Chiral anomaly induces superconducting baryon crystal

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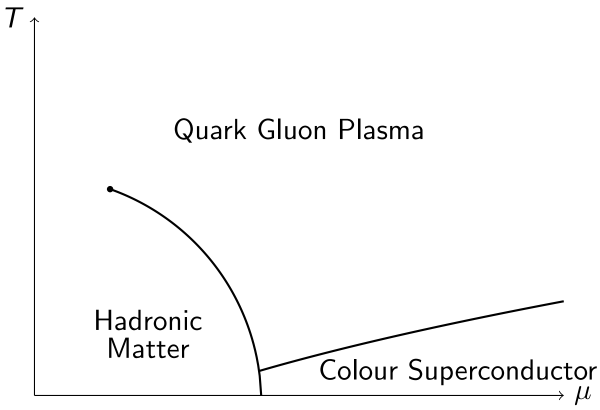
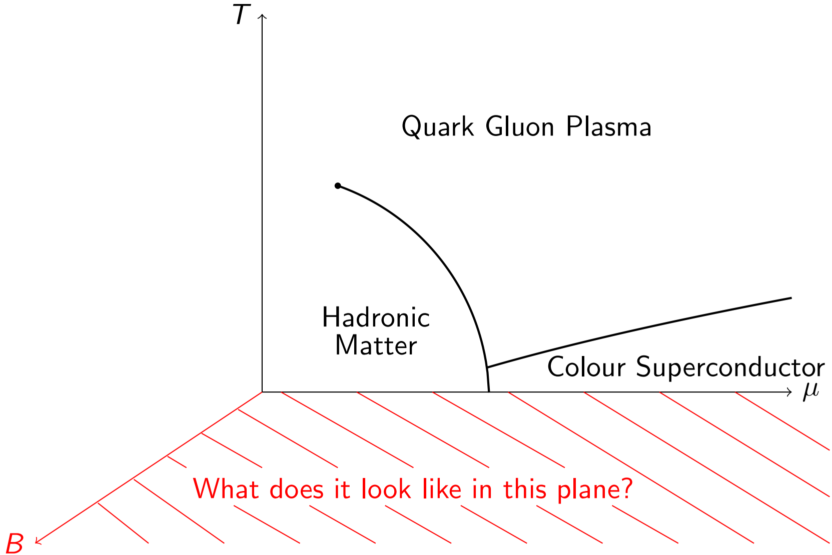
Schematic QCD Phase Diagram

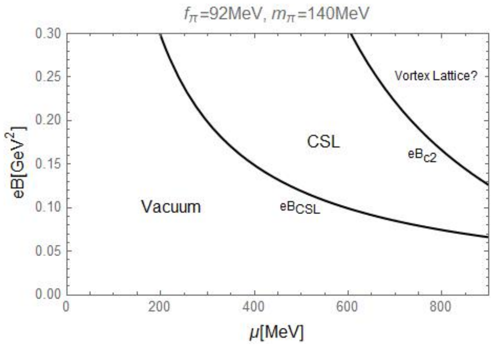


Schematic QCD Phase Diagram



The B-μ plane

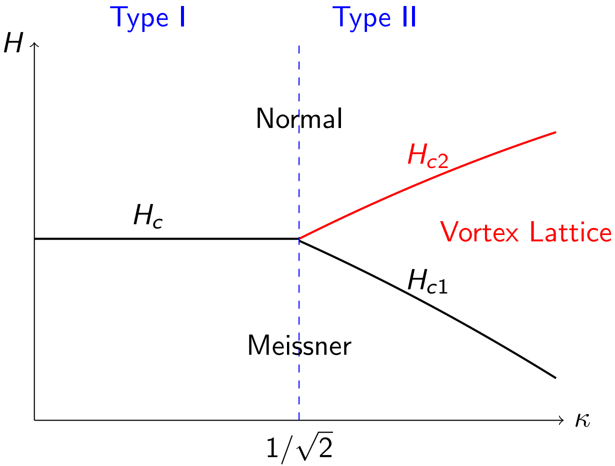
- Chiral anomaly induces a coupling $\propto \mu B \nabla \pi^0$ [1]
- The ground state at $\mu \lesssim 1\,\mathrm{GeV}$ and large B is an inhomogeneous phase of neutral pions (π^0) called the Chiral Soliton Lattice (CSL)[2]



