Neutrino Tomography of the Earth with ORCA Detector

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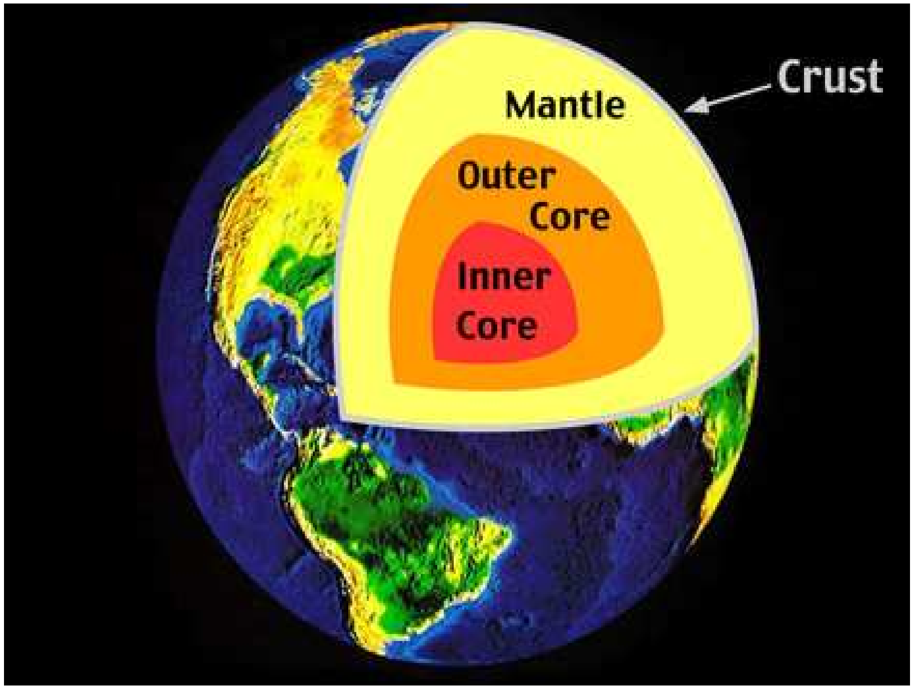
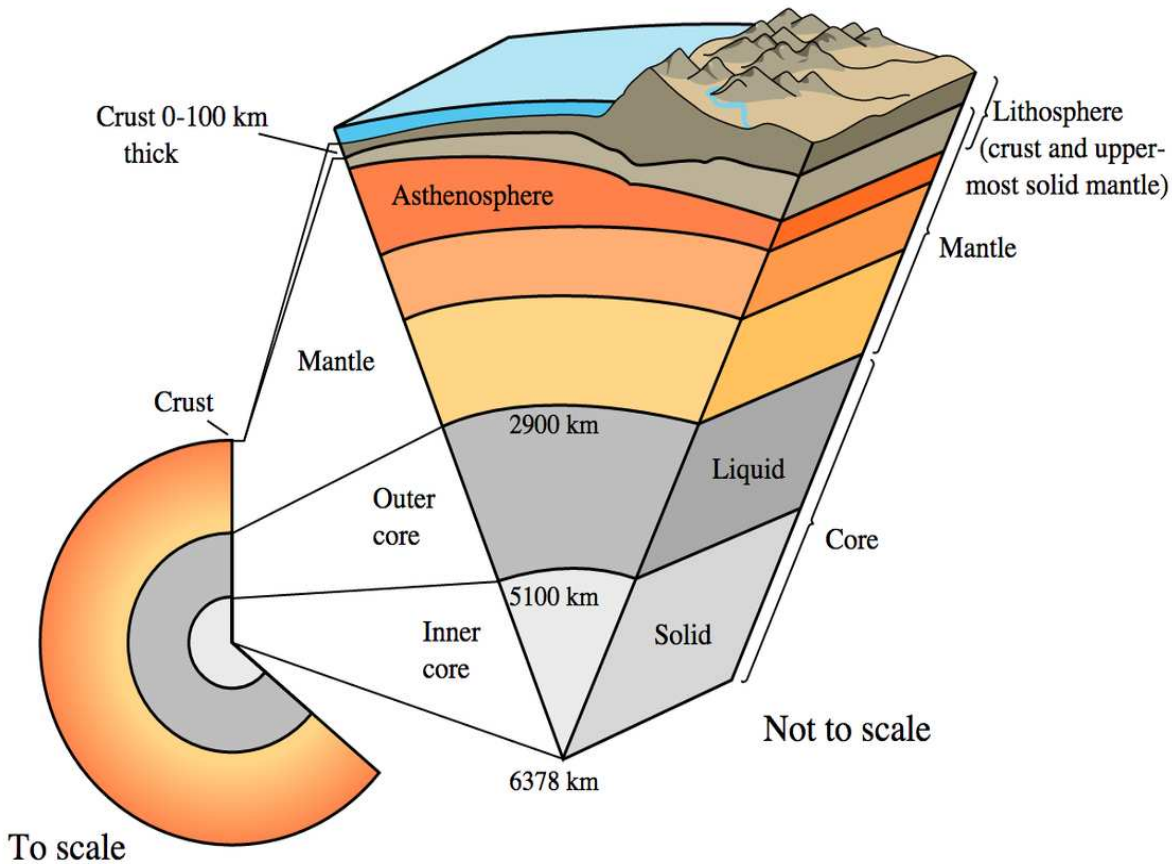
Based on: F. Capozzi and S.T.P., EPJ C82 (2022) 461 [arXiv:2111.13048]

Research in Neutrino Physics: we strive to understand at deepest level what are the origins of neutrino masses and mixing and what determines the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years. And we try to understand what are the implications of the remarkable discovery that neutrinos have mass, mix and oscillate for elementary particle physics, cosmology and for better understanding of the Earth, the Sun, the stars, formation of Galaxies, the Early Universe, i.e., for better deeper understanding of Nature in general.

The Earth




Crust 0-100 km thick
Lithosphere (crust and upper-most solid mantle)
Asthenosphere
Mantle
Mantle
Crust
2900 km
Liquid
Outer core
Core
5100 km
Inner core
Solid
Not to scale
6378 km
To scale

At present our knowledge about the interior composition of the Earth and its density structure is based primarily on seismological and geophysical data.

See, e.g., W.F. McDonough, "Treatise on Geochemistry: The Mantle and Core", vol. 2 (ed. R. W. Carlson, Elsevier-Pergamon, Oxford, 2003), p. 547.
B.L.N. Kennett, Geophys. J. Int. **132**, 374 (1998);
G. Masters and D. Gubbins, Phys. Earth Planet. Inter. **140**, 159 (2003).

These data were used to construct the Preliminary Reference Earth Model (PREM) of the density distribution of the Earth: A. M. Dziewonski and D. L. Anderson, "Preliminary reference earth model," Phys. Earth Planet. Interiors 25 (1981) 297.

In the PREM model, $\rho_{\rm E}$ is assumed to be spherically symmetric, $\rho_{\rm E} = \rho_{\rm E}(r)$, r being the distance from the Earth center, and there are two major density structures - the core and the mantle, and a certain number of substructures (shells or layers). The mantle has seven shells in the model, while the core is divided into an Inner Core (IC) and Outer Core (OC).

The change of $\rho_{\rm E}$ (N_e) from the mantle to the core, according to PREM, can well be approximated by a step function.

The Earth (PREM)

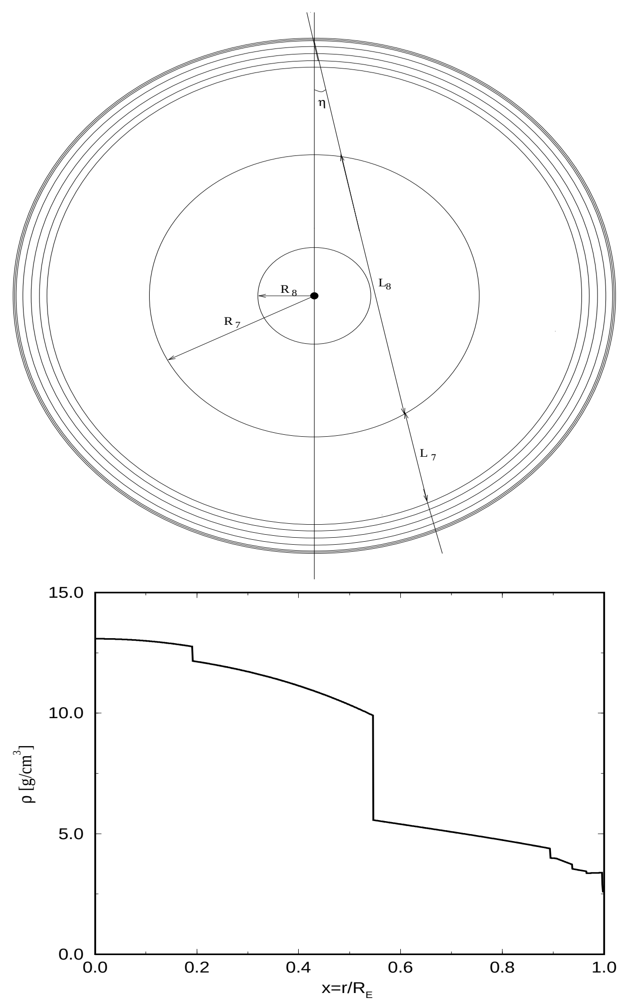


FIG. 1. Density profile of the Earth.

