PhD Defense

Modeling and Simulation of Active Plasma Lenses for High Brightness Electron Beams

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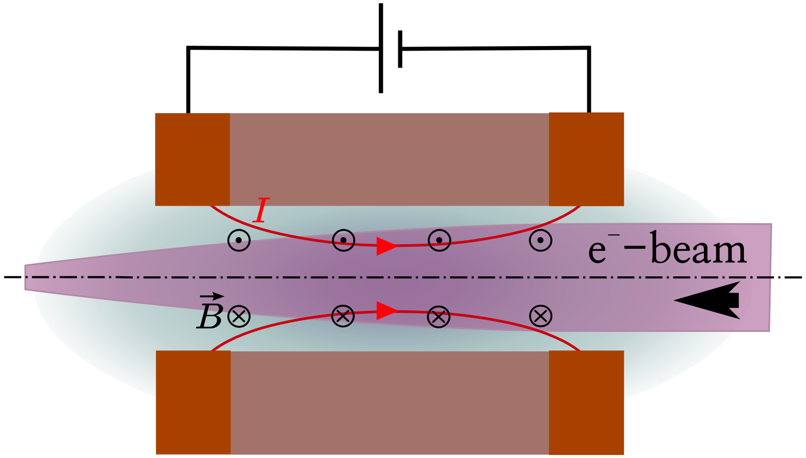
SPARC LAB

Contents

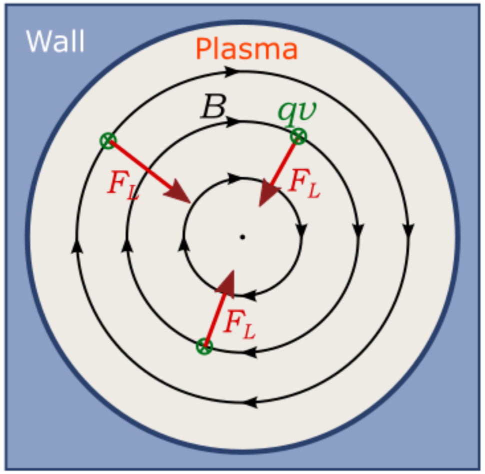
1.1. Active plasma lenses in accelerator physics

What is an active plasma lens*?

It is a device which can focus an electron beam, thanks to an azimuthal magnetic field generated by a discharge current induced in a gas-filled capillary

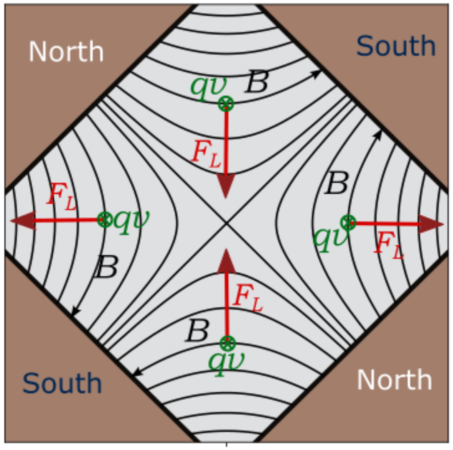


Scheme of principle of an active plasma lens



Schematic transverse view of an active plasma lens

Compare with quadrupole:



*W.K.H. Panofsky and W.R. Baker, Rev. Sci. Instr. 21, 445 (1950)

1.1. Active plasma lenses in accelerator physics

- Interesting features:
  - Magnetic field gradients even higher than those achievable in permanent magnet quadrupoles, up to several kT/m
  - Compact and capable of focusing a beam in both transverse planes
  - The focusing strength, K, scales as $1/\gamma$

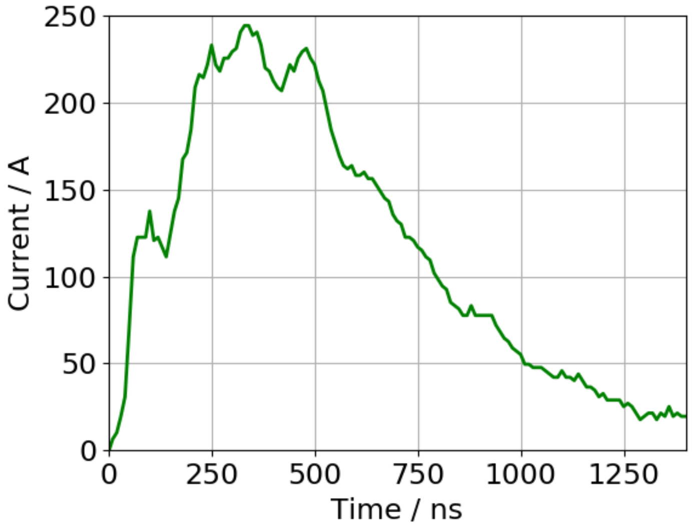
⇩

- Promising because...
  - High focusing gradient → deliver small spot (…*matched*) beams to accelerators of new conception
  - Compact and symmetric → miniaturization of accelerators → cost reduction
  - K, scales as $1/\gamma$ → Better than solenoids (scaling as $1/,\gamma^2$) competing with quadrupoles

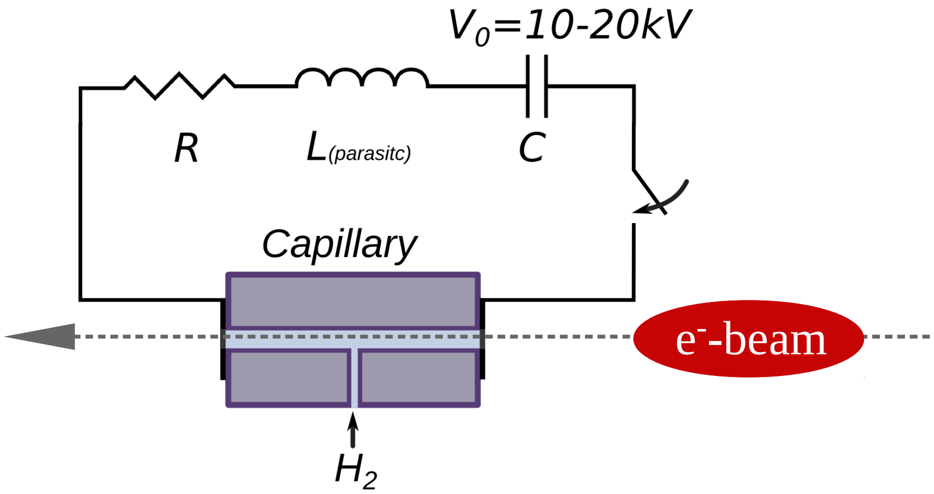
1.2. Real device

- Made of *printed plastic* or *sapphire* (with external support of printed plastic)
- Filled with hydrogen (≈10^{-6}-10^{-7}g/cm^3)
- Typical dimensions:
  - Diameter of the aperture: 1-2 mm
  - Length: 1-3 cm

