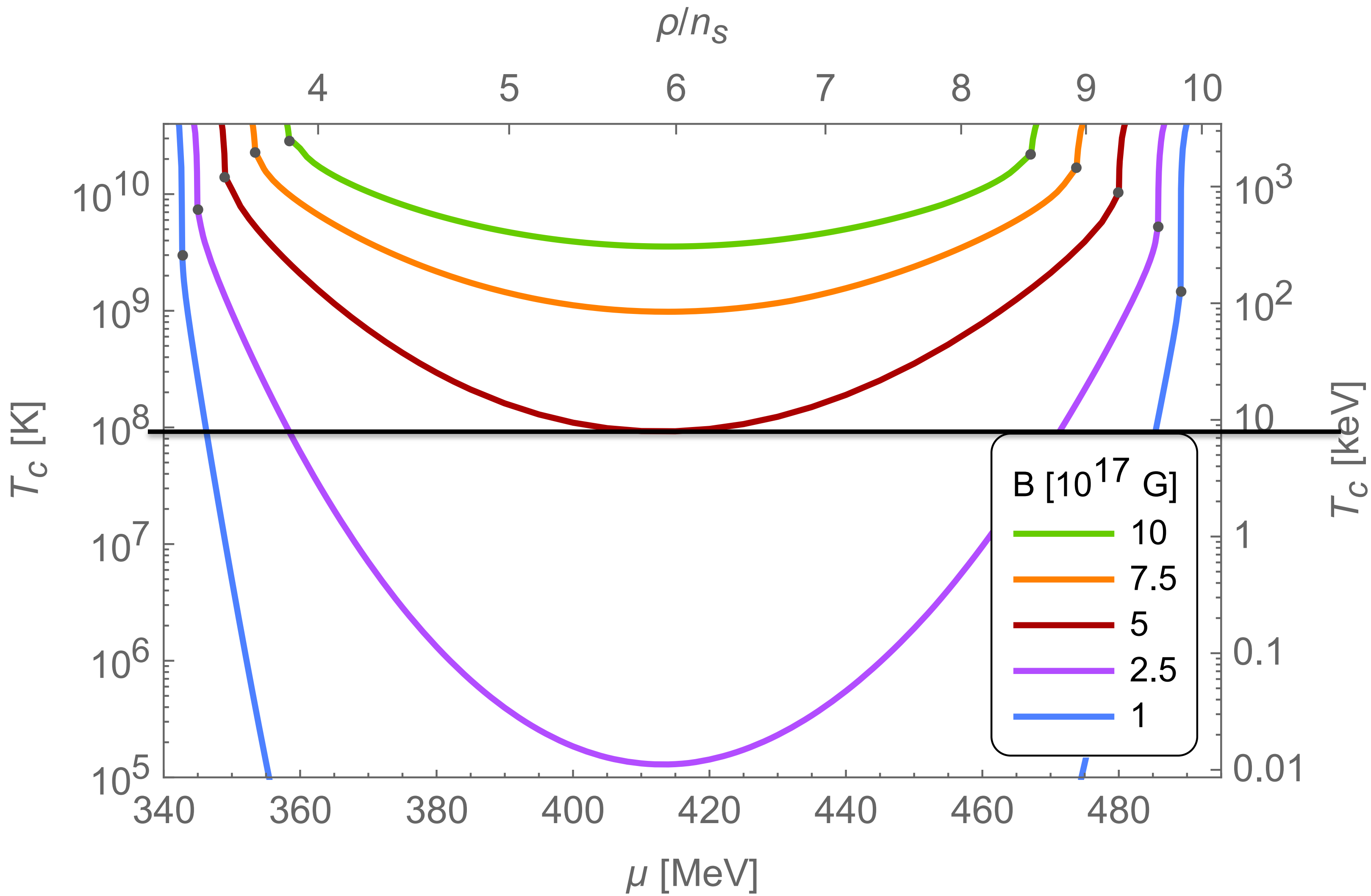
Gyory and VI, arXiv: 2203.14209

Relevance for NS



Gyory and VI, arXiv: 2203.14209

Relevance for NS even in the Remnant Mass Region



However,
Single modulated phases like DCDW and the Kink Crystal are unstable against thermal fluctuations.