$R_\oplus = 6371$ km; $R_{\rm IC} = 1221.5$ km; $R_{\rm C} = 3480$ km; $D_{\rm man} = 2891$ km;
$\bar{\rho}_{\rm man} = 4.45$ g/cm^3; $\bar{\rho}_{\rm C} = 10.99$ g/cm^3; $\bar{\rho}_{\rm IC} = 12.89$ g/cm^3; $\bar{\rho}_{\rm OC} = 10.90$ g/cm^3.

Spherically symmetric ρ_E: the ν trajectory through the Earth is specified by the nadir (zenith) angle θ_n (or θ_z).

For $\theta_n \leq 33.17^o$, or path lengths $L \geq 10660$ km, neutrinos cross the Earth core.

The path length for neutrinos which cross only the Earth mantle is given by $L = 2R_\oplus \cos\theta_n$, θ_n.

If neutrinos cross the Earth core, the lengths of the paths in the mantle, $2L^{\rm man}$, and in the core, $L^{\rm core}$, are determined by: $L^{\rm man} = R_\oplus \cos\theta_n - (R_c^2 - R_\oplus^2 \sin^2\theta_n)^{\frac{1}{2}}$, $L^{\rm core} = 2(R_c^2 - R_\oplus^2 \sin^2\theta_n)^{\frac{1}{2}}$.

$\rho_E(r)$ (N_e) changes relatively little around the quoted mean values along the trajectories of neutrinos which cross a substantial part of the Earth mantle, or the mantle, the outer core and the inner core.

The determination of the radial density distributions in the mantle and core, $\rho_{man}(r)$ and $\rho_c(r)$, from seismological and geophysical data is not direct and suffers from uncertainties.

B. A. Bolt, Q. J. R. Astron. Soc. **32**, 367 (1991).
B.L.N. Kennett, Geophys. J. Int. **132**, 374 (1998);
G. Masters and D. Gubbins, Phys. Earth Planet. Inter. **140**, 159 (2003).

It requires the knowledge, in particular, of the seismic wave speed velosity distribution in the interrior of the Earth, which depends on the pressure, tempertaure, composition and elastic properties of the Earth's interior that are not known with a good/high precision.
Often $\rho_{\text{E}}(r)$ is determined using an empirical relation between the seismic wave velosities and $\rho_{\text{E}}(r)$ (one example is the Birch low, which may fail at the higher densities of the core) and the so-called "Adams-Williamson eqaution" (from 1923). E. Williamson and L.H. Adams, J. Wash. Acad. Sci. 13 (1923) 413.

An approximate and perhaps rather conservative estimate of this uncertainty for $\rho_{\text{man}}(r)$ is $\sim 5\%$; for the core density $\rho_c(r)$ it is larger and can be significantly larger (Bolt:1991,Kennett:1998,Masters:2003).

**A precise knowledge of $\rho_{\mathrm{E}}(r)$ and of $\bar{\rho}_{\mathrm{man}}$, $\bar{\rho}_{\mathrm{C}}$ and $\bar{\rho}_{\mathrm{IC}}$, is essential for understanding the physical conditions and fundamental aspects of the structure and properties of the Earth's interior
(including the dynamics of mantle and core,
the bulk composition of the Earth's three structures,
the generation, properties and evolution of the Earth's magnetic field and the gravity field of the Earth)
(Bolt:1991,Yoder:1995,McDonough:2003,McDonough:2008zz).**

The thermal evolution of the Earth's core, in particular, depends critically on the density change across the inner core - outer core boundary (see, e.g., Baffet:1991).

An independent determination of $\rho_{\mathrm{E}}(r)$ and of $\bar{\rho}_{\mathrm{man}}$, $\bar{\rho}_{\mathrm{C}}$ and $\bar{\rho}_{\mathrm{IC}}$, is highly desirable and would be extremely useful.

A unique alternative method of determination of the density profile of the Earth is the neutrino tomography of the Earth.

The propagation of the active flavour neutrinos and antineutrinos ν_α and $\bar{\nu}_\alpha$, $\alpha = e, \mu, \tau$, in the Earth is affected by the Earth matter.

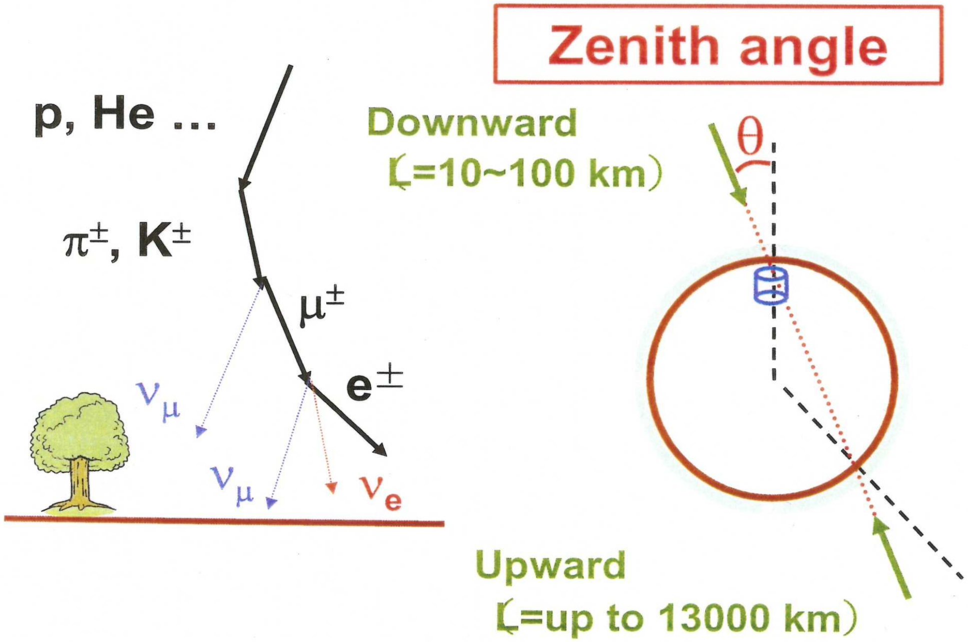
The original idea is based on the observation that
$\sigma(\nu_\alpha(\bar{\nu}_\alpha) + N)$ rises with energy.
For ν_α, $\bar{\nu}_\alpha$ with $E_\nu \gtrsim$ a few TeV, the inelastic scattering off protons and neutrons leads to absorption of νs and thus to attenuation of the initial ν flux.
The magnitude of the attenuation depends on the Earth matter density profile along the neutrino path.
Attenuation data for νs with different path-lengths in the Earth carry information about the matter density distribution in the Earth interior.
The absorption method of Earth tomography with accelerator neutrino beams, which is difficult (if not impossible) to realise in practice , was discussed first by Placci and Zavattini in 1973 and Volkova and Zatsepin in 1974, and later in grater detail in Nedyalkov:1981, Nedyalkov:1981yy, Nedyalkov:1982, Nedyalkov:1983, DeRujula:1983ya,
Wilson:1983an, Askar:1984, Borisov:1986sm, Borisov:1989kh,
Winter:2006vg, Kuo95, Jain:1999kp, Reynoso:2004dt.

Atmospheric νs are a perfect tool for performing Earth tomography:
i) consist of significant fluxes of ν_μ, ν_e, $\bar{\nu}_\mu$ and $\bar{\nu}_e$, produced in the interactions of cosmic rays with the Earth atmosphere,
ii) have a wide range of energies spanning the interval from a few MeV to multi-GeV to multi-TeV,
iii) being produced isotropically in the upper part of the Earth atmosphere at a height of ~ 15 km, they travel distances from ~ 15 km to 12742 km before reaching detectors located on the Earth surface, crossing the Earth along all possible directions and thus "scanning" the Earth interior.

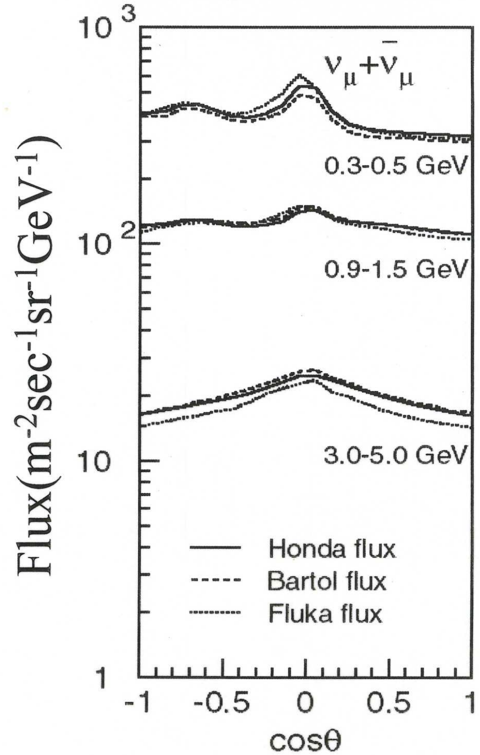
See, e.g., T.K. Gaisser and M. Honda, Ann. Rev. Nucl. Part. Sci. **52** (2002) 153 [arXiv:hep-ph/0203272]

The interaction rates that allow to get information about the Earth density distribution can be obtained in the currently taking data IceCube experiment and in the future experiments PINGU, ORCA (within KM3Net project), Hyper Kamiokande and DUNE, which are under construction.

