QCDF Amplitudes from SU(3) Symmetries

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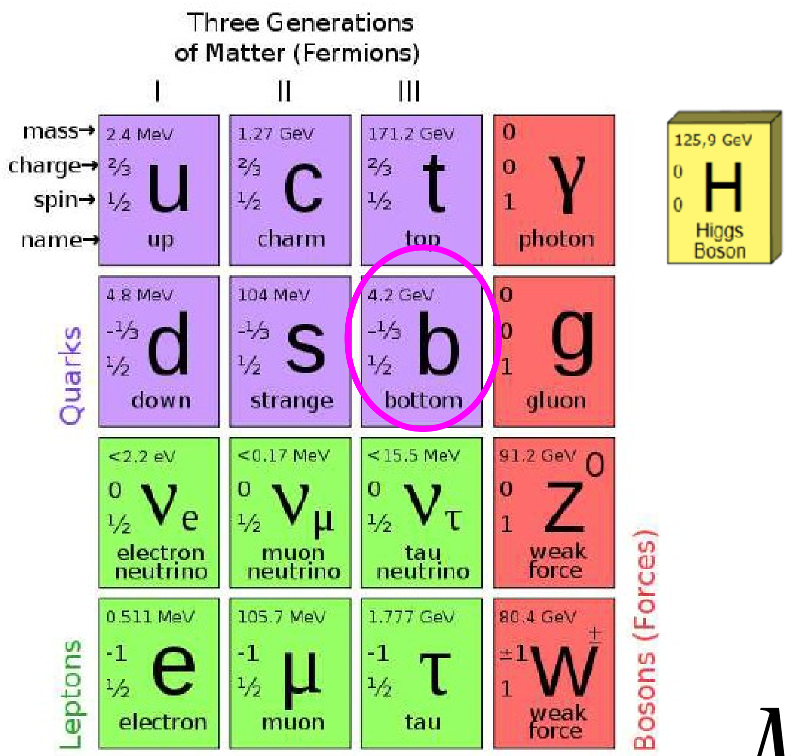
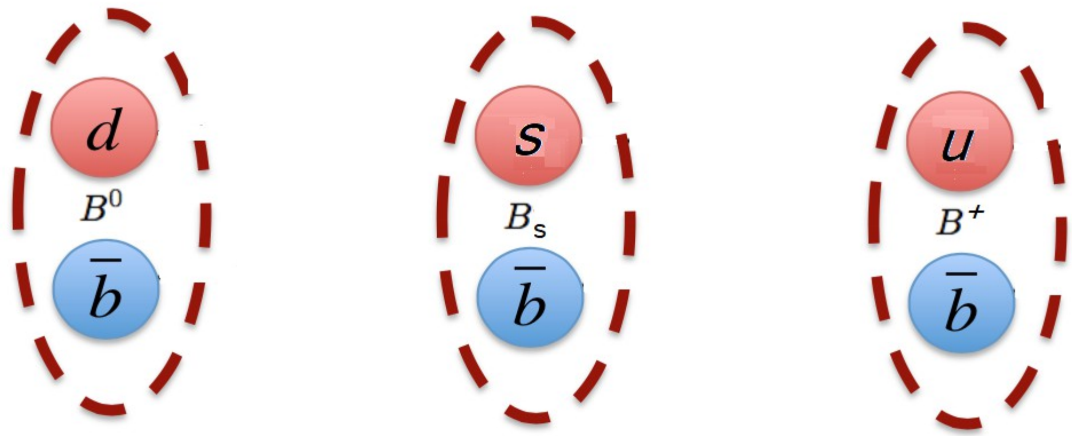
Non-leptonic B meson decays





Bound-states of b quarks

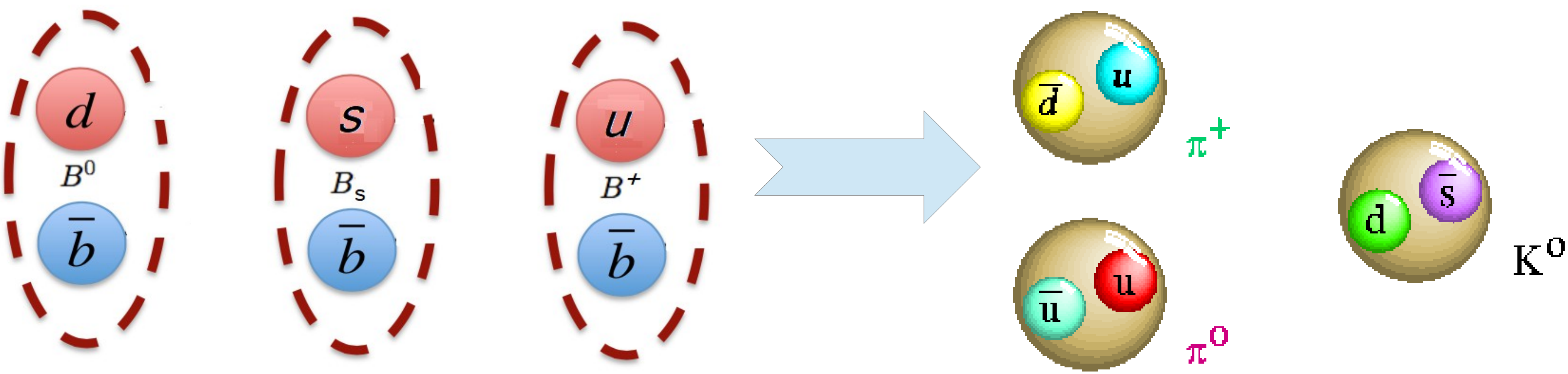
$$m_b \simeq 4 \ \ GeV$$

$$M_{B^0} \simeq M_{B_s} \simeq M_{B^+} \simeq 5\, GeV$$

5 times the mass of a Hydrogen atom

Non-leptonic B meson decays

We are interested in B meson decays into pairs of light pseudoscalar mesons



$$B \to PP$$

The light pseudoscalar mesons are bound states of light quarks $[u, d, s]$ (SU(3) symmetry)

$$B = (B^+, B_d^0, B_s^0)$$

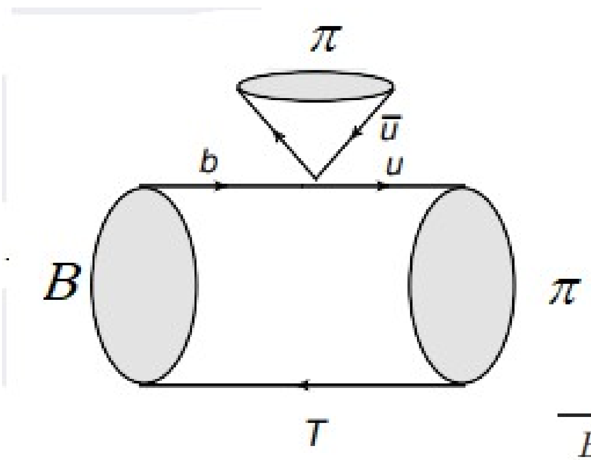
$$q_i \otimes \overline{q_j} \rightarrow 3 \otimes \bar{3} = 8 \oplus 1$$
$$i, j \in [u, d, s]$$

P

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & \eta_s + \eta'_s \end{pmatrix}$$

Non-leptonic B meson decays

We are interested in B meson decays into pairs of light pseudoscalar mesons



$$b \rightarrow u\,\bar{u}\,q \qquad q = d\,,\,s$$

Several possible decay channels

$B^- \to \pi^0\pi^-$	$\overline{B}^0 \to K^0\overline{K}^0$	$B^- \to \pi^0 K^-$	$\overline{B}_s^0 \to \pi^+\pi^-$
$B^- \to \pi^-\eta_8$	$\overline{B}^0 \to \eta_8\eta_8$	$B^- \to \pi^-\overline{K}^0$	$\overline{B}_s^0 \to \pi^0\pi^0$
$B^- \to \pi^-\eta_1$	$\overline{B}^0 \to \eta_8\eta_1$	$B^- \to K^-\eta_8$	$\overline{B}_s^0 \to \pi^0\eta_1$
$B^- \to K^0K^-$	$\overline{B}^0 \to \eta_1\eta_1$	$B^- \to K^-\eta_1$	$\overline{B}_s^0 \to K^+K^-$
$\overline{B}^0 \to \pi^+\pi^-$	$\overline{B}_s^0 \to \pi^0 K^0$	$\overline{B}^0 \to \pi^+K^-$	$\overline{B}_s^0 \to K^0\overline{K}^0$
$\overline{B}^0 \to \pi^0\pi^0$	$\overline{B}_s^0 \to \pi^-K^+$	$\overline{B}^0 \to \pi^0\overline{K}^0$	$\overline{B}_s^0 \to \eta_8\eta_8$
$\overline{B}^0 \to \pi^0\eta_8$	$\overline{B}_s^0 \to K^0\eta_8$	$\overline{B}^0 \to \overline{K}^0\eta_8$	$\overline{B}_s^0 \to \eta_8\eta_1$
$\overline{B}^0 \to \pi^0\eta_1$	$\overline{B}_s^0 \to K^0\eta_1$	$\overline{B}^0 \to \overline{K}^0\eta_1$	$\overline{B}_s^0 \to \eta_1\eta_1$
$\overline{B}^0 \to K^+K^-$			