- At $B \geq B_{c2}$ the CSL becomes unstable to charged pion fluctuations

[1] D. T. Son and M. A. Stephanov, Phys. Rev. D 77, 2008 (014021)
[2] T. Brauner and N. Yamamoto, JHEP, 132, 2017(04)

Superconductivity refresher



- The charged pion instability is reminiscent of the Vortex Lattice-Normal transition, but inverted

Lagrangian

We work in Chiral Perturbation Theory with two-flavours, starting from the Lagrangian

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\mathsf{EM}} + \mathcal{L}_{\mathsf{WZW}} \,,$$

where

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4} \mathrm{Tr}\left[\nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma\right] + \frac{m_\pi^2 f_\pi^2}{4} \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] \,,$$

$$\mathcal{L}_{\mathsf{EM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \,,$$

and the chiral anomaly is incorporated via the Wess-Zumino-Witten[3] term

$$\mathcal{L}_{\mathsf{WZW}} = -\left(A_\mu^B - \frac{e}{2} A_\mu\right) \frac{\epsilon^{\mu\nu\rho\lambda}}{24\pi^2} \mathrm{Tr}\Big[(\Sigma\nabla_\nu\Sigma^\dagger)(\Sigma\nabla_\rho\Sigma^\dagger)(\Sigma\nabla_\lambda\Sigma^\dagger) + \frac{3ie}{4} F_{\nu\rho} \tau_3 \left(\Sigma\nabla_\lambda\Sigma^\dagger + \nabla_\lambda\Sigma^\dagger\Sigma\right)\Big] \,.$$

[3] J. Wess and B. Zumino, Phys. Lett. B 37, 1971 & E. Witten, Nucl. Phys. B 223, 1983

Free energy

From the Lagrangian we obtain the free energy

$$\mathcal{F} = \frac{1}{V} \int dV \left(|\left(\nabla - i\left(e\boldsymbol{A} + \nabla\alpha\right)\right)\varphi|^2 - \left(\nabla\alpha\right)^2 |\varphi|^2 + \frac{\left(\nabla|\varphi|^2\right)^2}{2\left(f_\pi^2 - 2|\varphi|^2\right)} \right.$$
$$\left. - m_\pi^2 f_\pi \sqrt{f_\pi^2 - 2|\varphi|^2} \cos\alpha + \frac{f_\pi^2}{2}\left(\nabla\alpha\right)^2 + \frac{\boldsymbol{B}^2}{2} - \frac{e\mu}{4\pi^2}\boldsymbol{B}\cdot\nabla\alpha \right),$$

where φ (complex scalar field) and α (real scalar field) parameterise the pion fields, and the magnetic field $\boldsymbol{B} = \nabla \times \boldsymbol{A}$.

Equations of motion

From the Lagrangian/Free Energy we obtain the equations of motion for φ, $\boldsymbol{A}$ and α

$$0 = \left[\mathcal{D} + \frac{\nabla^2|\varphi|^2}{f_\pi^2 - 2|\varphi|^2} + \frac{\left(\nabla|\varphi|^2\right)^2}{(f_\pi^2 - 2|\varphi|^2)^2} + m_\pi^2 \cos\alpha \left(1 - \frac{f_\pi}{\sqrt{f_\pi^2 - 2|\varphi|^2}}\right)\right]\varphi\,,$$

$$\nabla \times \boldsymbol{B} = -ie\left(\varphi^*\nabla\varphi - \varphi\nabla\varphi^*\right) - 2e\left(e\boldsymbol{A} + \nabla\alpha\right)|\varphi|^2\,,$$

$$\nabla \cdot \left[\left(1 - \frac{2|\varphi|^2}{f_\pi^2}\right)\nabla\alpha\right] = m_\pi^2\sqrt{1 - \frac{2|\varphi|^2}{f_\pi^2}}\sin\alpha\,,$$

respectively, where

$$\mathcal{D} \equiv \nabla^2 - i\nabla\cdot\left(e\boldsymbol{A} + \nabla\alpha\right) - 2i\left(e\boldsymbol{A} + \nabla\alpha\right)\cdot\nabla - \left(e\boldsymbol{A} + \nabla\alpha\right)^2 + (\nabla\alpha)^2 - m_\pi^2\cos\alpha\,.$$