Atmospheric neutrinos



Zenith angle dist. of Atmospheric ν flux

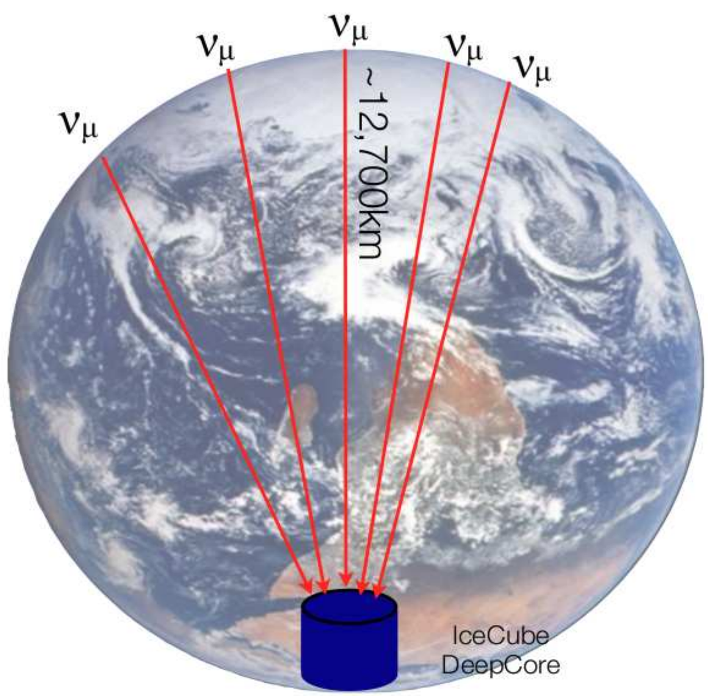


Eν > a few GeV

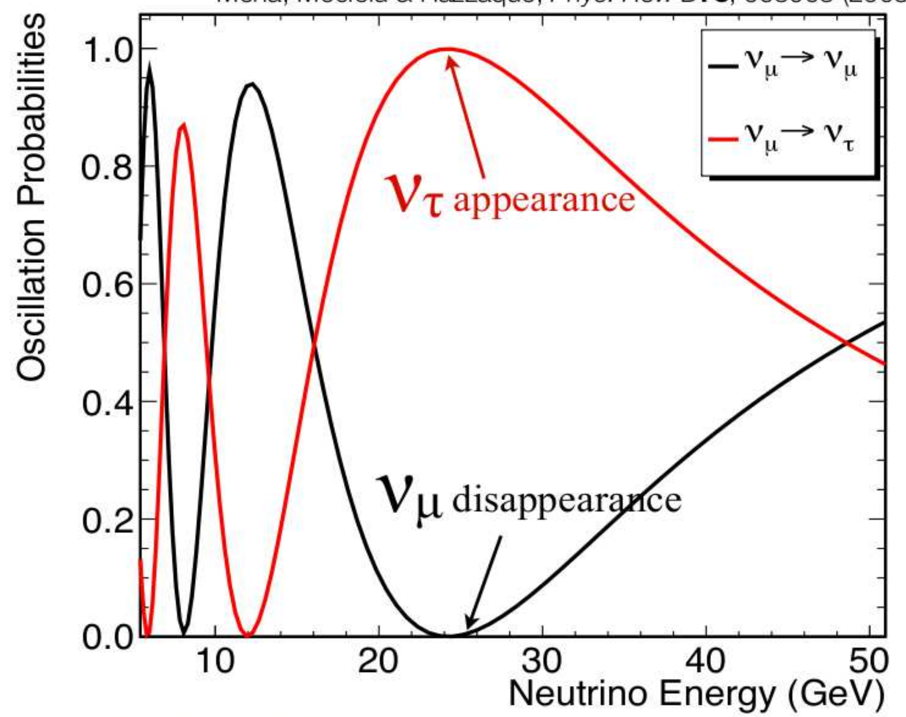
Up/Down Symmetry

Neutrino Oscillation Source

- Northern Hemisphere ν_μ oscillating over one earth radii produces ν_μ (ν_τ) oscillation minimum(maximum) at ~25 GeV
  - Covers all possible terrestrial baselines
  - "Beam" is free and never turns off



Mena, Mocioiu & Razzaque, *Phys. Rev.* D**78**, 093003 (2008)



The idea of using the absorption method of Earth tomography with atmospheric neutrinos was discussed first in M. C. Gonzalez-Garcia *et al.*, Phys. Rev. Lett. 100 (2008) 061802.

A. Donini, S. Palomares-Ruiz and J. Salvado, Nature Phys. 15 (2019) 37, using the IceCube zenith angle distribution data on multi-TeV (1.5-20.0 TeV) atmospheric ν_μ and $\bar{\nu}_\mu$ obtained information about $\rho_{\mathrm{E}}(r)$, which, although not very precise, broadly agrees with the PREM model.

"Weighted" the Earth with neutrinos: $M_\oplus^\nu = (6.0^{+1.6}_{-1.3}) \times 10^{24}$ kg, to be compared with the gravitationally determined value: $M_\oplus = (5.9722 \pm 0.0006) \times 10^{24}$ kg.

Marked the beginning of real experimental data driven neutrino tomography of the Earth.

The oscillations $\nu_\alpha \leftrightarrow \nu_\beta$ and $\bar{\nu}_\alpha \leftrightarrow \bar{\nu}_\beta$, $\alpha, \beta = e, \mu$, having $E \sim (0.1 - 15.0)$ GeV and traversing the Earth can be strongly modified by the Earth matter effects. These modifications depend on the Earth matter density (more precisely, the electron number density $N_e(r)$, see further) along the path of the νs. Thus, by studying the effects of Earth matter on the oscillations of, e.g., ν_μ and ν_e ($\bar{\nu}_\mu$ and $\bar{\nu}_e$) neutrinos traversing the Earth along different trajectories it is possible to obtain information about the Earth (electron number) density distribution.

Neutrino Oscillations in Matter (Earth mantle)

When neutrinos propagate in matter, they interact with the backgrpound of electrons, protons and neutrnos, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong \sin^2\theta_{23}\,\sin^2 2\theta^m_{13}\,\sin^2\frac{\Delta M^2_{31} L}{4E}$$

$\sin^2 2\theta^m_{13}$, ΔM^2_{31} depend on the matter potential $V_{eff} = \sqrt{2}\, G_F\, N_e$,

For antineutrinos V_{eff} has the opposite sign: $V_{eff} = -\sqrt{2}\, G_F, N_e$.

$\Delta m^2_{31} > 0$ (NO): $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced, $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

$\Delta m^2_{31} < 0$ (IO): $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced, $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$−suppressed

$$\sin^2 2\theta_{13}^m = \frac{\tan^2 2\theta_{13}}{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta_{13}},$$

$$\cos 2\theta_{13}^m = \frac{1-N_e/N_e^{res}}{\sqrt{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta_{13}}},$$

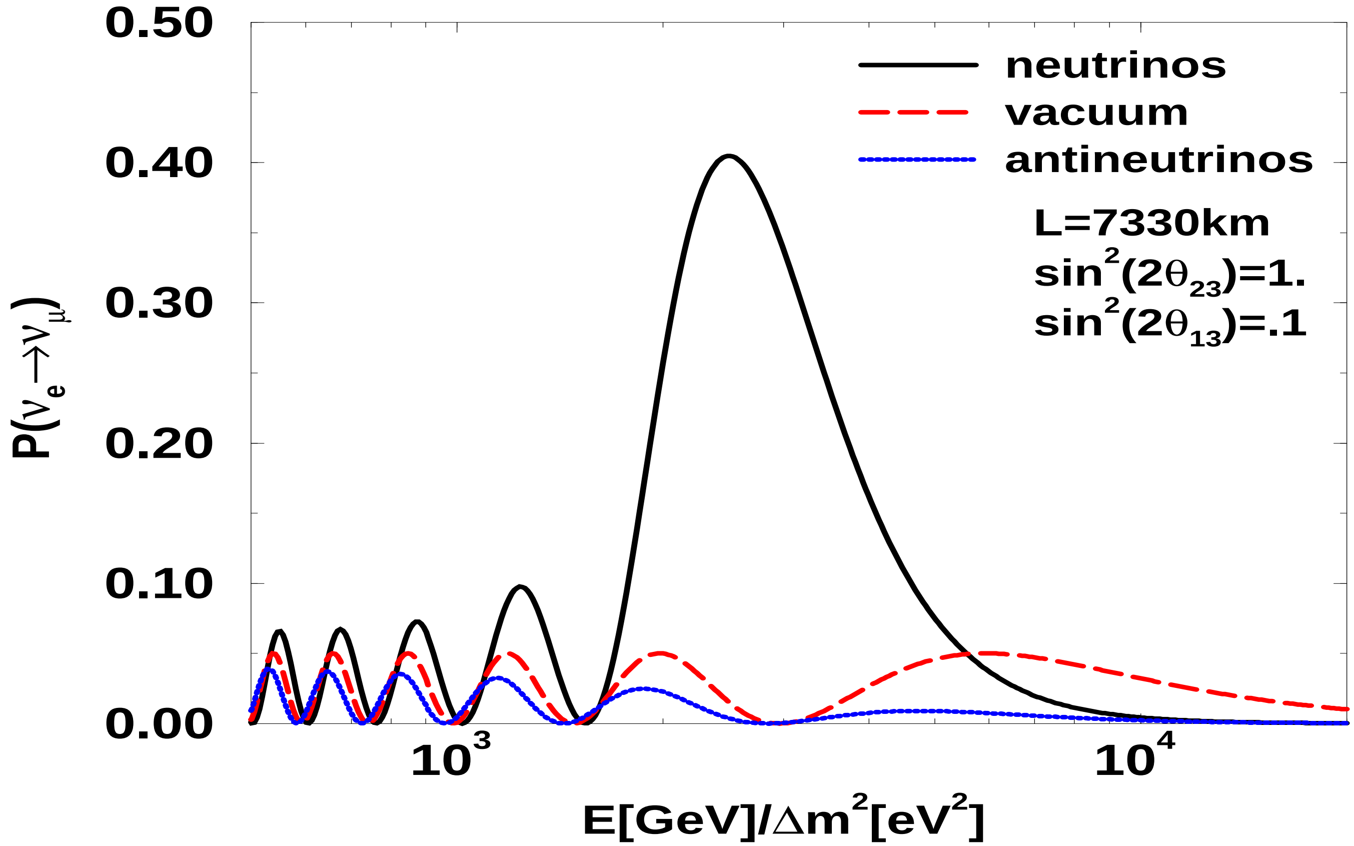
$$N_e^{res} = \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2E\sqrt{2}G_F} \cong$$

