From 3d dualities to 2d free field correlators and back

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work in progress with Hwang and Pasquetti

Infra-red dualities relate two different UV quantum field theories describing the same system in the IR.

We can test dualities by matching IR global symmetries and quantities that are invariant along the RG flow, such as anomalies and BPS operators.

The most famous duality is Seiberg duality for 4d $\mathcal{N}=1$ SQCD [Seiberg 1994]

$SU(N_c)$ with N_f flavors Q^i, $\tilde{Q}_i$ $\mathcal{W}=0$	$\longleftrightarrow$	$SU(N_f-N_c)$ with N_f flavors q^i, $\tilde{q}_i$ N_f^2 singlets ${M^i}_j$ and $\mathcal{W}={M^i}_j q^j \tilde{q}_i$

Global symmetry: $SU(N_f)\times SU(N_f)\times U(1)_B$

Operator map:

$$
\begin{aligned}
Q^i \tilde{Q}_j \quad &\leftrightarrow \quad {M^i}_j \\
\epsilon^{a_1 \cdots a_{N_c}} Q^{i_1}_{a_1} \cdots Q^{i_{N_c}}_{a_{N_c}} \quad &\leftrightarrow \quad \epsilon^{i_1 \cdots i_{N_c} j_1 \cdots j_{N_f-N_c}} \epsilon_{a_1 \cdots a_{N_f-N_c}} \tilde{Q}^{a_1}_{j_1} \cdots \tilde{Q}^{a_{N_f-N_c}}_{j_{N_f-N_c}} \\
\epsilon_{a_1 \cdots a_{N_c}} \tilde{Q}^{a_1}_{i_1} \cdots \tilde{Q}^{a_{N_c}}_{i_{N_c}} \quad &\leftrightarrow \quad \epsilon_{i_1 \cdots i_{N_c} j_1 \cdots j_{N_f-N_c}} \epsilon^{a_1 \cdots a_{N_f-N_c}} Q^{j_1}_{a_1} \cdots Q^{j_{N_f-N_c}}_{a_{N_f-N_c}}
\end{aligned}
$$

A revival of IR dualities took place in the last decade thanks to the advent of supersymmetric localization. [Witten 80s, Nekrasov 2003, Pestun 2007, ...]

The path integral of some protected observables "localizes" on the saddle-points of the classical action and reduces to a matrix integral

$$\mathcal{Z}(\mu; q) = \int \mathrm{d}\sigma \, \mathcal{Z}_{\text{cl}}(\sigma, \mu) \mathcal{Z}_{\text{1-loop}}(\sigma, \mu, q) \, .$$

When the theory has enough supersymmetry, the result is 1-loop exact.

Some supersymmetric partition functions and superconformal indices are independent from the gauge coupling and can be used to test IR dualities.

A duality implies a non-trivial integral identity between the localized supersymmetric partition functions of the dual theories.

Supersymmetric localization is a two steps process:

- define the theory on a compact space preserving supersymmetry to remove IR divergences; [Festuccia, Seiberg 2011]
- find the saddle-points (BPS locus) and compute 1-loop determinants

  $\Rightarrow$ special functions show up

Depending on the background on which the theory is defined different special functions appear

$\Rightarrow$ connection to mathematical analysis results on special functions and integral identities

Example: 4d $\mathcal{N}=1$ theories on $S^3\times S^1 \Rightarrow$ elliptic hyperbolic gamma functions

$$\Gamma_e(z)=\prod_{n,m=0}^{\infty}\frac{1-p^{n+1}q^{m+1}/z}{1-p^nq^mz}$$

The partition function on $S^3\times S^1$ of SQCD with $SU(N_c)$ gauge group, N_f flavors and $\mathcal{W}=0$ is (global symmetry $SU(N_f)_\mu\times SU(N_f)_{\tilde{\mu}}\times U(1)_\nu$)

