FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES IN NON-LEPTONIC B DECAYS

MATTHIAS NEUBERT

PRISMA+ CLUSTER OF EXCELLENCE & MITP — JOHANNES GUTENBERG UNIVERSITY MAINZ

FPCAPRI 2022 — ANACAPRI, 11 JUNE 2022

WORK IN PROGRESS WITH PHILIPP BÖER & MICHEL STILLGER

Cluster of Excellence PRISMA+
DFG EXC 2118/1
Precision Physics, Fundamental Interactions and Structure of Matter

MOTIVATION

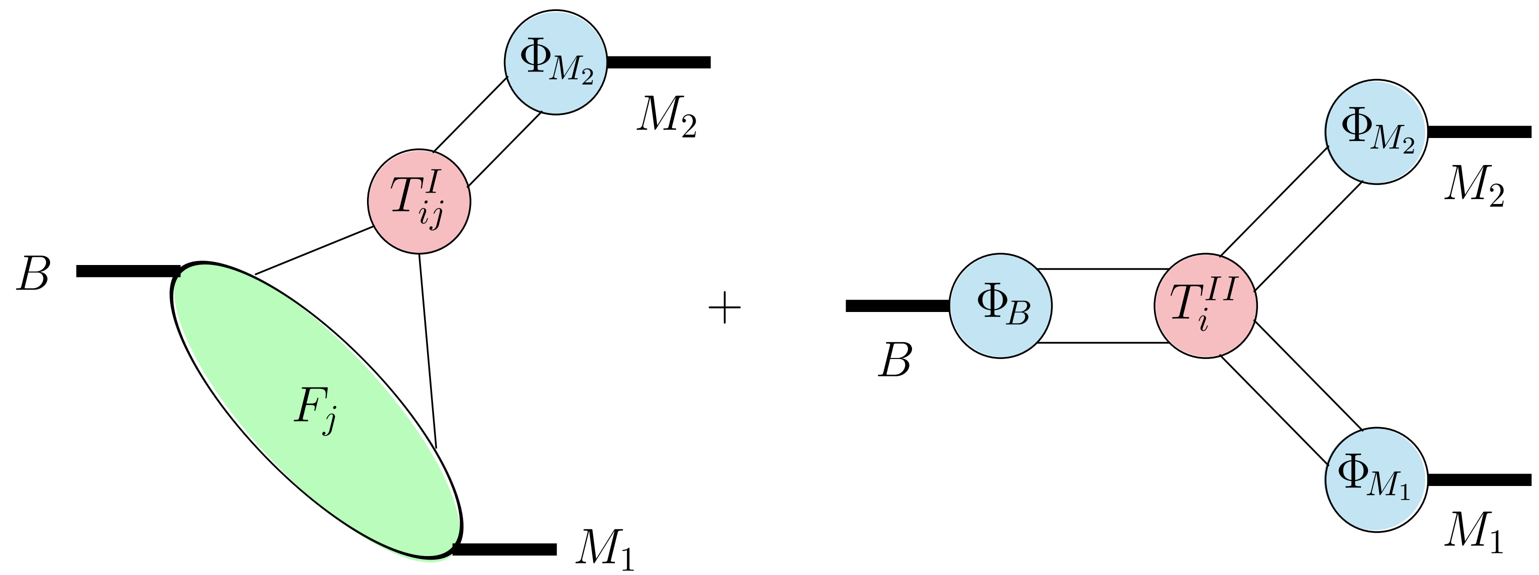
How to obtain a theory of power corrections for observables not admitting a (Euclidean) operator product expansion?

- highly relevant for phenomenology, e.g. LHC physics and heavy flavor physics, which rely on factorization theorems involving hadronic matrix elements of light-ray operators
  - collider physics: PDFs, generalized PDFs, …
  - heavy flavor physics: *B*-meson shape function, LCDAs, …
- 1990s: “CERN Renormalon Club”

MOTIVATION

QCD factorization approach for non-leptonic B decays (1999-2001)

M. Beneke, G. Buchalla, C.T. Sachrajda, MN (1999, 2000, 2001)



$$\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle = \sum_j F_j^{B\to M_1}(m_2^2) \int_0^1 du\, T_{ij}^I(u)\, \Phi_{M_2}(u) \;+\; (M_1 \leftrightarrow M_2)$$

$$+ \int_0^1 d\xi du dv\, T_i^{II}(\xi,u,v)\, \Phi_B(\xi)\, \Phi_{M_1}(v)\, \Phi_{M_2}(u)$$

MOTIVATION

SCET – Soft-Collinear Effective Theory (2000-2002)

C.W. Bauer, S. Fleming, D. Pirjol, I.W. Stewart (2000)
C.W. Bauer, D. Pirjol, I.W. Stewart (2001)
M. Beneke, A.P. Chapovsky, M. Diehl, T. Feldmann (2002)

- rigorous EFT framework for describing the interactions of energetic light particles
- solid basis for deriving soft-collinear factorization theorems
- theory of power corrections to factorization theorems?

MOTIVATION

QCD factorization approach

- does not allow for a consistent calculation of power-suppressed contributions to the decay amplitudes
- factorization at $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ is spoiled by **endpoint-divergent convolution integrals**, e.g.:

$$\int_0^1 \frac{dx}{x^2}\,\phi_\pi(x) \sim \int_0^1 \frac{dx}{x}$$

MOTIVATION

SCET factorization at next-to-leading power (NLP)

I. Moult, G. Vita, K. Yan (2019)
I. Moult, I.W. Stewart, G. Vita [+ H.X. Zhu] (2019)
M. Beneke, A. Broggio, S. Jaskiewicz, L. Vernazza (2019)
Z.L. Liu, MN (2019)

- subject of intense recent research
- endpoint divergences are a generic feature
- not clear how to deal with them; simplest case:

$$\int_0^1 \frac{dx}{x^{1+\epsilon}}\, H(x)\, \langle O(x) \rangle$$

hard function (Wilson coefficient) → $H(x)$

soft function (matrix element) → $\langle O(x) \rangle$

  - how to interpret addition $1/\epsilon$ poles from convolution integral?
  - how to renormalize such integrals?

