Tame the beast: physics at strong coupling

Domenico Orlando
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PLEASE... Don't Shoot the....
PIANO PLAYER
HE'S DOING THE BEST HE CAN!

Outline

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Anomalous magnetic moment of the electron (five loops QED)

$$a_e\Big|_{\text{theory}} = 0.0011596521816(7)$$

$$a_e\Big|_{\text{exp}} = 0.0011596521807(2)$$

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3cm

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You can never escape strong coupling

You can run but you can't hide

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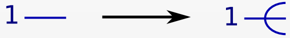
Take a quantum-mechanical system of N fermions ψ_a with quartic interaction [Stanford]

$$H = \sum_{a<b<c<d} J_{abcd} \psi_a \psi_b \psi_c \psi_d$$

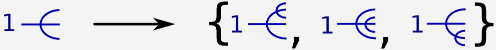
Time evolution makes operators "grow".

$$\psi(t) = e^{iHt} \psi e^{-i\psi t}$$

$$[H, \psi_1] = \sum_{1<b<c<d} J_{1bcd} \psi_b \psi_c \psi_d$$



$$[H, [H, \psi_1]]$$



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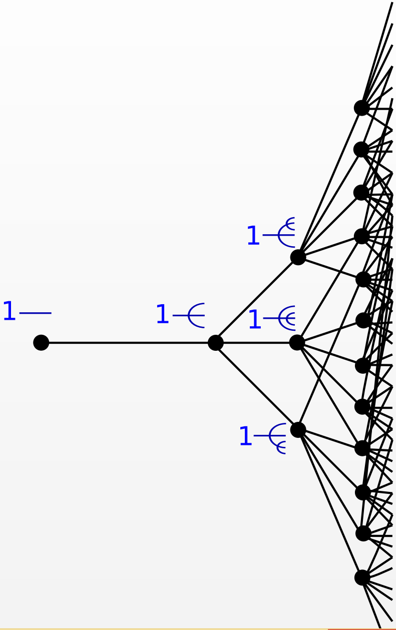
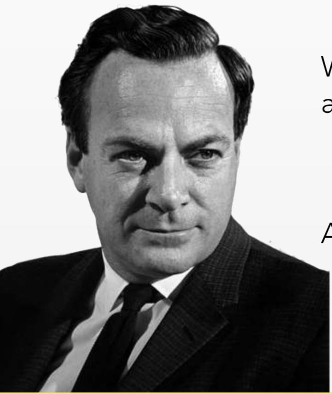


The growth of legs is exponential.
It cannot be controlled by the coupling J, no matter how small.
The perturbative expansion will break.
[Rubakov, arXiv:9511236]

Outline

Weak and strong coupling

It is easier to say what a weakly coupled system is.
A system is weakly coupled if the path integral can be approximated by a loop expansion around a leading trajectory γ.



We can decompose any observable into the contribution from γ and "the quantum contribution":

$$\mathcal{O} = \mathcal{O}_\gamma + \mathcal{O}_q$$

An observable is classical or quantum if $\mathcal{O}_\gamma \gtrless \mathcal{O}_q$.

The harmonic oscillator

The harmonic oscillator is the quintessential weakly-coupled system.
The classical EOM is

$$\ddot{\bar{q}} + \omega^2 \bar{q} = j(t)$$

If we decompose a generic trajectory as

$$q(t) = \bar{q}(t) + h(t)$$

we can write the amplitude associated to the classical solution

$$\exp[iS(\bar{q}, j)] \propto \langle t_f, q_f | t_i, q_i \rangle$$

and this is the only physical contribution to the generating functional

$$Z(j) = \int \mathrm{d}q_i \, \mathrm{d}q_f \, \langle 0 | t_f, q_f \rangle \, \langle t_f, q_f | t_i, q_i \rangle \, \langle t_i, q_i | 0 \rangle$$

The contribution of $h(t)$ is a simple prefactor, which is not physically meaningful.

ϕ^4 in the broken phase

Take a complex scalar φ with action

$$L = \partial_\mu \varphi^* \partial_\mu \varphi + \mu^2 |\varphi|^2 - \lambda |\varphi|^4$$

The classical solution is $\varphi = \sqrt{\mu^2/(2\lambda)} = \text{const.}$

The fluctuations around the classical solution are described by a Goldstone boson.

The loops are controlled by λ and $\mu\sqrt{\lambda}$.

Localizable quantum field theories

From this point of view, if the path integral of a (supersymmetric) theory can be computed via localization, the theory is weakly coupled.

The point is not so much if there is an action or if you find the leading trajectory with a Lagrangian equation of motion.