$$\mathcal{Z}^{(N_c)}_{\textbf{SQCD}}(\mu_i,\tilde{\mu}_i,\nu)=\frac{(q;q)_\infty^{N_c-1}(p;p)_\infty^{N_c-1}}{N_c!}\oint\prod_{a=1}^{N_c-1}\frac{\mathrm{d}z_a}{2\pi i\, z_a}\frac{1}{\prod_{a<b}^{N_c}\Gamma_e\left((z_a/z_b)^{\pm1}\right)}\times$$
$$\times\prod_{a=1}^{N_c}\prod_{i=1}^{N_f}\Gamma_e\left((pq)^{1/2-N_f}\nu\,\mu_iz_a\right)\Gamma_e\left((pq)^{1/2-N_f}\nu^{-1}\tilde{\mu}_iz_a^{-1}\right)\,,$$

where $\prod_{a=1}^{N_c}z_a=\prod_{i=1}^{N_f}\mu_i=\prod_{i=1}^{N_f}\tilde{\mu}_i=1$.

Seiberg duality translates into the integral identity [Dolan, Osborn 2008]

$$\mathcal{Z}^{(N_c)}_{\textbf{SQCD}}(\mu_i,\tilde{\mu}_i,\nu)=\prod_{i,j=1}^{N_f}\Gamma_e\left((pq)^{1/2}\mu_i\tilde{\mu}_j\right)\mathcal{Z}^{(N_f-N_c)}_{\textbf{SQCD}}(\mu_i^{-1},\tilde{\mu}_i^{-1},\nu)\,.$$

Dimensional reduction of dualities: from 4d to 3d

We can dimensionally reduce a theory from 4d to 3d by compactifying it on a circle S^1 and taking $r \to 0$.

There are many subtleties in studying the dimensional reduction of 4d dualities. [Aharony, Razamat, Seiberg, Willett 2013]

One of them is related to the fact that in 3d we have monopole operators. In the reduction some of them may be generated in the superpotential.

The presence of monopole superpotentials explains for example why in the resulting 3d theories we don't have some $U(1)$ axial symmetries that in 4d are anomalous.

The dimensional reduction of dualities can be conveniently analysed using partition functions

$$\lim_{r\to 0} \Gamma_e \left(e^{2\pi i r x}\right) \sim \Gamma_h\left(x\right) \quad \Rightarrow \quad \lim_{r\to 0} \mathcal{Z}_{S^3\times S^1} \sim \mathcal{Z}_{S^3}$$

Monopole superpotentials manifest as constraints on the parameters of the partition function that descend from requirements of anomaly cancellations in 4d.

3d dualities with monopole superpotentials

This led to consider more seriously 3d theories with monopole superpotentials and look for possible dualities that descend from 4d dualities [Benini, Benvenuti, Pasquetti 2017]

$U(N_c)$ with N_f flavors Q^i, $\tilde{Q}_i$ $\mathcal{W} = \mathfrak{M}^+ + \mathfrak{M}^-$	$\longleftrightarrow$	$U(N_f - N_c - 2)$ with N_f flavors q^i, $\tilde{q}_i$ N_f^2 singlets ${M^i}_j$ and $\mathcal{W} = \mathfrak{M}^+ + \mathfrak{M}^- + {M^i}_j q^j \tilde{q}_i$

This duality can be derived by dimensionally reducing and deforming Intriligator–Pouliot duality in 4d.

The equality of S^3 partition functions is obtained as a limit of the equality of $S^3 \times S^1$ partition functions of IP duality. [Rains 2003]

Dimensional reduction of dualities: from 3d to 2d

The moduli space of 3d theories consists of two branches:

- Higgs branch, VEV for scalar in the chiral multiplet, related to real masses.
- Coulomb branch, VEV for scalar in the vector multiplet, related to FI parameters.

When we consider the reduction of 3d dualities to 2d there are subtleties with non-compact directions of the moduli space. [Aharony, Razamat, Willett 2017]

In order to establish a connection with 2d CFT correlators, we are interested in the case in which all possible real mass deformations and FI parameters are turned on and we have isolated vacua.