Capillary, made of printed plastic



Discharge current time profile

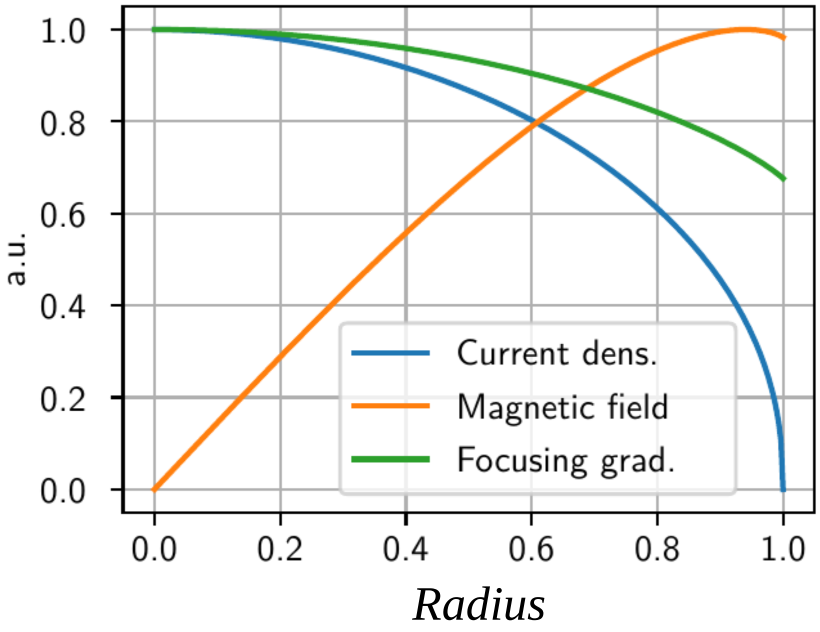


Electric circuit powering the gas discharge

- At SPARC_LAB electron density measurement (exploiting Stark Broadening effect) is implemented

* F. Filippi et al, J. Instrum, 11(09), C09015 (2016)

1.3. Issues related to active plasma lenses



- Focusing strength in APL is: $k \propto \frac{B(r)}{r}$
- For a radially uniform *k* **(desirable condition)** *we* need magnetic field with linear dependence on *r.*
- This is often not the case!
- Reason:
  - Capillary walls have cooling effect
    ▼
  - Plasma is hotter on axis
    ▼
  - Plasma electrical resistivity: $\eta \propto \frac{1}{T^{3/2}}$
    ▼
  - Current density concentrates on axis
    ▼
  - Magnetic field has a dropping profile

1.3. Issues related to active plasma lenses

- **Undesired passive plasma lensing** may occur.
  - If the electron density is in certain ranges, beam emittance grows significantly
  - This is very likely to happen on the plasma plums coming from the capillary extremities[1]

- **Scattering of beam electrons with plasma ions and neutrals** may be important (emittance growth), depending on the gas density and atomic number.

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[1] A. Marocchino et al, Appl. Phys. Lett, 111(18):184101, (2017)

2.1. Numerical model for capillary discharges

Simulating (conic) capillary discharges:

- 2D – axially symmetric geometry
- Fluid approach (→ kinetic approach is impractical)
- Local thermodynamic equilibrium conditions

- Description:
  - Lagrangian (grid cells moving with the fluid) → **DUED[1]:**

- Hydrodynamic model
- Ohmic heating due to curren flow
- Current density computed with *static current flow* approximation

Preliminary results obtained[3]

  - Eulerian (fluid moving through grid cells) → **PLUTO[2]**

- Resistive magneto-hydrodynamic model

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1 Atzeni et al., Comput. Phys. Commun. 169 (2005) 153
2 Mignone et al.,Astrophys J Suppl S, Vol. 170, Iss. 1 (2007), pp. 228-242
3 Brentegani et al., NIM – A, 909, (2018), pp 404-407

2.1. Numerical model for capillary discharges

Capillary walls → **Eulerian** description to avoid grid pathologies (PLUTO)

- Mass conservation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}) = 0,$$

- Momentum conservation

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}\mathbf{v}) = \frac{1}{\mu_0}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} - \boldsymbol{\nabla} p,$$

- Energy conservation

$$\begin{aligned}\frac{\partial \epsilon}{\partial t} + \boldsymbol{\nabla} \cdot (\epsilon \mathbf{v}) = &- \boldsymbol{\nabla} \cdot (p\mathbf{v}) + \boldsymbol{\nabla} \cdot (\kappa \boldsymbol{\nabla} T), \\ &- \boldsymbol{\nabla} \cdot \left[\left(\frac{\eta}{\mu_0^2}\boldsymbol{\nabla} \times \mathbf{B}\right) \times \mathbf{B}\right], \\ &+ \frac{1}{\mu_0}\boldsymbol{\nabla} \cdot [\mathbf{B}\,(\mathbf{B} \cdot \mathbf{v})],\end{aligned}$$

- Magnetic field evolution

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) - \boldsymbol{\nabla} \times \left[\frac{\eta}{\mu_0}(\boldsymbol{\nabla} \times \mathbf{B})\right].$$

- Equation of state for $p \leftrightarrow T \leftrightarrow \varepsilon_i$
- Saha's eq. for ionization degree

