An accurate evaluation of (anti-)neutrino scattering on nucleons

based on
G. Ricciardi, F. Vissani, NV
to appear

Natascia Vignaroli

University of Naples
"Federico II" and INFN Naples

FPCAPRI 2022 12/06/2022

Motivation

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad \text{(IBD)} \qquad \nu_e + n \rightarrow e^- + p$$

Knowing precisely the cross section of these processes is **essential**

at relatively low neutrino energies, for detectors that rely on water or Hydrocarbons (scintillators, Cherenkov light detectors).

Most accurate estimates available at the moment were obtained almost 20 years ago: Beacon, Vogel (1999) and Strumia, Vissani (2003) Phys. Lett. B 564

We aim to **update these results**:

Both the **central values** and the **uncertainties**

Motivation

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad \text{(IBD)} \qquad \nu_e + n \rightarrow e^- + p$$

Knowing precisely the cross section of these processes is **essential**

at relatively low neutrino energies, for detectors that rely on water or Hydrocarbons (scintillators, Cherenkov light detectors).

Most accurate estimates available at the moment were obtained almost 20 years ago: Beacon, Vogel (1999) and Strumia, Vissani (2003)

We aim to **update these results**:

Both the **central values** and the **uncertainties**

↓

We will try to obtain the most accurate values considering the recent progresses, in particular from neutrino detector experiments

Motivation

$$\bar{\nu}_e + p \to e^+ + n \quad \text{(IBD)} \qquad \nu_e + n \to e^- + p$$

Knowing precisely the cross section of these processes is **essential**

at relatively low neutrino energies, for detectors that rely on water or Hydrocarbons (scintillators, Cherenkov light detectors).

Most accurate estimates available at the moment were obtained almost 20 years ago: Beacon, Vogel (1999) and Strumia, Vissani (2003)

We aim to **update these results**:

Both the **central values** and the **uncertainties**

↓

Precision crucial for high-statistic experiments: reactor neutrinos as *Daya Bay*, supernova detectors as *SuperKamioKande*
Especially for the future ones: *Juno, HyperKamiokande*

The calculation: matrix element and weak hadronic currents

$$\mathcal{M} = \bar{v}_\nu \gamma^a (1-\gamma_5) v_e \cdot$$

$$\bar{u}_n \left(f_1 \gamma_a + g_1 \gamma_a \gamma_5 + i f_2 \sigma_{ab} \frac{q^b}{2M} + g_2 \frac{q_a}{M} \gamma_5 + f_3 \frac{q_a}{M} + i g_3 \sigma_{ab} \frac{q^b}{2M} \gamma_5 \right) u_p$$

The calculation: matrix element and weak hadronic currents

$$\mathcal{M} = \bar{v}_\nu \gamma^a (1-\gamma_5) v_e \cdot$$

$$\bar{u}_n \left(f_1 \gamma_a + g_1 \gamma_a \gamma_5 + i f_2 \sigma_{ab} \frac{q^b}{2M} + g_2 \frac{q_a}{M} \gamma_5 + f_3 \frac{q_a}{M} + i g_3 \sigma_{ab} \frac{q^b}{2M} \gamma_5 \right) u_p$$

$$\{f_1, f_2\} = \frac{\{1-(1+\xi)t/4M^2, \xi\}}{(1-t/4M^2)(1-t/M_V^2)^2} \qquad \xi = k_p - k_n = 3.706$$

This is preferable (and more accurate) for the energies we are interested in

$$\frac{F(Q^2)}{F(0)} \equiv 1 - \frac{\langle r^2 \rangle \, Q^2}{6} + \mathcal{O}(Q^4)$$

Y.H.Lin, H.W. Hammer and U. G. Meissner, Eur. Phys. J. A 57 (2021) no.8, 255

$$\sqrt{\langle r_1^{v\,2} \rangle} = 0.751^{+0.002}_{-0.001}{}^{+0.002}_{-0.003} \text{ fm} \qquad \sqrt{\langle r_2^{v\,2} \rangle} = 0.880 \pm 0.001 \pm 0.003 \text{ fm}$$

Known with precision (negligible source of uncertainty to the cross section)

The calculation: matrix element and weak hadronic currents

$$\mathcal{M} = \bar{v}_\nu \gamma^a (1-\gamma_5) v_e \cdot$$

$$\bar{u}_n \left(f_1 \gamma_a + g_1 \gamma_a \gamma_5 + i f_2 \sigma_{ab} \frac{q^b}{2M} + g_2 \frac{q_a}{M} \gamma_5 + f_3 \frac{q_a}{M} + i g_3 \sigma_{ab} \frac{q^b}{2M} \gamma_5 \right) u_p$$

Large source of uncertainty to the cross section from the axial form factor g_1

The calculation: matrix element and weak hadronic currents

$$\mathcal{M} = \bar{v}_\nu \gamma^a (1-\gamma_5) v_e \cdot$$

$$\bar{u}_n \left(f_1 \gamma_a + g_1 \gamma_a \gamma_5 + i f_2 \sigma_{ab} \frac{q^b}{2M} + g_2 \frac{q_a}{M} \gamma_5 + f_3 \frac{q_a}{M} + i g_3 \sigma_{ab} \frac{q^b}{2M} \gamma_5 \right) u_p$$

SCC

G-parity $G = C e^{i\pi I_2}$

$GV_\mu G^{-1} = V_\mu \,, \quad GA_\mu G^{-1} = -A_\mu$ First Class

$GV_\mu G^{-1} = -V_\mu \,, \quad GA_\mu G^{-1} = A_\mu$ Second Class

SCC (recently discussed by Ankovski; Giunti; Ivanov) previously neglected for neutrino-nucleon cross section.

