

Theory of Inclusive B Decays

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Outline

- Introduction and Motivation
- How do we make theoretical predictions?
- How well can we calculate?
- $|V_{cb}|$ and $\bar{B}
 ightarrow X_c \, \ell \, \bar{
 u}_\ell$
- $|V_{ub}|$ and $\bar{B} \to X_u \,\ell \, \bar{\nu}_\ell$
- $\bar{B} \to X_s \gamma$
- Conclusions

Introduction and Motivation

Motivation

- Flavor physics allows access to new physics at scales beyond reach of current colliders
- E.g. $K \bar{K}$ mixing, $B \bar{B}$ mixing probe scales above hundreds of TeV
- Consistent tension: Inclusive $|V_{cb}|, |V_{ub}| > \text{Exclusive } |V_{cb}|, |V_{ub}|$

Motivation: Theoretically Interesting

- Theoretically Interesting: test of basic QFT tools
- Factorization theorems
- Operator product expansion Example: $\bar{B} \rightarrow X_c \, \ell \, \bar{\nu}_\ell$ OPE is known to Perturbative: third order, Non-perturbative: fourth order
- Theoretically Interesting: window to non-perturbative physics



• At leading twist the $ar{B} o X_s \, \gamma$ photon spectrum is the B-meson pdf

How do we make theoretical predictions?

Effective Hamiltonian

• At energies $\ll m_W, m_Z, m_t$ effective Hamiltonian is known For review see [Buras, hep-ph/9806471] e.g. $\bar{B} \rightarrow X_s \gamma$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qs}^* \left(C_1 Q_1^q + C_2 Q_2^q + \sum_{i=3,...,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

- C_i calculable in perturbation theory
- Q_i operators with non-perturbative matrix elements

$$Q_{1}^{q} = (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad (q = u, c)$$

$$Q_{7\gamma} = \frac{-e}{8\pi^{2}}m_{b}\bar{s}\sigma_{\mu\nu}(1 + \gamma_{5})F^{\mu\nu}b$$

$$Q_{8g} = \frac{-g_{s}}{8\pi^{2}}m_{b}\bar{s}\sigma_{\mu\nu}(1 + \gamma_{5})G^{\mu\nu}b$$

Main problem

- Main problem: we know the operators but usually cannot calculate the matrix elements
- Strong interaction operators made of quarks and gluons
- Local: e.g. $\bar{q}(0)\cdots q(0)$
- Non-Local: e.g. $\bar{q}(0)\cdots q(tn)$ n light-cone vector
- What kind of objects do we encounter?

Non perturbative objects: $\langle f(p_f) | O | i(p_i) \rangle$

1) Decay constant: Local operator, $p_f = 0$

$$\langle 0|ar{d}\gamma^{\mu}(1-\gamma_5)u|\pi(p)
angle=if_{\pi}p^{\mu}$$

Also diagonal local matrix elements: $\langle \bar{B} | \bar{b} \, \vec{D}^2 \, b | \bar{B} \rangle = 2 M_B \mu_\pi^2$

2) Form factor: Local operator, $p_f - p_i = q$

$$\langle N(p_f)|\sum_{q} e_q \,\bar{q}\gamma^{\mu}q|N(p_i)\rangle = \bar{u}(p_f) \left[\gamma_{\mu}F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^N(q^2)q_{\nu}\right]u(p_i)$$
Flavor: $\langle D(p_f)|\bar{c}\gamma^{\mu}b|\bar{B}(p_i)\rangle = f_+(q^2)(p_i + p_f)^{\mu} + f_-(q^2)(p_i - p_f)^{\mu}$
PDF: Non-local operator, $p_f - p_i = 0$

$$\phi_q(\xi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \, e^{i\xi n \cdot pt} \langle N(p)|\bar{\psi}(0) \, [0, tn] \, \frac{\#}{2} \, \psi(tn)|N(p)\rangle$$
Flavor: $S(\omega) = \frac{1}{2\pi} \frac{1}{2M_B} \int_{-\infty}^{\infty} dt \, e^{i\omega t} \langle \bar{B}(v)|\bar{b}(0) \, [0, tn] b(tn)|\bar{B}(v)\rangle$

4) Non-local Form factor: Non-local operator, $p_i - p_f = q$

$$\langle K^{(*)}(p_f)|\bar{s}_L(0)\gamma^{
ho}\cdots \tilde{G}_{lphaeta}b_L(tn)|B(p_i)
angle$$

[Khodjamirian, Mannel, Pivovarov, Wang, JHEP 09, 089 (2010)]

3)

What to do with Non Perturbative Objects?