$$6.56 \times 10^6 \frac{\Delta m^2[\mathrm{eV}^2]}{E[\mathrm{MeV}]} \cos 2\theta \, \mathrm{cm}^{-3} \ \mathrm{N_A} \,,$$

$$\frac{\Delta M_{31}^2}{2E} \equiv \frac{\Delta m_{31}^2}{2E} \left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13}\right)^{\frac{1}{2}}$$

For $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$: $N_e \rightarrow (-N_e)$.

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ in the mantle (MSW)



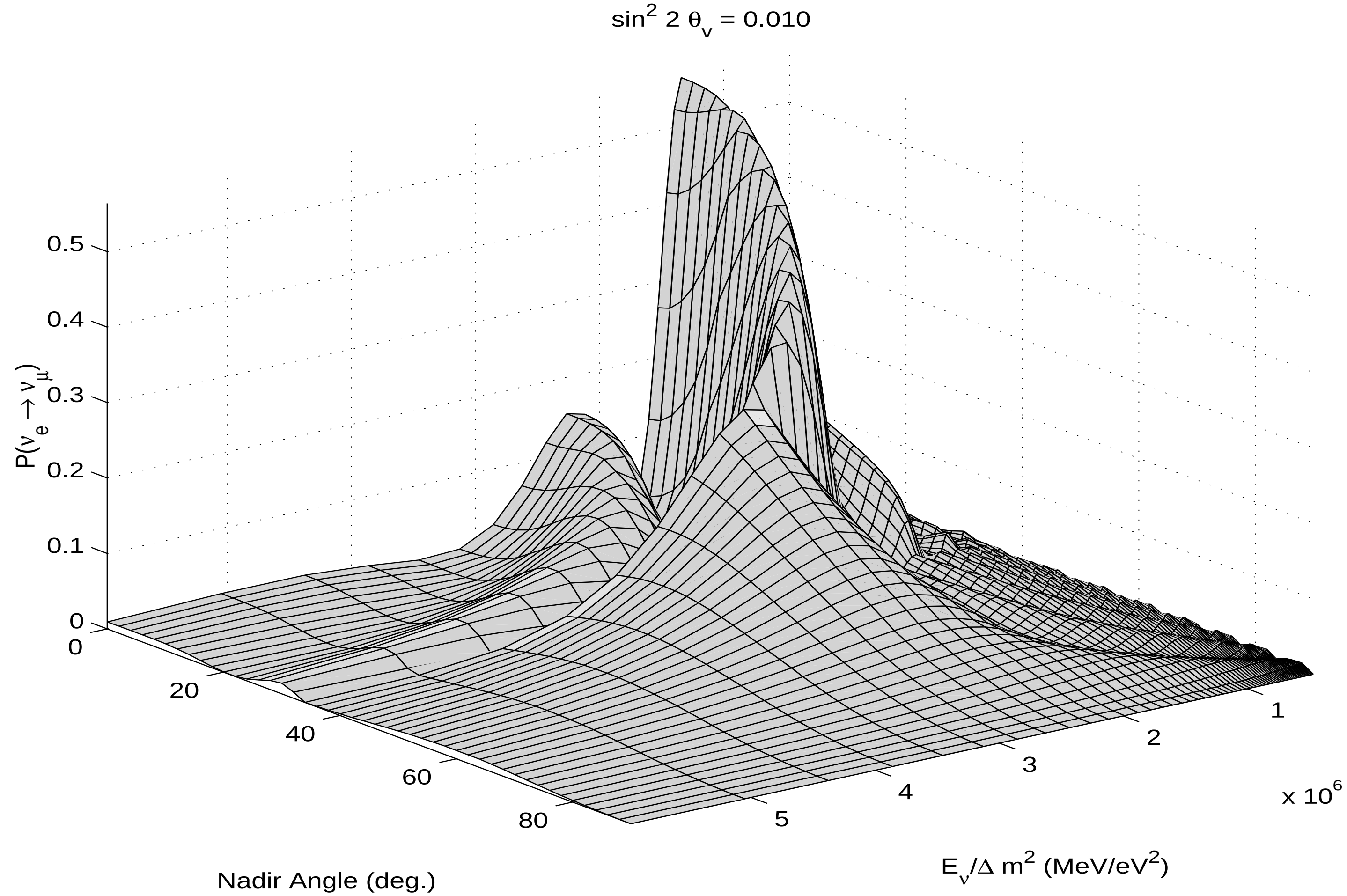
$\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2, $E^{res} = 6.25$ GeV; $P^{3\nu} = \sin^2\theta_{23} P^{2\nu}_m = 0.5 P^{2\nu}_m$;
$N_e^{res} \cong 2.3$ cm^{-3} N$_A$; $L^{res}_m = L^v / \sin 2\theta_{13} \cong 6250/0.32$ km; $2\pi L/L_m \cong 0.75\pi (\neq \pi)$.

I. Mocioiu, R. Shrock, 2000

Oscillations of Neutrinos Crossing the Earth Core

Resonance-like Amplification of Oscillations of Neutrinos Crossing the Earth Core

Earth matter effects in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (NOLR)

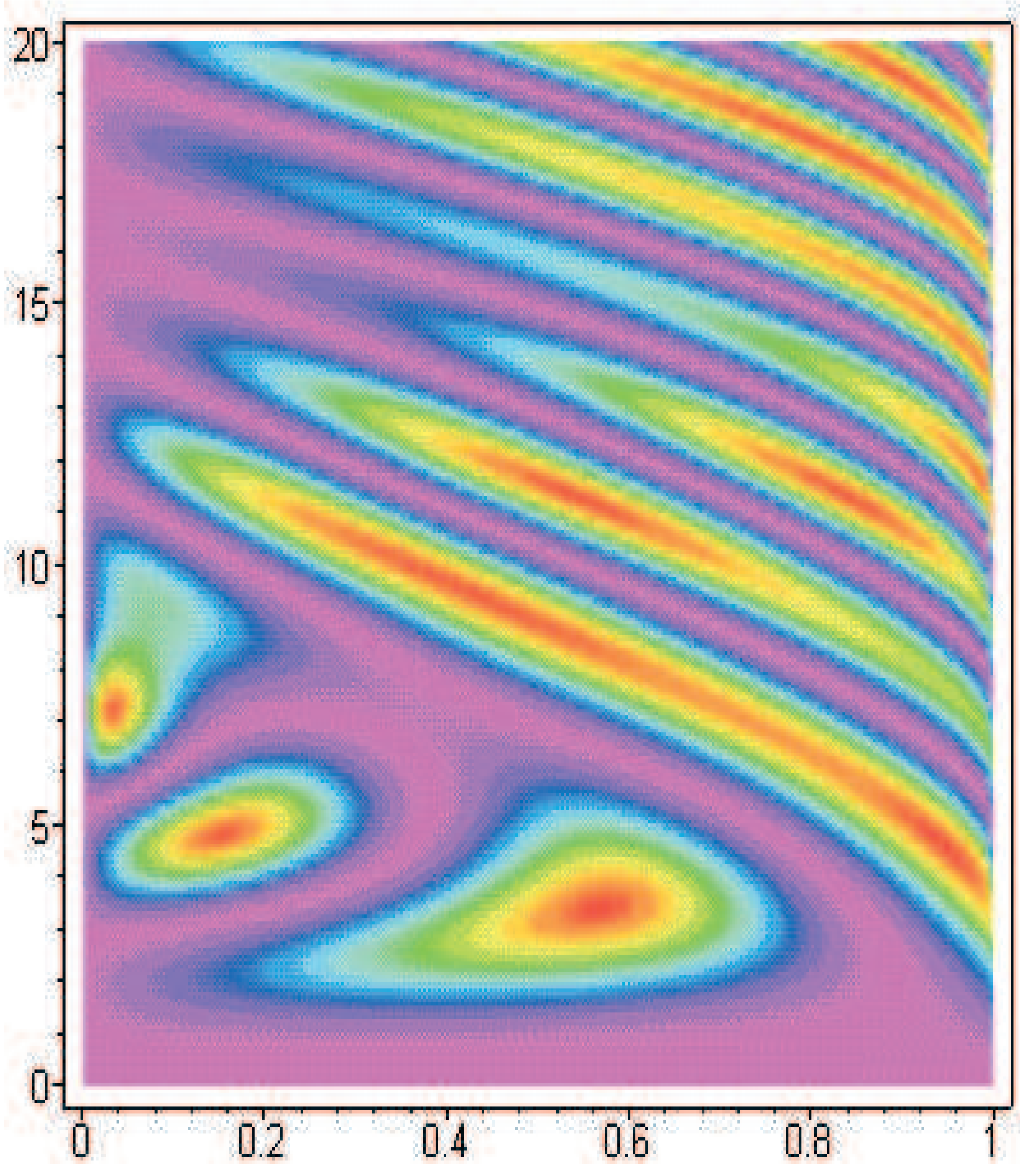


S.T.P., 1998;
M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \to \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)})$, $\theta_v \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m^2_{\rm atm}$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