$\Rightarrow$ **Disclaimer:** What we obtain are dualities for mass deformed theories!

We have to specify how these parameters scale with r in the 2d limit:

- **Higgs limit:** real masses are kept finite, while FI parameters become large $\Rightarrow$ Coulomb branch is lifted. The resulting theory is a 2d GLSM.
- **Coulomb limit:** FI parameters are kept finite, while real masses become large $\Rightarrow$ Higgs branch is lifted. The resulting theory is a 2d LG model.

From 3d to 2d using superconformal index

We can investigated the dimensional reduction of 3d dualities using supersymmetric partition functions on $S^2 \times S^1$ or $D_2 \times S^1$ ($q \sim \mathrm{e}^r$).

The superconformal index, that is the partition function on $S^2 \times S^1$, takes the form [Imamura, Yokoyama 2011; Kapustin, Willett 2011]

$$\mathcal{Z}_{S^2 \times S^1}(\mu; q) = \sum_m \frac{1}{|\mathcal{W}_m|} \oint \frac{dz}{2\pi i\, z} \mathcal{Z}_{\text{cl}}\, \mathcal{Z}_{\text{vec}}\, \mathcal{Z}_{\text{mat}}\,,$$

The 1-loop contribution is written in terms of q-Pochhammer symbols

$$(a; q)_\infty = \prod_{n=0}^{\infty} (1 - a q^n)$$

The Higgs limit is implemented using

$$\lim_{q \to 1} \frac{(q^x; q)_\infty}{(q^y; q)_\infty} = \frac{\Gamma(y)}{\Gamma(x)} (1-q)^{y-x}\,.$$

$\Rightarrow$ S^2 partition function of GLSM.

The Coulomb limit is implemented using

$$\lim_{q \to 1} \frac{(z\, q^x; q)_\infty}{(z\, q^y; q)_\infty} = (1-z)^{y-x}\,.$$

$\Rightarrow$ S^2 partition function of LG model.

From 3d monopole duality to complex integral

Our work originated from the observation that the Coulomb limit of the superconformal index results in a complex integral.

For example, the limit of the monopole duality

$U(N_c)$ with N_f flavors Q^i, $\tilde{Q}_i$
$\mathcal{W} = \mathfrak{M}^+ + \mathfrak{M}^-$

$\longleftrightarrow$

$U(N_f - N_c - 2)$ with N_f flavors q^i, $\tilde{q}_i$
N_f^2 singlets ${M^i}_j$ and
$\mathcal{W} = \mathfrak{M}^+ + \mathfrak{M}^- + {M^i}_j q^j \tilde{q}_i$

yields the following identity for complex integrals [Pasquetti, MS 2019]

$$\int \mathrm{d}^2\vec{z}_{N_c} \prod_{i<j}^{N_c} |z_i - z_j|^2 \prod_{i=1}^{N_c} \prod_{a=1}^{N_f} |z_i - \tau_a|^{2p_a} = \prod_{a=1}^{N_f} \gamma(1+p_a) \prod_{a<b}^{N_f} |\tau_a - \tau_b|^{2(1+p_a+p_b)} \times$$

$$\times \int \mathrm{d}^2\vec{z}_{N_f-N_c-2} \prod_{i<j}^{N_f-N_c-2} |z_i - z_j|^2 \prod_{i=1}^{N_f-N_c-2} \prod_{a=1}^{N_f} |z_i - \tau_a|^{-2(1+p_a)},$$

where $\mathrm{d}^2\vec{z}_n = \frac{1}{\pi\, n!} \prod_{i=1}^{n} \mathrm{d}^2 z_i$ and $\gamma(x) = \Gamma(x)/\Gamma(1-x)$ and provided that

$$\sum_{a=1}^{N_f} p_a = -N_c - 1\,.$$

Complex integrals of this form show up in the study of 2d CFT correlators in the free field realization.