MOTIVATION

Consistent solution based on *D*-dimensional refactorization conditions and plus-type subtractions of endpoint divergences

Z.L. Liu, MN (2019)
Z.L. Liu, B. Mecaj, MN, X. Wang (2020)

- formalism developed for the example of $H \to (b\bar{b})^* \to \gamma\gamma$, but technique is completely general
- factorization theorems contains (at least) two different terms with endpoint singularities, e.g.:

$$\int_0^1 \frac{dx}{x^{1+\epsilon}}\, H_1(x)\, \langle O_1(x) \rangle + \int_0^\infty \frac{dx}{x^{1+\epsilon}}\, H_2(x)\, \langle O_2(x) \rangle$$

- using SCET techniques, one can prove that in the singular regions the two integrands, defined in dimensional regularization, are **identical** (to all orders in α_s) up to power-suppressed terms

MOTIVATION

Consistent solution based on *D*-dimensional refactorization conditions and plus-type subtractions of endpoint divergences

Z.L. Liu, MN (2019)

- endpoint divergences can then be removed by a rearrangement of the integrands (in the bare theory):

integrand expanded in singular region

$$\int_0^1 \frac{dx}{x^{1+\epsilon}} \Big(H_1(x) \, \langle O_1(x) \rangle - [[H_1(x)]] \, [[\langle O_1(x) \rangle]] \Big)$$

$$+ \int_0^\infty \frac{dx}{x^{1+\epsilon}} \Big(H_2(x) \, \langle O_2(x) \rangle - [[H_2(x)]] \, [[\langle O_2(x) \rangle]] \Big)$$

- difference is a scaleless integral, which vanishes is dim reg
- one can prove that this rearrangement can be maintained after renormalization

Z.L. Liu, B. Mecaj, MN, X. Wang (2020)

MOTIVATION

Consistent solution based on *D*-dimensional refactorization conditions and plus-type subtractions of endpoint divergences

Z.L. Liu, MN (2019)
Z.L. Liu, B. Mecaj, MN, X. Wang (2020)

- recently, the same method has been applied to flavor non-singlet, off-diagonal contributions to thrust in the 2-jet region

  M. Beneke, M. Garny, Jaskiewicz, J. Strohm, R. Szafron (2022)

- goal of this project:

first application in context of *B* physics!

MOTIVATION

Consistent solution based on *D*-dimensional refactorization conditions and plus-type subtractions of endpoint divergences

Z.L. Liu, MN (2019)
Z.L. Liu, B. Mecaj, MN, X. Wang (2020)

- recently, the same method has been applied to flavor non-singlet, off-diagonal contributions to thrust in the 2-jet region

M. Beneke, M. Garny, Jaskiewicz, J. Strohm, R. Szafron (2022)

- goal of this project:

first application in context of *B* physics!

Good news:

MOTIVATION

Consistent solution based on *D*-dimensional refactorization conditions and plus-type subtractions of endpoint divergences

Z.L. Liu, MN (2019)
Z.L. Liu, B. Mecaj, MN, X. Wang (2020)

- recently, the same method has been applied to flavor non-singlet, off-diagonal contributions to thrust in the 2-jet region

M. Beneke, M. Garny, Jaskiewicz, J. Strohm, R. Szafron (2022)

- goal of this project:

first application in context of *B* physics!

Good news:

- power corrections to QCD factorization can be factorized in a systematic way, with regularized endpoint behavior …

MOTIVATION

Consistent solution based on *D*-dimensional refactorization conditions and plus-type subtractions of endpoint divergences

Z.L. Liu, MN (2019)
Z.L. Liu, B. Mecaj, MN, X. Wang (2020)

- recently, the same method has been applied to flavor non-singlet, off-diagonal contributions to thrust in the 2-jet region

M. Beneke, M. Garny, Jaskiewicz, J. Strohm, R. Szafron (2022)

- goal of this project:

first application in context of *B* physics!

Good news:

- power corrections to QCD factorization can be factorized in a systematic way, with regularized endpoint behavior …
- … but at the expense of a proliferation of nonperturbative functions

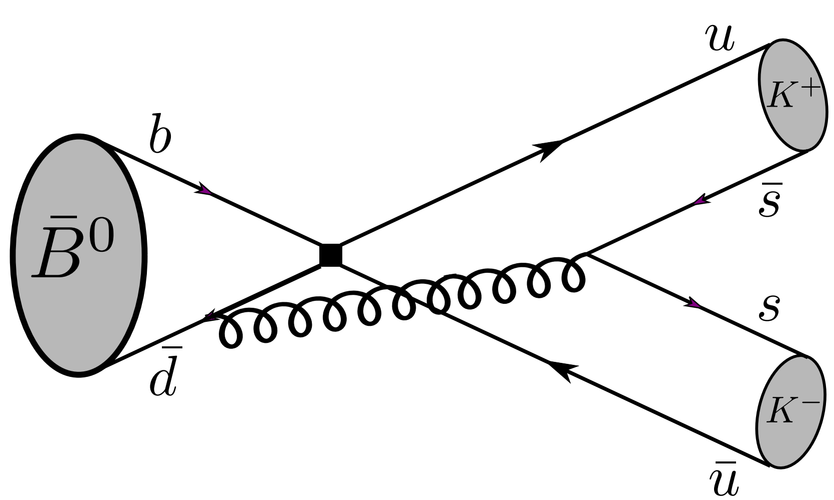
FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