Are they relevant?

The calculation: cross section (IBD)

$$\frac{d\sigma}{dt} = \frac{G_F^2 \boxed{\cos^2\theta_C}}{64\pi(s-m_p^2)^2}\overline{|\mathcal{M}^2|}$$

$$\overline{|\mathcal{M}^2|} = A_{\bar\nu}(t) - (s-u)B_{\bar\nu}(t) + (s-u)^2 C_{\bar\nu}(t)$$

$$\Delta = m_n - m_p \approx 1.293\,\mathrm{MeV} \qquad M = \frac{m_n+m_p}{2} \approx 938.9\,\mathrm{MeV}$$

$$\begin{aligned}
A_{\bar\nu} = (t-m_e^2)\Big[&8|f_1^2|(4M^2+t+m_e^2) + 8|g_1^2|(-4M^2+t+m_e^2) + 2|f_2^2|(t^2/M^2+4t+4m_e^2)\\
&+8m_e^2 t|g_2^2|/M^2 + 16\mathrm{Re}[f_1^* f_2](2t+m_e^2) + 32m_e^2\mathrm{Re}[g_1^* g_2]\Big]\\
-\Delta^2\Big[&(8|f_1^2| + 2t|f_2^2|/M^2)(4M^2+t-m_e^2) + 8|g_1^2|(4M^2-t+m_e^2) + 8m_e^2|g_2^2|(t-m_e^2)/M^2\\
&+16\mathrm{Re}[f_1^* f_2](2t-m_e^2) + 32m_e^2\mathrm{Re}[g_1^* g_2]\Big] - 64m_e^2 M\Delta\mathrm{Re}[g_1^*(f_1+f_2)] + A_{SC}
\end{aligned}$$

$$B_{\bar\nu} = 32t\mathrm{Re}[g_1^*(f_1+f_2)] + 8m_e^2\Delta(|f_2^2| + \mathrm{Re}[f_1^* f_2 + 2g_1^* g_2])/M + B_{SC}$$

$$C_{\bar\nu} = 8(|f_1^2| + |g_1^2|) - 2t|f_2^2|/M^2 + C_{SC}$$

The calculation: cross section (IBD)

For the SCC contribution (not considered in Strumia, Vissani 2003)
we find:

$$A_{SC} = -2\,t\,(4 - t/M^2)\left[4m_e^2|f_3^2| + |g_3^2|(t - m_e^2) + 8\Delta M \mathrm{Re}[g_3^* g_1] + \Delta^2|g_3^2|\right]$$
$$+\mathcal{O}(\Delta^3 M) + \mathcal{O}(\Delta\, m_e^2\, t/M) + \mathcal{O}(m_e^4)$$

$$B_{SC} = 8\,m_e^2\left[4\,\mathrm{Re}[f_1^* f_3] + \mathrm{Re}[f_2^* f_3]\,t/M^2 + 2\,\mathrm{Re}[g_1^* g_3] + \mathrm{Re}[g_2^* g_3]\,t/M^2 + \Delta|g_3^2|/M\right]$$
$$+16\Delta\mathrm{Re}[(f_1^* + f_2^*)g_3]\,t/M$$

$$C_{SC} = -2|g_3^2|\,t/M^2$$

Given this form, we expect the SCC contribution to be potentially
relevant only at higher energies → and for g_3 in particular

The calculation: cross section (ν on neutron)

$$\frac{d\sigma}{dt}(\nu n \to e^- p) = \frac{G_F^2 \cos^2\theta_C}{64\pi(s-m_p^2)^2}\left[A_\nu(t) + (s-u)B_\nu(t) + (s-u)^2 C_\nu(t)\right]$$

$$A_\nu = A_{\bar\nu} + 16\frac{\Delta}{M}\Big(8\mathrm{Re}[f_1^* f_3]M^2 m_e^2 + 2\mathrm{Re}[f_1^* f_3]m_e^4 - 2\mathrm{Re}[f_1^* f_3]m_e^2 t + 4\Delta\mathrm{Re}[f_1^* g_3]M m_e^2$$

$$+2\mathrm{Re}[f_2^* f_3]m_e^4 + 4\Delta\mathrm{Re}[f_2^* g_3]M m_e^2 - 4\mathrm{Re}[g_1^* g_3]M^2 m_e^2 + 8\mathrm{Re}[g_1^* g_3]M^2 t$$

$$+\mathrm{Re}[g_1^* g_3]m_e^4 + \mathrm{Re}[g_1^* g_3]m_e^2 t - 2\mathrm{Re}[g_1^* g_3]t^2 + 2\mathrm{Re}[g_2^* g_3]m_e^4\Big)$$

$$B_\nu = B_{\bar\nu} - 64\mathrm{Re}[f_1^* f_3]m_e^2 - 32\frac{\Delta}{M}\mathrm{Re}[f_1^* g_3]t - 16\frac{m_e^2}{M^2}\mathrm{Re}[f_2^* f_3]t - 32\frac{\Delta}{M}\mathrm{Re}[f_2^* g_3]t$$