ρ: mass density

$\mathbf{v}$: fluid velocity

p: thermal pressure

ϵ: total energy density

κ: thermal conductivity

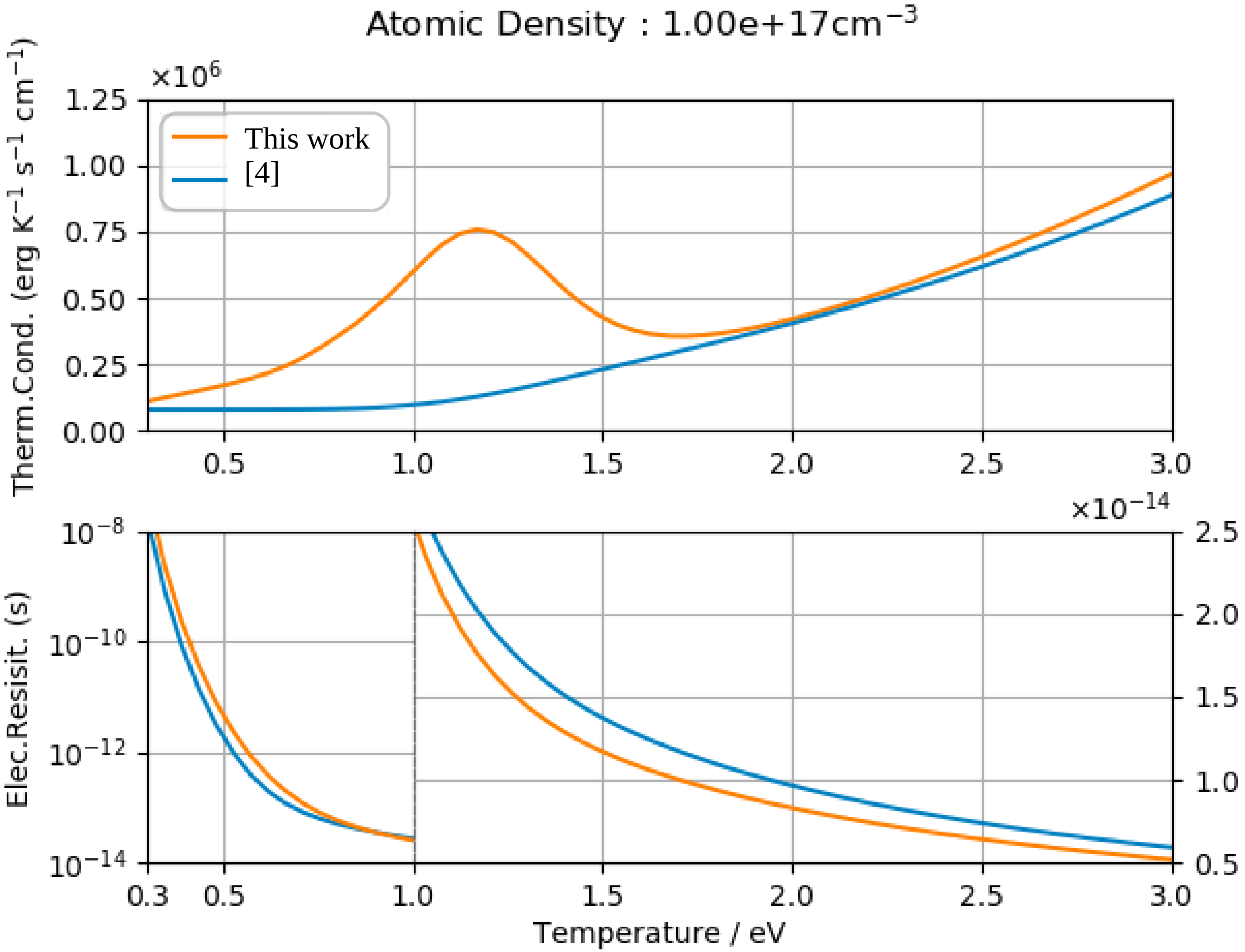
T: plasma temperature

η: electrical resistivity

$\mathbf{B}$:magnetic field

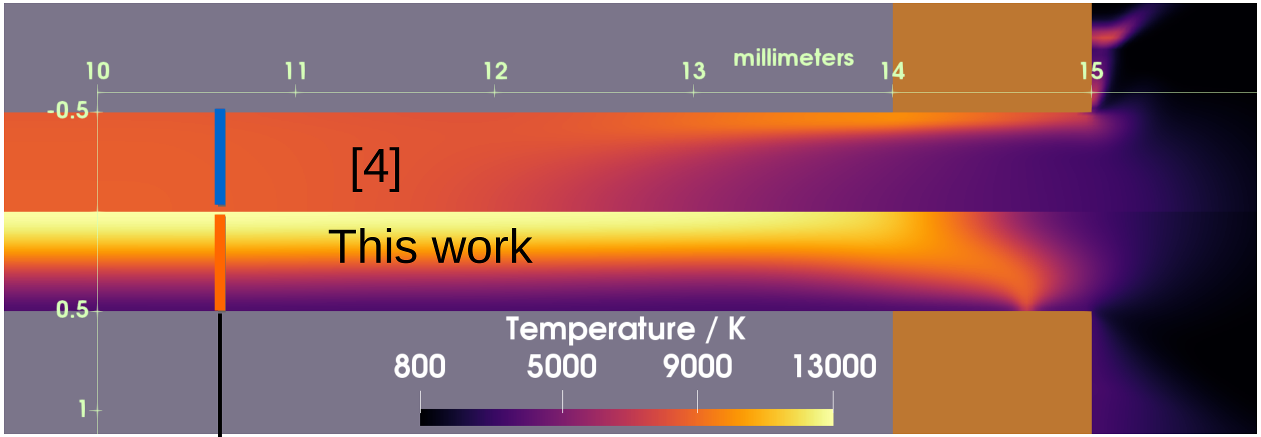
2.2. Accurate transport parameters

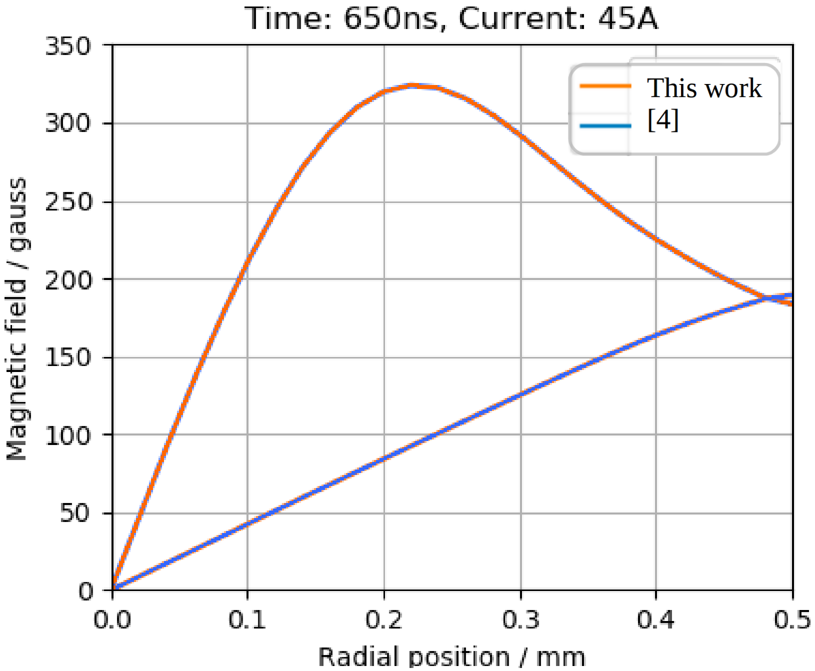
The transport parameters have been computed with a rigorous approach[1,2,3]



1. R. S. Devoto, Phys. Fluids 9, 1230 (1966)
2. J. F. J. Janssen, PhD, Department of Applied Physics, Eindhoven (2016)
3. M. Capitelli, D. Bruno, A. Laricchiuta, ‘Fundamental Aspects of Plasma Chemical Physics: Transport’ vol 74, Springer(2013)
4. Bobrova, N. A., et al., Simulations of a hydrogen-filled capillary discharge waveguide. Phys. Rev. E, 65 (2001), 016407

2.2. Accurate transport parameters





Line plot of magnetic field for fixed longitudinal position (16mm from capillary center)