Consider the decay $\bar{B}^0 \to K^+K^-$ as a study case

- pure weak annihilation mode: $(b\bar{d}) \to (u\bar{s}) + (s\bar{u})$, so the valence quarks of the *B* meson must be annihilated at the weak vertex
- amplitude starts at $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ relative to leading-order amplitudes
- QCD factorization yields a divergent result ($\bar{x} \equiv 1 - x$):

$$\mathcal{A}(\bar{B}^0 \to K^+K^-) \propto 4\pi\alpha_s\, f_B f_K^2 \int_0^1 dx \int_0^1 dy\, \Phi_{K^+}(x)\, \Phi_{K^-}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right]$$

diverges for $x \to 1$



FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

Consider the decay $\bar{B}^0 \to K^+K^-$ as a study case

- analysis of this decay involves a 3-step matching procedure:

SM → H_{eff} → SCET-1 → SCET-2

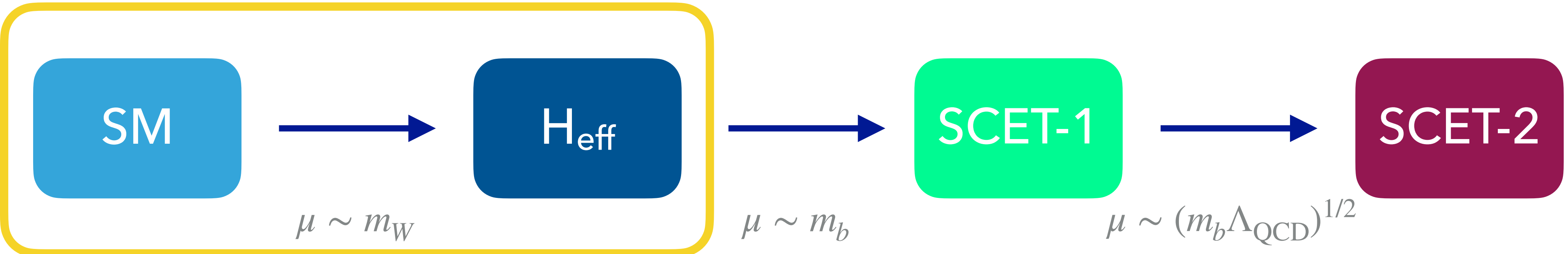
$\mu \sim m_W$ $\qquad$ $\mu \sim m_b$ $\qquad$ $\mu \sim (m_b\Lambda_{\text{QCD}})^{1/2}$

- rather than presenting a detailed technical discussion, I will illustrate the resulting structures using tree-level Feynman graphs
- resulting factorization theorem:

$$\mathcal{A}(\bar{B}^0 \to K^+K^-) = \sum_i H_i(m_b, \mu) \otimes J_i(m_b\Lambda_{\text{QCD}}, \mu) \otimes S_i(\Lambda_{\text{QCD}}, \mu)$$

↑ hard functions $\qquad$ ↑ jet functions $\qquad$ ↑ soft functions (nonperturbative)

FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

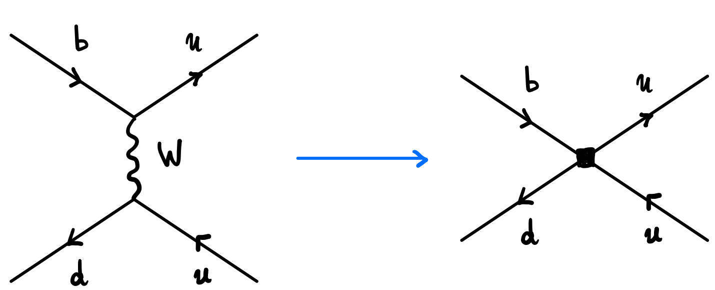


Step 1: effective weak Hamiltonian

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qd}^* \left[C_1\, Q_1^q + C_2\, Q_2^q + \sum_{i=3}^{6} C_i\, Q_i + C_{8g}\, Q_{8g} \right] + \text{h.c.}$$

- local operators:

$$Q_1^q = (\bar d q)_{V-A} (\bar q b)_{V-A} \qquad Q_2^q = (\bar d^i q^j)_{V-A} (\bar q^j b^i)_{V-A}$$

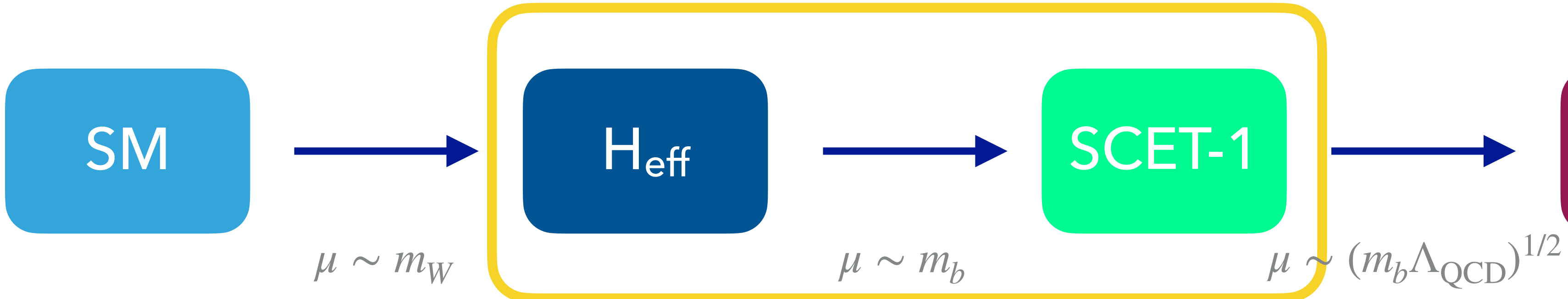


$$Q_3 = (\bar d b)_{V-A} \sum_q (\bar q q)_{V-A} \qquad Q_4 = (\bar d^i b^j)_{V-A} \sum_q (\bar q^j q^i)_{V-A}$$

$$Q_5 = (\bar d b)_{V-A} \sum_q (\bar q q)_{V+A} \qquad Q_6 = (\bar d^i b^j)_{V-A} \sum_q (\bar q^j q^i)_{V+A}$$