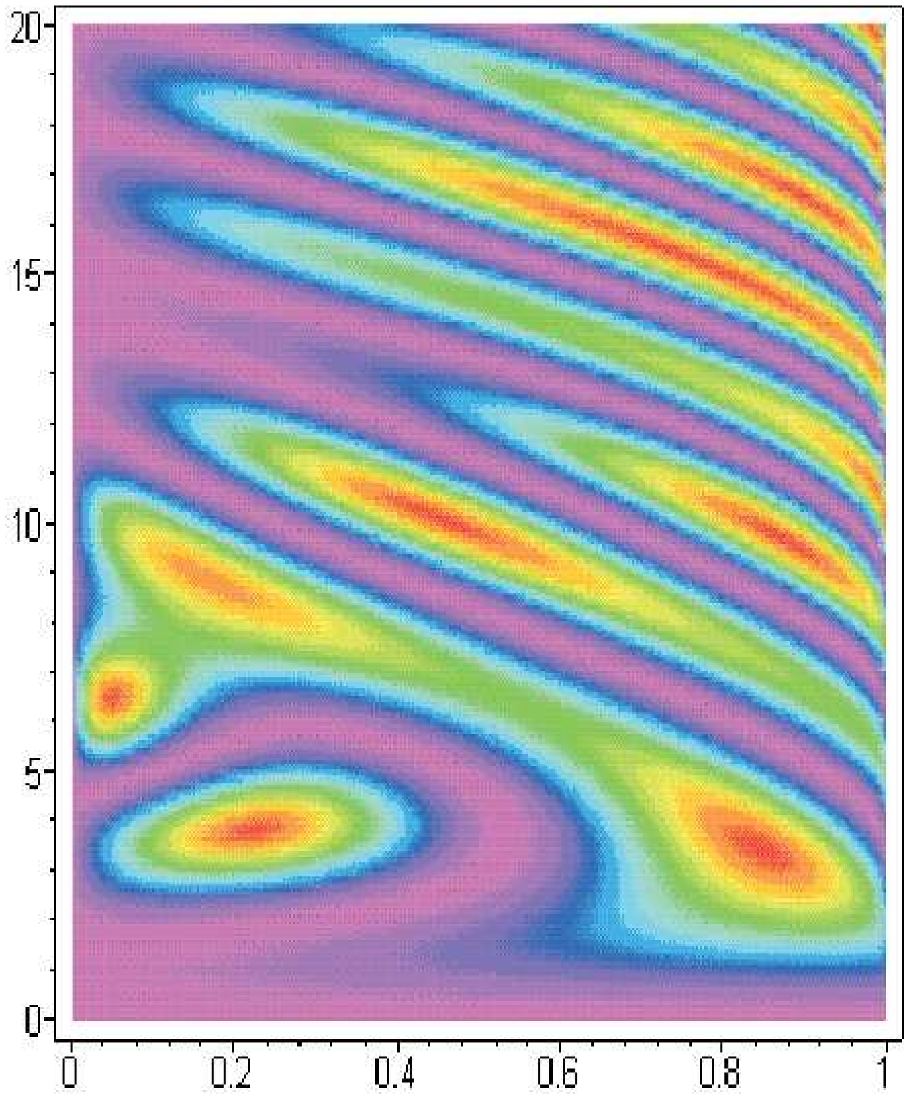
Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;

Vertical axis: $\Delta m^2/E$ $[10^{-7} eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)



The same for $\theta_n = 23^\circ$.
Vertical axis: $\Delta m^2/E$ $[10^{-7} eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$.

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

• **For Earth center crossing ν's ($\theta_n = 0$) and, e.g.** $\sin^2 2\theta_{13} = 0.01$**, NOLR occurs at** $E \cong 4$ **GeV** ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2)**.**

S.T.P., hep-ph/9805262

• **For the Earth core crossing ν's:** $P_{2\nu} = 1$ **due to NOLR when**

$$\tan \Phi^{\rm man}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}},$$

$$\tan \Phi^{\rm core}/2 \equiv \tan \phi'' = \pm \frac{\cos 2\theta'_m}{\sqrt{-\cos(2\theta''_m)\cos(2\theta''_m - 4\theta'_m)}}$$

$\Phi^{\rm man}$ ($\Phi^{\rm core}$) **- phase accumulated in the Earth mantle (core),**
θ'_m (θ''_m) **- the mixing angle in the Earth mantle (core).**

$P_{2\nu} = 1$ **due to NOLR for** $\theta_n = 0$ **(Earth center crossing ν's) at, e.g.** $\sin^2 2\theta_{13} = 0.034;\ 0.154$**,** $E \cong 3.5;\ 5.2$ **GeV** ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2)**.**

At the same time for E =3.5 GeV (5.2 Gev), the probability $P_{2\nu} \gtrsim 0.5$ **for the values of** $\sin^2 2\theta_{13}$ **from the interval** $0.02 \lesssim \sin^2 2\theta_{13} \lesssim 0.10$ **(**$0.04 \lesssim \sin^2 2\theta_{13} \lesssim 0.26$**).**

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).

The mantle-core enhancement of $P_m^{2\nu}$ (or $\bar{P}_m^{2\nu}$) is relevant, in particular, for the searches of sub-dominant $\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}$ (or $\bar{\nu}_{e(\mu)} \rightarrow \bar{\nu}_{\mu(e)}$) oscillations of atmospheric neutrinos having energies $E \gtrsim 2$ GeV and crossing the Earth core on the way to the detector.

S.T.P., hep-ph/9805262; M. Chizhov, S.T.P., hep-ph/9903424

The effects of Earth matter on the oscillations of atmospheric (and accelerator) neutrinos have not been observed so far; can be used for performing neutrino tomography of the Earth.

The fluxes of atmospheric $\nu_{e,\mu}$ of energy E, which reach the detector after crossing the Earth along a given trajectory specified by the value of θ_n, $\Phi_{\nu_{e,\mu}}(E,\theta_n)$, are given by the following expressions in the case of the 3-neutrino oscillations under discussion:

$$\Phi_{\nu_e}(E,\theta_n) \cong \Phi^0_{\nu_e}\left(1 + [s^2_{23} r - 1]\ P^{2\nu}_m\right),$$

$$\Phi_{\nu_\mu}(E,\theta_n) \cong \Phi^0_{\nu_\mu}\left(1 + s^4_{23}[(s^2_{23}\ r)^{-1} - 1]P^{2\nu}_m - 2c^2_{23}s^2_{23}\left[1 - Re\ \left(e^{-i\kappa}A^{2\nu}_m(\nu_\tau \rightarrow \nu_\tau)\right)\right]\right),$$

where $\Phi^0_{\nu_{e(\mu)}} = \Phi^0_{\nu_{e(\mu)}}(E,\theta_n)$ is the $\nu_{e(\mu)}$ flux in the absence of neutrino oscillations and

$$r \equiv r(E,\theta_n) \equiv \frac{\Phi^0_{\nu_\mu}(E,\theta_n)}{\Phi^0_{\nu_e}(E,\theta_n)} \quad .$$

s^2_{23}**: b.f.v.** 0.573 (0.575) **NO (IO);** 3σ **CL: (0.415-0.619).**

$r(E,\theta_n) \cong (2.6 \div 4.5)$ **for neutrinos giving the main contribution to the multi-GeV samples,** $E \cong (2 \div 10)$ **GeV.**

M. Honda, 1995.

Hyper Kamiokande (5SK), IceCube-PINGU, ANTARES-ORCA;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India); ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-);
not designed to detect ν_e and $\bar{\nu}_e$ induced events.

IceCube at the South Pole: PINGU

PINGU: 50SK; ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-, no μ charge identification); Challenge: $E_\nu \gtrsim 2$ GeV (?)

KM3Net in Mediteranian sea: ORCA (near Toulon)

Studies of atmospheric ν oscillations with DUNE.

Atmospheric ν experiments

Subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s^2_{23}\, P_{2\nu}\,, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c^2_{23}\, P_{2\nu}\,,$$
$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s^4_{23}\; P_{2\nu} - 2c^2_{23}s^2_{23}\; \left[1 - Re\; (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))\right]\;,$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m^2_{31}, \theta_{13}; E, \theta_n; N_e)$: 2-$\nu$ $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth,
$\nu'_\tau = s_{23}\, \nu_\mu + c_{23}\, \nu_\tau$; $\Delta m^2_{21} \ll |\Delta m^2_{31(32)}|$, $E_\nu \gtrsim 2$ GeV;

κ and $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ are known phase and 2-ν amplitude.