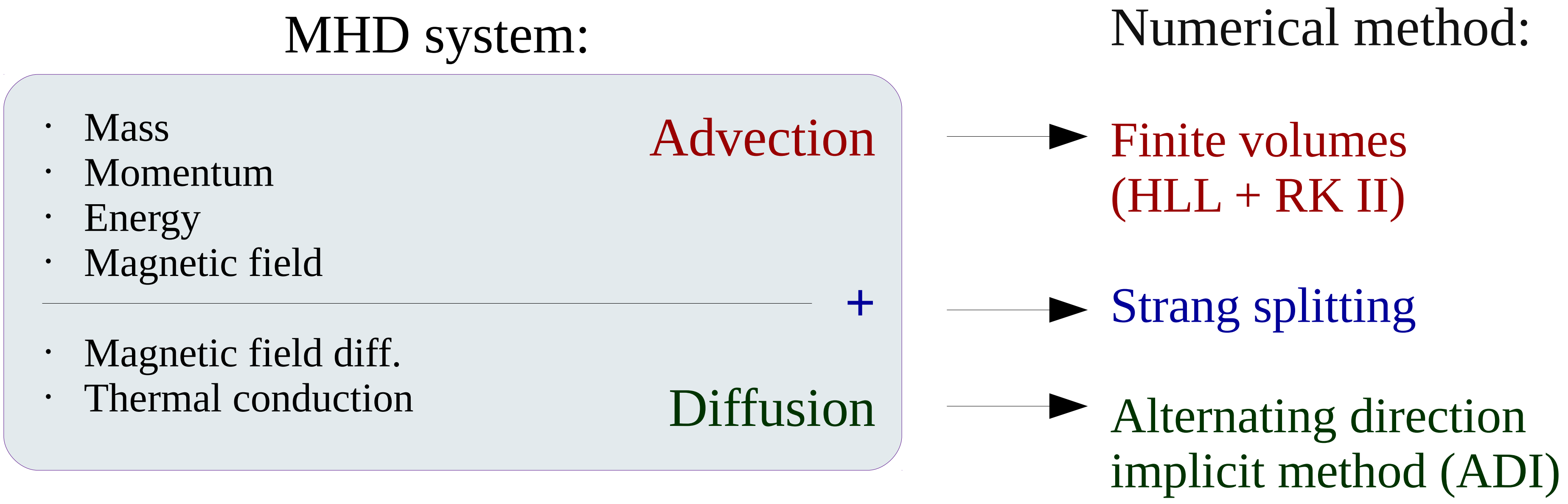
- The transport parameters have dramatic effects on the plasma behavior
- Changes in the temperature field are responsible for differences in the electrical resistivity

↓

- The magnetic field profile changes dramatically

4. Bobrova, N. A., et al., Simulations of a hydrogen-filled capillary discharge waveguide. Phys. Rev. E, 65 (2001), 016407

2.3. Semi-implicit algorithm



MHD system:

- Mass
- Momentum
- Energy
- Magnetic field

Advection

+

- Magnetic field diff.
- Thermal conduction

Diffusion

Numerical method:

Finite volumes (HLL + RK II)

Strang splitting

Alternating direction implicit method (ADI)

Douglas Rachford method*:

$$\frac{\hat{\Psi}^{n+1}-\Psi^{n}}{\Delta t}+D_r(\hat{\Psi}^{n+1})+D_z(\Psi^{n})=0$$

$$\frac{\Psi^{n+1}-\Psi^{n}}{\Delta t}+D_r(\hat{\Psi}^{n+1})+D_z(\Psi^{n+1})=0$$

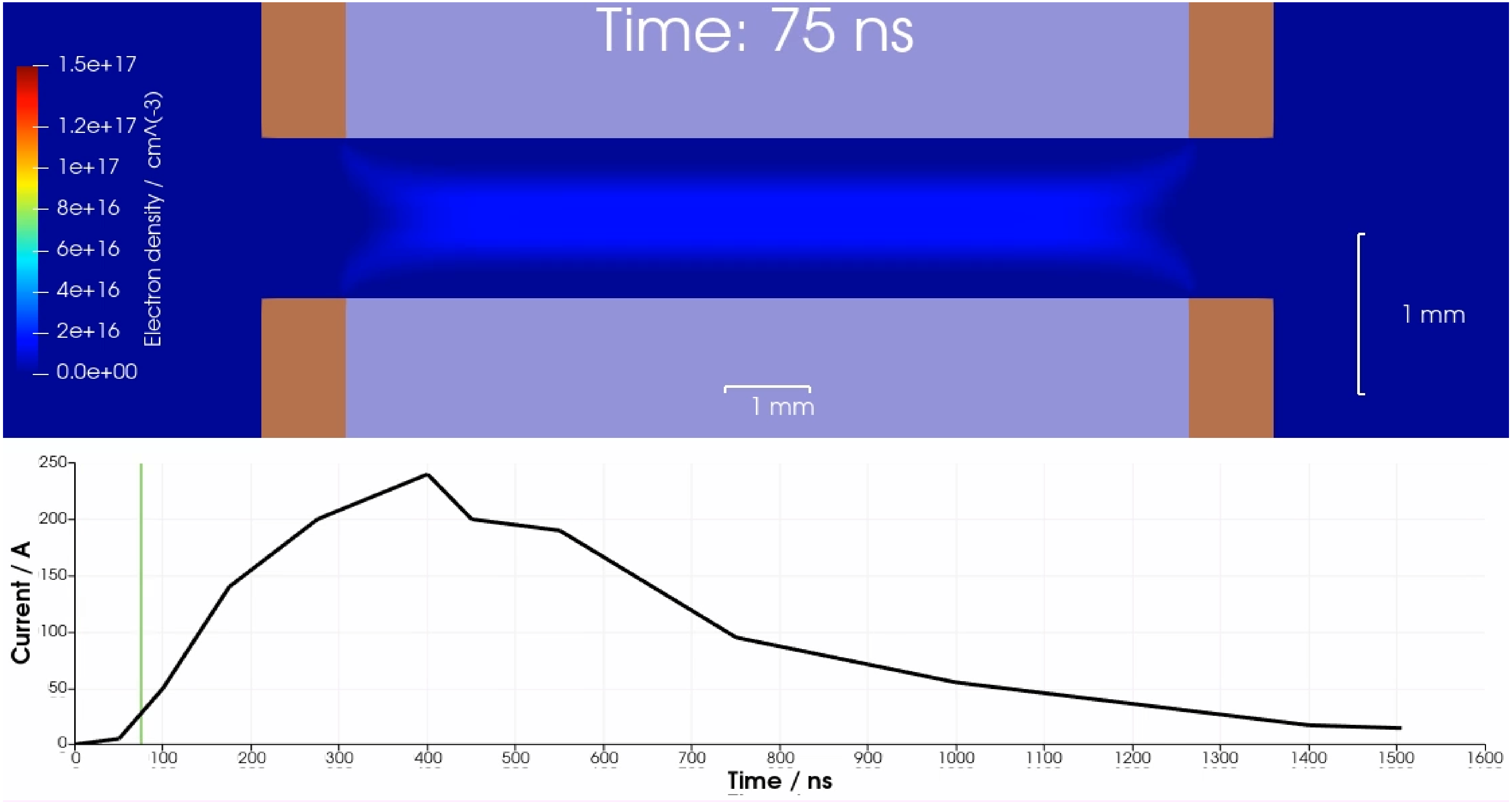
$$\Psi^{n} \longrightarrow \hat{\Psi}^{n+1} \longrightarrow \Psi^{n+1}$$

Explicit step in *z* + Implicit step in *r*   Explicit step in *r* + Implicit step in *z*

Solution of non-tridiagonal systems is not required!