$$-32\mathrm{Re}[g_1^* g_3]m_e^2 - 16\frac{m_e^2}{M^2}\mathrm{Re}[g_2^* g_3]t$$

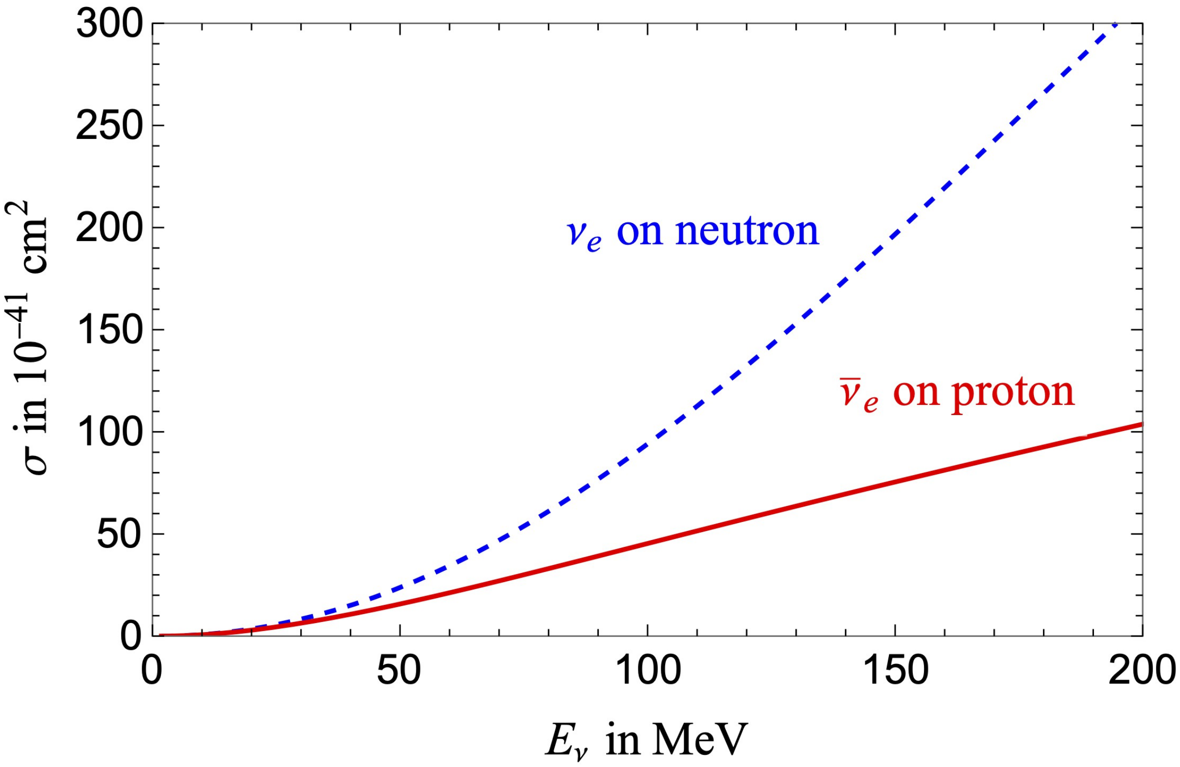
$$C_\nu = C_{\bar\nu}$$

We include radiative corrections (A. Kurylov, M. J. Ramsey-Musolf and P. Vogel, PRC 67(2003), 035502)

$$d\sigma(E_\nu, E_e) \to d\sigma(E_\nu, E_e)\left[1 + \frac{\alpha}{\pi}\left(6.00 + \frac{3}{2}\log\frac{m_p}{2E_e} + 1.2\left(\frac{m_e}{E_e}\right)^{1.5}\right)\right]$$

and Sommerfeld corrections

$$F(E_e) = \frac{\eta}{1 - \exp(-\eta)}, \text{ with } \eta = \frac{2\pi\alpha}{\sqrt{1 - m_e^2/E_e^2}}$$



Uncertainty at low energy

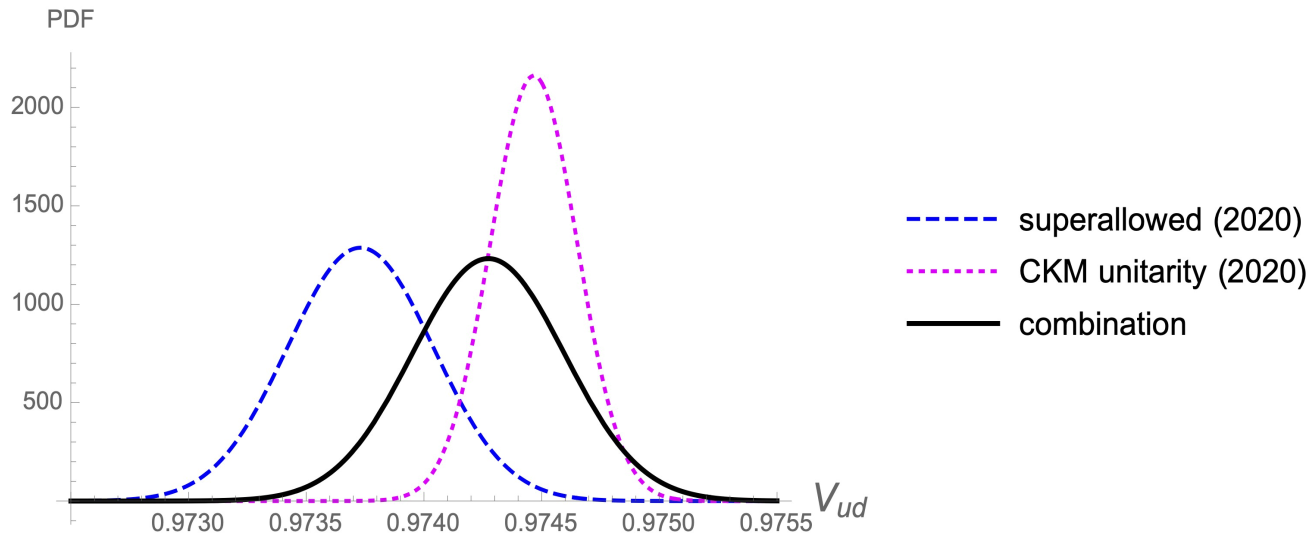
$$\frac{d\sigma}{dt} = \frac{G_F^2 \cos^2\theta_C}{64\pi(s-m_p^2)^2}\overline{|\mathcal{M}^2|}$$

