The Infrared Triangle

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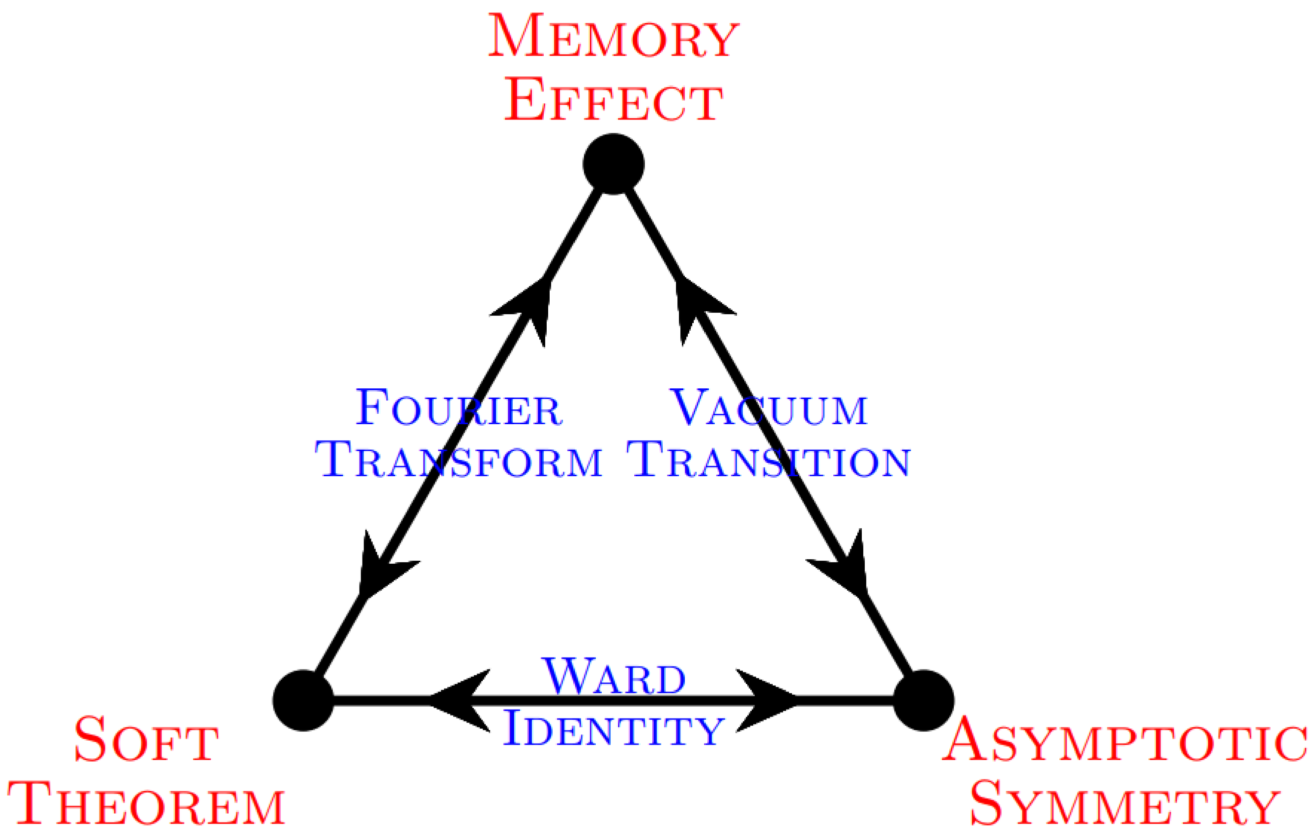
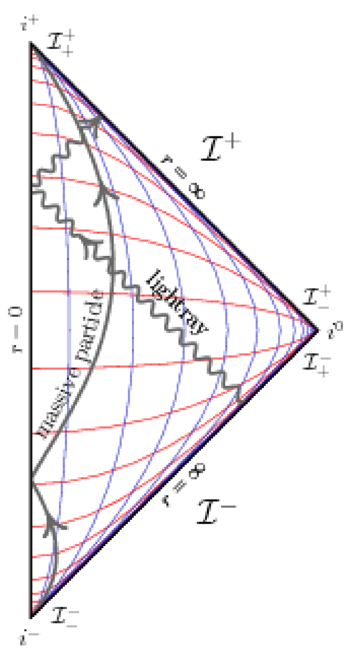
1. Introduction



Over the last few years a triangular equivalence relation was discovered connecting three apparently different topics: asymptotic symmetries, soft theorems and memory effects. This equivalence relation governs the infrared dynamics of all physical theories. We can draw a triangle potentially in every theory with a massless particle, for example in QED, QCD, SUSY and String Theory [1]. Each of these triangles comes with its own peculiarities and we need to study them individually case by case. This new perspective has led to better conceptual understanding of each old corner, discovery of new corners, and new research programs searching for and exploring new triangles. I chose to study this equivalence relation in the context of gravity where the triangle was firstly discovered and I examined in particular its generalization to higher dimensions [2,3].