The fluctuations wash out the long-range order at finite T!

Hidaka, Kamikado, Kanazawa & Noumi, PRD 92, 2015, 034003
Lee et al. PRD 92, 2015, 0304024

Topology Ensures No Landau-Peierls Instability

$$\mathcal{F}[M(x)] = \mathcal{F}_0 + v_z^2(\partial_z\theta)^2 + v_\perp^2(\partial_\perp\theta)^2 + \zeta^2(\partial_z^2\theta + \partial_\perp^2\theta)^2 \qquad \theta = qmu$$

$$\langle M \rangle = m e^{iqz} e^{-\langle (qu)^2 \rangle /2}$$

$$\begin{aligned} \langle q^2 u^2 \rangle &= \frac{1}{(2\pi)^2} \int_0^\infty dk_\perp k_\perp \int_{-\infty}^\infty dk_z \frac{T}{m^2(v_z^2 k_z^2 + v_\perp^2 k_\perp^2 + \zeta^2 k^4)} \\ &\simeq \frac{T}{4\pi m \sqrt{v_z^2 v_\perp^2}}. \end{aligned}$$

No Landau-Peierls instability.
Stable at arbitrarily low Ts

$$v_\perp^2 = \boxed{a_{4.2} + m^2 a_{6.2} + 2q^2 a_{6.4}} + q b_{5,3}$$

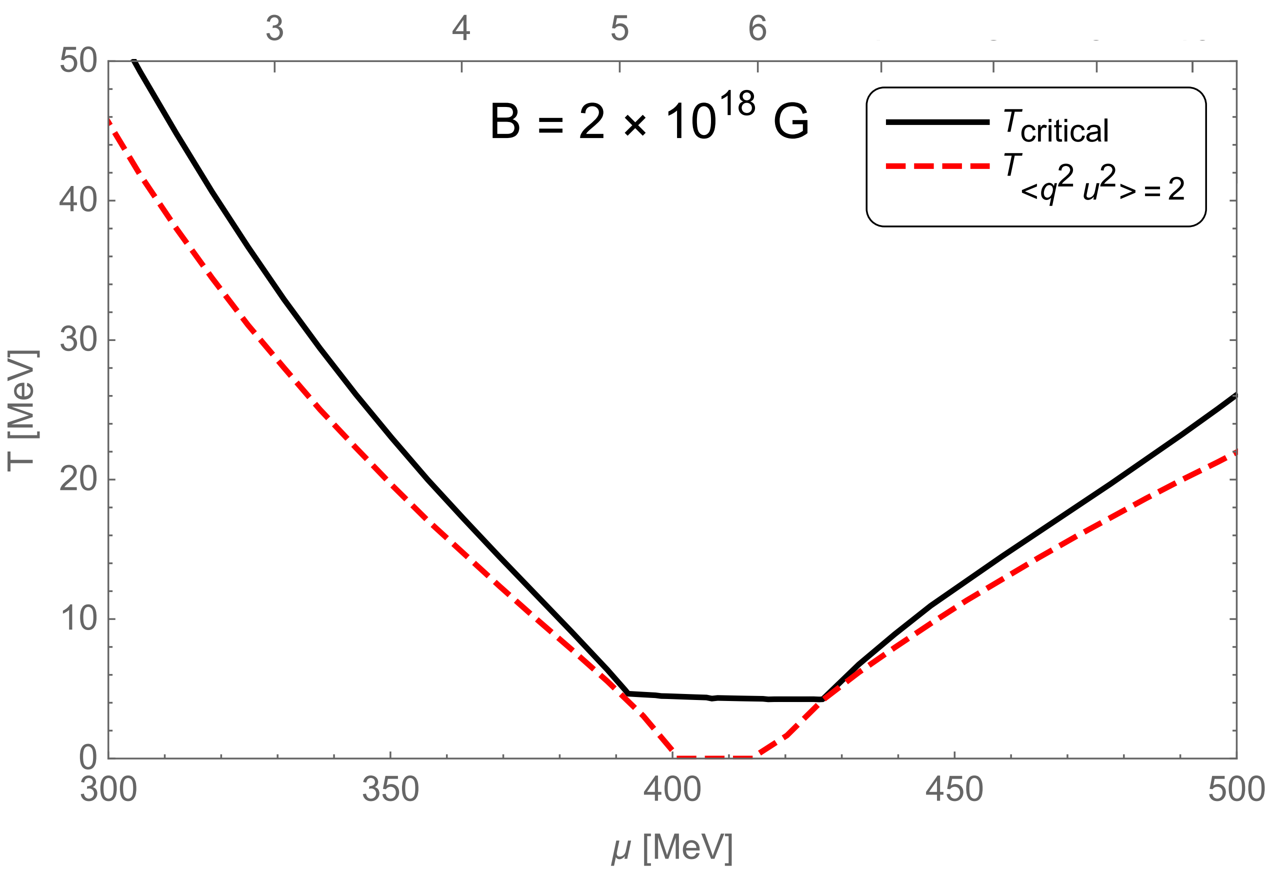
At B=0, this term is zero

Topology is essential for the absence of LP instability!

Ferrer and VI, PRD 102, 014010, 2020

Fluctuations Not Relevant at NS Temperatures