Topological decomposition

Consider the process $B \to PP$

where P is a charmless pseudoscalar meson

The physical amplitude can be decomposed as

$$\mathcal{A}^{TDA} = i\frac{G_F}{\sqrt{2}}\left[\mathcal{T}^{TDA} + \mathcal{P}^{TDA}\right]$$

$$\lambda_p^{(q)} = V_{pb}V_{pq}^* \qquad \lambda_u^{(q)} \qquad \lambda_t^{(q)} \qquad q = d, s$$

$$\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$$

Topological decomposition

$$
\begin{aligned}
\mathcal{T}^{TDA} = {} & \underline{T}\ B_i(M)^i_j \bar{H}^{jl}_k (M)^k_l + \underline{C}\ B_i(M)^i_j \bar{H}^{lj}_k (M)^k_l + \underline{A}\ B_i \bar{H}^{il}_j (M)^j_k (M)^k_l \\
& + \underline{E}\ B_i \bar{H}^{li}_j (M)^j_k (M)^k_l + \underline{T_{ES}} B_i \bar{H}^{ij}_l (M)^l_j (M)^k_k + \underline{T_{AS}} B_i \bar{H}^{ji}_l (M)^l_j (M)^k_k \\
& + \underline{T_S} B_i (M)^i_j \bar{H}^{lj}_l (M)^k_k + \underline{T_{PA}} B_i \bar{H}^{li}_l (M)^j_k (M)^k_j + \underline{T_P} B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l \\
& + \underline{T_{SS}} B_i \bar{H}^{li}_l (M)^j_j (M)^k_k,
\end{aligned}
$$

SU(3) Flavour $\quad [u, d, s]$

$$
B = (B^+, B^0_d, B^0_s) \qquad M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & \eta_s + \eta'_s \end{pmatrix}
$$

$$
\bar{H}^{12}_1 = \lambda_u^{(d)}, \quad \bar{H}^{13}_1 = \lambda_u^{(s)},
$$

Topological decomposition

$$
\begin{aligned}
\mathcal{T}^{TDA} = {} & \underline{T}\; B_i(M)^i_j \bar{H}^{jl}_k (M)^k_l + \underline{C}\; B_i(M)^i_j \bar{H}^{lj}_k (M)^k_l + \underline{A}\; B_i \bar{H}^{il}_j (M)^j_k (M)^k_l \\
& + \underline{E}\; B_i \bar{H}^{li}_j (M)^j_k (M)^k_l + \underline{T_{ES}} B_i \bar{H}^{ij}_l (M)^l_j (M)^k_k + \underline{T_{AS}} B_i \bar{H}^{ji}_l (M)^l_j (M)^k_k \\
& + \underline{T_S} B_i (M)^i_j \bar{H}^{lj}_l (M)^k_k + \underline{T_{PA}} B_i \bar{H}^{li}_l (M)^j_k (M)^k_j + \underline{T_P} B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l \\
& + \underline{T_{SS}} B_i \bar{H}^{li}_l (M)^j_j (M)^k_k,
\end{aligned}
$$

T :Color allowed tree.

C : Color-suppressed tree.

E : W-exchange diagram.

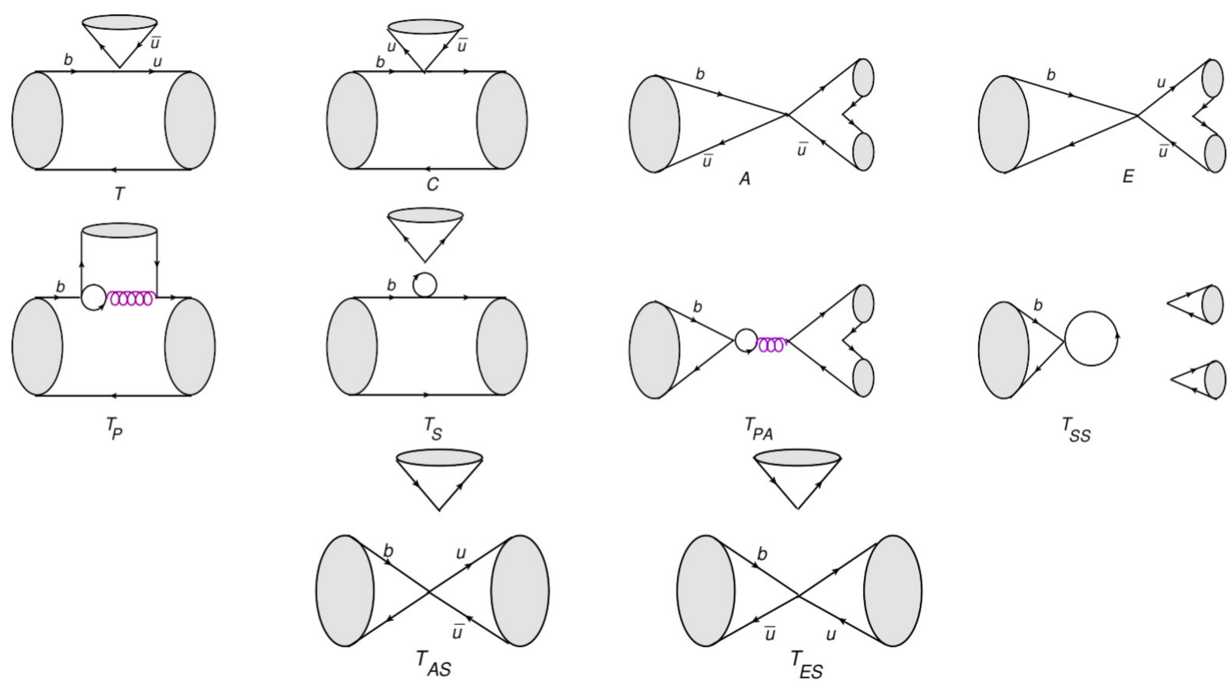
P : QCD-penguin.

S : QCD-singlet penguin.

A : Annihilation.

Topological decomposition

$$
\begin{aligned}
\mathcal{T}^{TDA} = {} & \underline{T}\ B_i(M)^i_j \bar{H}^{jl}_k (M)^k_l + \underline{C}\ B_i(M)^i_j \bar{H}^{lj}_k (M)^k_l + \underline{A}\ B_i \bar{H}^{il}_j (M)^j_k (M)^k_l \\
& + \underline{E}\ B_i \bar{H}^{li}_j (M)^j_k (M)^k_l + \underline{T_{ES}} B_i \bar{H}^{ij}_l (M)^l_j (M)^k_k + \underline{T_{AS}} B_i \bar{H}^{ji}_l (M)^l_j (M)^k_k \\
& + \underline{T_S} B_i (M)^i_j \bar{H}^{lj}_l (M)^k_k + \underline{T_{PA}} B_i \bar{H}^{li}_l (M)^j_k (M)^k_j + \underline{T_P} B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l \\
& + \underline{T_{SS}} B_i \bar{H}^{li}_l (M)^j_j (M)^k_k,
\end{aligned}
$$