The result that you get is non-perturbative with respect to the parameters in the action you started with, but this is so also for the harmonic oscillator.

The path integral is literally the sum over contributions from a discrete set of trajectories.

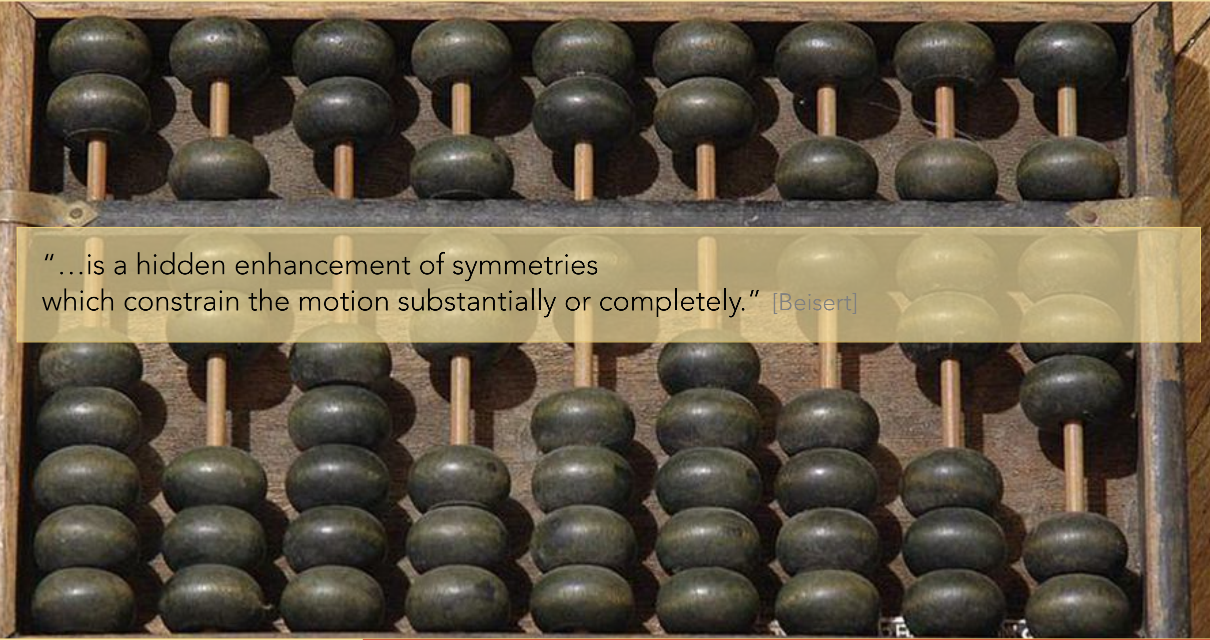
Outline

Anna Karenina principle

Happy families are all alike;
every unhappy family is unhappy in its own way.

Use the symmetry

Integrability



"…is a hidden enhancement of symmetries
which constrain the motion substantially or completely." [Beisert]

Integrability

Some systems have an infinite tower of independent integrals of motion.
We can pack them in a Lax pair, $M(u)$ and $L(u)$, function of a complex parameter u.
The complete set of EOM is the Lax equation

$$\frac{\mathrm{d}}{\mathrm{d}t} L(u) = [L(u), M(u)] \qquad \forall u \in \mathbb{C}$$

The eigenvalue spectrum of $L(u)$ is conserved in time.
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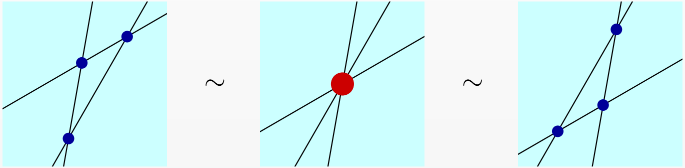
There is no special Hamiltonian among the infinite possible ones.
There is no special classical solution.

Factorized scattering

How is this possible? How can all the dynamics be encoded by a Riemann surface?
The reason is that these systems are very constrained.
The main constraint comes from factorizability of the interactions



This is why typically integrable systems live in $1 + 1$ dimensions.
There is *no room* for a *real* three-particle interaction.

Bootstrap

Bootstrap

The idea of bootstrap is to constrain directly the observables imposing consistency conditions dictated by the symmetries.
The more the symmetry, the stronger the constraints.

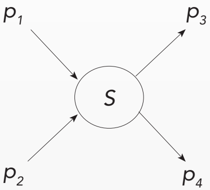
One never needs to separate a “classical” from a “quantum” part.

There is no need for the notion of leading trajectory.
Actually, there is no need for a Lagrangian, which is after all a classical object.