$$\langle M\rangle = m e^{iqz} e^{-\langle (qu)^2\rangle/2} \qquad \langle q^2u^2\rangle/2 \simeq \frac{T}{8\pi m\sqrt{v_z^2 v_\perp^2}}.$$



Threshold temperature

$$T_{\langle q^2u^2\rangle} \simeq 4\pi m\sqrt{v_z^2 v_\perp^2}\,\langle q^2u^2\rangle$$

Thanks to the topology $v_\perp \neq 0$ and $T_{<q^2u^2>}$ is tens of MeV, hence the phase is stable against thermal fluctuations at Ts and densities relevant for NS

Ferrer, Gyory, and VI, in preparation

MDCDW Satisfies NS Heat Capacity Limit Constraint

$$\frac{C}{\tilde{T}_8} > 3.1 \times 10^{36}\ \mathrm{erg\,K}^{-1} \left(\frac{\tilde{T}_7}{7}\right)^{-2} \left(\frac{E}{7.5 \times 10^{43}\ \mathrm{erg}}\right) \qquad C \approx 2.5 \times 10^{36}\ \ \mathrm{erg\,K}^{-1} \tilde{T}_8$$

Cumming, et. Al, PRD 95, 025806, 2017

For the MDCDW

$$C_V^{\mathrm{MDCDW}} = \sum_{f=u,d} \frac{|e_f B| N_c}{(2\pi)^2} \sum_{l,\xi,\epsilon} \int_{-\infty}^{\infty} dp \left(\frac{|E^f_{l,\xi,\epsilon} - \mu|}{2T}\right)^2 sech^2\left(\frac{|E^f_{l,\xi,\epsilon} - \mu|}{2T}\right)$$