SU(3)-Irreducible decomposition

$$
\begin{aligned}
\mathcal{T}^{IRA} = {} & \underline{A_3^T} B_i(\bar{H}_{\bar{3}})^i(M)^j_k(M)^k_j + \underline{C_3^T} B_i(M)^i_j(M)^j_k(\bar{H}_{\bar{3}})^k + \underline{B_3^T} B_i(\bar{H}_3)^i(M)^k_k(M)^j_j \\
& + \underline{D_3^T} B_i(M)^i_j(\bar{H}_{\bar{3}})^j(M)^k_k + \underline{A_6^T} B_i(H_6)^{ij}_k(M)^l_j(M)^k_l + \underline{C_6^T} B_i(M)^i_j(\bar{H}_6)^{jl}_k(M)^k_l \\
& + \underline{B_6^T} B_i(\bar{H}_6)^{ij}_k(M)^k_j(M)^l_l + \underline{A_{15}^T} B_i(\bar{H}_{\overline{15}})^{ij}_k(M)^l_j(M)^k_l + \underline{C_{15}^T} B_i(M)^i_j(\bar{H}_{\overline{15}})^{jk}_l(M)^l_k \\
& + \underline{B_{15}^T} B_i(\bar{H}_{\overline{15}})^{ij}_k(M)^k_j(M)^l_l.
\end{aligned}
$$

SU(3) irreducible decomposition

$$
\bar{H}^{ij}_k = \frac{1}{8}(H_{\overline{15}})^{ij}_k + \frac{1}{4}(H_6)^{ij}_k - \frac{1}{8}(H_{\overline{3}})^i\delta^j_k + \frac{3}{8}(H_{\overline{3}'})^j\delta^i_k
$$

SU(3)-Irreducible decomposition

$$\mathcal{T}^{IRA} = \underline{A_3^T} B_i(\bar{H}_{\bar{3}})^i (M)^j_k (M)^k_j + \underline{C_3^T} B_i (M)^i_j (M)^j_k (\bar{H}_{\bar{3}})^k + \underline{B_3^T} B_i (\bar{H}_3)^i (M)^k_k (M)^j_j$$
$$+ \underline{D_3^T} B_i (M)^i_j (\bar{H}_{\bar{3}})^j (M)^k_k + \underline{A_6^T} B_i (H_6)^{ij}_k (M)^l_j (M)^k_l + \underline{C_6^T} B_i (M)^i_j (\bar{H}_6)^{jl}_k (M)^k_l$$
$$+ \underline{B_6^T} B_i (\bar{H}_6)^{ij}_k (M)^k_j (M)^l_l + \underline{A_{15}^T} B_i (\bar{H}_{\overline{15}})^{ij}_k (M)^l_j (M)^k_l + \underline{C_{15}^T} B_i (M)^i_j (\bar{H}_{\overline{15}})^{jk}_l (M)^l_k$$
$$+ \underline{B_{15}^T} B_i (\bar{H}_{\overline{15}})^{ij}_k (M)^k_j (M)^l_l.$$

Topological to SU(3)

X.-G. He and W. Wang: 1803.04227

$$A_3^T = -\frac{A}{8} + \frac{3E}{8} + T_{PA}, \qquad B_3^T = T_{SS} + \frac{3T_{AS} - T_{ES}}{8},$$

$$C_3^T = \frac{1}{8}(3A - C - E + 3T) + T_P, \qquad D_3^T = T_S + \frac{1}{8}(3C - T_{AS} + 3T_{ES} - T)$$

$$A_6^T = \frac{1}{4}(A - E), \qquad B_6^T = \frac{1}{4}(T_{ES} - T_{AS}),$$

$$C_6^T = \frac{1}{4}(-C + T), \qquad A_{15}^T = \frac{A + E}{8},$$

$$B_{15}^T = \frac{T_{ES} + T_{AS}}{8}, \qquad C_{15}^T = \frac{C + T}{8},$$

SU(3) amplitudes from data

The physical amplitudes can be expressed as linear combinations of the SU(3) sub-amplitudes

Channel	A_3^T	C_3^T	A_6^T	C_6^T	A_{15}^T	C_{15}^T	B_3^T	B_6^T	B_{15}^T	D_3^T
$B^- \to \pi^0\pi^-$	0	0	0	0	0	$4\sqrt{2}$	0	0	0	0
$B^- \to K^0K^-$	0	1	1	-1	3	-1	0	0	0	0
$B^0 \to \pi^+\pi^-$	2	1	-1	1	1	3	0	0	0	0
$B^0 \to \pi^0\pi^0$	2	1	-1	1	1	-5	0	0	0	0
$B^0 \to K^+K^-$	2	0	0	0	2	0	0	0	0	0
$B^0 \to K^0\bar{K}^0$	2	1	1	-1	-3	-1	0	0	0	0
$B_s \to \pi^0K^0$	0	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{5}{\sqrt{2}}$	0	0	0	0
$B_s \to \pi^-K^+$	0	1	-1	1	-1	3	0	0	0	0
$B^- \to \pi^0K^-$	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{3}{\sqrt{2}}$	$\frac{7}{\sqrt{2}}$	0	0	0	0
$B^- \to \pi^-K^0$	0	1	1	-1	3	-1	0	0	0	0
$B^0 \to \pi^+K^-$	0	1	-1	1	-1	3	0	0	0	0
$B^0 \to \pi^0K^0$	0	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{5}{\sqrt{2}}$	0	0	0	0
$B_s \to \pi^+\pi^-$	2	0	0	0	2	0	0	0	0	0
$B_s \to \pi^0\pi^0$	2	0	0	0	2	0	0	0	0	0
$B_s \to K^+K^-$	2	1	-1	1	1	3	0	0	0	0
$B_s \to K^0\bar{K}^0$	2	1	1	-1	-3	-1	0	0	0	0

SU(3) amplitudes from data

Extract the SU(3) amplitudes by fitting to data

$$\Gamma(\bar{B} \to M_1 M_2) = \frac{S}{16\pi M_B} |\mathcal{A}_{B\to M_1 M_2}|^2$$

S=1 if *$M_1 \neq M_2$* *S=1/2* *if* *$M_1 = M_2$*

Observables:

Branching fractions $$\mathcal{B}(\bar{B} \to \bar{f}) = \frac{1}{2}\tau_B \Big[\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)\Big]$$

CP Asymmetries $$\mathcal{A}_{\rm CP}(\bar{B} \to \bar{f}) = \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)}$$

SU(3) amplitudes from data

Perform a χ^2 fit $\chi^2 = \sum_i \left(\frac{\mathcal{O}_i^{\rm Theo} - \mathcal{O}_i^{\rm Exp}}{\sigma_i^{\rm Exp}} \right)^2$

10 Tree complex amplitudes

$$A_3^T \,, \quad C_3^T \,, \quad A_6^T \,, \quad C_6^T \,, \quad A_{15}^T \,, \quad C_{15}^T \,, \quad B_3^T \,, \quad B_6^T \,, \quad B_{15}^T \,, \quad D_3^T$$

and 10 Penguin complex amplitudes (replace T for P above)

The combinations $C_6^T - A_6^T$ and $B_6^T + A_6^T$ always appear together (analogously for penguins)

Redefine

$$C_6^T - A_6^T \to C_6^T \qquad C_6^P - A_6^P \to C_6^P$$

$$B_6^T + A_6^T \to B_6^T \qquad B_6^P + A_6^P \to B_6^P$$

Absorb a global phase by taking C_3^P as a real parameter

35 parameters + θ_{FKS} = 36 parameters to fit.

SU(3) amplitudes from data

Fit for the modulus and and phases of the relevant parameters.

Use random sampling to obtain the best fit point with 10^9 points:

- Calculate the χ^2 function for 10^6 points assuming a flat probability distribution.
- Select the best 5 points leading to the minimum χ^2.
- Use these partial minimums as starting points for the Sequential Least Square Programming algorithm, SLSQP.
- Repeat 10^3 times to get the overall minimum.

To obtain the 65 % C.L regions apply a likelihood ratio test using Wilk's theorem.