* Douglas, J. and Rachford, H. H. Transactions of the American mathematical Society, 82 (1956), 421

3.1. Comparison with electron density measurements



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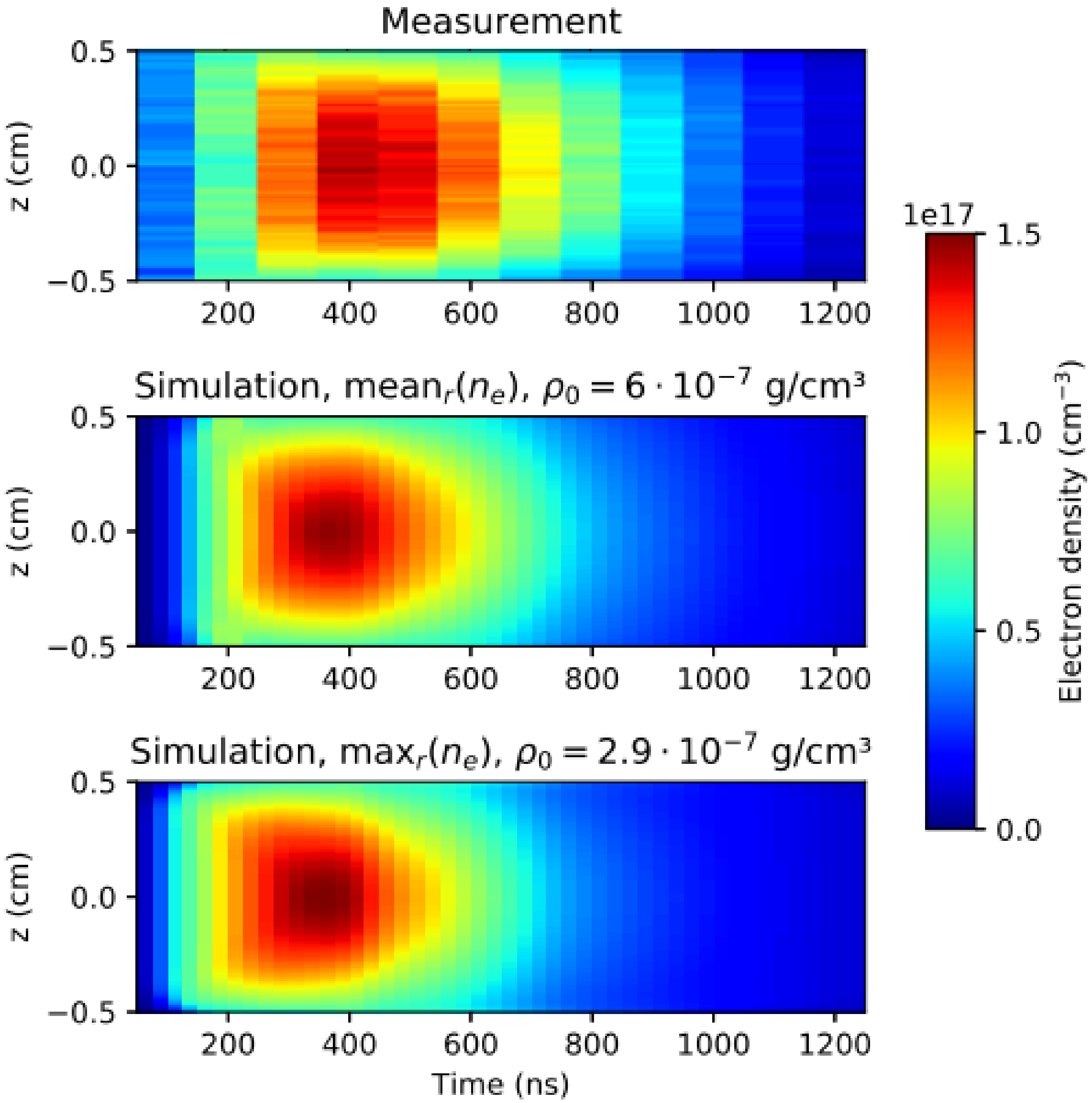
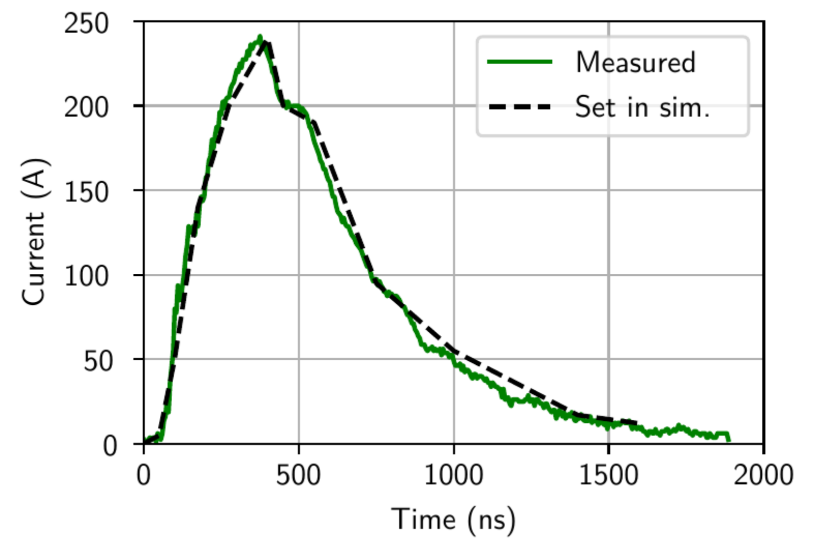


At SPARC_LAB the electron density in discharges is measured exploiting Stark Broadening effect[1]

Diameter	1 mm
Length	1 cm
Initial gas density	$2.9 \cdot 10^{-7}$, $6 \cdot 10^{-7}$ g/cm^3

Current profile



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[1] Filippi, Ph.D. thesis, Università di Roma la Sapienza (2017).