Cabibbo angle

Super-allowed charged current transitions → $V_{\rm ud}(\text{s.a.}) = 0.9737(3)$

$V_{\rm us} = 0.2245(8)$ $V_{\rm ub} = 3.82(24)\times 10^{-3}$ → $V_{\rm ud}(\text{unit}) = 0.9745(2)$

CKM unitarity



$$V_{\rm ud} = 0.9743(3)$$

Uncertainty at low energy

Axial coupling

$$\lambda = -\frac{g_1(0)}{f_1(0)}$$

Direct measure from polarized neutrons

We include the last most precise measure from Perkeo III (2019) and the 8 previous measurements, also the four obtained before 2002 (which have potential systematic problems, but enlarging their errors by a factor of 2)

PDG prescription to combine different measures not perfectly consistent with each other
All errors are enlarged by the scale factor S

$$S = \sqrt{\frac{\chi^2_{\rm min}}{N-1}}$$

$$\lambda = 1.2760(5)$$

Uncertainty at low energy

Neutron lifetime (constraint)

$$\frac{1}{\tau_{\rm n}} = \frac{V_{\rm ud}^2\,(1+3\lambda^2)}{4906.4 \pm 1.7{\rm s}}$$

$$\tau_{\rm n}({\rm SM}) = 878.38 \pm 0.89\ {\rm s}$$

Two methods for τ_n measurements:

1) ultra-cold neutrons are trapped and their number is measured over time (*tot*)
2) using beam neutrons, single channel decay rates are measured (*beam*)

$$\tau_{\rm n}({\rm tot}) = 878.52 \pm 0.46\ {\rm s}\ (N = 9 \text{ and } S = 1.8)$$

$$\tau_{\rm n}({\rm beam}) = 888.0 \pm 2.0\ {\rm s}\ (N = 2 \text{ and } S = 0.3)$$

Uncertainty at low energy

Neutron lifetime (constraint)

$$\frac{1}{\tau_{\mathrm{n}}} = \frac{V_{\mathrm{ud}}^2\,(1+3\lambda^2)}{4906.4 \pm 1.7\mathrm{s}}$$

$$\tau_{\mathrm{n}}(\mathrm{SM}) = 878.38 \pm 0.89\ \mathrm{s}$$

Two methods for τ_n measurements:

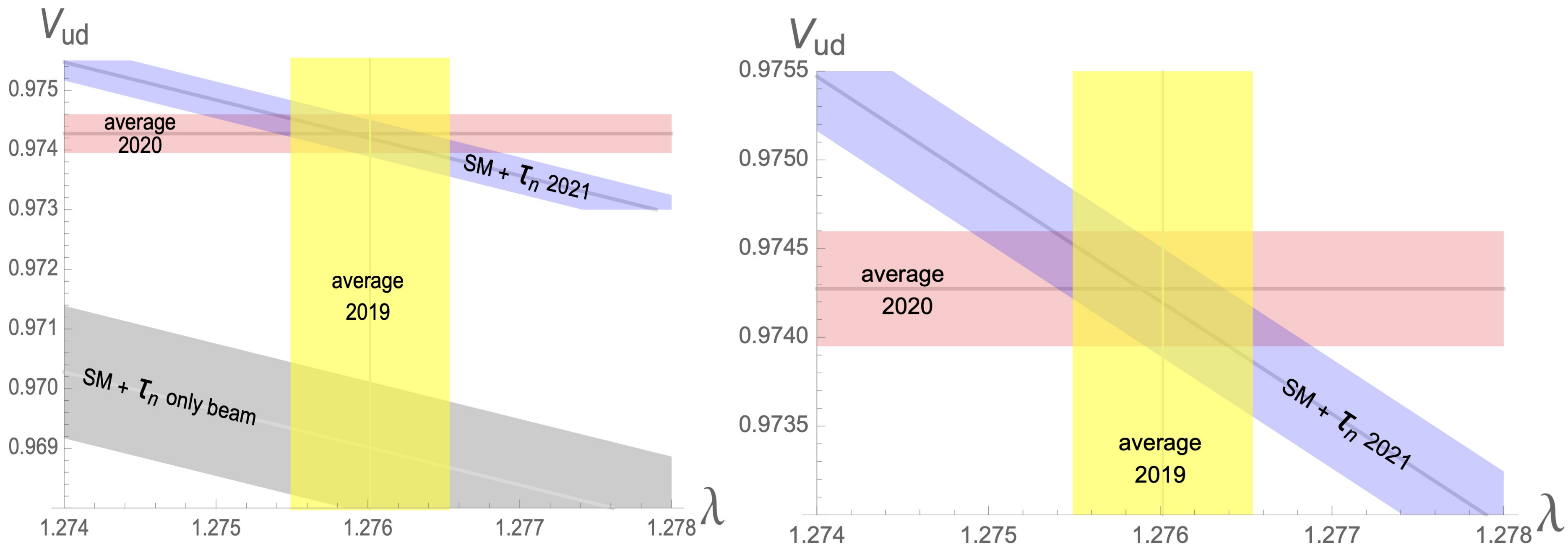
1) ultra-cold neutrons are trapped and their number is measured over time (*tot*)
2) using beam neutrons, single channel decay rates are measured (*beam*)