This is not surprising, since we know many instances of gauge/CFT correspondences, where a dictionary between exact quantities in supersymmetric gauge theories computed via localization and CFT correlators or conformal blocks can be established. [Alday, Gaiotto, Tachikawa 2009; Aganagic, Haouzi, Kozcaz, Shakirov 2013 ...]

The mass deformations we turned on in 3d are needed to establish such a dictionary, since they are mapped to momenta and insertion points of the CFT correlators.

Integral identities like the one obtained from the 2d Coulomb limit of the monopole duality have been intensively used to manipulate free field correlators.

Question:

Can we *uplift* other identities for complex integrals known in the CFT literature to new genuine (not mass deformed) IR dualities in 3d?

Liouville free field correlators

Lioville theory is a rational CFT that admits a Lagrangian description in terms of a single scalar field ϕ with potential $\mathrm{e}^{2b\phi}$.

Correlators of k vertex operators $V_\alpha(z,\bar{z}) = \mathrm{e}^{2\alpha\phi}$ exhibit poles when the momenta satisfy the screening quantization condition [Goulian, Li 1991]

$$\alpha \equiv \alpha_1 + \cdots + \alpha_k = Q - Nb, \qquad N \in \mathbb{N}\,.$$

The residue in turn takes the form of a free field Dotskenko-Fateev correlator with N screening charges:

$$\underset{\alpha=Q-Nb}{\mathrm{res}} \langle V_{\alpha_1}(z_1) V_{\alpha_2}(z_2) \cdots V_{\alpha_k}(z_k) \rangle = (-\pi\mu)^N I_N(\alpha_1, \cdots, \alpha_k) =$$

$$= (-\pi\mu)^N \prod_{i<j}^{k} |z_i - z_j|^{-4\alpha_i\alpha_j} \int \prod_{i=1}^{N} \prod_{j=1}^{k} |x_i - z_j|^{-4b\alpha_k} \prod_{i<j}^{N} |x_i - x_j|^{-4b^2} \mathrm{d}^2 \vec{x}_N\,.$$

$\rightsquigarrow$ suggests a $U(N)$ gauge theory with one adjoint chiral, k flavors and singlets.

Liouville free field correlators: 3-point function

In the case of the 3-pt. function, one can evaluate this integral by iterating the basic integral identity [Fateev, Litvinov 2007]

$$I_N(\alpha_1,\alpha_2,\alpha_3) = \prod_{k=1}^{N}\left(\frac{\gamma(-kb^2)}{\gamma(-b^2)}\right)\prod_{j=0}^{N-1}\frac{1}{\gamma(2b\alpha_1+jb^2)\gamma(2b\alpha_2+jb^2)\gamma(2b\alpha_3+jb^2)}\,.$$

This can be written in a form that depends parametrically on N so to lift the quantization condition and reconstruct the correlator for arbitrary value of the momenta

$$\Upsilon(x+b) = \gamma(bx)b^{1-2bx}\Upsilon(x)$$
$$\Upsilon(x+b^{-1}) = \gamma(b^{-1}x)b^{2b^{-1}x-1}\Upsilon(x)\,.$$

The result leads to the famous DOZZ formula [Dorn, Otto 1994; Zamolodchikov, Zamolodchikov 1996]

$$C(\alpha_1,\alpha_2,\alpha_3) = \left(\pi\mu\gamma(b^2)b^{2-2b^2}\right)^{\frac{(Q-\alpha)}{b}}\frac{\Upsilon'(0)\prod_{k=1}^{3}\Upsilon(2\alpha_k)}{\Upsilon(\alpha-Q)\prod_{k=1}^{3}\Upsilon(\alpha-2\alpha_k)}\,.$$

3d uplift of Liouville 3-pt. function

The 3d uplift of the evaluation formula for the free field correlator of the Liouville 3-pt. function is a recently proposed duality: [Benvenuti 2018]