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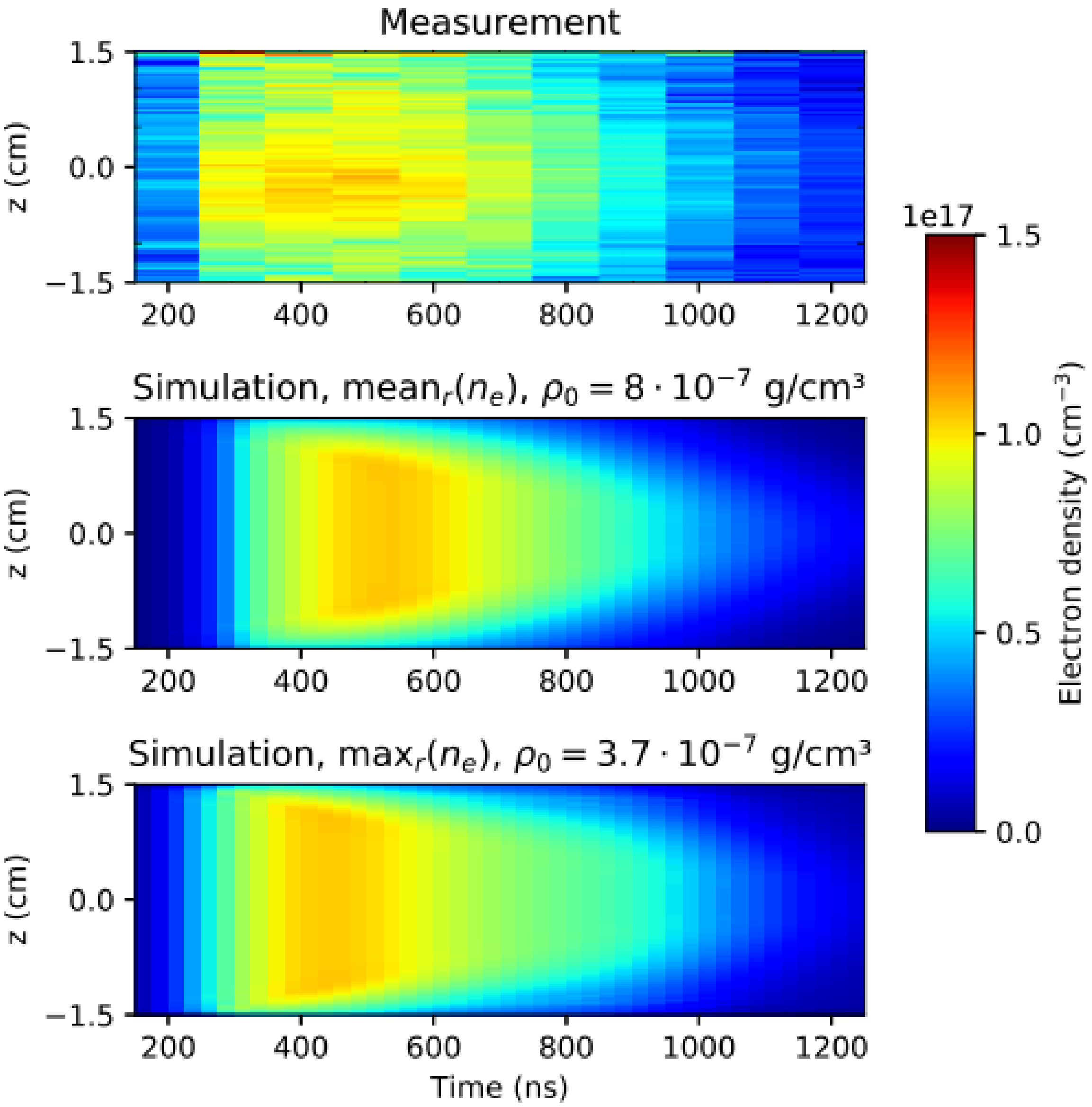
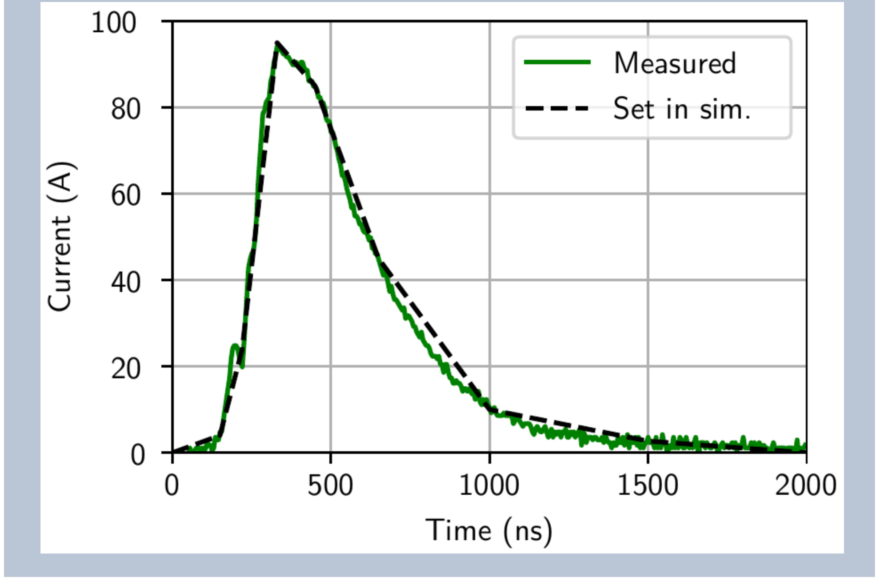

Measurement

Simulation, $\text{mean}_r(n_e)$, $\rho_0 = 8 \cdot 10^{-7}$ g/cm³

Simulation, $\max_r(n_e)$, $\rho_0 = 3.7 \cdot 10^{-7}$ g/cm³

At SPARC_LAB the electron density in discharges is measured exploiting Stark Broadening effect[1]

Diameter	*1 mm*
Length	*3 cm*
Initial gas density	$8 \cdot 10^{-7}, 3.7 \cdot 10^{-7}$ g/cm^3
Current profile	



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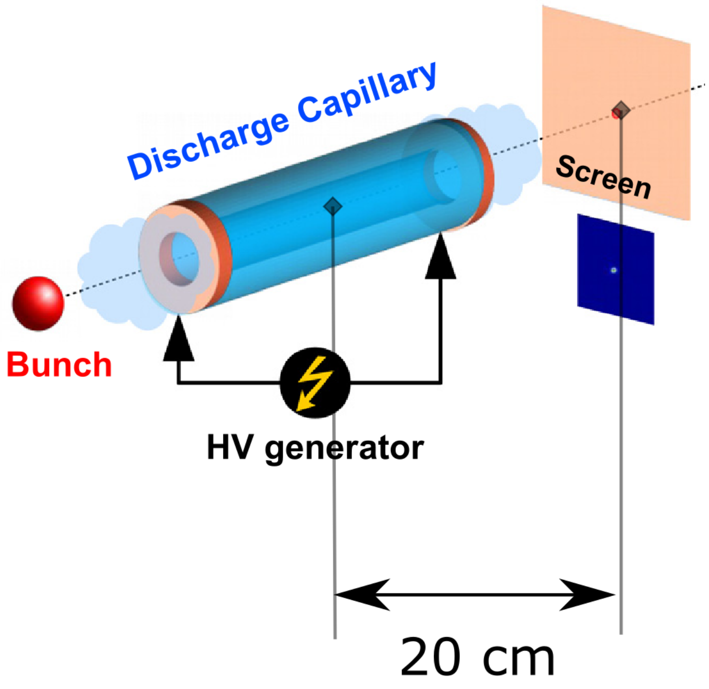
1. Filippi, Ph.D. thesis, Università di Roma la Sapienza (2017).