- What to do with the Non Perturbative Objects?
- 1) Calculate using some non perturbative method, e.g. Lattice
- 2) Extract carefully from experiment
- 3) Use symmetries
- 4) When all else fails, model
- For example
- 1) f_B calculated from Lattice QCD
- 2) ϕ_q extracted from fits to DIS
- 3) SU(3) flavor for $B \rightarrow PP$
- 4) Non-perturbative error for $\bar{B} \rightarrow X_s \gamma$, $|V_{ub}|$
 - Since $m_b\sim 5~{\rm GeV}\Rightarrow$ two expansion parameters for b-quark decays
 - $\alpha_s(m_b) \sim 0.2$
 - $\Lambda_{\text{QCD}}/\textit{m}_{b}\sim0.1$

How well can we calculate?

How well can we calculate?

- Questions:
- What is the current "state of the art"?
- Can the theoretical prediction be improved?
- Will it lead to smaller error bars?
- Examples:
- $|V_{cb}|$ and $\bar{B}
 ightarrow X_c \, \ell \, \bar{
 u}_\ell$
- $|V_{ub}|$ and $\bar{B} \to X_u \,\ell \, \bar{\nu}_\ell$
- $\bar{B} \to X_s \gamma$
- See also "Challenges in Semileptonic B Decays" Workshop (April 2022, Barolo, Italy) https://indico.cern.ch/event/851900/

$|V_{cb}|$ and $\bar{B} o X_c \, \ell \, \bar{ u}_\ell$

$|V_{cb}|$ and $\bar{B} \to X_c \,\ell \, \bar{\nu}_\ell$

• Semileptonic $b \rightarrow c$ transition

$$\mathcal{H}_{\mathsf{eff}} = rac{G_{\textit{F}}}{\sqrt{2}} C_1(\mu) V_{cb} \, ar{\ell} \gamma_\mu (1-\gamma^5)
u_\ell \, ar{c} \gamma^\mu (1-\gamma^5) b$$

• Using the optical theorem can calculate $\bar{B} o X_c \, \ell \, \bar{
u}_\ell$ as an OPE

$$\Gamma\sim c_0\langle {\cal O}_0
angle+c_2^jrac{\langle {\cal O}_2^j
angle}{m_b^2}+\cdots$$

- $c_0 \langle O_0
 angle$ is a free quark decay. At tree level same as $\mu o e \, ar
 u_e
 u_\mu$
- c_i^J perturbative in α_s
- $\langle O_i \rangle$ are non perturbative, can be extracted from experiment
- $\langle O_0
 angle = \langle ar{B} | ar{b} b | ar{B}
 angle = 1$
- $\langle O_2^{\text{kin.}} \rangle = \langle \bar{B} | \bar{b} (iD)^2 b | \bar{B} \rangle \Rightarrow \mu_\pi^2$
- $\langle O_2^{\text{mag.}} \rangle = \langle \bar{B} | \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b | \bar{B} \rangle \Rightarrow \mu_G^2$ can be extracted from $M_{B^*} M_B$

$|V_{cb}|$ and $\bar{B} ightarrow X_c \, \ell \, \bar{ u}_\ell$

• Using the optical theorem can calculate $ar{B} o X_c \, \ell \, ar{
u}_\ell$ as an OPE

$$\Gamma \sim c_0 \langle O_0 \rangle + c_2^j \frac{\langle O_2^j \rangle}{m_b^2} + c_3^j \frac{\langle O_3^j \rangle}{m_b^3} + c_4^j \frac{\langle O_4^j \rangle}{m_b^4} + c_5^j \frac{\langle O_5^j \rangle}{m_b^5} + \cdots$$