Theory A: $U(N)$ gauge theory with one adjoint Φ, one flavor P, $\tilde{P}$, N singlets β_j

$$\mathcal{W} = \sum_{j=1}^{N} \beta_j \mathrm{Tr}\Phi^j \,.$$

Theory B: WZ model with $3N$ singlets α_j, $T_j^{\pm}$

$$\hat{\mathcal{W}} = \sum_{i,j,l=1}^{N} \alpha_i T_j^+ T_{N-l+1}^- \delta_{i+j+l,2N+1} \,.$$

Global symmetry: $U(1)_{top} \times U(1)_\Phi \times U(1)_P$

Operator map:

$$\begin{aligned} \mathfrak{M}^+_{\Phi^s} \quad &\leftrightarrow \quad T^+_{s+1} \\ \mathfrak{M}^-_{\Phi^s} \quad &\leftrightarrow \quad T^-_{N-s} \\ \mathrm{Tr}\left(\tilde{P}\Phi^s P\right) \quad &\leftrightarrow \quad \alpha_{s+1}, \qquad s = 0, \cdots, N-1 \,. \end{aligned}$$

Analytic proof of S^3 partition function identity and perturbative match of the $S^2 \times S^1$ index.

Liouville free field correlators: (k+3)-point function

Also the free field correlator for 3 primaries and k degenerate operators can be rewritten in a form suitable for analytic continuation using the basic integral identities [Fateev, Litvinov 2007]

$$\langle V_{-\frac{b}{2}}(z_1)\dots V_{-\frac{b}{2}}(z_k)V_{\alpha_1}(0)V_{\alpha_2}(1)V_{\alpha_3}(\infty)\rangle = \Omega_k^N(\alpha_1,\alpha_2,\alpha_3)\prod_{a=1}^{k}|z_a|^{2b\alpha_1}|z_a-1|^{2b\alpha_2}\times$$

$$\times\prod_{a<b}^{k}|z_a-z_b|^{-b^2}\int\prod_{a=1}^{k}|x_a|^{2A}|x_a-1|^{2B}K_k^C(x_1,..,x_k|z_1,..,z_k)\prod_{a<b}^{k}|x_a-x_b|^{-4b^2}\,\mathrm{d}^2\vec{x}_k\ ,$$

where the kernel function of Fateev and Litvinov

$$K_k^{\Delta}(m_1,..,m_k|t_1,..,t_k) = \frac{\gamma(-kb^2)}{\gamma^k(-b^2)}\prod_{a<b}^{k}|t_a-t_b|^{2+4b^2}\prod_{a=1}^{k}|m_a-t_k|^{2\Delta}\times$$

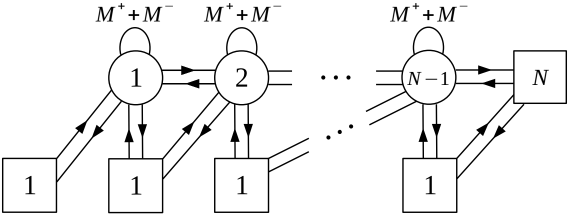
$$\times\int\prod_{i<j}^{k-1}|\tau_i-\tau_j|^2\prod_{i=1}^{k-1}|\tau_i-t_k|^{-2\Delta+2b^2}\prod_{a=1}^{k}|\tau_i-m_a|^{-2-2b^2}K_{k-1}^{\Delta+b^2}(\tau_1,..,\tau_{k-1}|t_1,..,t_{k-1})\,d^2\vec{\tau}_{k-1}$$

is invariant under the exchange of the two sets of parameters

$$K_k^{\Delta}(m_1,..,m_k|t_1,..,t_k) = K_k^{\Delta}(t_1,..,t_k|m_1,..,m_k)\,.$$

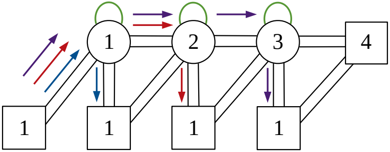
$M[SU(N)]$ theory

We can uplift the kernel of Fateev and Litvinov to a $3d$ $\mathcal{N}=2$ quiver gauge theory with monopole superpotential [Pasquetti, MS 2019]