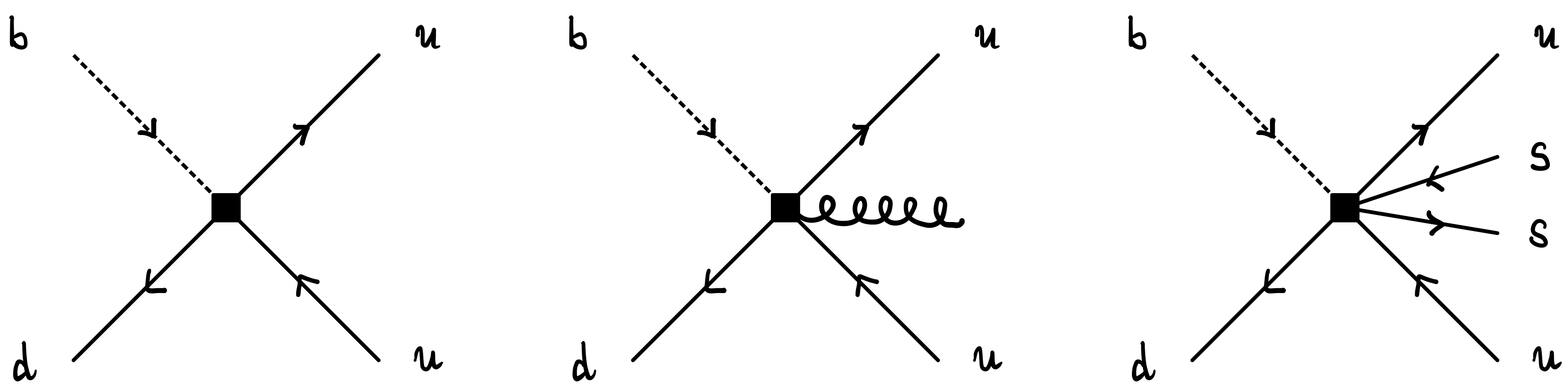
$$Q_{8g} = -\frac{g_s}{8\pi^2}\, m_b\, \bar d \sigma_{\mu\nu} (1+\gamma_5) t^a b\, G^{\mu\nu\, a}$$

FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES



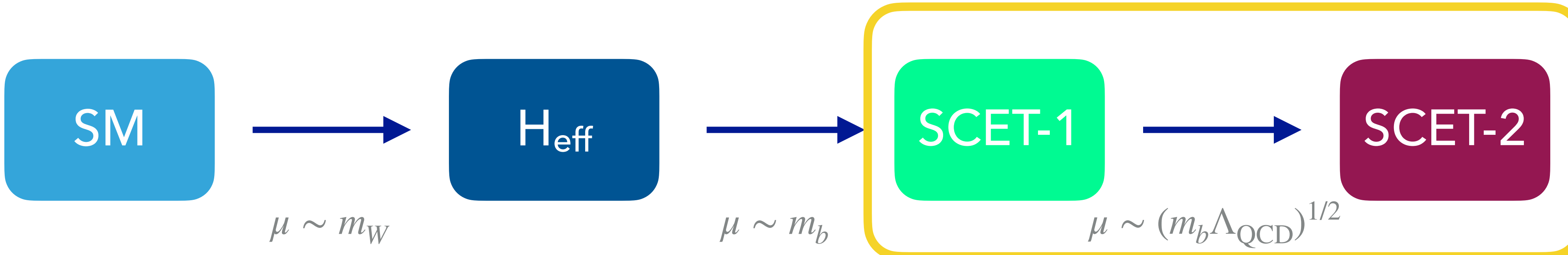
Step 2: matching onto SCET-1

- relevant operators: 4-quark operators, 4-quark operators with an extra gluon, 6-quark operators



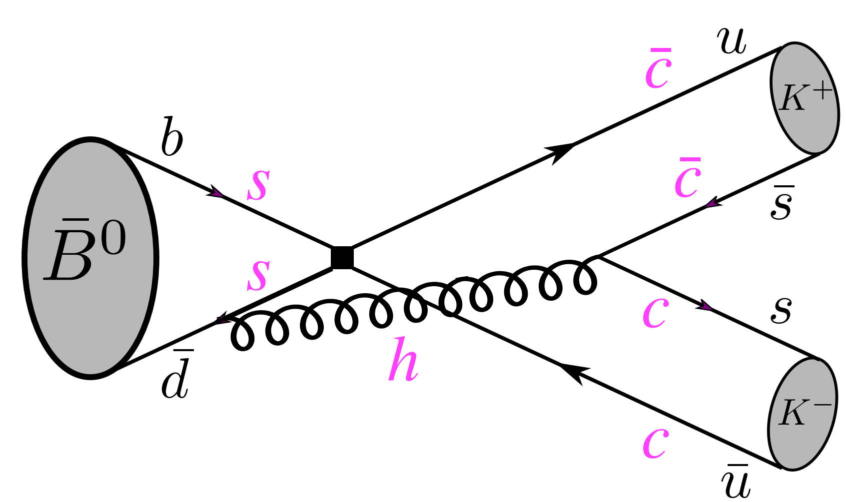
- matching coefficients define the hard functions

FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

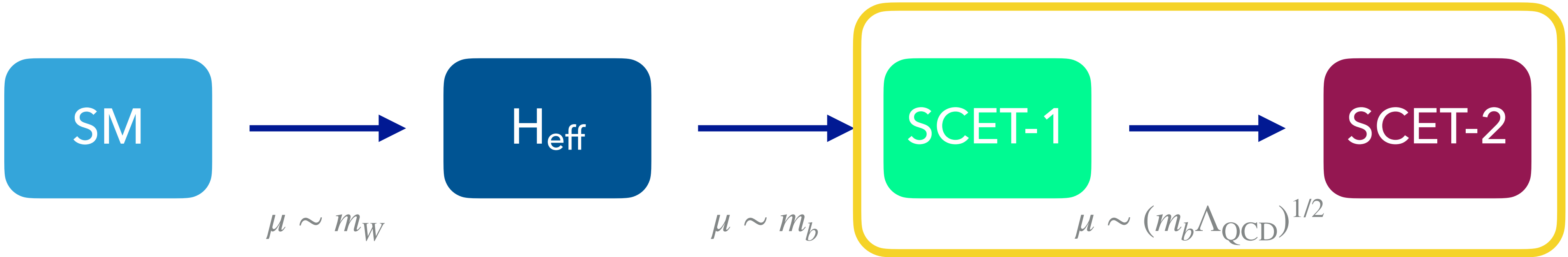


Step 2: matching onto SCET-2

- relevant operators:
  - **6-quark operators**
  - 6-quark operators plus final-state gluon
  - 8-quark operators
  - 8-quark operators plus initial-state gluon
  - 8-quark operators plus final-state gluon
- matching coefficients define the jet functions

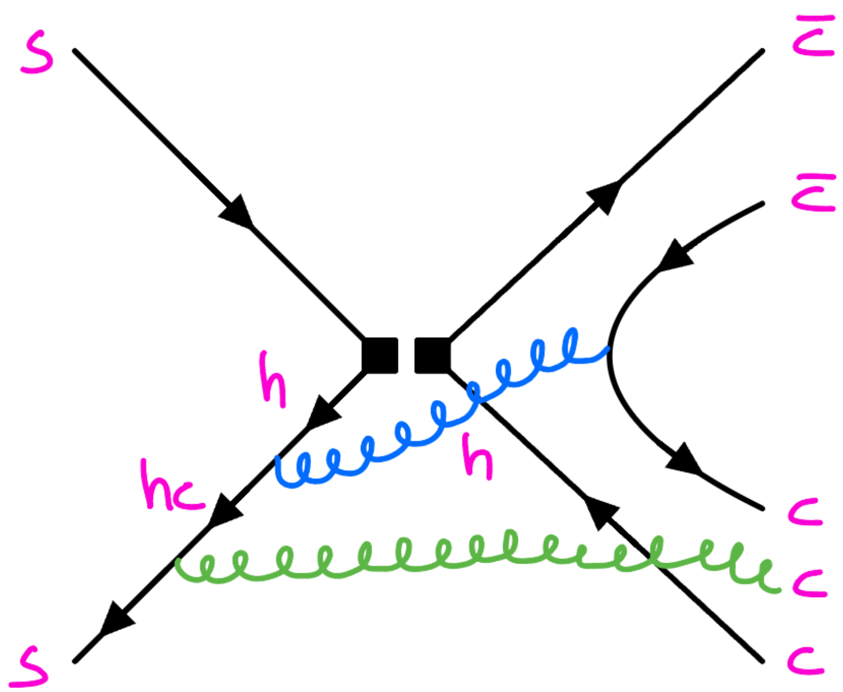


FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

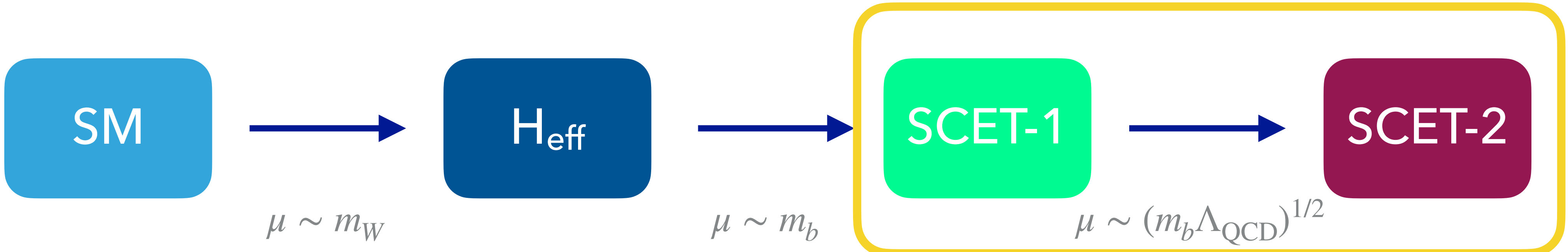


Step 2: matching onto SCET-2

- relevant operators:
  - 6-quark operators
  - **6-quark operators plus final-state gluon**
  - 8-quark operators
  - 8-quark operators plus initial-state gluon
  - 8-quark operators plus final-state gluon
- matching coefficients define the jet functions