NO: $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ **matter enhanced,** $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ **- suppressed**

IO: $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ **matter enhanced,** $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$**−suppressed**

No charge identification (SK, HK, IceCube-PINGU, ANTARES-ORCA); event rate (DIS regime): $[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$

Charge identification: INO; event rate (DIS regime): $\sigma(\nu_l + N \rightarrow l^- + X)$, $\sigma(\bar{\nu}_l + N \rightarrow l^+ + X)$

The Earth tomography based on the study of the effects of Earth matter on the oscillations of atmospheric neutrinos with different path-lengths in the Earth is discussed in:

W. Winter, Nucl. Phys. B 908 (2016) 250 (PINGU, ORCA; 7 layers, densities in these layers varied independently; the Earth mass and the Earth hydrostatic equilibrium constraints; systematic errors).

S. Bourret et al. [KM3NeT], J. Phys. Conf. Ser. 888 (2017) 012114 (ORCA; systematic errors not accounted for).

A. Kumar and S. Kumar Agarwalla, JHEP 08 (2021) 139 (INO).

K. J. Kelly, P. A. N. Machado, I. Martinez-Soler and Y. F. Perez-Gonzalez, JHEP 05 (2022) 187 (DUNE).

Early studies:

S. Choubey, P. Ghoshal and S.T.P., studies performed in the period 2008 - 2011 (HK, LAr), unpublished.

S. Choubey and S.T.P., studies performed in 2014 (PINGU), unpublished.

Composition of the Earth core (N_e):

C. Rott, A. Taketa and D. Bose, Sci. Rep. 5 (2015) 15225 (generic detector).

S. Bourret *et al.*, PoS ICRC2019 (2020) 1024 (ORCA).

Important Constraints

The total Earth mass constraint:

$$M_\oplus = (5.9722 \pm 0.0006) \times 10^{24}\ \text{kg}$$

Earth hydrostatic equilibrium constraint:

$$\rho_{man} \leq \rho_{OC} \leq \rho_{IC}.$$

Earth momentum of inertia constraint:

$$I_\oplus = (8.01736 \pm 0.00097) \times 10^{37}\ \text{kg}\,\text{m}^2.$$

$$I_\oplus = 0.330745\, M_\oplus\, R_\oplus^2.$$

The Earth matter effects in the ν oscillations depend on

$$V = \sqrt{2}\, G_{\mathrm{F}}\, N_e$$

$$N_e^{(E)}(r) = \rho_{\mathrm{E}}(r)\, Y_e / m_{\mathrm{N}},$$

Y_e **- electron fraction number (or** Z/A **factor).**

For isotopically symmetric matter $Y_e = 0.5$.

Earth core composition models: $Y_e^{oc} = 0.466 - 0.471$.

Earth mantle composition models: $Y_e^{oc} = 0.490 - 0.496$.

We used: $Y_e^{man} = 0.490$ **and** $Y_e^{c} = 0.467$.

Aspects of the Analysis

We change the PREM density in a given layer $\rho_i(r)$, $i = \mathrm{IC}, \mathrm{OC}, \mathrm{man}$, by a factor $(1+\kappa_i)$, κ_i is r-independent real constant:

$$\rho_i(r) \rightarrow (1+\kappa_i)\rho_i(r).$$

We will present results on sensitivity of ORCA to

$$\Delta\rho_i = 100\%\,((1+\kappa_i)\rho_i(r) - \rho_i(r))/\rho_i(r) = 100\%\,\kappa_i.$$

The Earth mass constraint, i.e., $M_\oplus$ should remain unchanged when varying $\rho_i(r)$, is accounted for.

When we consider, e.g., variation of $\rho_{\mathrm{IC}}(r)$, we study two cases: we compensate it by the corresponding change of density of i) the outer core $\rho_{\mathrm{OC}}(r)$, and ii) of the mantle $\rho_{\mathrm{man}}(r)$.
We proceed in a similar way when we analyse the sensitivity of ORCA to the OC, core and mantle densities, $\rho_{\mathrm{OC}}(r)$, $\rho_{\mathrm{C}}(r)$, and $\rho_{\mathrm{man}}(r)$.

In order to assess the effect of the Earth total mass constraint we obtained results on the ORCA's sensitivity to the mantle, outer core, inner core and total core densities without imposing this external constraint.

Simulation of Events in ORCA

For the unoscillated fluxes of atmospheric ν_μ, ν_e, and $\bar{\nu}_\mu$, $\bar{\nu}_e$, $\Phi_\alpha(\theta_n, E)$ and $\bar{\Phi}_\alpha(\theta_n, E)$, we use azimuth-averaged double differential $d^2\Phi_\alpha/(d\cos\theta_n dE)$ and $d^2\bar{\Phi}_\alpha/(d\cos\theta_n dE)$ updated fluxes from M. Honda *et al.*, arXiv:1502.0391 (i.e., the "solar minimum, without mountain over the detector" fluxes at the Frejus cite from the tables given in the web site quoted in arXiv:1502.0391).
The "source" of atmospheric neutrinos is assumed to be a layer located at 15 km above the Earth surface.

The principal observables in ORCA detector - the double differential event spectra in the neutrino energy E and nadir angle θ_n are calculated using the methods developed and described in F. Capozzi et al., arXiv:1503.01999 and arXiv:1708.03022.

The relevant detection characteristics of ORCA - the energy and angular resolutions and the dependence of the effective volumes for the different types/classes of events on the initial neutrino energy - are taken from S. Adrian-Martinez *et al.* [KM3Net], arXiv:1601.07459, and correspond to the benchmark (9 m vertical spacing) configuration of the ORCA experiment.

We consider $E \in [2, 100]$ GeV and $\theta_n/\pi \in [0, 0.5]$. These two ranges are divided into 20 equally-spaced bins (linearly for θ_n and logarithmically for

E), for a total of 400 bins for cascade events and an equal number for track events.

Four sets of systematic uncertainties are taken into account in the calculations: minimal, optimistic, default and conservative.

In the analysis δ and $\sin^2\theta_{23}$ are kept fixed to certain values. We have obtained results for $\delta = 3\pi/2$ and eleven values of $\sin^2\theta_{23}$ from the interval [0.40,0.60], which belong to, and essentially span, the 3σ range of allowed values of $\sin^2\theta_{23}$ obtained in the latest global neutrino data analyses.

Results

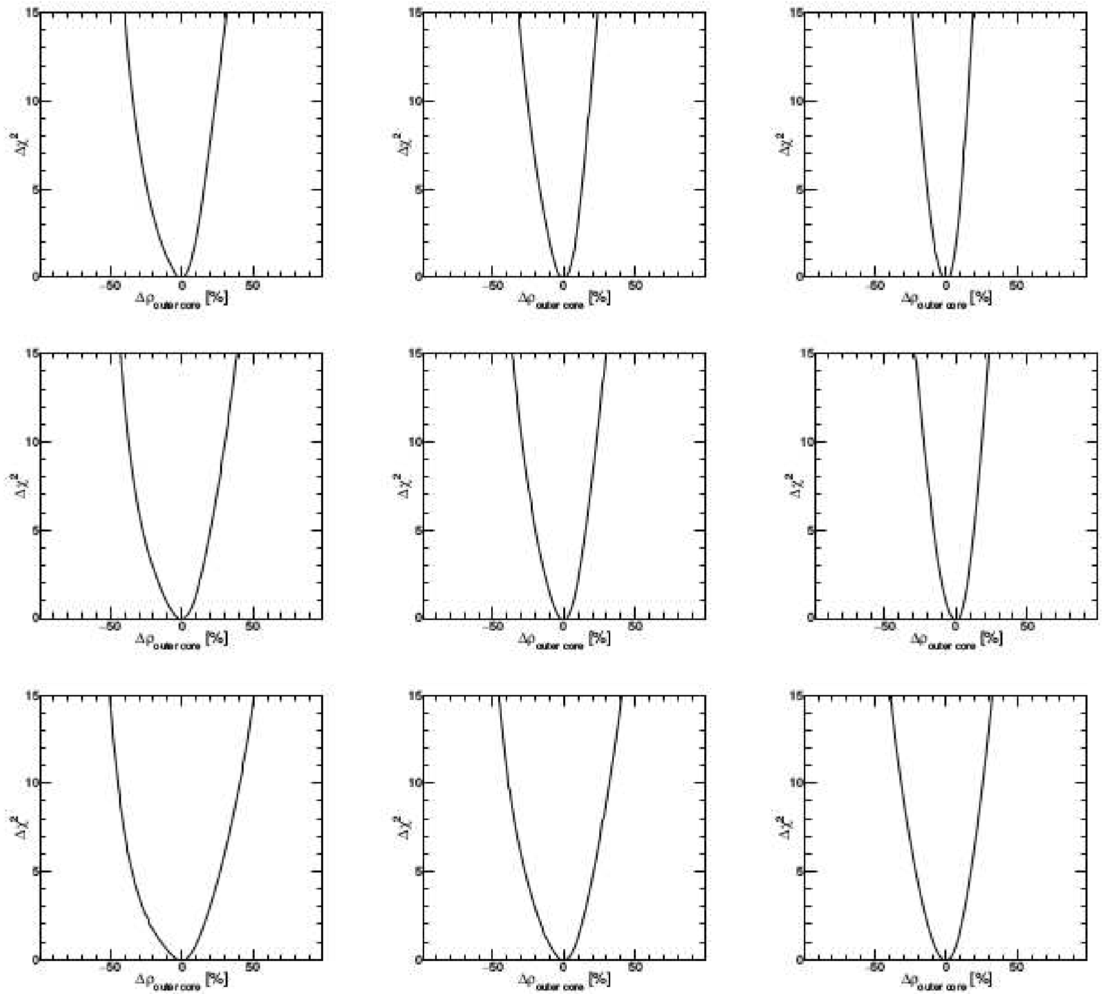


Figure 1: Sensitivity to the OC density in the case of NO spectrum and 10 years of data. The Earth total mass constraint is implemented by compensating the OC density variation with a corresponding mantle density change. The results shown are for $\sin^2\theta_{23} = 0.42, 0.50, 0.58$ (left, center and right panels) and in the cases of "minimal", "optimistic" and "conservative" systematic errors (top, middle and bottom panels). See text for further details.