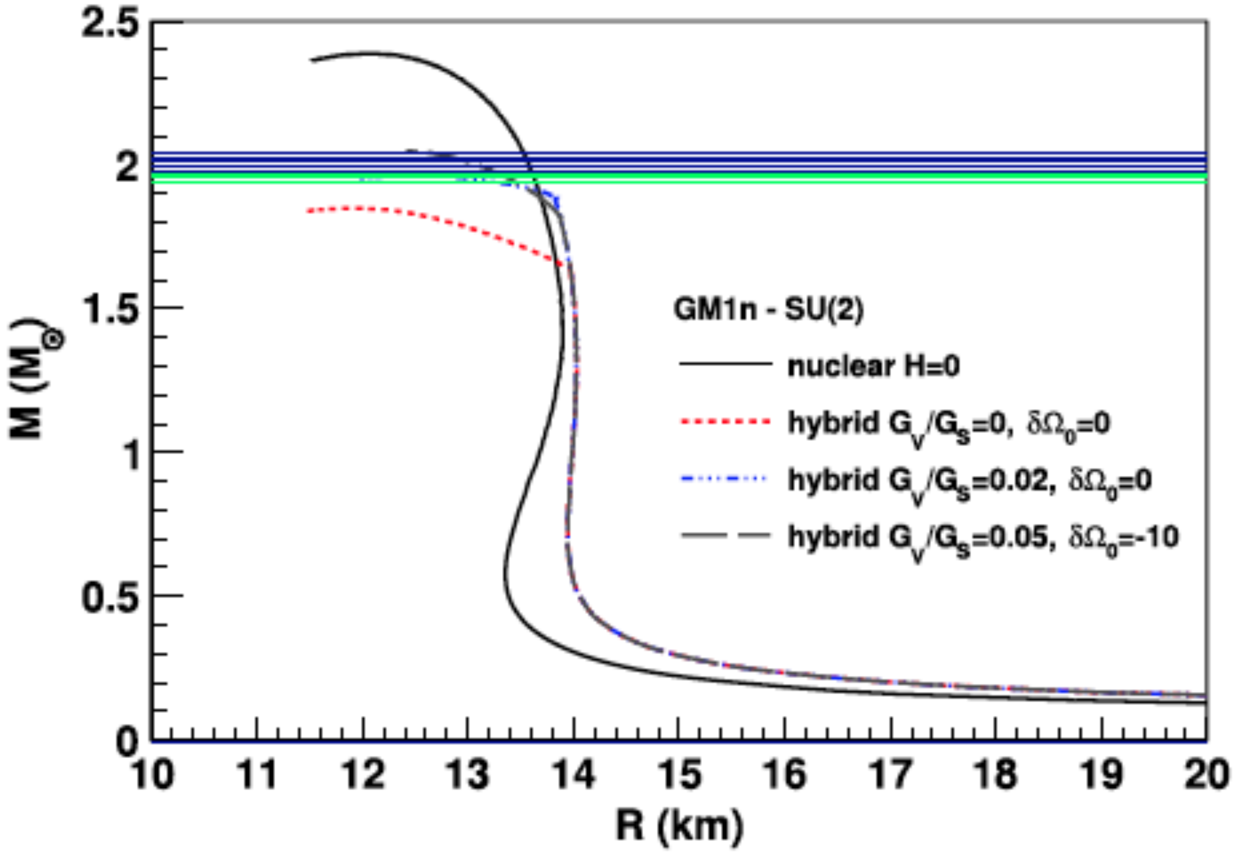
$$C_V^{\mathrm{MDCDW}} \simeq \sum_{f=u,d} \frac{4|e_f B| N_c}{(2\pi)^2} \sum_{l,\xi,\epsilon} \int_{-\infty}^{\infty} dp \left(\frac{|E^f_{l,\xi,\epsilon} - \mu|}{2T}\right)^2 e^{-|E^f_{l,\xi,\epsilon} - \mu|/T}$$

$$C_V^{\mathrm{MDCDW}} \simeq \frac{4\pi^2}{3} n_q k_B \left(\frac{T}{T_F}\right) \longrightarrow \tilde{C}_V^{\mathrm{MDCDW}} = C_V^N \times V_{\mathrm{NS}} = 0.4 \times 10^{38}\ \mathrm{erg/K}$$

Main contribution comes from the LLL. Here once again topology is fundamental for NS applications!