SU(3) amplitudes from data

Best fit point (modulus in GeV^3)

$$|A_3^T| = 0.029, \quad \delta_{A_3^T} = -3.083, \quad |C_3^T| = 0.258, \quad \delta_{C_3^T} = -0.105,$$
$$|C_6^T| = 0.235, \quad \delta_{C_6^T} = -0.079, \quad |A_{15}^T| = 0.029, \quad \delta_{A_{15}^T} = -3.083,$$
$$|C_{15}^T| = 0.151, \quad \delta_{C_{15}^T} = 0.061, \quad |B_3^T| = 0.034, \quad \delta_{B_3^T} = 3.087$$
$$|B_6^T| = 0.033, \quad \delta_{B_6^T} = -0.286, \quad |B_{15}^T| = 0.008, \quad \delta_{B_{15}^T} = -1.892$$
$$|D_3^T| = 0.055, \quad \delta_{D_3^T} = 2.942,$$

$$|A_3^P| = 0.014, \quad \delta_{A_3^P} = -1.328, \quad |C_6^P| = 0.145, \quad \delta_{C_6^P} = -2.881,$$
$$|A_{15}^P| = 0.003, \quad \delta_{A_{15}^P} = 2.234, \quad |C_{15}^P| = 0.003, \quad \delta_{C_{15}^P} = -0.608,$$
$$|B_3^P| = 0.043, \quad \delta_{B_3^P} = 2.367, \quad |B_6^P| = 0.099, \quad \delta_{B_6^P} = 0.353,$$
$$|B_{15}^P| = 0.031, \quad \delta_{B_{15}^P} = -0.690, \quad |D_3^P| = 0.030, \quad \delta_{D_3^P} = 0.477,$$
$$|C_3^P| = 0.008, \quad \theta_{FKS} = 0.628.$$

Annihilation amplitudes below 10%.

$$\chi^2/d.o.f. = 0.851$$

Fit-Results: Branching fractions

Channel	Branching ratio in units of 10^{-6}		Channel	Branching ratio in units of 10^{-6}	
	Experimental	Theoretical		Experimental	Theoretical
$B^- \to \pi^0\pi^-$	5.5 ± 0.4	$6.04^{+2.42}_{-2.51}$	$B^- \to \eta\pi^-$	4.02 ± 0.27	$3.80^{+1.25}_{-1.55}$
$B^- \to K^0K^-$	1.31 ± 0.17	$1.36^{+0.17}_{-0.16}$	$B^- \to \eta'\pi^-$	2.7 ± 0.9	$3.55^{+4.49}_{-1.67}$
$\bar{B}^0 \to \pi^+\pi^-$	5.12 ± 0.19	$6.31^{+0.61}_{-0.50}$	$\bar{B}^0 \to \eta\pi^0$	0.41 ± 0.17	$0.41^{+8.90}_{-4.08}$
$\bar{B}^0 \to \pi^0\pi^0$	1.59 ± 0.26	$1.01^{+1.30}_{-0.51}$	$\bar{B}^0 \to \eta'\pi^0$	1.2 ± 0.6	$1.20^{+3.62}_{-1.19}$
$\bar{B}^0 \to K^+K^-$	0.078 ± 0.015	$0.13^{+0.08}_{-0.07}$	$\bar{B}_s \to \eta K^0$	Not available	$0.13^{+0.11}_{-0.08}$
$\bar{B}^0 \to K^0\bar{K}^0$	1.21 ± 0.16	$1.13^{+0.83}_{-0.91}$	$\bar{B}_s \to \eta' K^0$	Not available	$6.65^{+1.48}_{-1.65}$
$\bar{B}_s \to \pi^-K^+$	5.8 ± 0.7	$7.75^{+0.63}_{-0.09}$	$B^- \to \eta K^-$	2.4 ± 0.4	$2.34^{+1.39}_{-1.67}$
$B^- \to \pi^0K^-$	12.9 ± 0.5	$12.78^{+1.75}_{-1.94}$	$B^- \to \eta' K^-$	70.4 ± 2.5	$70.82^{+11.16}_{-11.53}$
$B^- \to \pi^-\bar{K}^0$	23.7 ± 0.8	$23.85^{+2.23}_{-2.31}$	$\bar{B}^0 \to \eta K^0$	1.23 ± 0.27	$1.38^{+1.15}_{-0.36}$
$\bar{B}^0 \to \pi^+K^-$	19.6 ± 0.5	$19.47^{+1.72}_{-2.24}$	$\bar{B}^0 \to \eta' K^0$	6.6 ± 0.4	$6.65^{+1.48}_{-1.65}$
$\bar{B}^0 \to \pi^0\bar{K}^0$	9.9 ± 0.5	$10.17^{+2.00}_{-2.30}$	$\bar{B}_s \to \eta\pi^0$	$< 10^3$	$31.15^{+39.05}_{-31.14}$
$\bar{B}_s \to \pi^+\pi^-$	0.7 ± 0.1	$0.57^{+0.40}_{-0.42}$	$\bar{B}_s \to \eta'\pi^0$	Not available	$11.13^{+74.75}_{-11.12}$
$\bar{B}_s \to \pi^0\pi^0$	< 210	$0.28^{+0.20}_{-0.21}$	$\bar{B}^0 \to \eta\eta$	< 1	$0.30^{+0.70}_{-0.30}$
$\bar{B}_s \to K^+K^-$	26.6 ± 2.2	$20.63^{+6.80}_{-8.09}$	$\bar{B}_s \to \eta\eta$	$< 1.5 \times 10^3$	$2.58^{+36.53}_{-2.57}$
$\bar{B}_s \to K^0\bar{K}^0$	20 ± 6	$24.64^{+18.84}_{-21.14}$	$\bar{B}^0 \to \eta'\eta'$	< 1.7	$1.14^{+0.57}_{-1.07}$
$\bar{B}_s \to \pi^0K^0$	Not available	$0.71^{+1.47}_{-0.27}$	$\bar{B}_s \to \eta'\eta'$	33 ± 7	$33.00^{+24.52}_{-31.74}$
			$\bar{B}^0 \to \eta'\eta$	< 1.2	$0.61^{+0.59}_{-0.60}$
			$\bar{B}_s \to \eta'\eta$	Not available	$0.61^{+0.59}_{-0.60}$