S-matrix bootstrap

Scattering of the lightest particle in a massive Lorentz-invariant QFT in $d = 1 + 1$

$$\begin{cases} p_i^2 = m^2 \\ s = (p_1 + p_2)^2 \\ t = 4m^2 - s \\ u = 0 \end{cases}$$



Extend the S matrix to an analytic function of s and ask for

- analyticity $S(s)^* = S(s^*)$
- crossing symmetry $S(s) = S(t)$ (identical particles)
- unitarity $|S(s)|^2 \leq 1$ (in the physical region)
- analytic structure (branch cuts, poles)

Conformal bootstrap

The approach is particularly useful for CFTs.
Take a CFT and write a four-point function

$$\langle \varphi(0)\,\varphi(1)\,\varphi(z,\bar{z})\,\varphi(\infty)\rangle = G(z,\bar{z})$$

Ask for:

- OPE and block decomposition
- unitarity
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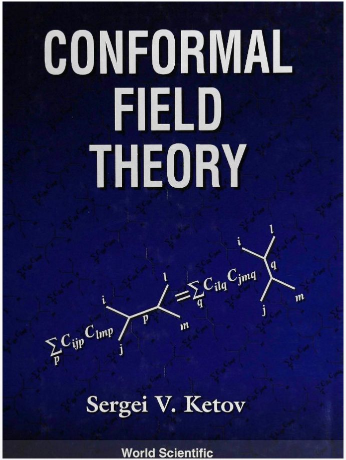

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Special sectors

Let me go back to the definition of weak coupling.
The path integral is approximated by a loop expansion around a leading trajectory r.

$$\mathcal{O} = \mathcal{O}_r + \mathcal{O}_q$$

There is an intermediate possibility.

Weaker condition: only some observables are *classical* with respect to some *local* classical trajectory.

A weakly coupled theory

A strongly coupled theory

"Locally weakly coupled"

The $O(2)$ model

$O(2)$ vector model in three dimensions.
The infrared Wilson–Fisher fixed point of ϕ^4.

It is a CFT. There are no dimensionful parameters and everything *naturally* of order one.
There is no preferred classical trajectory.

- In nature: ^4He.
- Simplest example of spontaneous symmetry breaking.
- Not accessible in perturbation theory. Not accessible in $4-\varepsilon$. Not accessible in large N.
- Hard on the lattice.
- Bootstrap.

A symmetry

This system has a global $O(2)$symmetry .
And unlike gauge symmetries, global symmetries are important intrinsic properties of the system.

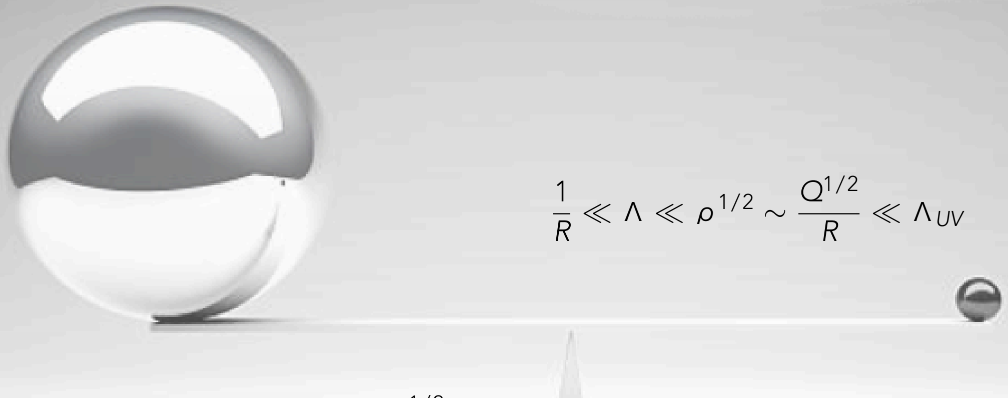
We can slice the Hilbert space into sectors of fixed $O(2)$ charge.
In each of this sectors there will be a dominating classical solution.

$$\varphi(t, x) = a_0 e^{i\mu t}$$

This solution breaks the symmetry and the fluctuations around it are controlled by Goldstone fields.