- $1/m_b^0$: One operator
- $1/m_b$: No operators
- 1/m_b²: Two operators
 [Blok, Koyrakh, Shifman, Vainshtein PRD 49, 3356 (1994)]
 [Manoar, Wise PRD 49, 1310 (1994)]
- 1/m_b³: Two operators
 [Gremm, Kapustin, PRD 55, 6924 (1997)]
- [Mannel, Turczyk, Uraltsev JHEP 1011, 109 (2010)]:
- $1/m_b^4$: Nine operators
- $1/m_b^5$: Eighteen operators
- All above: c_i^j at $\mathcal{O}(\alpha_s^0)$. Are these all the possible operators?

Interlude

- Are these all the possible operators?
- Question answered in [Gunawardna, GP JHEP 1707 137 (2017)]
- List such operators, in principle, to arbitrary dimension
- NRQED and NRQCD bilinear ops., in principle, to arbitrary dimension
- See blackboard talk later this week
- See also [Kobach, Pal PLB 772 225 (2017)] using Hilbert series
- Are these all the possible operators? No.
- For $1/m_b^0$, $1/m_b^2$, $1/m_b^3$ these are all the possible operators
- $1/m_b^4$: 9 operators at $\mathcal{O}(lpha_s^0) \Rightarrow 11$ operators at $\mathcal{O}(lpha_s)$ or higher
- $1/m_b^{5}$: 18 operators at $\mathcal{O}(\alpha_s^0) \Rightarrow$ 25 operators at $\mathcal{O}(\alpha_s)$ or higher
- These are unknown but extremely small For example: $\alpha_s \left(\Lambda_{\rm QCD}/m_b\right)^4 \sim 0.2 \cdot (0.1)^4 \sim 10^{-5}$

Power corrections

• $1/m_b^4$, $1/m_b^5$ matrix elements extracted from $\bar{B} \to X_c \ell \bar{\nu}_\ell$ [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

Table 2

Default fit results: the second and third columns give the central values and standard deviations.

m ^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_{c}(3 \text{ GeV})$	0.987	0.013	r_2	-0.063	0.037
μ_{π}^2	0.432	0.068	r_3	-0.017	0.025
μ_G^2	0.355	0.060	r_4	-0.002	0.025
ρ_D^3	0.145	0.061	r_5	0.001	0.025
$\begin{array}{c} \mu_{\pi}^2 \\ \mu_{G}^2 \\ \rho_{D}^3 \\ \rho_{LS}^3 \\ \overline{m}_1 \end{array}$	-0.169	0.097	r_6	0.016	0.025
\overline{m}_1	0.084	0.059	r_7	0.002	0.025
\overline{m}_2	-0.019	0.036	r_8	-0.026	0.025
\overline{m}_3	-0.011	0.045	r ₉	0.072	0.044
\overline{m}_4	0.048	0.043	r ₁₀	0.043	0.030
\overline{m}_5	0.072	0.045	r_{11}	0.003	0.025
\overline{m}_6	0.015	0.041	r ₁₂	0.018	0.025
\overline{m}_7	-0.059	0.043	r ₁₃	-0.052	0.031
\overline{m}_8	-0.178	0.073	r ₁₄	0.003	0.025
\overline{m}_9	-0.035	0.044	r ₁₅	0.001	0.025
χ^2/dof	0.46		r_{16}	0.001	0.025
BR(%)	10.652	0.156	r ₁₇	-0.028	0.025
10 ³ V _{cb}	42.11	0.74	r ₁₈	-0.001	0.025

• "The higher power corrections have a minor effect on $|V_{cb}|$... There is a -0.25% reduction in $|V_{cb}|$ "