Ferrer, VI, and Sanson, PRD 103, 123013, 2021

Isospin Asymmetric Magnetic DCDW Compatible with 2 $M_\odot$



Carignano, Ferrer, VI, Paulucci, Phys.Rev. D92 (2015) no.10, 105018

Outlook:

- *Explore the feasibility of the MDCDW for NSB mergers*
- *Check the MDCDW phase against Tidal Deformability constraints. Is topology important there too?*
- *Compare MDCDW with color superconducting phases. Inhomogeneous ones?*
- *Compatibility with new multimessenger NS observations?*

Auxiliary Slides

Nontrivial Topology of the MDCDW Phase

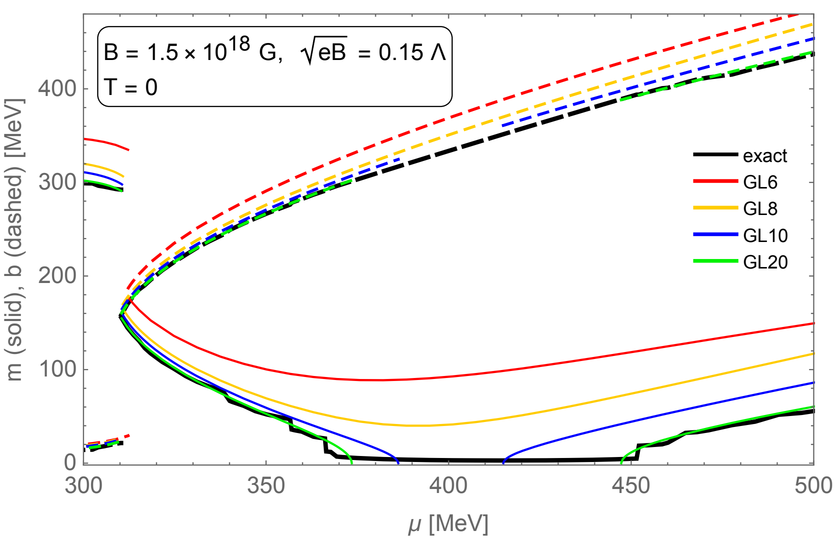
Topology emerges due to the LLL spectral asymmetry

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_T(B,\mu,T) + \frac{m^2}{4G}.$$