Scales

- We look at a finite box of typical length R
- The $U(1)$ charge Q fixes a second scale $\rho^{1/2} \sim Q^{1/2}/R$



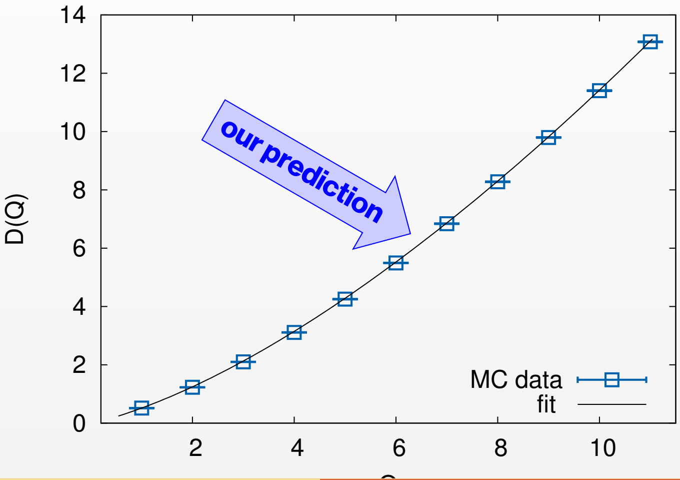
$$\frac{1}{R} \ll \Lambda \ll \rho^{1/2} \sim \frac{Q^{1/2}}{R} \ll \Lambda_{UV}$$

Scale separation: for $\Lambda \ll \rho^{1/2}$ the effective action is weakly coupled.
Under perturbative control in powers of $Q^{-1/2}$.

Conformal dimensions

Dimension of the lowest operator of charge Q:

$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$



Large-charge universality class

There are *universality classes* of systems at large charge.

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What fields appear in the spectrum of fluctuations?

Remember: even if the full theory has Lorentz invariance, fixing the charge breaks it. The Goldstones are in general non-relativistic.

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Are there moduli?

The $O(N)$ vector model, the Reissner–Nordström black hole, fermions at unitarity, asymptotically safe QCD all behave in a similar way.

$N = 2, d = 4$ SCFT

Another very nice example is $\mathcal{N} = 2$ four-dimensional SCFTs with Coulomb branch of dimension one (think of $SU(2)$ super Yang–Mills with four flavors).
In this case we can use the R-symmetry.

The dimension of the lowest operator at fixed R-charge is fixed by the BPS condition.
Three-point function of the Coulomb branch operators

$$\left\langle \Phi^{n_1}(x_1)\Phi^{n_2}(x_2)\bar{\Phi}^{n_1+n_2}(x_3)\right\rangle = \frac{C^{n_1,n_2,n_1+n_2}}{|x_1 - x_3|^{2n_1 D}|x_2 - x_3|^{2n_2 D}}$$

The OPE of Φ with itself is regular, so we can set $x_2 = x_1$ and the three-point function is actually a two-point function.

$$C^{n',n-n',n} = |x_1 - x_2|^{2nD}\left\langle \Phi^n(x_1)\bar{\Phi}^n(x_2)\right\rangle = e^{q_n - q_0}$$

$Q = nD$ is the controlling parameter (it's the R-charge).

The classical trajectory

The dynamics on the moduli space is described by the Coulomb branch operator $\Phi(x)$. If we fix the R-charge there is also here a dominating classical trajectory for large Q

$$\phi(x) = \frac{e^{t/r}}{2\pi r} Q^{1/2}$$

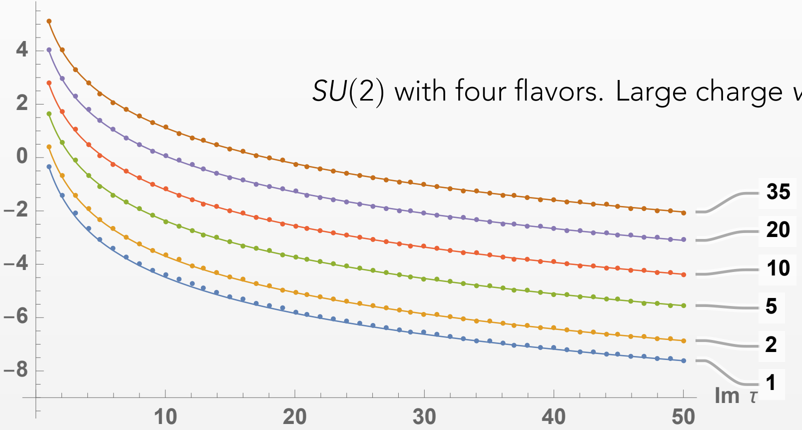
Thanks to supersymmetry, if there is a marginal coupling, the fluctuations are fixed by the Toda lattice equation

$$\partial_\tau \partial_{\bar{\tau}} q_n = e^{q_{n+1} - q_n} - e^{q_n - q_{n-1}}$$

Comparison with localization

We find a closed formula for the three-point function, that depends only on the a anomaly coefficient