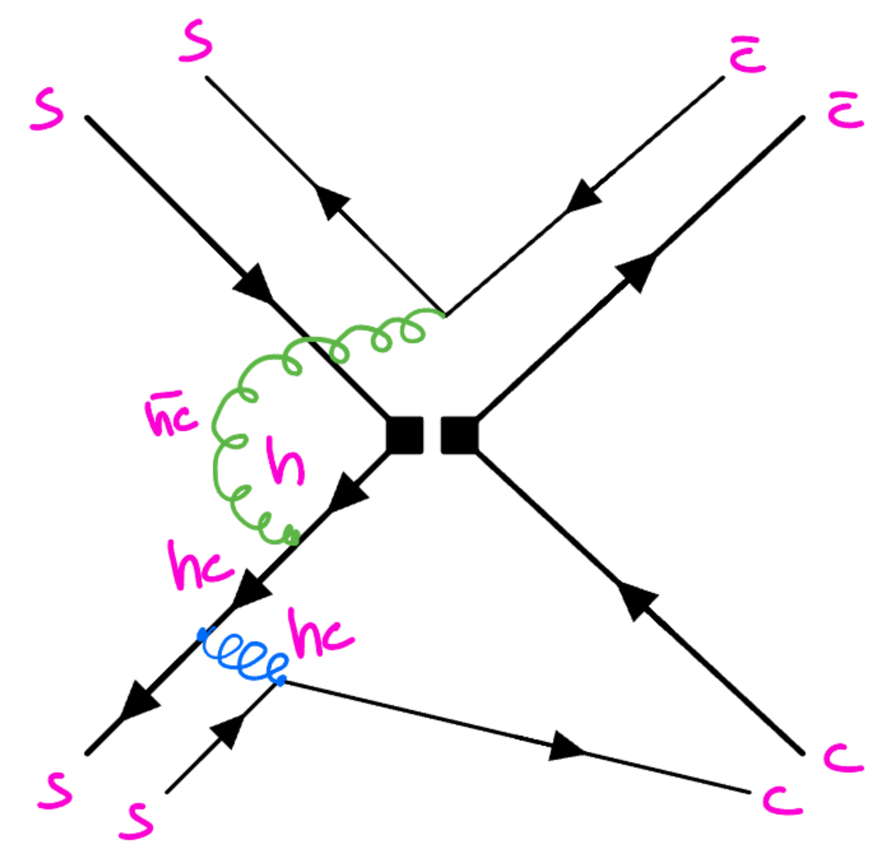


FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

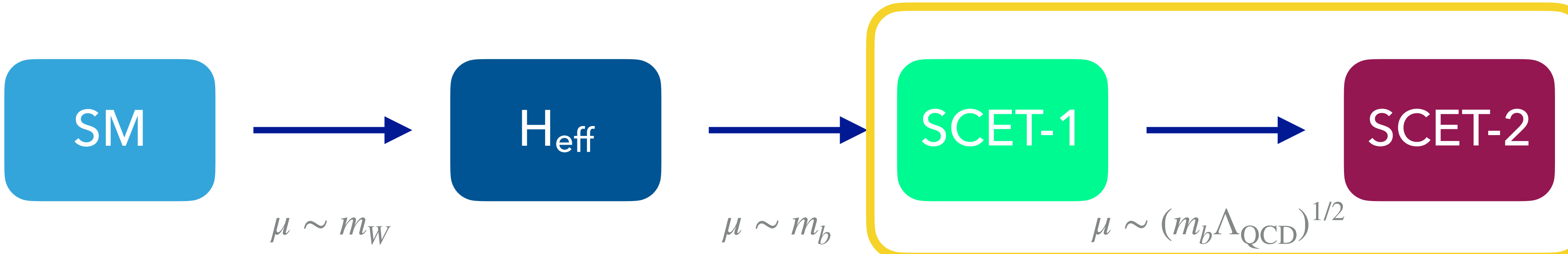


Step 2: matching onto SCET-2

- relevant operators:
  - 6-quark operators
  - 6-quark operators plus final-state gluon
  - **8-quark operators**
  - 8-quark operators plus initial-state gluon
  - 8-quark operators plus final-state gluon
- matching coefficients define the jet functions