3.2. Comparison with beam focusing measurements

- Reproducing experimental electron beam focusing by 90A-peak discharge[1]



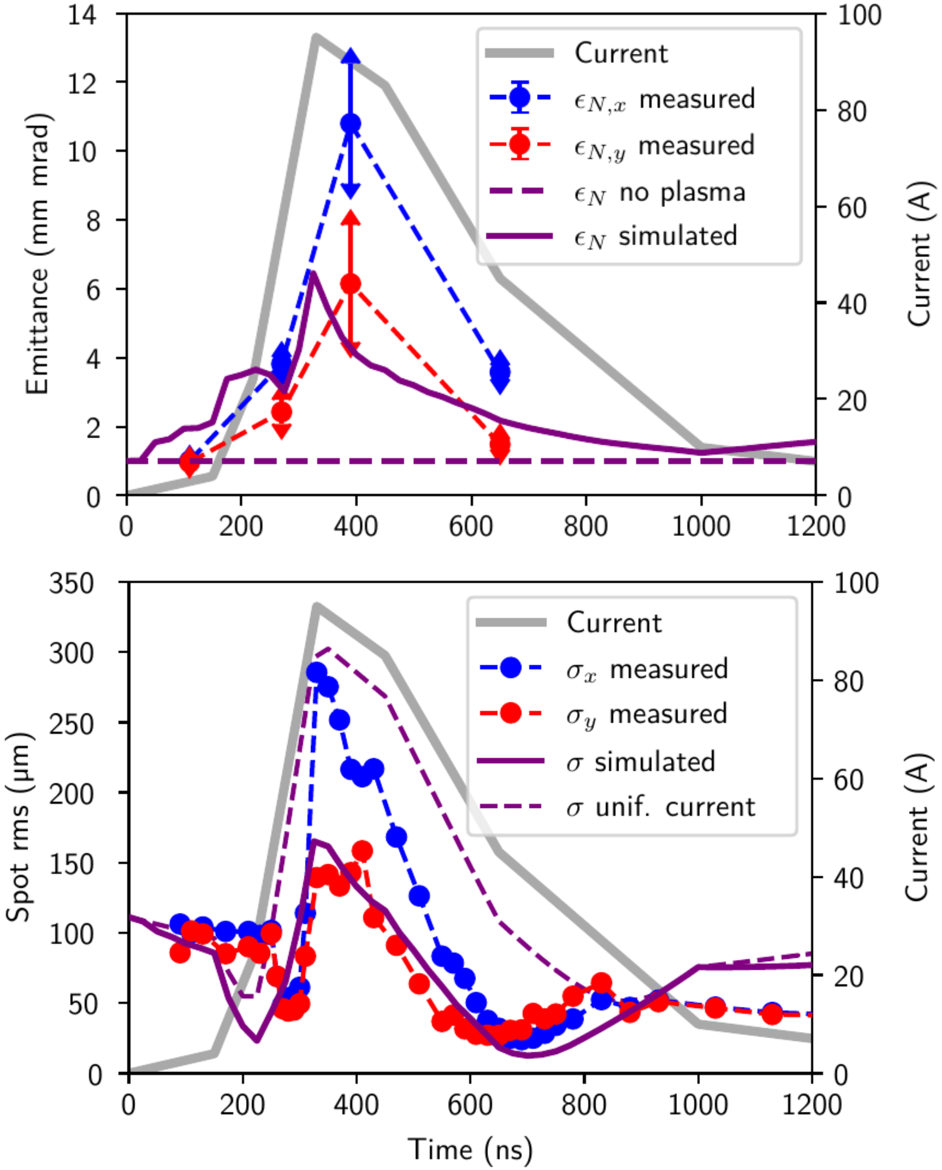
Capillary	
Diameter	1 mm
Length	3 cm
Initial gas pressure	~ 100 mbar
Current profile	90A-peak, 1μs duration

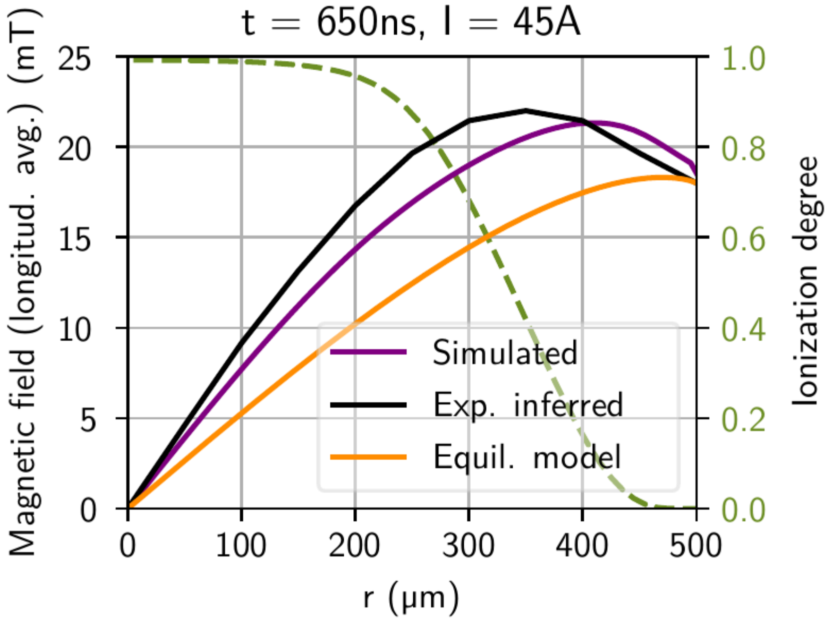
Electron bunch	
Charge	50 pC
Energy	126 MeV
Energy spread (rms)	50 keV
Norm. rms emittance	1 mm mrad
Duration	1.1 ps
Spot at capillary entrance (rms)	130 μm

1. R. Pompili et al, Appl. Phys. Lett, 110(10):104101, (2017)

3.2. Comparison with beam focusing measurements

- Reproducing experimental electron beam focusing by 90A-peak discharge[1]





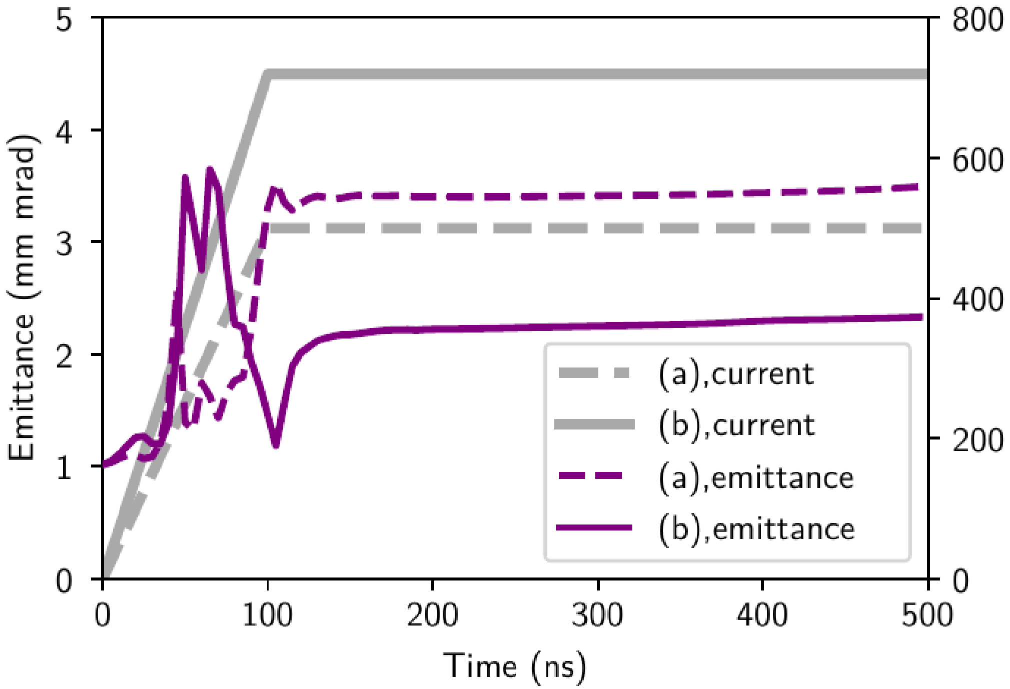
1. R. Pompili et al, Appl. Phys. Lett, 110(10):104101, (2017)