$$\Omega_{anom}^{f} = -\frac{N_c|e_f B|}{(2\pi)^2} q\mu$$

$$\rho_B^A = 3\frac{|e|}{4\pi^2} qB$$

Anomalous baryon number density



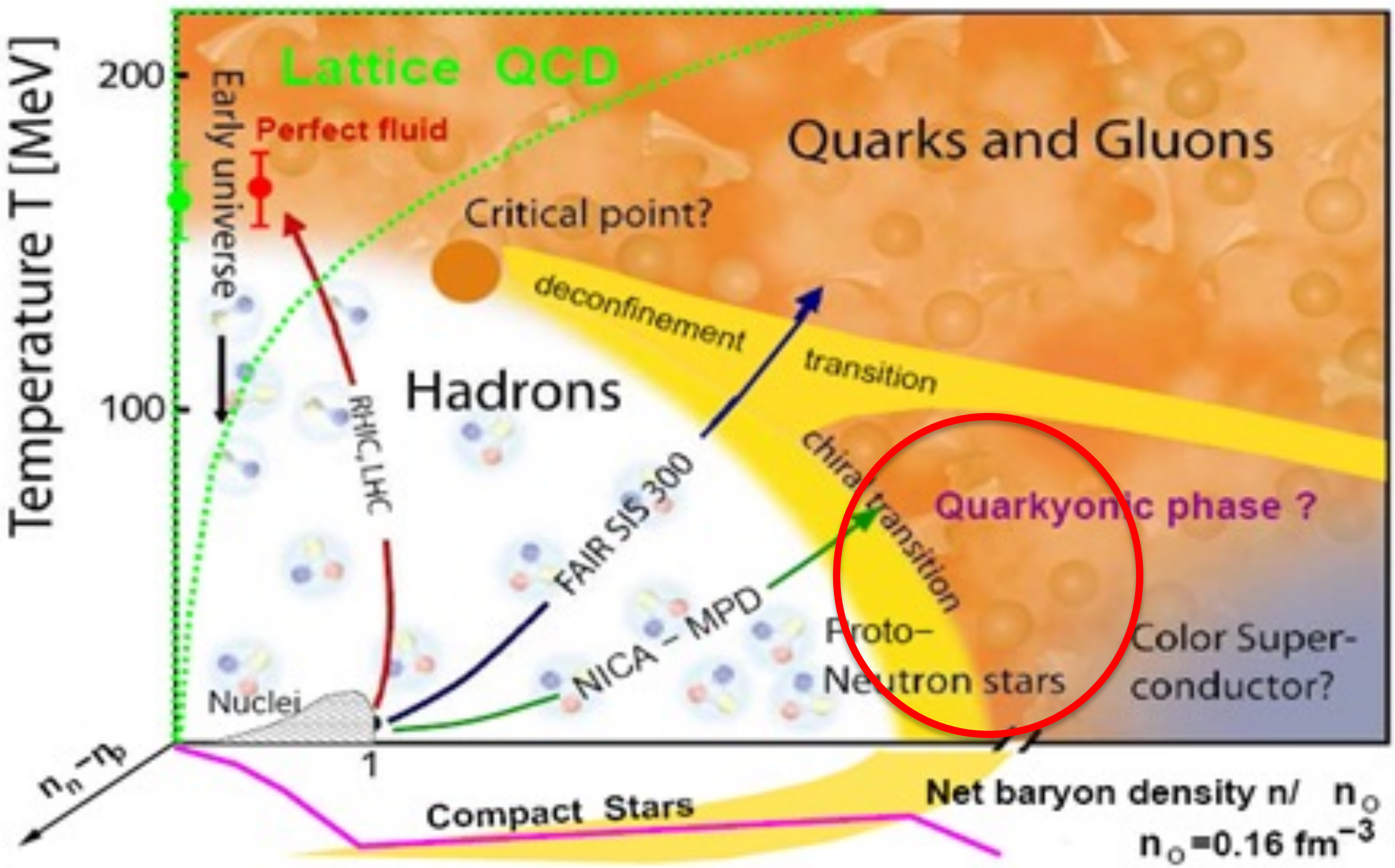
The anomaly makes the MDCDW energetically favored over the homogeneous condensate.

Solution exists even at low μ

Condensate never disappears!

Gyory and VI, arXiv: 2203.14209

Region of Interest for Inhomogeneous Phases



Low Energy Theory of the DCDW Phase

Lee et al. PRD 92 (2015) 034024

B=0

Symmetry: $SU(2)_L \times SU(2)_R$, plus spatial rotations and translation

$$\Omega_{GL}(\phi(x)) = \alpha_2(\phi\cdot\phi) + \alpha_{4,1}(\phi\cdot\phi)^2 + \alpha_{4,2}(\nabla\phi\cdot\nabla\phi) + \alpha_{6,1}(\nabla^2\phi\cdot\nabla^2\phi) + \alpha_{6,2}(\nabla\phi\cdot\nabla\phi)(\phi\cdot\phi) + \alpha_{6,3}(\phi\cdot\phi)^3 + \alpha_{6,4}(\phi\cdot\nabla\phi)^2 + \ldots$$

Consider a general fluctuation of the condensate. The phonon and the axial isospin rotation about the third axis (neutral pion) are locked. There are 3 NG modes.

$$\phi = (\Delta+\delta)\begin{pmatrix} \cos(qz+\beta_3)\cos\beta_2\cos\beta_1 \\ \cos(qz+\beta_3)\cos\beta_2\sin\beta_1 \\ \cos(qz+\beta_3)\sin\beta_2 \\ \sin(qz+\beta_3) \end{pmatrix}$$

Substituting $\phi(x)$ in the free-energy and expanding in powers of the fluctuations and their derivatives, one obtains the low-energy theory of the fluctuations.