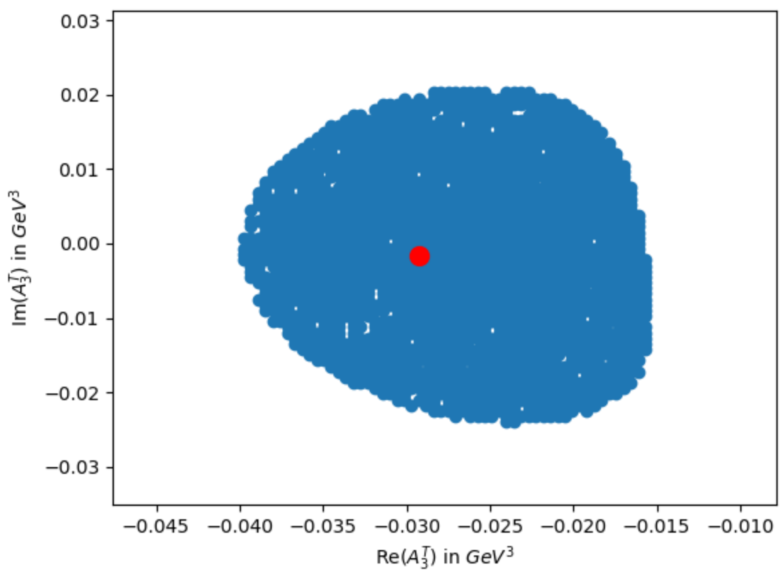
Experimental results from PDG Live

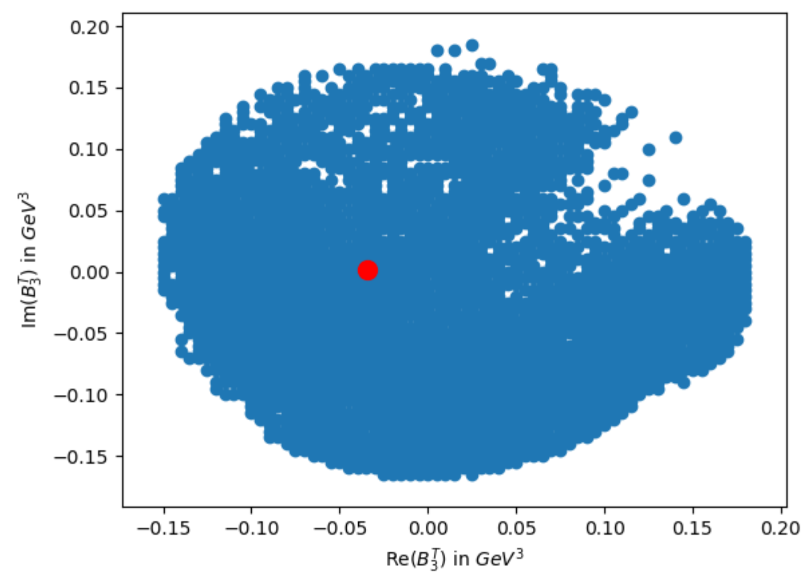
Fit-Results: CP Asymmetries

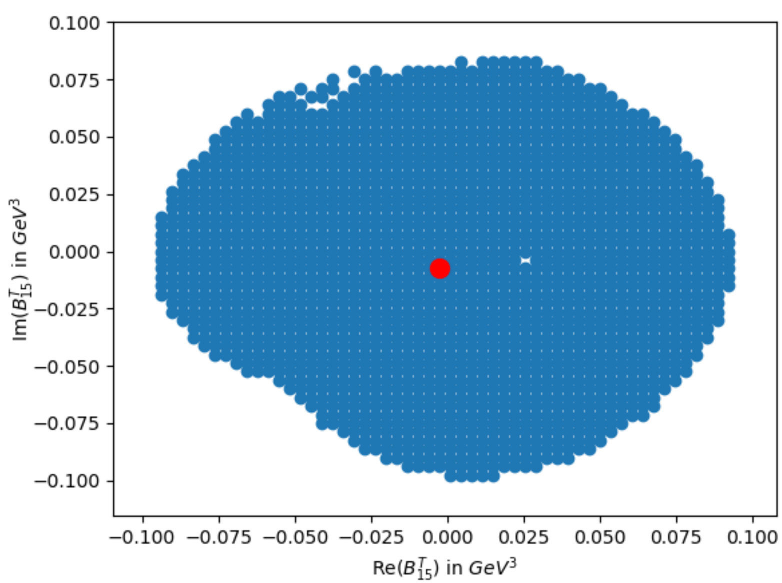
Channel	CP asymmetries in percent		Channel	CP asymmetries in percent	
	Experimental	Theoretical		Experimental	Theoretical
$B^- \to \pi^0\pi^-$	3 ± 4	$5.45^{+22.02}_{-20.60}$	$B^- \to \eta\pi^-$	-14 ± 7	$-11.37^{+14.49}_{-26.90}$
$B^- \to K^0K^-$	4 ± 14	$18.82^{+36.93}_{-30.83}$	$B^- \to \eta'\pi^-$	6 ± 16	$4.71^{+59.79}_{-57.97}$
$\bar{B}^0 \to \pi^+\pi^-$	32 ± 4	$35.01^{+3.19}_{-22.29}$	$\bar{B}_s \to \eta K^0$	< 0.1	$0.10^{+0.00}_{-100.07}$
$\bar{B}^0 \to \pi^0\pi^0$	33 ± 22	$-10.58^{+40.69}_{-89.40}$	$\bar{B}_s \to \eta' K^0$	Not available	$-0.58^{+100.57}_{-79.58}$
$\bar{B}^0 \to K^0\bar{K}^0$	-60 ± 70	$-6.88^{+85.39}_{-81.37}$	$B^- \to \eta K^-$	-37 ± 8	$-42.23^{+42.23}_{-16.00}$
$\bar{B}_s \to \pi^-K^+$	22.1 ± 1.5	$20.84^{+2.39}_{-2.57}$	$B^- \to \eta' K^-$	0.4 ± 1.1	$0.63^{+3.98}_{-4.30}$
$B^- \to \pi^0K^-$	3.7 ± 2.1	$3.72^{+7.19}_{-4.35}$	$\bar{B}^0 \to \eta K^0$	Not available	$-0.01^{+40.07}_{-0.02}$
$B^- \to \pi^-K^0$	-1.7 ± 1.6	$-1.08^{+1.76}_{-2.32}$	$\bar{B}^0 \to \eta' K^0$	-6 ± 4	$0.03^{+4.82}_{-11.69}$
$\bar{B}^0 \to \pi^+K^-$	-8.3 ± 0.4	$-8.38^{+8.38}_{-1.01}$	$\bar{B}^0 \to \eta\pi^0$	Not available	$-27.39^{+127.11}_{-72.58}$
$\bar{B}^0 \to \pi^0\bar{K}^0$	0 ± 13	$-0.97^{+19.35}_{-3.20}$	$\bar{B}^0 \to \eta'\pi^0$	Not available	$-43.67^{+143.63}_{-56.33}$
$\bar{B}_s \to K^+K^-$	-14 ± 11	$-10.58^{+10.58}_{-3.60}$	$\bar{B}_s \to \eta\pi^0$	Not available	$0.88^{+94.98}_{-98.70}$
$\bar{B}_s \to \pi^+\pi^-$	Not available	$17.56^{+11.84}_{-38.25}$	$\bar{B}_s \to \eta'\pi^0$	Not available	$1.57^{+77.56}_{-95.66}$
$\bar{B}_s \to \pi^0\pi^0$	Not available	$17.56^{+11.84}_{-38.25}$	$\bar{B}^0 \to \eta\eta$	Not available	$3.46^{+96.50}_{-103.45}$
$\bar{B}_s \to K^0\bar{K}^0$	Not available	$0.31^{+5.07}_{-4.59}$	$\bar{B}_s \to \eta\eta$	Not available	$14.29^{+76.81}_{-113.09}$
$\bar{B}^0 \to K^+K^-$	Not available	$-78.45^{+161.99}_{-20.78}$	$\bar{B}^0 \to \eta'\eta'$	Not available	$42.41^{+57.55}_{-142.41}$
$\bar{B}_s \to \pi^0K^0$	Not available	$13.74^{+29.49}_{-113.73}$	$\bar{B}_s \to \eta'\eta'$	Not available	$-2.05^{+15.29}_{-13.44}$
			$\bar{B}^0 \to \eta'\eta$	Not available	$-12.32^{+112.32}_{-87.67}$
			$\bar{B}_s \to \eta'\eta$	Not available	$3.43^{+96.36}_{-103.22}$

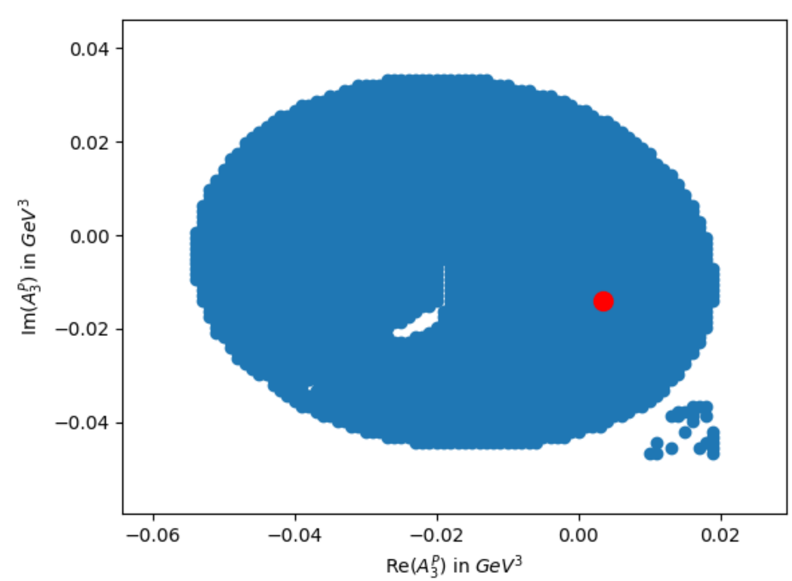
Experimental results from PDG Live

SU(3) Confidence Regions







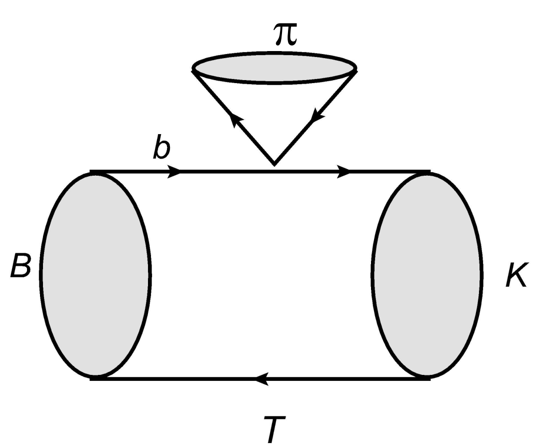


QCF Factorization decomposition

The topological and SU(3) invariant descriptions are just parametrizations of the decay amplitudes