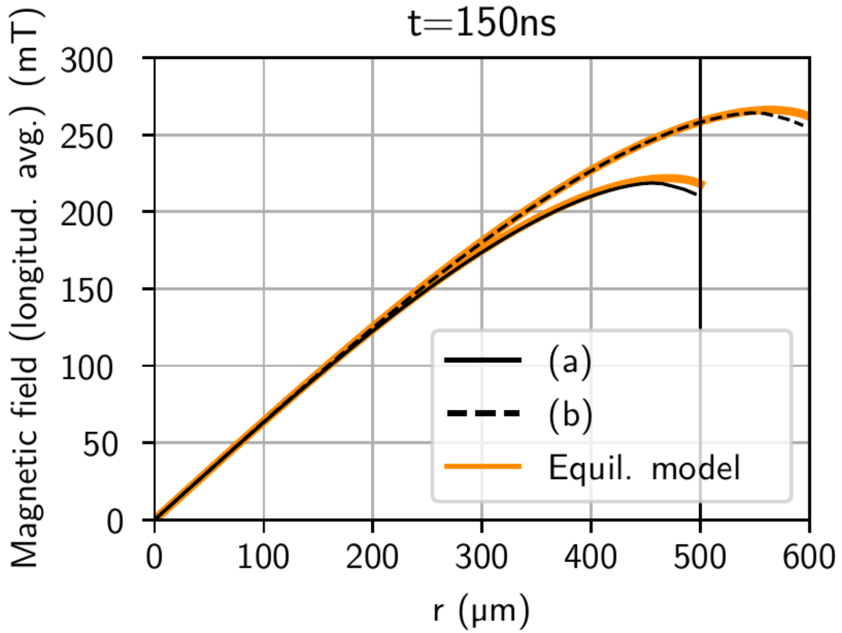
3.3. 1mm diameter *Vs* 1.2mm diameter capillary

	Case (a)	*Case (b)*
Diameter	1 mm	1.2 mm
Length	1.2 cm	1.2 cm
Initial gas density	$2.5 \cdot 10^{-7}$ g/cm^3	$2.5 \cdot 10^{-7}$ g/cm^3
Current profile	Flat top, 500 A	Flat top, 720 A

Electron bunch as before: 50 pC, 126 MeV, 50 keV, 1 mm mrad, 1.1 ps,130 μm

- **Result:** lower emittance growth for (b)



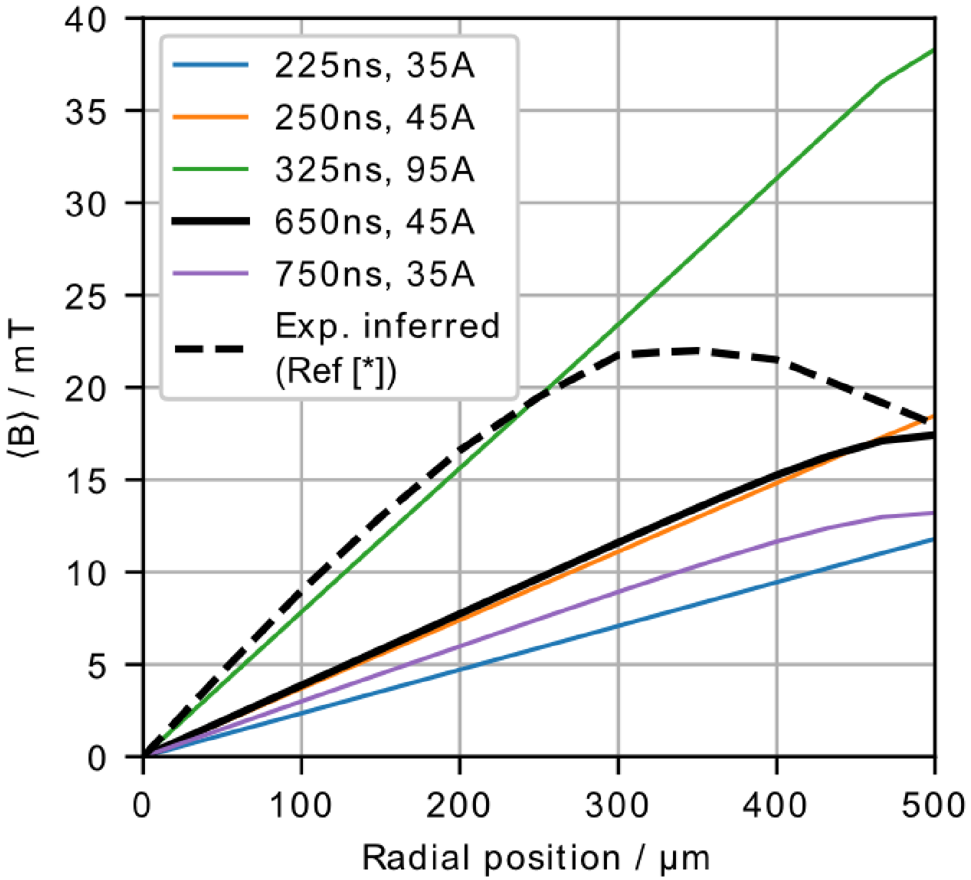


4. Conclusion and outlook

- I have shown the working principle of an active plasma lens
  - Active plasma lenses are **promising technologies**
  - Issues: magnetic field quality, passive focusing, beam scattering
- The need for 2D simulations is being addressed
  - **Hydrodynamic**, Lagrangian model with joule heating driven by a static current flow
  - **Magneto-hydrodynamic** model with Eulerian approach
  - With semi-implicit numerical method
  - Use of accurate **transport parameters** is necessary
- Main results:
  - Successful **comparison** of computed electron density **with Stark-Broadening measurements**
  - Successful **comparison with experiment of beam focusing** with an APL
  - First studies on the effect of capillary geometrical properties on the beam quality preservation

- SLIDE INTENZIONALMENTE VUOTA

APPENDIX: preliminary results with DUED



- In the approximation of thin lens, what matters for the focusing is average magnetic field:

$$\langle B(r)\rangle = \frac{\int_{-\infty}^{+\infty} B_\theta(r,z)\mathrm{d}z}{L_{\mathrm{cap}}}$$

- We can compare the integrated field at time 650ns (45A) with a field previously experimentally inferred, for the very same discharge[*] and for the timing corresponding to a current of 45A with descending slope.
- It is clear that the profiles do not match**

- Possible explanations:
  - The lack of the treatment of a self consistent magnetic field
  - The choice of both the initial (flat) profile of gas density distribution in the simulation
  - Plasma electrical resistivity, thermal conductivity not accurate enough
  - Not included the passive lensing effect

*. R.Pompili et al., Appl. Phys. Lett. 110, 104101 (2017)

**. E. Brentegani, et al.. NIM-A, ttps://doi.org/10.1016/j.nima.2018.03.012 (2018), In Press.