$$\tau_{\mathrm{n}}(\mathrm{tot}) = 878.52 \pm 0.46\ \mathrm{s}\ (N = 9 \text{ and } S = 1.8)$$

$$\tau_{\mathrm{n}}(\mathrm{beam}) = 888.0 \pm 2.0\ \mathrm{s}\ (N = 2 \text{ and } S = 0.3)$$

Uncertainty at low energy

Combination



We decide to not including beam measurements, and we exploit the resulting relation:

$$V_{\rm ud} = \frac{2.36323(75)}{\sqrt{1+3\lambda^2}}$$ (blue band)

Uncertainty at low energy

NO CORRELATION
(HYPER-CONSERVATIVE)

$$\begin{cases} V_{\rm ud} = 0.97427(32) \\ \lambda = 1.27601(52) \\ \rho = 0 \end{cases}$$

$\delta\sigma(V_{\rm ud}) = 0.66$ ‰

$\delta\sigma(\lambda) = 0.68$ ‰

$\delta\sigma = 0.94$ ‰

USING τ_n INFORMATION
(FULL TREATMENT)

$$\begin{cases} V_{\rm ud} = 0.97425(26) \\ \lambda = 1.27597(42) \\ \rho = -0.53 \end{cases}$$

$\delta\sigma(V_{\rm ud}) = 0.53$ ‰

$\delta\sigma(\lambda) = 0.55$ ‰

$\delta\sigma = 0.52$ ‰

Strumia, Vissani (2003) estimated δσ = 0.4%

Improvement by at least a factor 4!

Uncertainty at high energy

Dipole approximation

$$\{f_1, f_2\} = \frac{\{1 - (1+\xi)t/4M^2, \xi\}}{(1 - t/4M^2)(1 - t/M_V^2)^2}\,, \quad g_1 = \frac{g_1(0)}{(1 - t/M_A^2)^2}\,, \quad g_2 = \frac{2M^2 g_1}{m_\pi^2 - t}$$

Expansion in terms of radii

$$\frac{g_1(Q^2)}{g_1(0)} \equiv 1 - \frac{\langle r_{\rm A}^2\rangle\, Q^2}{6} + \mathcal{O}(Q^4)$$

$$M_{\rm A}^2 \equiv -2\frac{{g_1}'(0)}{g_1(0)} = \frac{12}{\langle r_{\rm A}^2\rangle}$$

Uncertainty at high energy: g_1

A. Bodek, S. Avvakumov, R. Bradford, and H. Budd,
Eur. Phys. J. C 53, 349-354 (2008)

$Experiment$ [13] ν_μd $\rightarrow \mu^-$ p p_s	QE evnts	Q^2 range GeV/c^2	$\overline{E}_\nu$ GeV	Vector FF used [7,8]	$-g_a, M_V^2$ used	M_A (published)	ΔM_A FF,RC	$M_A^{updated}$ GeV/c^2
$Mann_{73}$	166	$.05-1.6$	0.7	$Bartl, G_{en}=0$	$1.23,.84^2$	$0.95 \pm .12$		
$Barish_{77}$	500	$.05-1.6$	0.7	$Ollsn, G_{en}=0$	$1.23,.84^2$	$0.95 \pm .09$	$-.026, .002$	
$Miller_{82,77,73}$	1737	$.05-2.5$	0.7	$Ollsn, G_{en}=0$	$1.23,.84^2$	$1.00 \pm .05$	$-.030, .002$	$0.972 \pm .05$
$Baker_{81}$	1138	$.06-3.0$	1.6	$Ollsn, G_{en}=0$	$1.23,.84^2$	$1.07 \pm .06$	$-.028, .002$	$1.044 \pm .06$
$Kitagaki_{83}$	362	$.11-3.0$	20	$Ollsn, G_{en}=0$	$1.23,.84^2$	$1.05^{+.12}_{-.16}$	$-.025, .001$	$1.026^{+.12}_{-.16}$
$Kitagaki_{90}$	2544	$.10-3.0$	1.6	$Ollsn, G_{en}=0$	$1.254,.84^2$	$1.070^{+.040}_{-0.045}$	$-.036, .002$	$1.036^{+.040}_{-0.045}$
$Allasia_{90}$	552	.1-3.75	20	$dipole, G_{en}=0$	$1.2546, .84^2$	$1.080 \pm .08$	$-.080, .002$	$1.002 \pm .08$
Av. ν_μd [?,13]	5780	above		$BBBA2007_{25}$	1.267, .71	$1.051 \pm .026$	$\theta_\mu^-, E_\mu, \theta, P_p$	$1.016 \pm .026$
π $electrprd.$ [12]								$1.014 \pm .016$
$\overline{\nu}_\mu$H $\rightarrow \mu^-$ n [14]	13	0-1.0	1.1	$dipole, G_{en}=0$	$1.23, .84^2$	0.9 ± 0.35	$-.070, 0.01$	$.831 \pm 0.35$
$\overline{\nu}_\mu$H $\rightarrow \mu^-$ n [14]	13	0-1.0	1.1	$BBBA2007_{25}$	1.267, .71	σ_{QE}	θ_μ^+, E_μ	1.04 ± 0.40
$Average\ all$								$1.014 \pm .014$