A first principle technique to perform these calculations is QCD-Factorization

Beneke et al: 9905312   *Beneke et al: 0308039*



Naive Factorization

$$\langle K\pi | Q | B \rangle \sim F_{B\to K} f_\pi$$

QCF Factorization decomposition

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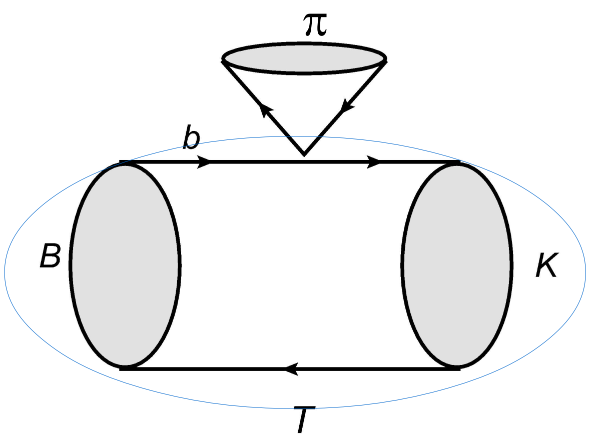

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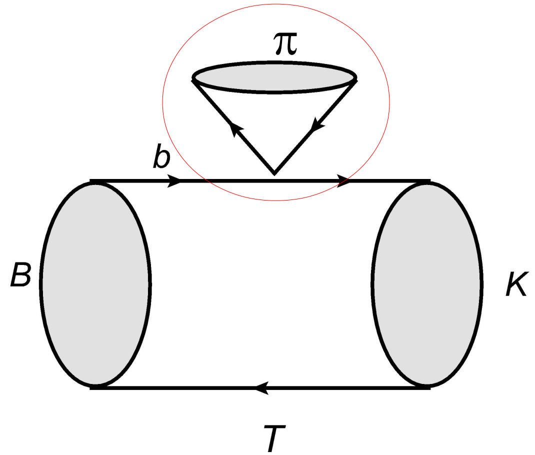


Naive Factorization

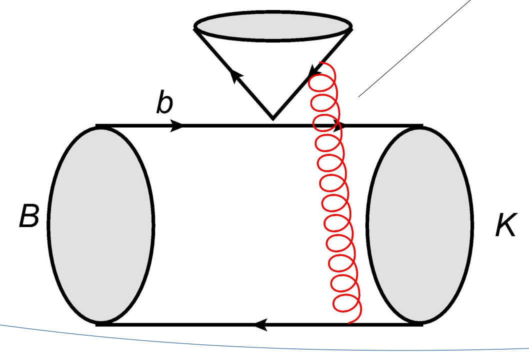
$$\langle K\pi | Q | B \rangle \sim F_{B \to K} f_{\pi}$$

QCF Factorization decomposition

Naive factorization special case of

$$\langle M_1 M_2|\hat{Q}_i|B\rangle = \sum_j F_j^{B\to M_1}(0)\int_0^1 du T_{ij}^I(u)\Phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$

$$+\int_0^1 d\xi du dv T_i^{II}(\xi,u,v)\Phi_B(\xi)\Phi_{M_1}(v)\Phi_{M_2}(u).$$



QCD-Factorization offers a systematic way to disentangle short from long distance physics considering $\Lambda_{QCD} \ll m_b$

QCF Factorization decomposition

$$\mathcal{A}^{\mathrm{QCDF}} = i\frac{G_F}{\sqrt{2}} \sum_{p=u,c} A_{M_1 M_2} \Big\{ BM_1 \left(\alpha_1 \delta_{pu} \hat{U} + \alpha_4^p \hat{I} + \alpha_{4,EW}^p \hat{Q}\right) M_2 \,\Lambda_p$$

$$+ BM_1 \Lambda_p \cdot \mathrm{Tr}\left[\left(\alpha_2 \delta_{pu} \hat{U} + \alpha_3^p \hat{I} + \alpha_{3,EW}^p \,\hat{Q}\right) M_2\right]$$

$$+ B \left(\beta_2 \delta_{pu} \hat{U} + \beta_3^p \hat{I} + \beta_{3,EW}^p \,\hat{Q}\right) M_1 M_2 \Lambda_p$$

$$+ B\Lambda_p \cdot \mathrm{Tr}\left[\left(\beta_1 \delta_{pu} \hat{U} + \beta_4^p \hat{I} + b_{4,EW}^p \,\hat{Q}\right) M_1 M_2\right]$$

$$+ B \left(\beta_{S2} \delta_{pu} \hat{U} + \beta_{S3}^p \hat{I} + \beta_{S3,EW}^p \,\hat{Q}\right) M_1 \Lambda_p \cdot \mathrm{Tr} M_2$$

$$+ B\Lambda_p \cdot \mathrm{Tr}\left[\left(\beta_{S1} \delta_{pu} \hat{U} + \beta_{S4}^p \hat{I} + b_{S4,EW}^p \hat{Q}\right) M_1\right] \cdot \mathrm{Tr} M_2 \Big\}$$

$$\Lambda_p = \begin{pmatrix} 0 \\ \lambda_p^{(d)} \\ \lambda_p^{(s)} \end{pmatrix}, \qquad \hat{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

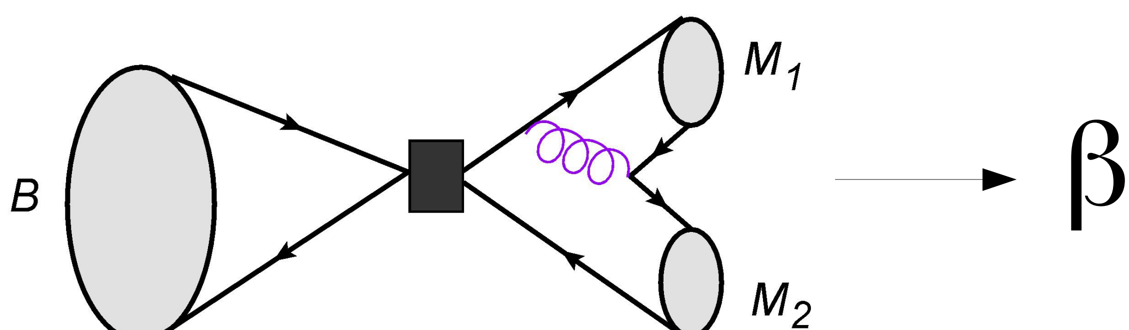
$$\hat{Q} = \frac{3}{2} Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}, \qquad \hat{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$A_{M_1 M_2} = M_B^2 \, F_0^{B \to M_1}(0) \, f_{M_2}$$

Beneke et al: 0308039

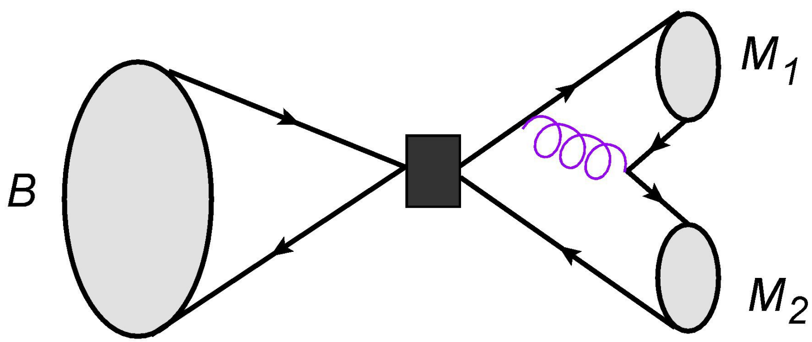
QCF Factorization decomposition

$$
\begin{aligned}
\mathcal{A}^{\mathrm{QCDF}} = i\frac{G_F}{\sqrt{2}} \sum_{p=u,c} A_{M_1 M_2} \Big\{ & BM_1 \left(\alpha_1 \delta_{pu} \hat{U} + \alpha_4^p \hat{I} + \alpha_{4,EW}^p \hat{Q}\right) M_2 \Lambda_p \\
& + BM_1 \Lambda_p \cdot \mathrm{Tr}\left[\left(\alpha_2 \delta_{pu} \hat{U} + \alpha_3^p \hat{I} + \alpha_{3,EW}^p \hat{Q}\right) M_2\right] \\
& + B\left(\beta_2 \delta_{pu} \hat{U} + \beta_3^p \hat{I} + \beta_{3,EW}^p \hat{Q}\right) M_1 M_2 \Lambda_p \\
& + B\Lambda_p \cdot \mathrm{Tr}\left[\left(\beta_1 \delta_{pu} \hat{U} + \beta_4^p \hat{I} + b_{4,EW}^p \hat{Q}\right) M_1 M_2\right] \\
& + B\left(\beta_{S2} \delta_{pu} \hat{U} + \beta_{S3}^p \hat{I} + \beta_{S3,EW}^p \hat{Q}\right) M_1 \Lambda_p \cdot \mathrm{Tr} M_2 \\
& + B\Lambda_p \cdot \mathrm{Tr}\left[\left(\beta_{S1} \delta_{pu} \hat{U} + \beta_{S4}^p \hat{I} + b_{S4,EW}^p \hat{Q}\right) M_1\right] \cdot \mathrm{Tr} M_2 \Big\}
\end{aligned}
$$



Weak annihilation contributions are non-factorizable

QCF Factorization decomposition



$$\longrightarrow \quad \beta$$

Weak annihilation contributions are non-factorizable ⇨ One of the main drawbacks of QCDF

These contributions are power suppressed Λ_{QCD}/m_b

To address this problem educated Ansatz are made

$$X_A = \left(1+\rho_A e^{i\phi_A}\right)\ln\frac{m_B}{\Lambda_h}$$

$$0 < \rho_A < 1$$

$$\Lambda_h \approx \mathcal{O}(\Lambda_{QCD})$$

WHAT CAN WE LEARN ABOUT THE ANNIHILATION CONTRIBUTIONS FROM DATA?