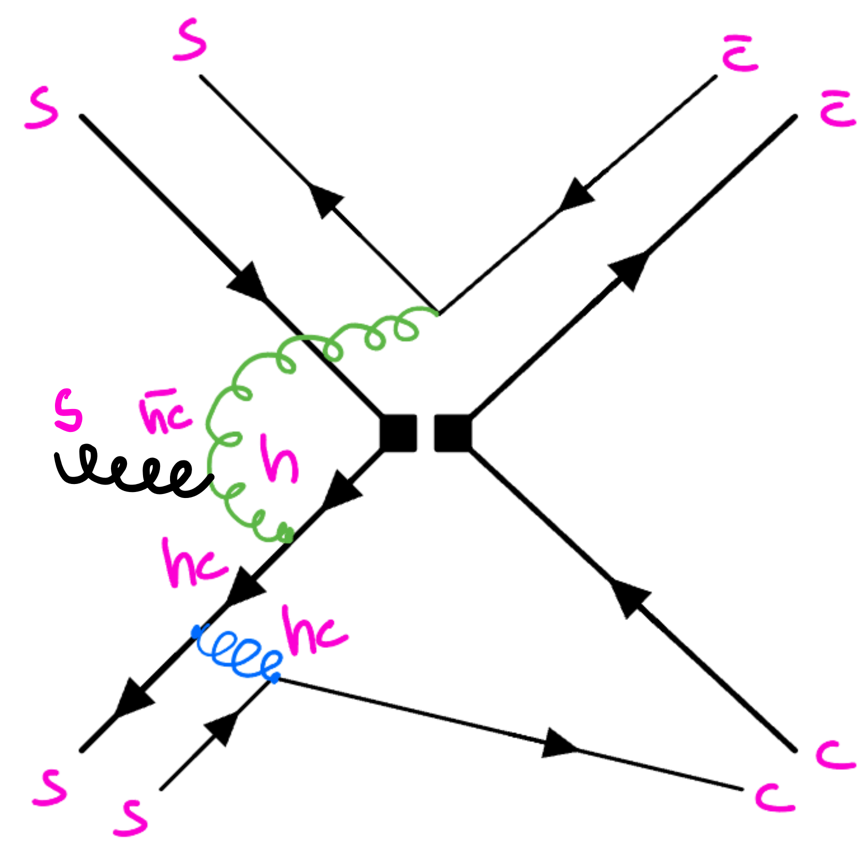


FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

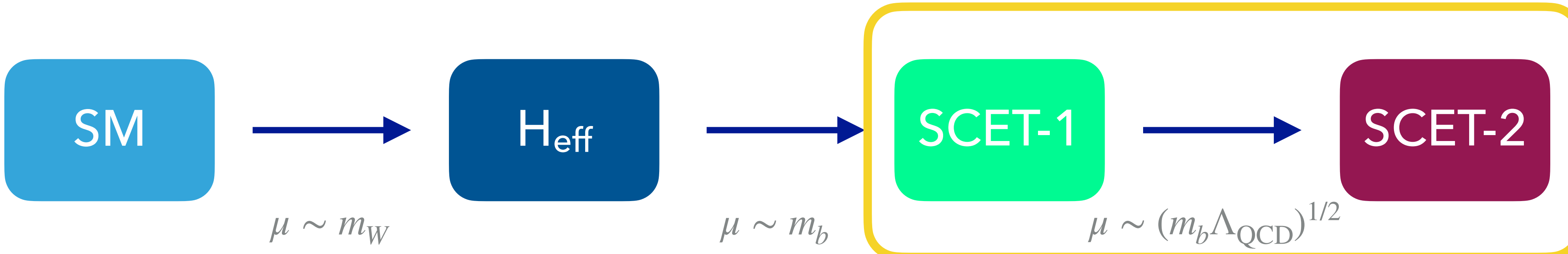


Step 2: matching onto SCET-2

- relevant operators:
  - 6-quark operators
  - 6-quark operators plus final-state gluon
  - 8-quark operators
  - **8-quark operators plus initial-state gluon**
  - 8-quark operators plus final-state gluon
- matching coefficients define the jet functions