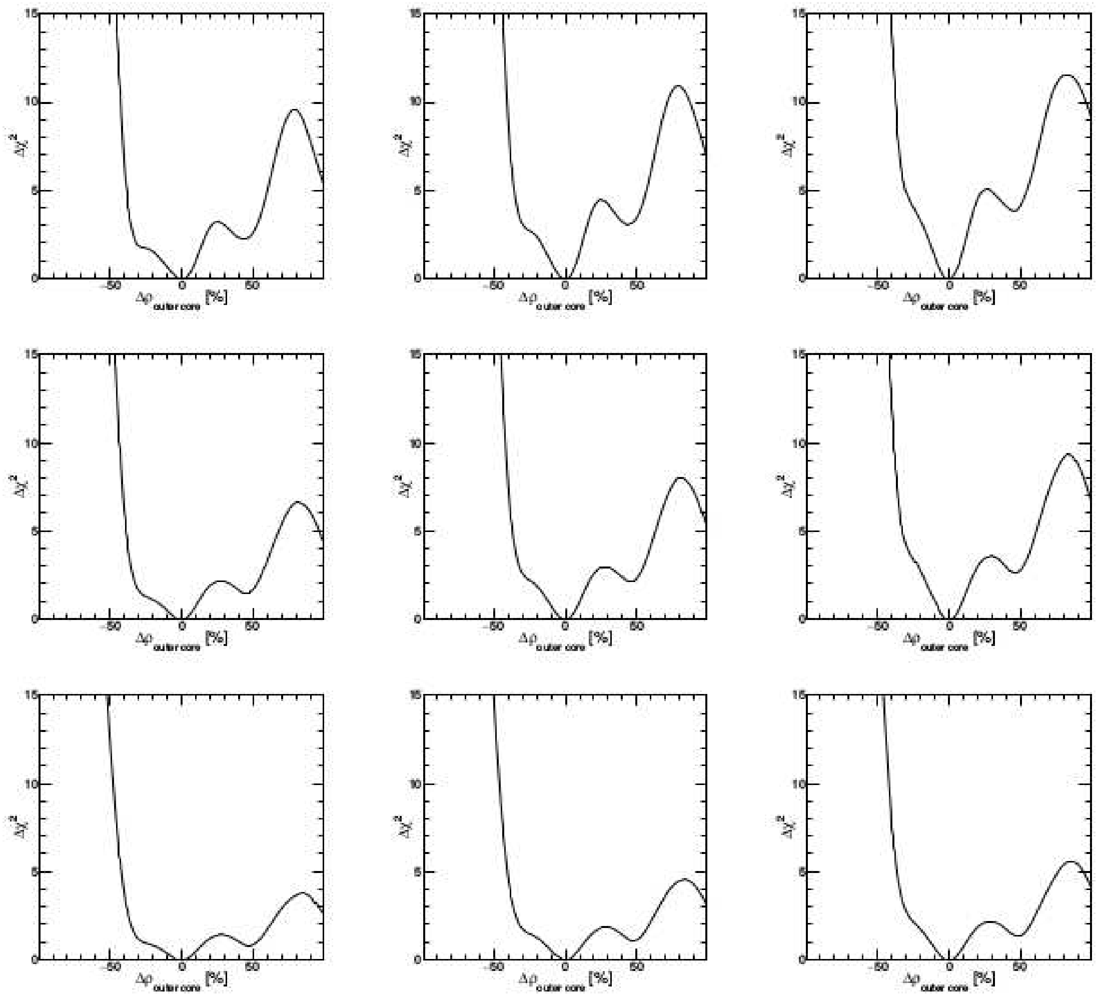


Figure 3: The same as in Fig. [1], but without implementing the Earth total mass constraint. The results shown are for $\sin^2\theta_{23} = 0.42,\ 0.50,\ 0.58$ (left, center and right panels) and in the cases of "minimal", "optimistic" and "conservative" systematic errors (top, middle and bottom panels). See text for further details.

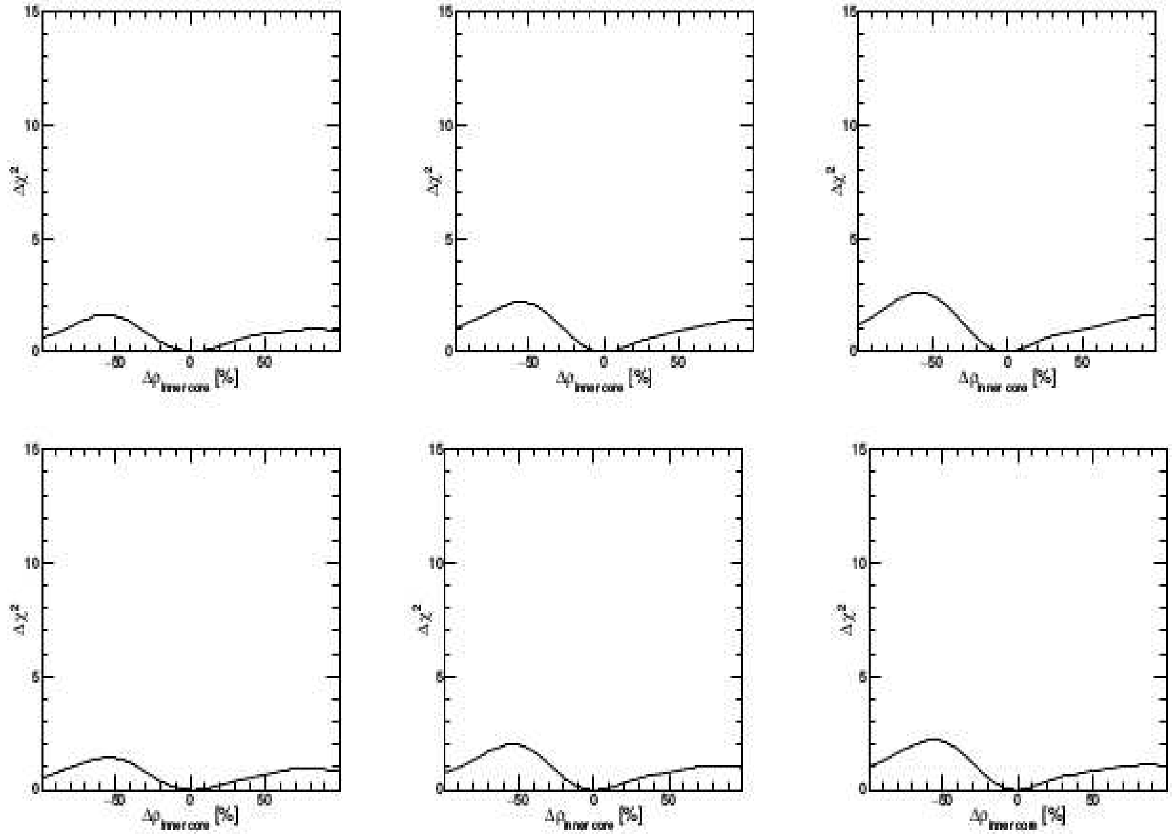


Figure 4: Sensitivity to the IC density. The Earth total mass constraint i) is implemented by compensating the IC density variation with a corresponding mantle density change (top panels), ii) is not implemented (bottom panels). The results shown are for $\sin^2\theta_{23} = 0.42$, 0.50, 0.58 (left, center and right panels) and in the case of "minimal" systematic errors. See text for further details.