State of the art: $|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$

• What is the current "state of the art"? As of 2021

$$\Gamma \sim c_0 \langle O_0 \rangle + c_2^j \frac{\langle O_2^j \rangle}{m_b^2} + c_3^j \frac{\langle O_3^j \rangle}{m_b^3} + c_4^j \frac{\langle O_4^j \rangle}{m_b^4} + c_5^j \frac{\langle O_5^j \rangle}{m_b^5} + \cdots$$

- c_0 known at $\mathcal{O}(\alpha_s^0), \mathcal{O}(\alpha_s^1), \mathcal{O}(\alpha_s^2), \mathcal{O}(\alpha_s^3)$ for selected observables
- c_2^{J} known at $\mathcal{O}(\alpha_s^0), \mathcal{O}(\alpha_s^1)$
- c_3^j known at $\mathcal{O}(lpha_s^0), \mathcal{O}(lpha_s^1)$ for selected observables
- c_4^j known at $\mathcal{O}(lpha_s^0)$
- c_5^j known at $\mathcal{O}(\alpha_s^0)$
- State of the art Inclusive |V_{cb}| = 42.16(51) · 10⁻³ [Bordone, Capdevila, Gambino, PLB 822, 136679 (2021)]
- HFLAV 2021: Exclusive $|V_{cb}| = 38.90(53) \cdot 10^{-3}$
- Exclusive/Inclusive $|V_{cb}|$ puzzle remains
- Can the theoretical prediction be improved? Yes, c^j₃ at O(α¹_s) fully differential
- Will it lead to smaller error bars? Probably

$|V_{ub}|$ and $\bar{B} o X_u \,\ell \, \bar{\nu}_\ell$

$|V_{ub}|$ and $\bar{B} \to X_u \,\ell \, \bar{\nu}_\ell$



- $|V_{ub}|$ plays a role in the unitarity triangle fit Like $|V_{cb}|$, $|V_{ub}|$ inclusive is larger than $|V_{ub}|$ exclusive
- PDG August 2021 review
- Inclusive $|V_{ub}| = (4.13 \pm 0.12^{+0.13}_{-0.14} \pm 0.18) \cdot 10^{-3}$
- Exclusive $|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \cdot 10^{-3}$

$|V_{ub}|$ and $\bar{B} \to X_u \,\ell \, \bar{\nu}_\ell$: Framework

• If we could measure total $\Gamma(\bar{B} \to X_u \, | \, \bar{\nu})$ we could use a local OPE

$$d\Gamma \sim \sum_{i,j} c_i^j rac{\langle O_i^j
angle}{m_b^i}$$

 c_i^j perturbative, $\langle O_i^j
angle$ non-perturbative **numbers**

- Since $\Gamma(\bar{B} \to X_c \ell \, \bar{\nu}_\ell) \gg (\bar{B} \to X_u \ell \, \bar{\nu}_\ell)$ total rate **cannot** be measured Need to cut the charm background: e.g. $M_X^2 < M_D^2 \sim m_b \Lambda_{QCD}$
- Not inclusive enough for local OPE, but non-local OPE still possible

$M_X^2 \sim m_b^2$	local OPE	("OPE region")
$M_X^2 \sim m_b \Lambda_{ m QCD}$	Non local OPE	("end point region")
$M_X^2 \sim \Lambda_{ m QCD}^2$	No inclusive description	("resonance region")

$|V_{ub}|$ and $\bar{B} \to X_u \,\ell \, \bar{\nu}_\ell$: Framework

Need to cut the charm background: e.g. $M_X^2 < M_D^2 \sim m_b \Lambda_{\rm QCD}$

Not inclusive enough for local OPE, but non-local OPE still possible

$$d\Gamma \sim H \cdot J \otimes S + \mathcal{O}\left(rac{1}{m_b}
ight)$$

- Can factorize perturbative coefficient into hard H and jet J functions
- S is a non-perturbative "shape function" (B-meson PDF)
- At leading power in Λ_{QCD}/m_b , S is $\bar{B} \rightarrow X_s \gamma$ photon spectrum