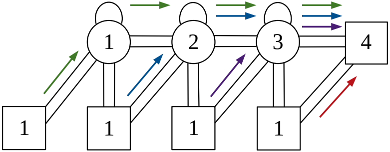


- the UV global symmetry $SU(N)_M \times \frac{\prod_{a=1}^{N} U(1)_{T_a}}{U(1)} \times U(1)_{m_A} \times U(1)_\Delta$ is enhanced in the IR to $SU(N)_M \times SU(N)_T \times U(1)_{m_A} \times U(1)_\Delta$;
- self-dual up to the exchange $SU(N)_M \leftrightarrow SU(N)_T$;
- flows to the $T[SU(N)]$ theory [Gaiotto, Witten 2009] when a real mass deformation for $U(1)_\Delta$ is turned on.

- Re-arrange gauge invariant operators into representations of the enhanced $SU(N)_T$ symmetry



Operator $\mathcal{M}$ in the adjoint representation of $SU(N)_T$.



Operator Π in the bifundamental representation of $SU(N)_M \times SU(N)_T$. Similarly we can construct a $\tilde{\Pi}$.

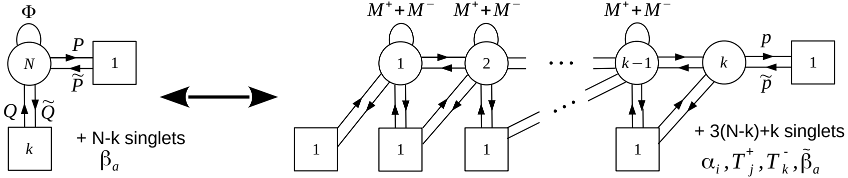
- Check the self-duality of $M[SU(N)]$ by:
    - mapping operators

$$Q\tilde{Q} \leftrightarrow \mathcal{M}, \qquad \Pi \leftrightarrow \tilde{\Pi}\,;$$

    - matching S^3 partition functions and $S^2 \times S^1$ index.

Rank stabilization duality

We can uplift the identity for the $(k+3)$-pt. free field correlator to a new 3d duality. This involves a theory constructed using $M[SU(N)]$ as a building block of which we gauge some of its global symmetries



Tests:

- Global symmetry $U(k)_Q \times U(1)_{top} \times U(1)_\Phi \times U(1)_P$
- Match of operators
- Analytic proof of S^3 partition function identity and perturbative match of the index for low k

Further developments

- Many other results on free field correlators can be uplifted to new 3d dualities

  $\Rightarrow$ from Toda theories we obtain dualities for quiver gauge theories

  [Hwang, Pasquetti, MS work in progress]

- We can further uplift some of the results from 3d to 4d:
  - The $M[SU(N)]$ theory has a 4d ancestor $E[USp(2N)]$ [Pasquetti, Razamat, MS, Zafrir 2019]
  - $E[USp(2N)]$ can be used as a building block gauging its symmetries $\Rightarrow$ relation to compactifications of the 6d rank-N E-string theory on tori with fluxes for its E_8 global symmetry [Pasquetti, Razamat, MS, Zafrir 2019]
  - other new dualities in 4d [Hwang, Pasquetti, MS work in progress]

Derivation of 3d monopole duality from 4d

- Start from Intriligator–Pouliot duality in 4d

$$\begin{array}{c} USp(2N_c) \text{ with } N_f \text{ flavors } Q^i,\ \tilde{Q}_i \\ \mathcal{W} = 0 \end{array} \quad \longleftrightarrow \quad \begin{array}{c} USp(2N_f - 2N_c - 4) \text{ with } N_f \text{ flavors } q^i,\ \tilde{q}_i \\ N_f(N_f-1)/2 \text{ singlets } M^i{}_j \text{ and} \\ \mathcal{W} = \mathfrak{M} \sum_{i<j}^{N_f} M^i{}_j q^j \tilde{q}_i \end{array}$$