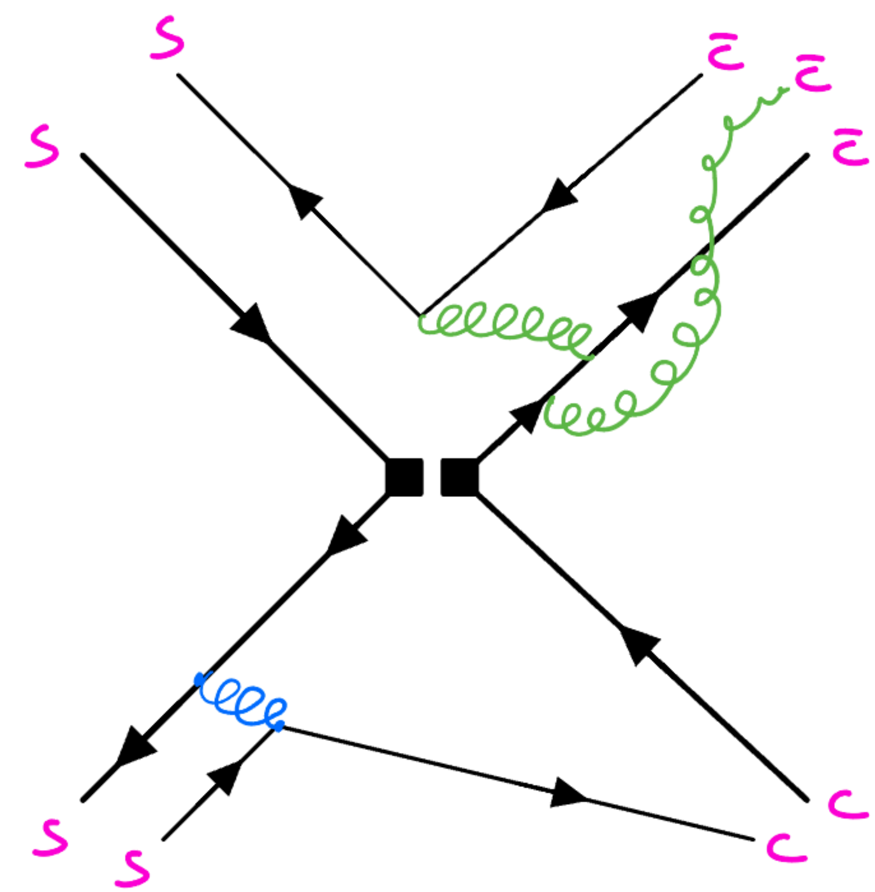


FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

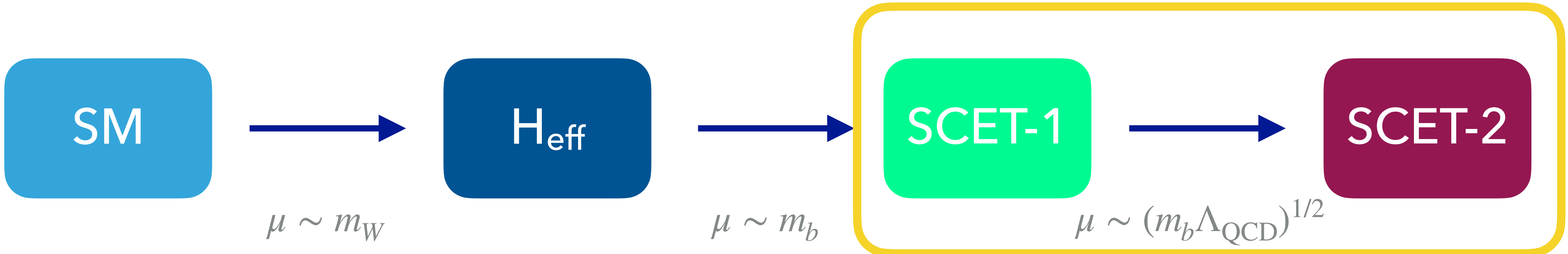


Step 2: matching onto SCET-2

- relevant operators:
  - 6-quark operators
  - 6-quark operators plus final-state gluon
  - 8-quark operators
  - 8-quark operators plus initial-state gluon
  - **8-quark operators plus final-state gluon**
- matching coefficients define the jet functions

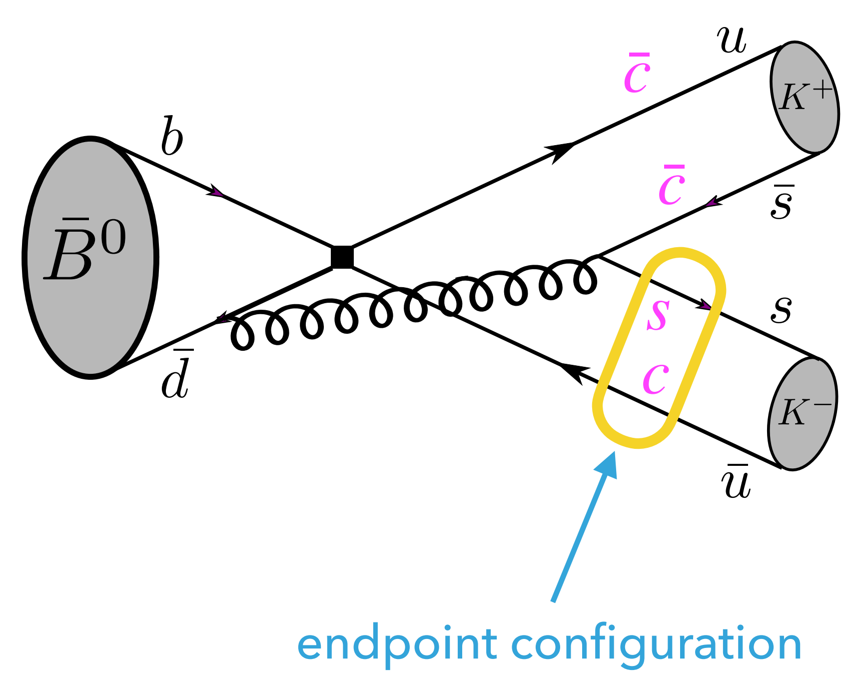


FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES

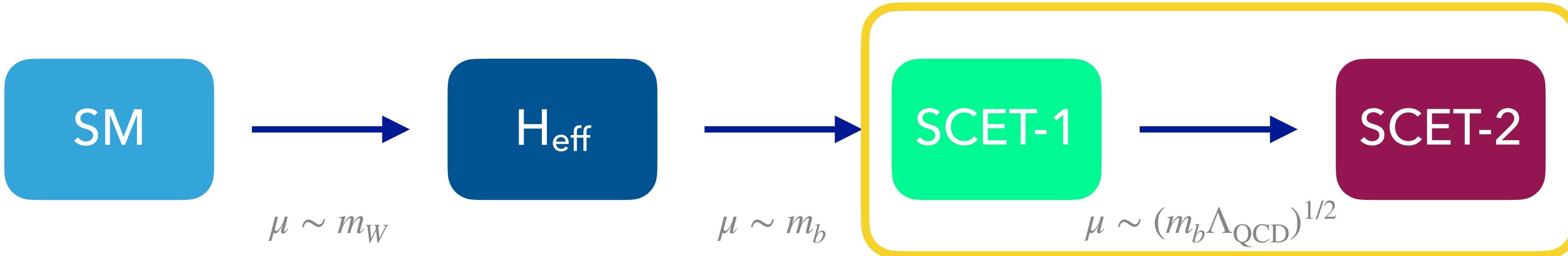


Step 2: **matching onto SCET-2**

- another relevant operator:
  - **endpoint configuration, where one of the kaons contains a soft quark**
- needed to cancel the endpoint divergences of other operators in the sum over all contributions!



FACTORIZATION OF WEAK ANNIHILATION AMPLITUDES



Step 2: **matching onto SCET-2**

- relevant hadronic quantities:
  - 2- and 3-particle LCDAs (twist-2 and twist-3) of the kaons
  - leading 2-particle LCDA of the *B* meson A.G. Grozin, MN (1996)
  - 4- and 5-particle DAs of the *B* meson, with field localized on two different light cones
  - soft $B \to K$ form factor for the endpoint contribution
- multi-particle *B*-meson amplitudes and soft form factor are new objects!

CONCLUSIONS

- first systematic extension of QCD factorization in nonleptonic B-meson decays to $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$
- endpoint divergences can be regularized and removed using the approach developed by us in 2019–2020
- SCET framework allows for resummation of all large logarithms based on RG equations (not discussed here)
- at NLP several new soft functions appear – DAs and form factors – which so far have not been studied
- expect that same soft functions will arise in NLP factorization theorems for other $B \to M_1 M_2$ decay amplitudes

Thank you!