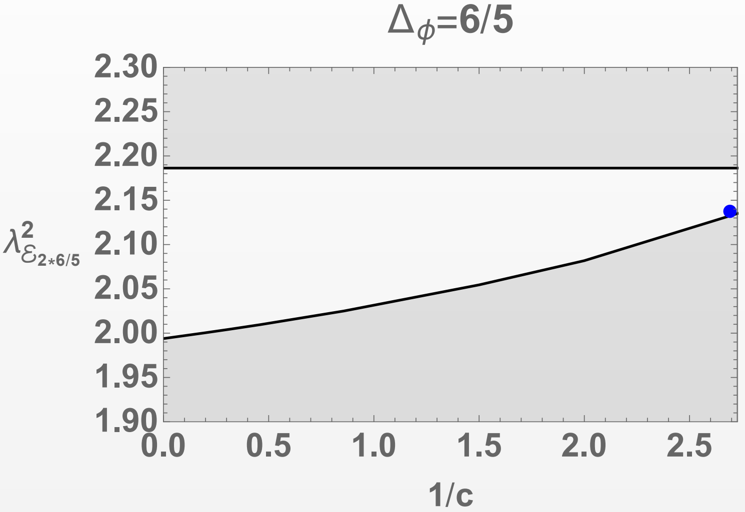
$$q_n = k_0(\tau, \bar{\tau}) + nf(\tau, \bar{\tau}) + \log(\Gamma(2n + \alpha + 1))$$



SU(2) with four flavors. Large charge *vs.* localization

Comparison with bootstrap

For strongly coupled theories one can use bootstrap to place bounds on the three-point coefficients with $n = 1$.



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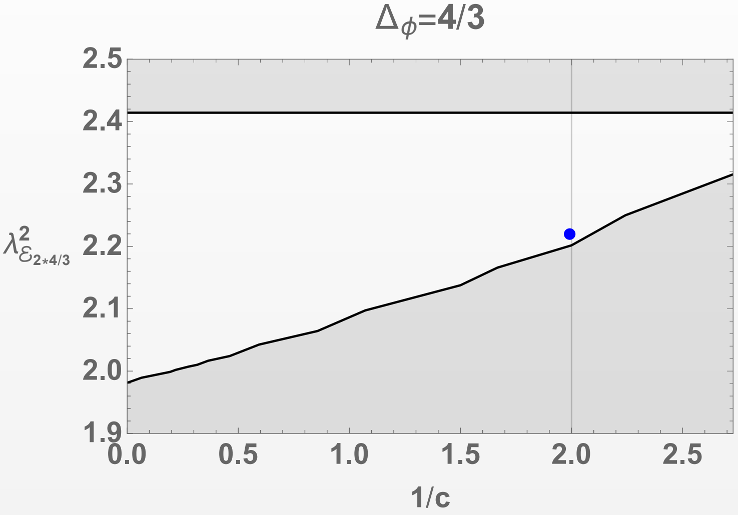

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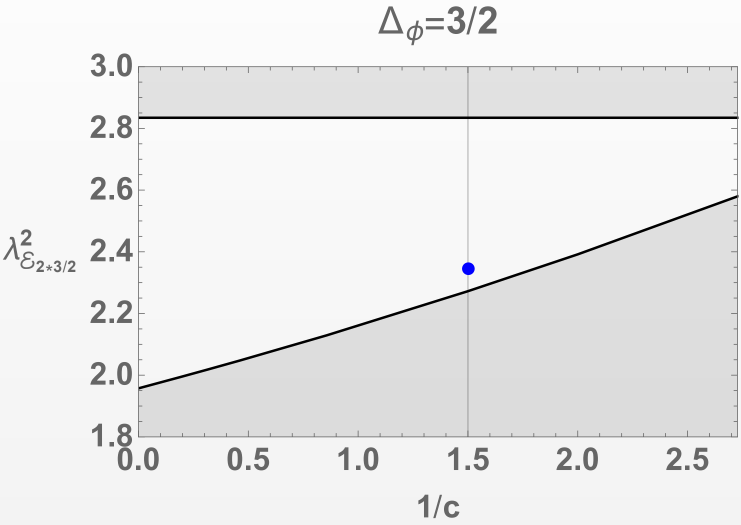

Outline

Conclusions

In conclusion

- Strongly coupled dynamics is out there. No matter how weakly coupled your system is, at some point the perturbative expansion will break.
- We can use symmetries to study strongly-coupled systems. This works also when we don't even have a Lagrangian.
- A given strongly-coupled theory can be locally weakly coupled.
- Some observables can still be accessible with perturbative methods.
- *Classical* and *quantum* are relative terms, which depend on a given *locally* leading trajectory in the path integral.