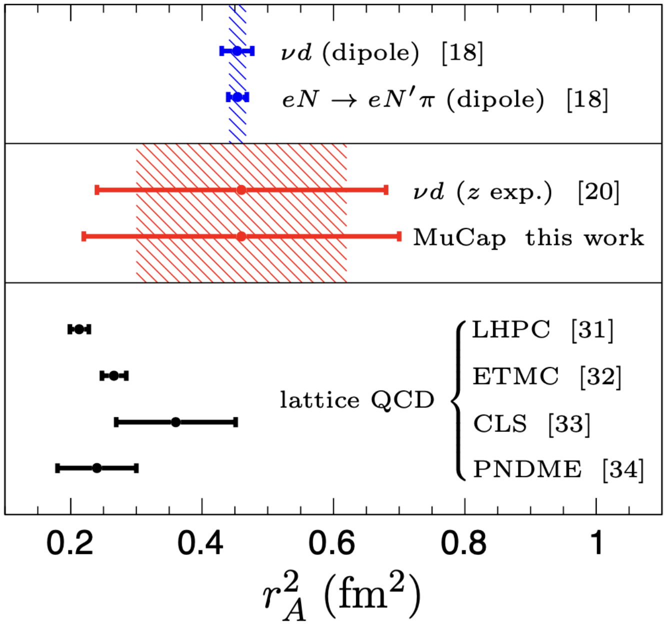
$$r_A^2 = 0.454 \pm 0.012\ \text{fm}^2$$

Uncertainty at high energy

R. J. Hill, P. Kammel, W. J. Marciano, and A. Sirlin,
Rept. Prog. Phys. 81 (2018) no.9, 096301

$$g_1(q^2) = \frac{1}{\pi}\int_{t_{cut}}^{\infty} dt' \frac{\mathrm{Im} g_1(t'+i0)}{t'-q^2} \qquad z(q^2, t_{cut}, t_0) = \frac{\sqrt{t_{cut}-q^2}-\sqrt{t_{cut}-t_0}}{\sqrt{t_{cut}-q^2}+\sqrt{t_{cut}-t_0}} \qquad g_1(q^2) = \sum_{k=0}^{k_{max}} a_k\, z(q^2)^k$$

Description	r_A^2 (fm^2)	Source/Reference
νd (dipole)	0.453(23)	[18]
$eN \to eN'\pi$ (dipole)	0.454(14)	[18]
average	0.454(13)	
νC (dipole)	0.26(7)	[21]
νd (z exp.)	0.46(22)	[20]
MuCap	0.46(24)	this work
average	0.46(16)	
lattice QCD	0.213(6)(13)(3)(0)	[31]
	0.266(17)(7)	[32]
	$0.360(36)^{+80}_{-88}$	[33]
	0.24(6)	[34]



$$r_A^2 = \begin{cases} 0.454 \pm 0.012\ \mathrm{fm}^2 \\ \boxed{0.46 \pm 0.16\ \mathrm{fm}^2} \end{cases}$$