Applying Abrikosov's expansion near B_{c2}

- To find solutions analytically near B_{c2}, we follow Abrikosov's original paper[4] and expand in the small parameter $\epsilon \sim \sqrt{B - B_{c2}}$ like so

$$\varphi = \varphi_0 + \delta\varphi + \dots\,, \quad \boldsymbol{A} = \boldsymbol{A}_0 + \delta\boldsymbol{A} + \dots\,, \quad \alpha = \alpha_0 + \delta\alpha + \dots\,,$$

where

$$\boldsymbol{A}_0,\, \alpha_0 \sim \epsilon^0\,, \qquad \varphi_0 \sim \epsilon^1\,, \qquad \delta\boldsymbol{A},\, \delta\alpha \sim \epsilon^2\,, \qquad \delta\varphi \sim \epsilon^3$$

- We expand the free energy and equations of motion (with $\boldsymbol{B}_0 = \nabla \times \boldsymbol{A}_0$ and $\delta\boldsymbol{B} = \nabla \times \delta\boldsymbol{A}$)
- For simplicity, we solve the equations of motion in the chiral limit (i.e. $m_\pi = 0$)

[4] A.A. Abrikosov, JETP, 5, p.1174, 1957(06)

Solutions of the expanded equations of motion

- The solutions at ϵ^0 are

$$\boldsymbol{B}_0 = B_{c2}\hat{\boldsymbol{e}}_z\,, \qquad \alpha_0(z) = \frac{e\mu}{4\pi^2 f_\pi^2} B_{c2} z$$

- The equation for φ_0 is the same as the Schrödinger equation for a particle in a magnetic field with the usual Landau level result[5]
- Choosing the ground state solution,

$$\varphi_0(x,y) = \sum_{n=-\infty}^{\infty} C_n e^{inqy} e^{-\frac{eB_{c2}}{2}\left(x-\frac{nq}{eB_{c2}}\right)^2}\,,$$

where relations between different C_n determine the configuration of the lattice

[5]See M. Tinkham, Introduction to Superconductivity. Dover Publications, New York, 2004

Solutions of expanded equations of motion

- The correction to the magnetic field becomes

$$\delta \boldsymbol{B}(x,y) = \left[\langle B \rangle - B_{c2} + e\left(\langle |\varphi_0(x,y)|^2\rangle - |\varphi_0(x,y)|^2\right)\right]\hat{\boldsymbol{e}}_z\,,$$

  where we've introduced a spatial average defined as

$$\langle f(x,y,z)\rangle = \frac{1}{V}\int f(x,y,z)dV\,,$$

  for a function $f(x,y,z)$ over the volume V
- The other equation at order ϵ^2 is solved by

$$\delta\alpha(z) = \frac{e\mu}{4\pi^2 f_\pi^2}\left(\langle B\rangle - B_{c2}\right)z$$

Free energy result

We do not solve the $\delta\varphi$ equation but use it instead to show that

$$e\langle|\varphi_0|^2\rangle = \frac{\langle B\rangle - B_{c2}}{(2\kappa^2 - 1)\,\beta + 1}\,, \quad \text{where} \quad \beta = \frac{\langle|\varphi_0|^4\rangle}{\langle|\varphi_0|^2\rangle^2}\,,$$

and $2\kappa^2 = B_{c2}/ef_\pi^2$.

With $\mathcal{F}$ expanded to fourth order, we obtain

$$\Delta\mathcal{F} = -\frac{1}{2}\frac{\left(\langle B\rangle - B_{c2}\right)^2}{(2\kappa^2 - 1)\,\beta + 1}\,,$$

which is the difference in free energy between our constructed phase and the "CSL" phase.