$$\mathcal{V}_\delta = M^2\delta^2 + a_{6,4}\Delta^2(\nabla\delta)^2 + 4a_{6,1}q^2(\nabla_z\delta)^2 + a_{6,1}(\nabla^2\delta)^2,$$

$$\mathcal{L} = (\partial_0\delta)^2 + \Delta^2(\partial_0\vec{\beta}_U)^2 + \Delta^2(\partial_0\beta_3)^2 - (\mathcal{V}_\delta + \mathcal{V}_{\delta\beta} + \mathcal{V}_\beta),$$

$$\mathcal{V}_{\delta\beta} = 4q\Delta[a_{6,2}\Delta^2\delta - 2a_{6,1}\nabla^2\delta]\nabla_z\beta_3,$$

$$\mathcal{V}_\beta = a_{6,1}\Delta^2(\nabla^2\vec{\beta}_U + q^2\vec{\beta}_U)^2 + a_{6,1}\Delta^2[(\nabla^2\beta_3)^2 + 4q^2(\nabla_z\beta_3)^2].$$

(Lack of) Stability of the DCDW Phase

The spectra of the pions have soft modes in the transverse directions

$$\omega_-^2 \simeq a_{6,1}[u_{z-}^2 k_z^2 + (\vec{k}^2)^2] - A\vec{k}^2 k_z^2 - Bk_z^4,$$

Which in turn leads to infrared divergencies in the second order fluctuations

$$\Delta^2\langle\beta_3^2(x)\rangle \simeq \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\frac{T}{\omega_-^2}$$

The DCDW phase is not stable against the thermal fluctuations of the condensate. There is no true long-range order at nonzero temperature

Lee et al. PRD 92 (2015) 034024

Magnetic DCDW Phase

B≠0 Symmetry: $(U(1)_L \times U(1)_R)_f$, spatial rotation about z and translation

$$\phi^T = (\sigma, \pi)$$

transforms as a 2-D vector under O(2) rotations

Breaking of symmetry: translation and chiral, but they are locked like in the zero-B case.

$$\phi(x) = \phi_0(z + u(x))e^{i\pi} = \Delta e^{iq(z+u(x))}e^{i\pi} = \phi_0(z)e^{i(qu+\pi)}$$

We can then consider only one, say the phonon u(x).

MDCDW

Described by Dirac Hamiltonian

$$H_f = -i\gamma^0\gamma^i(\partial_i + ie_f A_i + i\frac{e_f}{|e_f|}\gamma_5\partial_i\theta) + \gamma^0 m$$

Axion term in the electromagnetic action

$$S = -\kappa \int d^4x \epsilon^{\mu\alpha\nu\beta} A_\alpha \partial_\nu A_\beta \partial_\mu \theta$$

Topology is associated to asymmetry of the LLL states in the MDCDW.

$$\sigma_{xy}^{anom} = e^2 q/4\pi^2$$

Anomalous Hall conductivity

Ferrer and VI, '15,'17,'18

Weyl Semimetals

Described by Dirac Hamiltonian

$$H(\mathbf{k}) = \gamma^0\gamma^i(k_i - b_i\gamma^5) + m\gamma^0 + b_0\gamma^5.$$

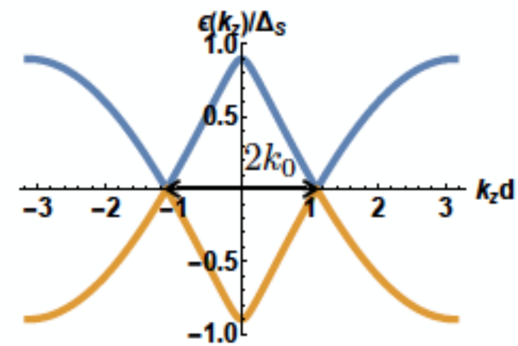
Axion term in the electromagnetic action

$$S = -\frac{e^2}{4\pi^2}\int dt\, d^3r\, b_\mu \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta.$$

Topology is associated to band structure with nodes of opposite chirality separated by 2b in momentum space

$$\sigma_{xy} = \frac{e^2}{h}\frac{2|b|}{2\pi^2}$$

Anomalous Hall conductivity



Burkov, '17