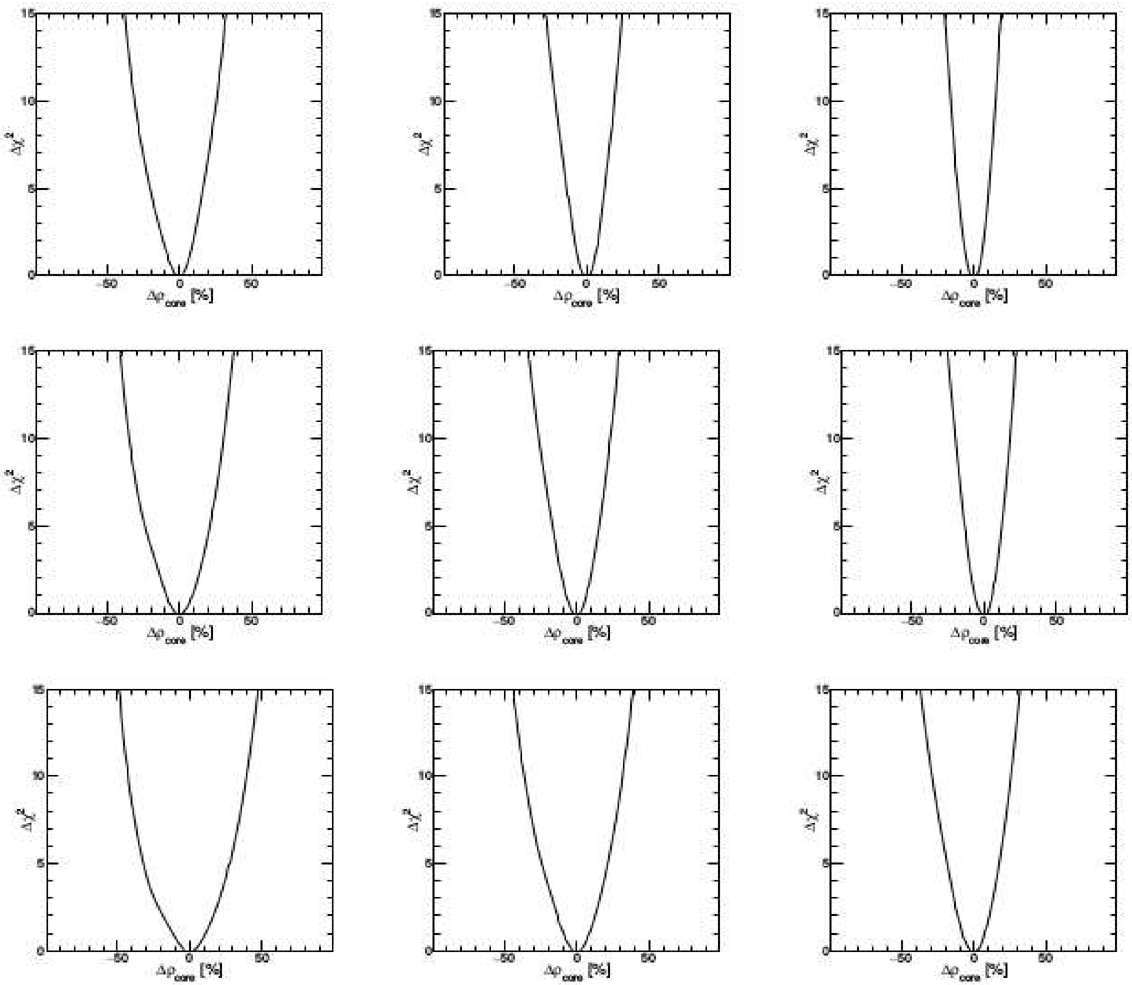


Figure 5: Sensitivity to the core density in the case of NO spectrum and 10 years of data. The Earth total mass constraint is implemented by compensating the core density variation with a corresponding mantle density change. The results shown are for $\sin^2\theta_{23} = 0.42,\ 0.50,\ 0.58$ (left, center and right panels) and in the cases of "minimal", "optimistic" and "conservative" systematic errors (top, middle and bottom panels). See text for further details.

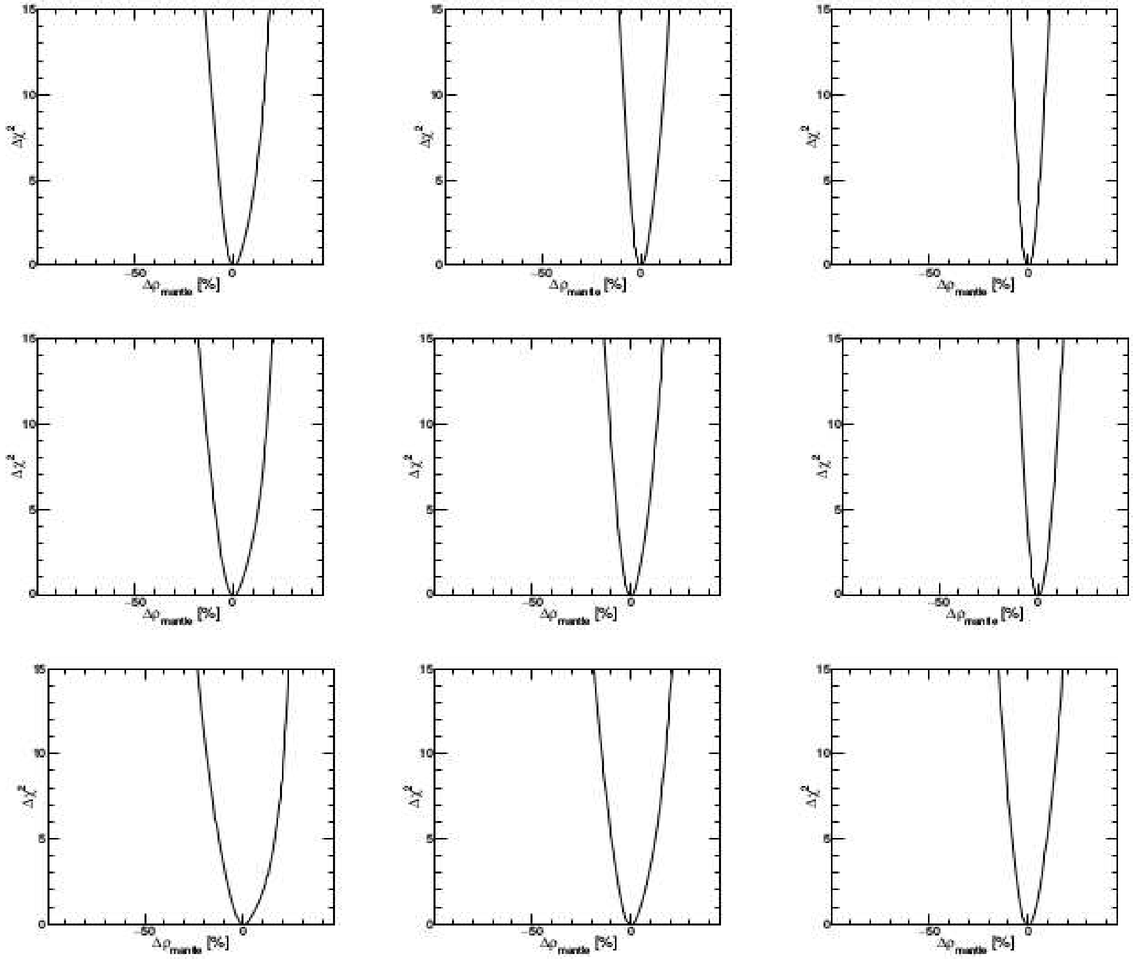


Figure 7: Sensitivity to the mantle density in the case of NO spectrum and 10 years of data. The Earth total mass constraint is implemented by compensating the mantle density variation with a corresponding OC density change. The results shown are for $\sin^2\theta_{23} = 0.42$, 0.50, 0.58 (left, center and right panels) and in the cases of "minimal", "optimistic" and "conservative" systematic errors (top, middle and bottom panels). See text for further details.

The sensitivity of ORCA to the densities of the outer core, core and mantle depend strongly
1. on the value of $\sin^2\theta_{23}$,
2. on the type of systematic errors employed in the analysis,
3. on whether the total Earth mass constraint is implemented or not, and
4. on the way the compensation of the density variation in a given layer by a change of density in another layer is implemented, i.e., on the choice of the "compensating" layer, when the total Earth mass constraint is imposed.
It depends also strongly on the type of neutrino mass spectrum.

"Most favorable" NO case: of "minimal" systematic errors, $\sin^2\theta_{23} = 0.58$ and implemented Earth mass constraint (by varying mantle (OC) density), ORCA can determine, e.g., the OC (mantle) density at 3σ C.L. after 10 years of operation with an uncertainty of (-18%)/+12% (of (-6%)/+8%).

"Most disfavorable" NO case with implemented Earth mass constraint but "conservative" systematic errors and $\sin^2\theta_{23} = 0.42$, the uncertainty reads (-43%)/+39% ((-17%/+20%); for for $\sin^2\theta_{23} = 0.50$ and 0.58 it is noticeably smaller: (-37)%/+30% and (-30%)/+24% ((-13%)/+17% and (-11%/+14%)).

The Earth hydrostatic equilibrium constraints were taken into account

and there effects were accounted for; in certain cases results remain unchanged, in other cases they change somewhat, but not dramatically.

The uncertainties in the determination of the outer core, total core and mantle densities by ORCA in the case of NO spectrum, according to our results, are considerably larger if the total Earth mass constraint is not implemented in the analysis, or if it is implemented but the inner core is used as a "compensation" layer.

We find also that the sensitivity of ORCA to the outer core, core and mantle densities is significantly worse for the IO neutrino mass spectrum than for the NO spectrum.

Conclusion.

The neutrino tomography of the Earth is a promissing powerful alternative method of obtaining valuable information about the Earth interior. It is at initial stage of development.

The ORCA experiment has the potential of making unique pioneering contributions to the studies of the Earth interior with atmospheric neutrinos, i.e., to the neutrino tomography of the Earth.