We have analytically constructed a phase which is preferred above B_{c2}!

β and lattice configurations

- To minimise $\mathcal{F}$ we must minimise β which can vary depending on the periodicity condition $C_n = C_{n+N}$, where N is an integer
- To explore a continuum of geometries[6], we set $N = 2$ and $C_0 = \pm i C_1$

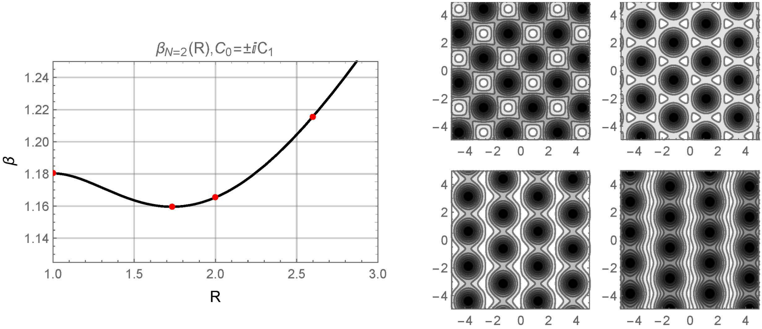


Figure: $R = q^2/\pi$. *Left*: Red dots correspond to contour plots on the right. *Right*: $|\varphi_0|^2$ in the x-y plane. Dark regions correspond to vortices.

[6]W.H. Kleiner, L.M. Roth and S.H. Autler, Phys. Rev. 133 5A A1226, 1964

Charged pion condensate and baryon number density

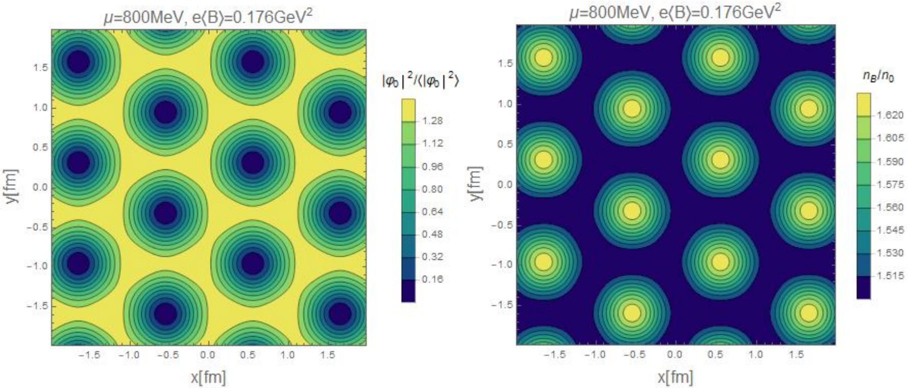


Figure: Charged pion vortex lattice (left) and local baryon number density (right).

A superconducting crystal with "baryon tubes"!

Summary

- The preferred phase above B_{c2} is a charged pion vortex lattice coexisting with a neutral pion superflow
- Further analysis of β parameter reveals the most preferred configuration of vortices is hexagonal
- The baryon number density is not only non-zero but also inhomogeneous, exhibiting the periodic hexagonal configuration of the charged pion vortex lattice
- Our phase is a 2D superconducting baryon crystal

Outlook

- In the more physical scenario where $m_\pi \neq 0$, a 3D crystalline structure is expected
- Our calculation is confined near B_{c2} - what does the lattice look like away from the transition?
- At $\mu \sim 1\,\mathrm{GeV}$ actual baryons are expected to emerge, thus their inclusion would lead to a more realistic calculation
- The CSL[7] and charged pion superconductivity[8] also emerge in the B-μ_I plane - can we extend our results to this plane?[9]

[7] T. Brauner, G. Filios, and H. Kolešová, JHEP 12, p. 29, 2019
[8] P. Adhikari, T. D. Cohen, and J. Sakowitz, Phys. Rev. C 91, 2015 (4)
[9] M. S. Grønli and T. Brauner, Eur. Phys. J. C 82, 2022(4)