- Reduce the duality to 3d

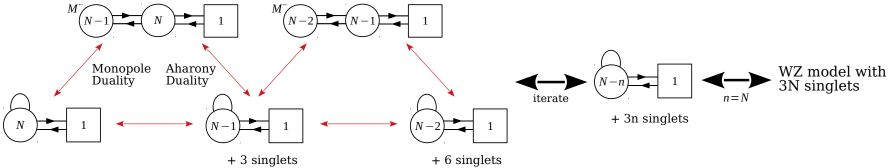
$$\begin{array}{c} USp(2N_c) \text{ with } N_f \text{ flavors } Q^i,\ \tilde{Q}_i \\ \mathcal{W} = \mathfrak{M} \end{array} \quad \longleftrightarrow \quad \begin{array}{c} USp(2N_f - 2N_c - 4) \text{ with } N_f \text{ flavors } q^i,\ \tilde{q}_i \\ N_f(N_f-1)/2 \text{ singlets } M^i{}_j \text{ and} \\ \mathcal{W} = \sum_{i<j}^{N_f} M^i{}_j q^j \tilde{q}_i \end{array}$$

- Give a VEV to some of the scalars in the vector multiplet to break $USp(2N_c)$ to $U(N_c)$ and perform a real mass deformation to keep some of the flavors massless

3d uplift of Liouville 3-pt. function

Analogies with the CFT picture:

- This duality can be derived iterating some basic dualities



The same procedure can be employed to prove the equality of partition functions, similarly to the proof of the evaluation of the free field integral.

- We can also make sense of the analytic continuation in N of the dual 3d theories $\Rightarrow 5d\ \mathcal{N}=1\ T_2$ theory.

  The 3d theory appears as a co-dimension two defect theory in 5d for quantized values of one of the parameters of the T_2 theory (geometric transition).

There exist two main classes of dualities in 3d:

- Mirror dualities, which swap Higgs and Coulomb branch (real masses $\leftrightarrow$ FI)

  $\Rightarrow$ we have to take opposite limits on the two sides of the duality

  $\Rightarrow$ we end up with 2d Mirror Symmetry between a GLSM and a LG model (Hori–Vafa duality)

- Seiberg-like dualities, which don't swap Higgs and Coulomb branch

  $\Rightarrow$ we have to take the same limit on the two sides of the duality

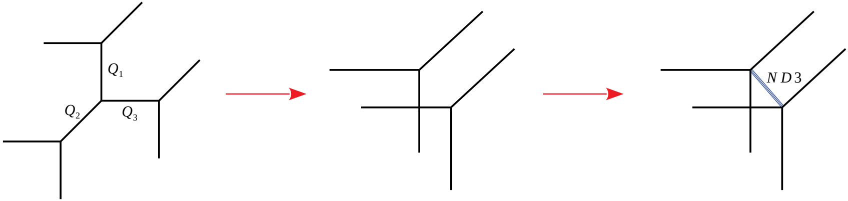
  - Higgs limit gives a similar Seiberg-like duality in 2d between GLSMs
  - Coulomb limit gives a duality between 2d LG models.