2. Asymptotic symmetries

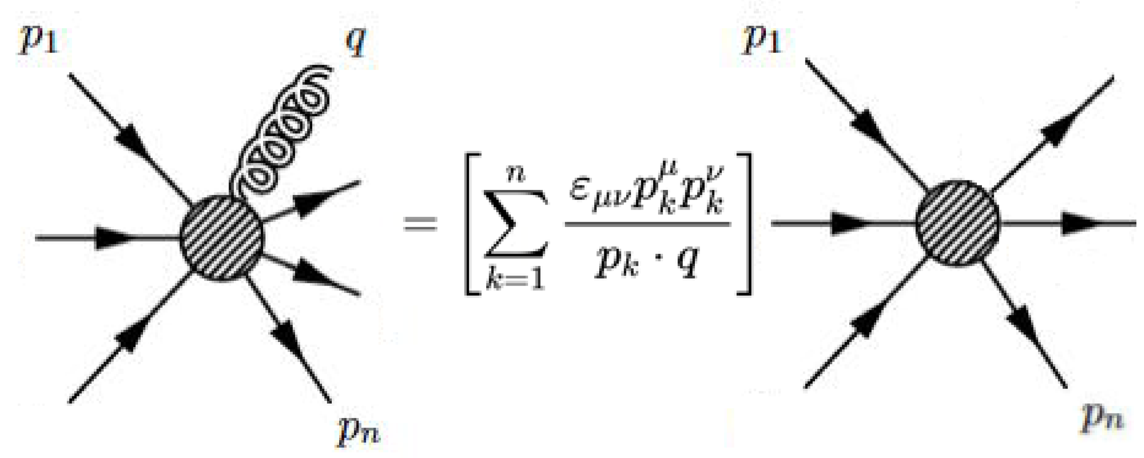
In gravity asymptotic symmetries are transformations that leave invariant the asymptotic structure of spacetime mapping an asymptotically flat metric (at null infinity) into an other asymptotically flat metric.



The asymptotic group is infinite-dimensional. After computing the charges of these symmetries, we can write their Ward Identity which is exactly the soft graviton theorem.

3. Soft theorems

The soft graviton theorem is a universal formula relating scattering amplitudes that differ only by the addition of a graviton whose energy is taken to zero



The formula is blind to the spin and any other quantum numbers of the particles involved in the scattering. In the expression, there is a pole in the soft energy of the graviton. The soft theorem has a nice and simple derivation with Feynman diagrams. This theorem is connected to a memory effect simply by a Fourier transform.

4. Memory effect

According to the gravitational memory effect, the passage of a gravitational wave produces a permanent shift in the distance between two detectors. This effect is an exciting experimental prospect for the coming years. The memory effect is a vacuum transition between two different states related by an asymptotic transformation.

5. Subleading triangles

There are multiple soft graviton theorems depending on which order in the soft energy we study the scattering

$$M_{n+1}(q;p_1,\dots,p_n) = \left[S^{(0)} + S^{(1)} + S^{(2)}\right] M_n(p_1,\dots,p_n) + \dots$$

where

$$S^{(0)} \equiv \sum_{k=1}^{n} \frac{\varepsilon_{\mu\nu} p_k^\mu p_k^\nu}{p_k \cdot q} \sim \frac{1}{\omega} \qquad S^{(1)} \equiv -i \sum_{k=1}^{n} \frac{\varepsilon_{\mu\nu} p_k^\mu q_\rho J_k^{\rho\nu}}{p_k \cdot q} \sim \omega^0$$

$$S^{(2)} \equiv -\frac{1}{2} \sum_{k=1}^{n} \frac{\varepsilon_{\mu\nu} q_\rho J_k^{\rho\mu} q_\lambda J_k^{\lambda\nu}}{q \cdot p_k} \sim \omega$$

Each soft theorem has its own IR triangle with different memory effects and asymptotic symmetries. I examined the subleading soft graviton theorem and I showed its connection to asymptotic symmetries in even dimensions higher than four [3]. I wrote the soft theorem as a Ward Identity of asymptotic symmetries

$$< out|Q\mathcal{S} - \mathcal{S}Q|in >= 0$$

6. Future directions and applications

- Discovering new asymptotic symmetries, soft theorems and memory effects. Results in one area can be translated into what are often new results in other areas.
- Implications for the Black Hole information paradox: from asymptotic symmetries we have an infinite number of charges that must be conserved, constraining the black hole state.
- New experimental prospects for the coming future in gravitational wave physics: memory effects will hopefully be measured by next generation detectors.
- Loop corrections to soft theorems potentially mean quantum anomalies for asymptotic symmetries. Only the leading soft theorem doesn't have loop corrections.
- Generalization to higher dimensions: we need a renormalization for the charges of asymptotic symmetries in order to avoid divergences.
- Celestial Holography: the IR triangle paved the way to a duality between the gravitational S-matrix and correlators in a conformal field theory living on the celestial sphere.

7. References

[1] A. Strominger, "Lectures on the Infrared Structure of Gravity and Gauge Theory," [arXiv:1703.05448 [hep-th]].

[2] D. Kapec, V. Lysov, S. Pasterski and A. Strominger, "Higher-dimensional supertranslations and Weinberg's soft graviton theorem," Ann. Math. Sci. Appl. **02** (2017), 69-94 [arXiv:1502.07644 [gr-qc]].

[3] D. Colferai and S. Lionetti, "Asymptotic symmetries and the subleading soft graviton theorem in higher dimensions," Phys. Rev. D **104** (2021) no.6, 064010 [arXiv:2005.03439 [hep-th]].