Uncertainty at high energy

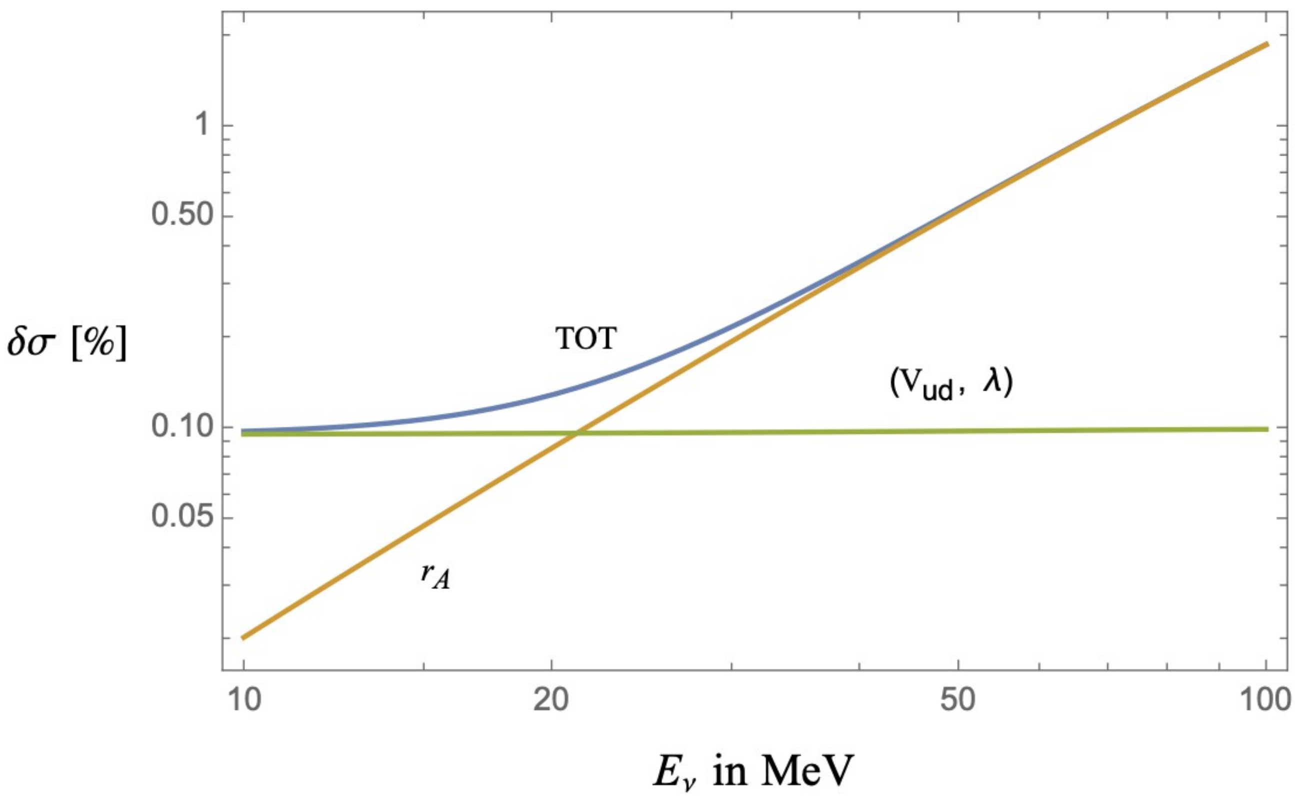
Induced error from g_2 is negligible

Expected, since g_2 enters in the cross section always with a suppression factor m^2_e.
If we consider a variation by an order of magnitude in g_2, the corresponding modification of the cross section is below the 0.1 permil level

SCC can be safely neglected

Taking into account the largest possible values for the SCC form factors compatible with data, f_3=4.4 f_1 and g_3= 0.4 g_1 (M. Day and K. S. McFarland, Phys. Rev. D 86 (2012), 053003), we evaluate that
the contributions to the total cross section are at most at the level of 0.3‰ (Ev/50 MeV), and, as expected, are dominated by g_3

Summary of uncertainties (conservative)



Positron spectrum in Super-Kamiokande

$$\frac{dS_e}{dE_e} = N_p \int_{E_\nu^{max}}^{E_\nu^{min}} dE_\nu \frac{dF}{dE_\nu}(E_\nu) \frac{d\sigma}{dE_e}(E_\nu, E_e)\, \epsilon(E_e)$$

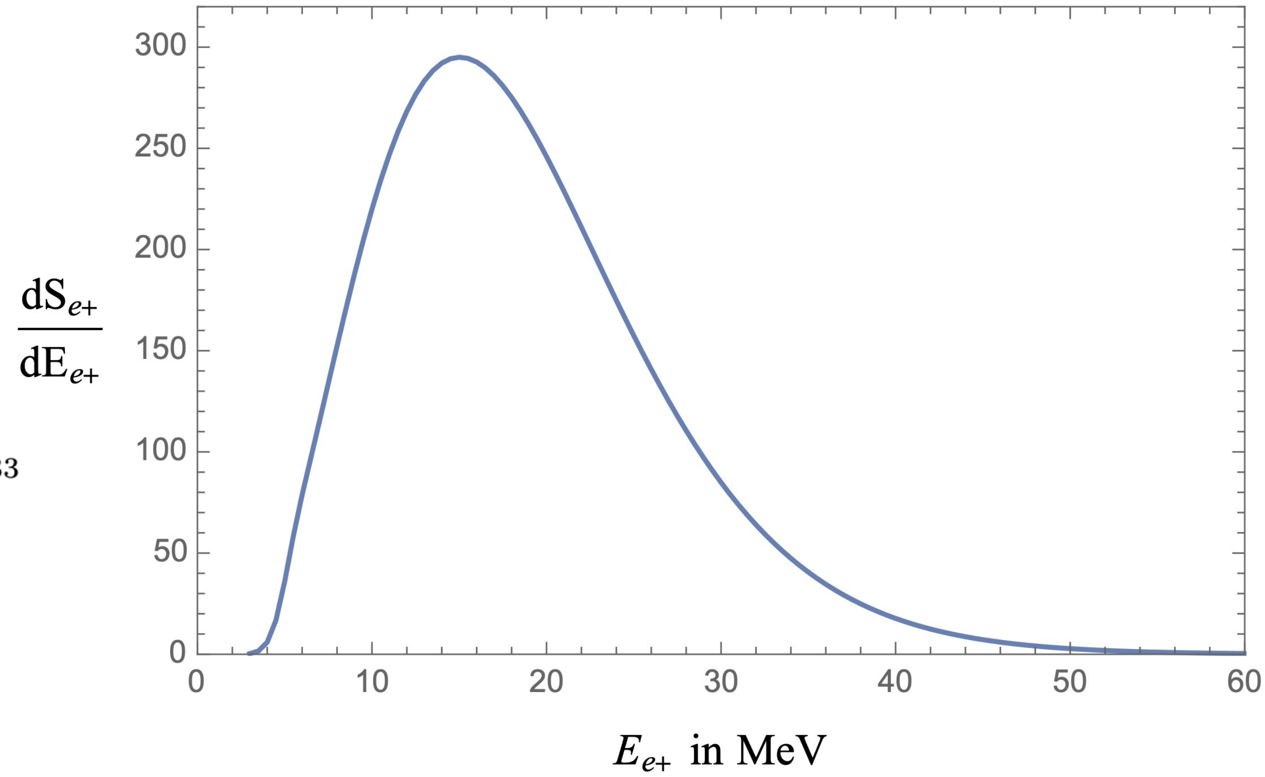
Vissani, J. Phys. G 42, 013001 (2015);
Vissani, Rosso, Symmetry 13, no.10, 1851 (2021)