Analytic continuation and geometric transition

We can perform analytic continuation in the rank N

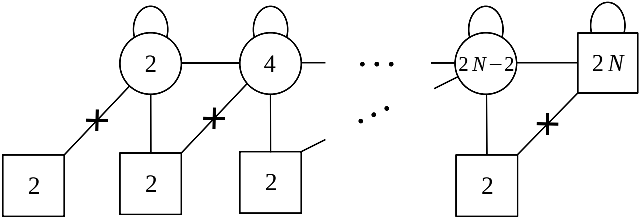
$$\mathcal{Z}_{WZ}^{S_b^3} = \operatorname*{Res}_{N\in\mathbb{N}} \frac{S_3'(0)\, S_3(-2i\mu+2i\tau)\, S_3\left(\frac{Q}{2}\pm i\zeta - i\mu - 2i(N-1)\tau\right)}{S_3(-2iN\tau)\, S_3(-2i\mu-2i(N-1)\tau)\, S_3\left(\frac{Q}{2}\pm i\zeta - i\mu + 2i\tau\right)} = \operatorname*{Res}_{N\in\mathbb{N}} \mathcal{Z}_{T_2}^{S^5}\,.$$

The result is the partition function of the $5d$ $\mathcal{N}=1$ T_2 theory and the quantization condition on N corresponds to geometric transition to the 3d defect theory.



$E[USp(2N)]$ theory

The $FM[SU(N)]$ theory has a $4d$ $\mathcal{N}=1$ ancestor with $USp(2n)$ gauge groups, called $E[USp(2N)]$ [Pasquetti, Razamat, MS, Zafrir 2019]

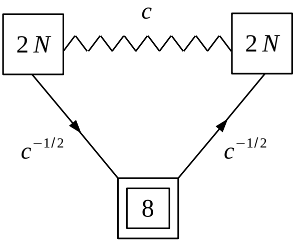


- the partition function on $S^3 \times S^1$ coincides with Rains' interpolation kernel;
- the UV global symmetry $USp(2N)_x \times \prod_{a=1}^{N} SU(2)_{y_a} \times U(1)_t \times U(1)_c$ is enhanced in the IR to $USp(2N)_x \times USp(2N)_y \times U(1)_t \times U(1)_c$;
- self-dual up to the exchange $USp(2N)_x \leftrightarrow USp(2N)_y$;
- flows to $FM[SU(N)]$ upon dimensional reduction and a real mass deformation.

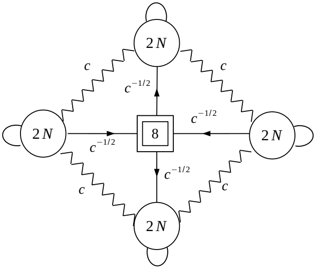
E-string surprise

We can use the SCFT which $E[USp(2N)]$ flows to as a building block to construct more complicated theories by gauging its two $USp(2N)$ global symmetries

These $4d$ $\mathcal{N}=1$ theories arise from the compactification of the $6d$ $(1,0)$ rank-N E-string theory on a Riemann surface with flux for the E_8 of its $E_8 \times SU(2)_L$ global symmetry.



Tube with flux $z=\frac{1}{2}$ for a $U(1)$ whose commutant in E_8 is E_7.



Torus with flux $z=2$. The global symmetry $SU(8)\times U(1)_c \times U(1)_t$ is expacted to get enhanced to $E_7 \times U(1) \times SU(2)_L$.

Symmetry enhancements from the superconformal index

- $4d$ superconformal index a.k.a. $S^3 \times S^1$ partition function with the superconformal R-charge

$$\mathcal{I} = 1 + \cdots + (\text{marginal operators} - \text{conserved currents})pq + \cdots$$

Looking at the pq term one can deduce possible global symmetry enhancements (even for the Cartan!).

- $3d$ superconformal index a.k.a. $S^2 \times S^1$ partition function with the superconformal R-charge ($x^2 = q$ from before)

$$\mathcal{I} = 1 + \cdots + (\text{marginal operators} - \text{conserved currents})x^2 + \cdots$$

If the theory has $\mathcal{N} \geq 3$ supersymmetry

$$\mathcal{I} = 1 + \cdots + (\text{global symmetry conserved currents})x + \cdots$$

Combining these two pieces of information one can deduce supersymmetry enhancements. [Garozzo, Lo Monaco, Mekareeya, MS 2019]