Recent work: SIMBA Collaboration

- At leading power in $\Lambda_{\rm QCD}/m_b$, S is $\bar{B} o X_s \gamma$ photon spectrum
- Recent extraction by the SIMBA (Analysis of B-Meson Inclusive Spectra) Collaboration [Bernlochner, Lacker, Ligeti, Stewart, F. Tackmann, K. Tackmann, PRL 127, 102001 (2021)]



$|V_{ub}|$ and $\bar{B} \to X_u \,\ell \, \bar{\nu}_\ell$: Framework

• At subleading power in $\Lambda_{\rm QCD}/m_b$

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \cdots$$

- Several subleading shape functions (SSF) appear (s_i) ("higher twist")
- Different linear combinations for $ar{B} o X_u \, \ell \, ar{
 u}_\ell$ and $ar{B} o X_s \, \gamma$
- $\bar{B} \rightarrow X_s \gamma$ has unique SSF ("resolved photon contributions")
- Shape functions moments are related to universal matrix elements: E.g. leading shape function: 1^{st} moment $\leftrightarrow m_b$, 2^{nd} moment $\leftrightarrow \mu_{\pi}^2$
- Different theoretical frameworks for $|V_{ub}|$ extractions:
- Use similar perturbative inputs, currently $\mathcal{O}(\alpha_s)$
- Differ in how they extract (or model) S
- Differ in how they treat power corrections

Recent work: Inclusive $|V_{ub}|$ from Belle data

- Current extractions used
- BLNP [Lange, Neubert, GP, PRD 72, 073006, (2005)]
- DGE [Andersen, Gardi, JHEP 01, 097, (2006)]
- GGOU [Gambino, Giordano, Ossola, Uraltsev, JHEP 10, 058, (2007)]
- ADFR [Aglietti, Di Lodovico, Ferrera, Ricciardi, EPJC 59, 831, (2009)]
- Recent work: Inclusive $|V_{ub}|$ from Belle data
 - [L. Cao et al. [Belle], PRD 104, 012008 (2021)]



• See also Francesco Tenchini's talk on Saturday

Recent work: Inclusive $|V_{ub}|$ from Belle data

Recent work: Inclusive |V_{ub}| from Belle data
 [L. Cao *et al.* [Belle], PRD **104**, 012008 (2021)]



- State of the art: theoretical framework developed before 2010
- Can the theoretical prediction be improved?
- Yes, many NNLO calculations are known:
- H, J at $\mathcal{O}(\alpha_s^2)$, j_i/m_b at $\mathcal{O}(\alpha_s)$, resolved photon contributions
- Not fully combined yet [Gunawardana, Lange, Mannel, Olschewsky, Vos, GP, to appear]
- Will it lead to smaller error bars? Not necessarily

 $\bar{B} \to X_s \gamma$

$\bar{B} \to X_s \gamma$

- $\bar{B} \rightarrow X_s \gamma$ BSM probe. PDG 2021: Br = $(3.49 \pm 0.19) \cdot 10^{-4}$
- 2015 SM prediction of branching ratio $(3.36 \pm 0.23) \cdot 10^{-4}$ [M. Misiak *et al.*, PRL **114**, 221801 (2015)]
- Largest uncertainty ~ 5% is non-perturbative from "resolved photons" At Λ_{QCD}/m_b [Benzke, Lee, Neubert, GP JHEP 1008, 099 (2010)]:



- Top line $Q_{7\gamma} Q_{8g}$, Bottom left: $Q_{8g} Q_{8g}$, Bottom right: $Q_1 Q_{7\gamma}$ "with field localized on two different light cones" Precursor to Matthias Neubert's talk on Saturday
- SM CP asymmetry dominated by $Q_1^q Q_{7\gamma} : -0.6\% < A_{X_s\gamma}^{SM} < 2.8\%$ [Benzke, Lee, Neubert, GP PRL **106**, 141801 (2011)] PDG 2021: $A_{X_s\gamma} = 1.5\% \pm 1.1\%$. Can we improve this?