$$\frac{dF}{dE_\nu} = \frac{\varepsilon}{4\pi D^2} \frac{E_\nu^2\, e^{-E_\nu/T}}{6T^4}$$

$$\varepsilon = 5 \times 10^{52} \text{ erg and } T = 4 \text{ MeV}$$

$$N_p = 2(1 - \Upsilon_D) \frac{\pi r^2 h \times \rho_{\text{water}}}{m_{\text{H}_2\text{O}}} = 2.167 \times 10^{33}$$

Efficiency ε from Kamiokande II



Positron spectrum in Super-Kamiokande

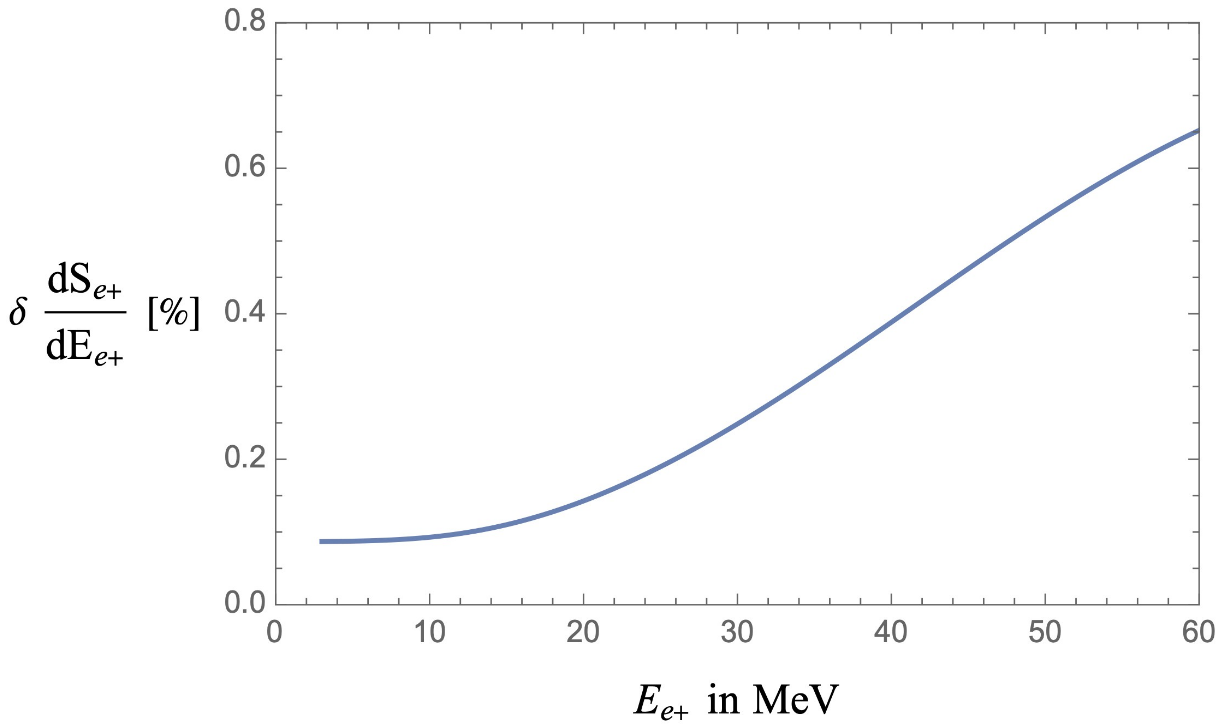
$$\frac{dS_e}{dE_e} = N_p \int_{E_\nu^{max}}^{E_\nu^{min}} dE_\nu \frac{dF}{dE_\nu}(E_\nu) \frac{d\sigma}{dE_e}(E_\nu, E_e)\, \epsilon(E_e)$$

$$\frac{dF}{dE_\nu} = \frac{\varepsilon}{4\pi D^2} \frac{E_\nu^2\, e^{-E_\nu/T}}{6T^4}$$

$$\varepsilon = 5 \times 10^{52} \text{ erg and } T = 4 \text{ MeV}$$

$$N_p = 2(1 - \Upsilon_D) \frac{\pi r^2 h \times \rho_{\text{water}}}{m_{\text{H}_2\text{O}}} = 2.167 \times 10^{33}$$

Efficiency ε from Kamiokande II



Conclusions

- We have given an accurate evaluatios of the cross sections for neutrino scattering on nucleons, which we hope to be a useful outcome for current and future neutrino experiments
- Central values of parameters have been updated (since 2003)
- Evaluation of the uncertainties:

  At low energy overall uncertainty at the 1 permil level (from Cabibbo angle and axial coupling)

  At higher energies, the uncertainty grows up to the percent level (from the uncertainty associated to the axial form factor)
- We find that the impact of second-class currents on the cross section is negligible