CAN WE PROFIT FROM THE SU(3) INVARIANT FITS?

TO ACHIEVE THIS FIRST ESTABLISH A DICTIONARY BETWEEN SU(3) AND THE QCDF DECOMPOSITION OF THE PHYSICAL AMPLITUDES

QCF Factorization-Topological Equivalence

Equivalence between the QCF and the topological amplitudes

Decompose the matrix *Q* in terms of *U* and *I* $\hat{Q} = \frac{3}{2}\hat{U} - \frac{1}{2}\hat{I}$

Use the $\lambda_u^{(q)}$ and $\lambda_t^{(q)}$ factors $\Lambda_t = -\Lambda_u - \Lambda_c$

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$$\tilde{\hat{C}}_r = \left[\tilde{T} + \frac{3}{2}\tilde{P}_2^u - \frac{3}{2}\tilde{P}_2^c\right]\hat{U} \otimes \Lambda_u + \left[\tilde{P}_1^u - \tilde{P}_1^c - \frac{1}{2}\left\{\tilde{P}_2^u - \tilde{P}_2^c\right\}\right]\hat{I} \otimes \Lambda_u$$

$$-\frac{3}{2}\tilde{P}_2^c\hat{U} \otimes \Lambda_t - \left[\tilde{P}_1^c - \frac{\tilde{P}_2^c}{2}\right]\hat{I} \otimes \Lambda_t,$$

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Equivalence between the QCF and the topological amplitudes

Decompose the matrix *Q* in terms of *U* and *I* $\hat{Q} = \frac{3}{2}\hat{U} - \frac{1}{2}\hat{I}$

Use the $\lambda_u^{(q)}$ and $\lambda_t^{(q)}$ factors $\Lambda_t = -\Lambda_u - \Lambda_c$

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$$-\frac{3}{2}\tilde{P}_2^c\hat{U} \otimes \Lambda_t - \left[\tilde{P}_1^c - \frac{\tilde{P}_2^c}{2}\right]\hat{I} \otimes \Lambda_t,$$

$$(\tilde{\tilde{C}}_r)_k^{ij} = \left[\tilde{T} + \frac{3}{2}\tilde{P}_2^u - \frac{3}{2}\tilde{P}_2^c\right]\hat{U}_k^i(\Lambda_u)^j + \left[\tilde{P}_1^u - \tilde{P}_1^c - \frac{1}{2}\left\{\tilde{P}_2^u - \tilde{P}_2^c\right\}\right]\delta_k^i(\Lambda_u)^j$$

$$-\frac{3}{2}\tilde{P}_2^c\hat{U}_k^i(\Lambda_t)^j - \left[\tilde{P}_1^c - \frac{\tilde{P}_2^c}{2}\right]\delta_k^i(\Lambda_t)^j$$

QCF Factorization-Topological Equivalence

Equivalence between the QCF and the topological amplitudes

Decompose the matrix *Q* in terms of *U* and *I* $\hat{Q} = \frac{3}{2}\hat{U} - \frac{1}{2}\hat{I}$

Use the $\lambda_u^{(q)}$ and $\lambda_t^{(q)}$ factors $\Lambda_t = -\Lambda_u - \Lambda_c$

$$\tilde{\tilde{C}}_r = \left[\tilde{T} + \frac{3}{2}\tilde{P}_2^u - \frac{3}{2}\tilde{P}_2^c\right]\hat{U}\otimes\Lambda_u + \left[\tilde{P}_1^u - \tilde{P}_1^c - \frac{1}{2}\left\{\tilde{P}_2^u - \tilde{P}_2^c\right\}\right]\hat{I}\otimes\Lambda_u$$

$$-\frac{3}{2}\tilde{P}_2^c\hat{U}\otimes\Lambda_t - \left[\tilde{P}_1^c - \frac{\tilde{P}_2^c}{2}\right]\hat{I}\otimes\Lambda_t,$$

$$(\tilde{\tilde{C}}_r)^{ij}_k = \left[\tilde{T} + \frac{3}{2}\tilde{P}_2^u - \frac{3}{2}\tilde{P}_2^c\right]\hat{U}^i_k(\Lambda_u)^j + \left[\tilde{P}_1^u - \tilde{P}_1^c - \frac{1}{2}\left\{\tilde{P}_2^u - \tilde{P}_2^c\right\}\right]\delta^i_k(\Lambda_u)^j$$

$$-\frac{3}{2}\tilde{P}_2^c\hat{U}^i_k(\Lambda_t)^j - \left[\tilde{P}_1^c - \frac{\tilde{P}_2^c}{2}\right]\delta^i_k(\Lambda_t)^j$$

The connection between the topological decomposition and the QCD-factorization is established through

$$U^i_k(\Lambda_u)^j = \bar{H}^{ij}_k, \qquad U^i_k(\Lambda_t)^j = \tilde{H}^{ij}_k, \qquad (\Lambda_t)^i = \tilde{H}^i.$$

QCF Factorization-Topological Equivalence

We consider the following results

$$\alpha_3^u = \alpha_3^c = \alpha_3, \quad \alpha_{3,EW}^u = \alpha_{3,EW}^c = \alpha_{3,EW}, \quad \beta_i^u = \beta_i^c = \beta_i, \quad b_i^u = b_i^c = b_i$$

$$|\alpha_{4,EW}^c - \alpha_{4,EW}^u| < 10^{-3} \qquad |\alpha_4^c - \alpha_4^u| \sim 2\%$$

NLO

NNLO

Bell, Beneke, Huber, Li:2002.03262

QCDF to topological transformation rules

$$T = A_{M_1M_2}\alpha_1, \quad C = A_{M_1M_2}\alpha_2, \quad E = A_{M_1M_2}\beta_1,$$

$$A = A_{M_1M_2}\beta_2, \quad T_{AS} = A_{M_1M_2}\beta_{S1}, \quad T_{ES} = A_{M_1M_2}\beta_{S2},$$

$$S = -A_{M_1M_2}\left[\alpha_3 + \beta_{S3} - \frac{\alpha_{3,EW}}{2} - \frac{\beta_{S3,EW}}{2}\right],$$

$$P = -A_{M_1M_2}\left[\alpha_4^c + \beta_3 - \frac{\alpha_{4,EW}^c}{2} - \frac{\beta_{3,EW}}{2}\right],$$

$$A_{M_1M_2} = (1.25 \pm 0.17)\ \mathrm{GeV}^3$$

Further details on the χ^2-fit

Best QCDF fit point (modulus in GeV^3)

$$A_{M_1 M_2}\alpha_1 = 1.072 + 5.596\times 10^{-5} i, \qquad A_{M_1 M_2}\alpha_2 = 0.136 + 0.073i,$$

$$A_{M_1 M_2}\beta_1 = -0.117 - 0.007i, \qquad A_{M_1 M_2}\beta_2 = A_{M_1 M_2}\beta_1,$$

$$A_{M_1 M_2}\beta_{S1} = -0.074 - 0.0112i, \qquad A_{M_1 M_2}\beta_{S2} = 0.054 - 0.049i,$$

$$A_{M_1 M_2}\alpha_{3,EW} = -0.193 - 0.045i, \qquad A_{M_1 M_2}\alpha^c_{4,EW} = 0.181 + 0.053i,$$

$$A_{M_1 M_2}\beta_{3,EW} = 0.005 - 0.006i, \qquad A_{M_1 M_2}b_{4,EW} = A_{M_1 M_2}\beta_{3,EW},$$