$\bar{B} \to X_s \gamma$

- At Λ_{QCD}/m_b : resolved photons from $Q_{7\gamma} Q_{8g}$, $Q_{8g} Q_{8g}$, $Q_1 Q_{7\gamma}$
- $Q_{7\gamma} Q_{8g}$ constrained by isospin asymmetry $\bar{B}^{0/\pm} \rightarrow X_s \gamma$ uncertainty reduced by a Belle measurement [Watanuki *et al.* [Belle Collaboration] PRD **99**, 032012 (2019)]
- $Q_{8g}-Q_{8g}$ is hard to improve, but small
- $Q_1 Q_{7\gamma}$ depends on a non-perturbative function $g_{17}(\omega, \omega_1)$ whose moments can be extracted from $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$ OPE
- 2010 analysis only had 2 non-zero moments [Benzke, Lee, Neubert, GP, JHEP 1008, 099 (2010)]

 $\langle \omega^0 \, \omega_1^0 \, g_{17} \rangle = 0.237 \pm 0.040 \,\, {\rm GeV}^2, \quad \langle \omega^1 \, \omega_1^0 \, g_{17} \rangle = 0.056 \pm 0.032 \,\, {\rm GeV}^3$

• 2019 analysis added 6 non-zero moments [Gunawardna, GP JHEP 11 141 (2019)]

$$\begin{split} &\langle \omega^0 \, \omega_1^2 \, g_{17} \rangle = 0.15 \pm 0.12 \, \, \text{GeV}^4, \quad \langle \omega^2 \, \omega_1^0 \, g_{17} \rangle = 0.015 \pm 0.021 \, \, \text{GeV}^4 \\ &\langle \omega^3 \, \omega_1^0 \, g_{17} \rangle = 0.008 \pm 0.011 \, \, \text{GeV}^5, \quad \langle \omega^1 \, \omega_1^1 \, g_{17} \rangle = 0.073 \pm 0.059 \, \, \text{GeV}^4 \end{split}$$

 $\langle \omega^2 \, \omega_1^1 \, g_{17} \rangle = -0.034 \pm 0.016 \,\, {\rm GeV^5}, \quad \langle \omega^1 \, \omega_1^2 \, g_{17} \rangle = 0.027 \pm 0.014 \,\, {\rm GeV^5}.$

Data from [Gambino, Healey, Turczyk PLB 763, 60 (2016)]

$\bar{B} \to X_s \gamma$

- Using moments model $\mathit{Q}_1 \mathit{Q}_{7\gamma}$ resolved photon
- New estimate of uncertainty: Total rate ↓ 50%, CP asymmetry ↑ 33% [Gunawardna, GP JHEP 11 141 (2019)]
- 2015 SM prediction of branching ratio (3.36 ± 0.23) · 10⁻⁴
 [M. Misiak *et al.*, PRL **114**, 221801 (2015)]
- 2020 SM prediction of branching ratio $(3.40 \pm 0.17) \cdot 10^{-4}$ [Misiak, Rehman, Steinhauser, JHEP **06**, 175 (2020)]
- Using different models, including some Λ_{QCD}^2/m_b^2 corrections and larger m_c range, a smaller reduction was found in [Benzke, Hurth PRD **102** 114024 (2020)]
- Can the theoretical prediction be improved?

Yes, m_c can be better controlled by an NLO analysis of $Q_1-Q_{7\gamma}$

• Will it lead to smaller error bars? Not necessarily

Conclusions

Conclusions

- Flavor physics probes very high scales and advanced theoretical tools
- This decade will be very exciting with, e.g., LHCb and Belle II data
- Puzzles and tensions motivate further theoretical work
- A big challenge is controlling non-perturbative effects
- Discussed "state of the art" of
- $|V_{cb}|$ and $\bar{B} \to X_c \,\ell \, \bar{\nu}_\ell$
- $|V_{ub}|$ and $\bar{B} \to X_u \,\ell \, \bar{\nu}_\ell$
- $\bar{B} \to X_s \gamma$
- Future: improve theory, but not necessarily smaller error bars
- More work to do. Thank You!