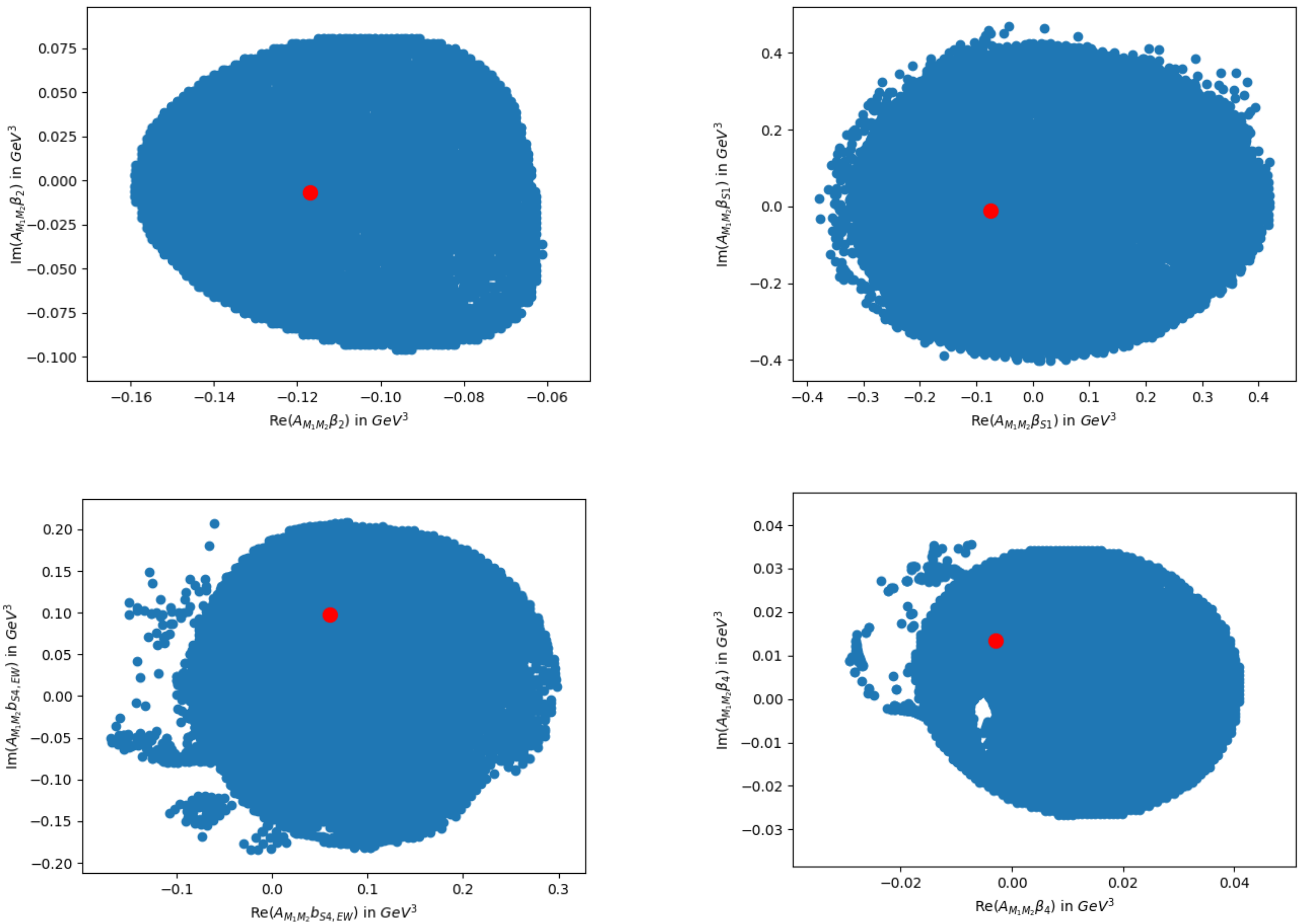
$$A_{M_1 M_2}\beta_{S3,EW} = -0.188 + 0.007i, \qquad A_{M_1 M_2}b_{S4,EW} = 0.061 + 0.098i,$$

$$A_{M_1 M_2}\beta_4 = -0.003 + 0.013i, \qquad A_{M_1 M_2}\beta_{S4} = 0.031 - 0.030i,$$

$$A_{M_1 M_2}\left(\alpha_3 + \beta_{S3}\right) = 0.230 + 0.067i, \qquad A_{M_1 M_2}\left(\alpha_4 + \beta_3\right) = -0.242 - 0.062i$$

Obtained by mapping the SU(3)-fit results into the QCDF amplitudes.

QCF Factorization confidence regions



Summary and Outlook

- We have established a set of transformation rules between the QCD factorization and the topological representation of physical amplitudes.

- By fitting to data we have determined bounds for different QCDF amplitudes.

- The real and imaginary components of the weak annihilation amplitudes as allowed by data can be between 4% and 30%.

- SU(3) symmetry asummed so far.

- Introduce SU(3) breaking by fitting to data the weak annihilation amplitudes combining NLO and NNLO results for independent channels

Further details on the χ^2-fit

Constraints from QCDF

Taking into account $\alpha_1(\pi\pi) = 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i.$

Beneke Huber et al: 0911.3655

We impose $\Re(\alpha_1) = 1.000^{+0.138}_{-0.138}$

In addition we require

$$T_{PA} = T_{SS} = T_S = 0, \quad |T_P| < 10\%$$

Phenomenological constraints

$$Br(B_s \to \pi^0\pi^0) < 2.10 \times 10^{-4}, \qquad Br(B_s \to \eta\pi^0) < 10^{-3},$$
$$Br(B^0 \to \eta\eta) < 10^{-6}, \qquad Br(B^0 \to \eta'\eta') < 1.7 \times 10^{-6},$$
$$Br(B^0 \to \eta'\eta) < 1.2 \times 10^{-6}, \qquad A_{CP}(B_s \to \eta K^0) < 10^{-3}.$$

PDG Live

SU(3) amplitudes from data

Include η contributions in the Feldmann–Kroll–Stech scheme

θ_{FKS} mixing angle *T. Feldmann et al: 9802409*

Channel	A_3^T	C_{3T}^T	A_6^T	C_6^T	A_{15}^T	C_{15}^T	B_3^T	B_6^T	B_{15}^T	D_3^T
$B^- \to \eta_q \pi^-$	0	$\sqrt{2}$	$\sqrt{2}$	0	$3\sqrt{2}$	$2\sqrt{2}$	0	$\sqrt{2}$	$3\sqrt{2}$	$\sqrt{2}$
$B^- \to \eta_s \pi^-$	0	0	0	1	0	-1	0	1	3	1
$B^0 \to \eta_q \pi^0$	0	-1	-1	0	5	2	0	-1	5	-1
$B^0 \to \eta_s \pi^0$	0	0	0	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	$\frac{5}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$B_s \to \eta_q K^0$	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$
$B_s \to \eta_s K^0$	0	1	-1	0	-1	-2	0	-1	-1	1
$B^- \to \eta_q K^-$	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{\sqrt{2}}$	$\frac{5}{\sqrt{2}}$	0	$\sqrt{2}$	$3\sqrt{2}$	$\sqrt{2}$
$B^- \to \eta_s K^-$	0	1	1	0	3	-2	0	1	3	1
$B^0 \to \eta_q K^0$	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$
$B^0 \to \eta_s K^0$	0	1	-1	0	-1	-2	0	-1	-1	1
$B_s \to \eta_q \pi^0$	0	0	-2	0	4	0	0	-2	4	0
$B_s \to \eta_s \pi^0$	0	0	0	$-\sqrt{2}$	0	$2\sqrt{2}$	0	$-\sqrt{2}$	$2\sqrt{2}$	0
$B^0 \to \eta_q \eta_q$	1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2	-1	1	1
$B^0 \to \eta_q \eta_s$	0	0	0	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	$2\sqrt{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$B^0 \to \eta_s \eta_s$	1	0	1	0	-1	0	1	1	-1	0
$B_s \to \eta_q \eta_q$	1	0	0	0	1	0	2	0	2	0
$B_s \to \eta_q \eta_s$	0	0	0	0	0	$\sqrt{2}$	$2\sqrt{2}$	0	$-\sqrt{2}$	$\sqrt{2}$
$B_s \to \eta_s \eta_s$	1	1	0	0	-2	-2	1	0	-2	1