Constraints on Boundary-localized Interactions

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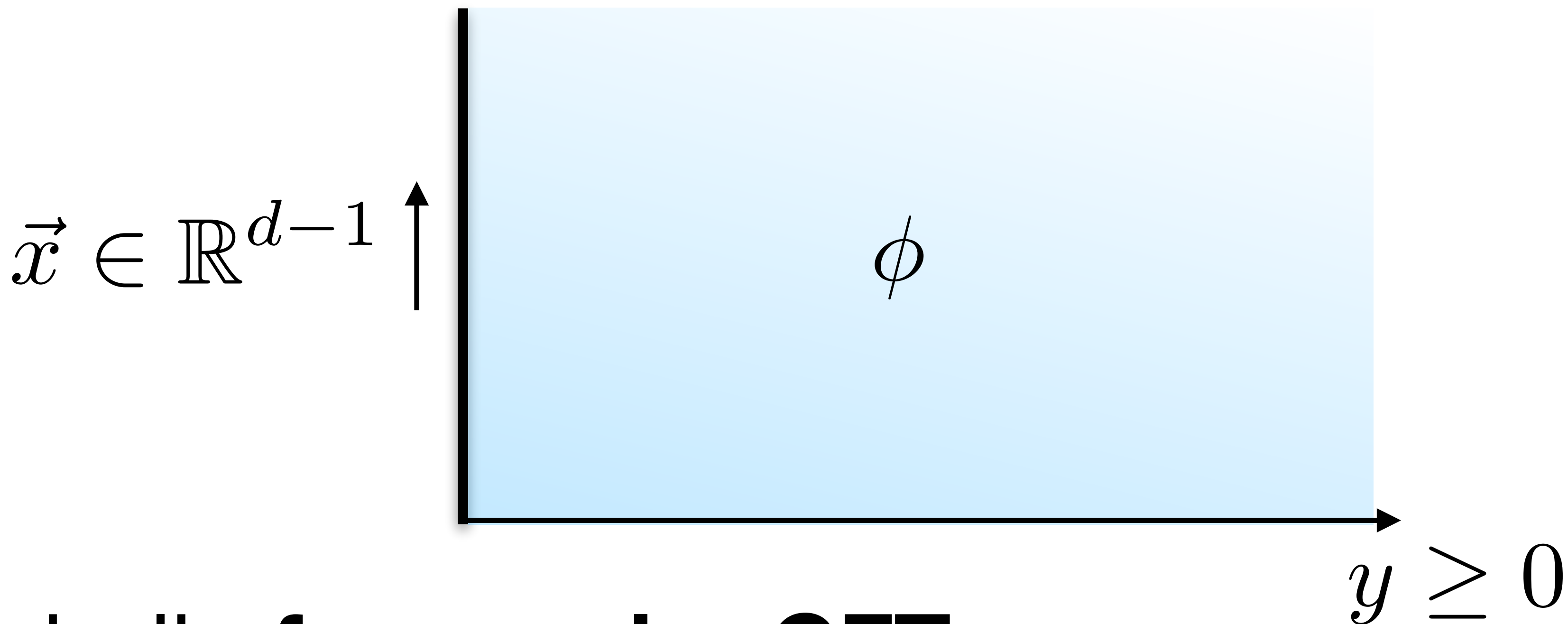
L.D., D. Gaiotto, E. Lauria, J. Wu, 1902.09567
C. Behan, L.D., E. Lauria, B. Van Rees, in progress

Conformal boundary conditions for free conformal field theories: free bulk, interacting boundary.

- Boundary phase transitions in systems described by free fields in the bulk.
- In the grand program of charting the space of possible CFTs, this is a corner that is simple enough, yet rich enough;
- Neat example: 4d Maxwell field;
- Task: phrasing it in a way that is amenable to numerical bootstrap;

Introduction: Free Bulk & Free Boundary



In the bulk: **free scalar CFT**

$$\Box\phi = 0 \quad \Rightarrow \quad \phi(y, \vec{x}) \underset{y \to 0}{\sim} (\hat{\phi}(\vec{x}) + \dots) + y(\widehat{\partial_y\phi}(\vec{x}) + \dots)$$

Stationary action (w/out interactions on the boundary):

$\hat{\phi}(\vec{x}) = 0$	or	$\widehat{\partial_y\phi}(\vec{x}) = 0$
Dirichlet		**Neumann**

Introduction: Free Bulk & Free Boundary

Similar for **free Dirac CFT** $\gamma^\mu \partial_\mu \Psi(y, \vec{x}) = 0$

$$\widehat{\psi_\pm}(\vec{x}) = \frac{\mathbb{1} \pm \gamma^y}{2} \Psi(y, \vec{x}) \Big|_{y=0}$$

$\widehat{\psi_+}(\vec{x}) = 0$	or	$\widehat{\psi_-}(\vec{x}) = 0$

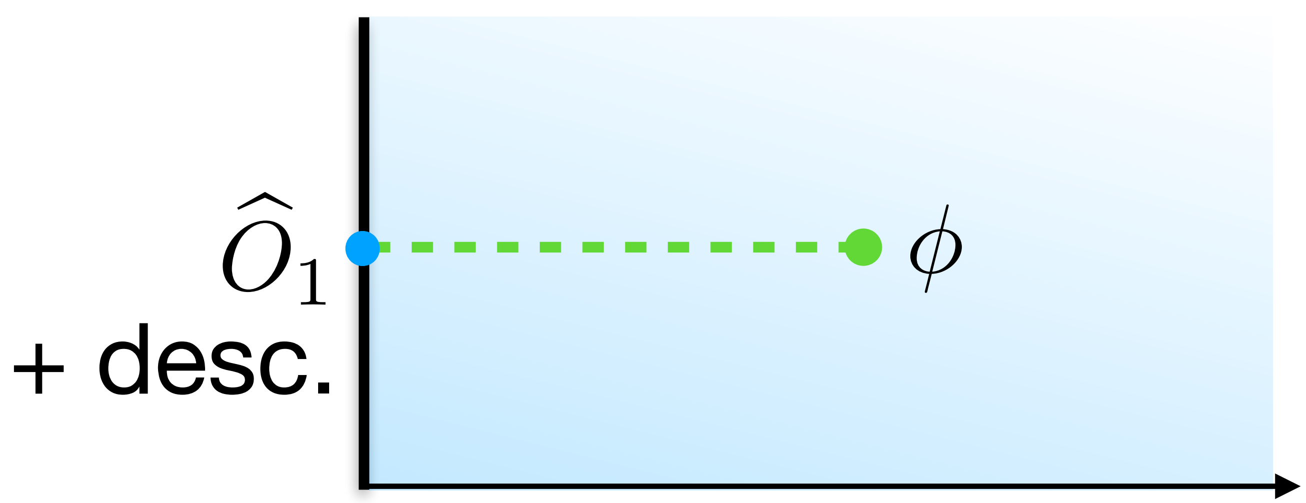
or **free higher-form CFT**, e.g. in 4d $\partial^\mu F_{\mu\nu} = 0 = \partial^\mu \tilde{F}_{\mu\nu}$

$$\hat{J}_a(\vec{x}) = \frac{1}{g^2} F_{ya}(y, \vec{x})\big|_{y=0} \;, \; \hat{I}_a(\vec{x}) = \frac{1}{4\pi i} \epsilon_{abc} F^{bc}(y, \vec{x})\big|_{y=0}$$

$\hat{J}_a(\vec{x}) = 0$	or	$\hat{I}_a(\vec{x}) = 0$

Introduction: Free Bulk & Free Boundary

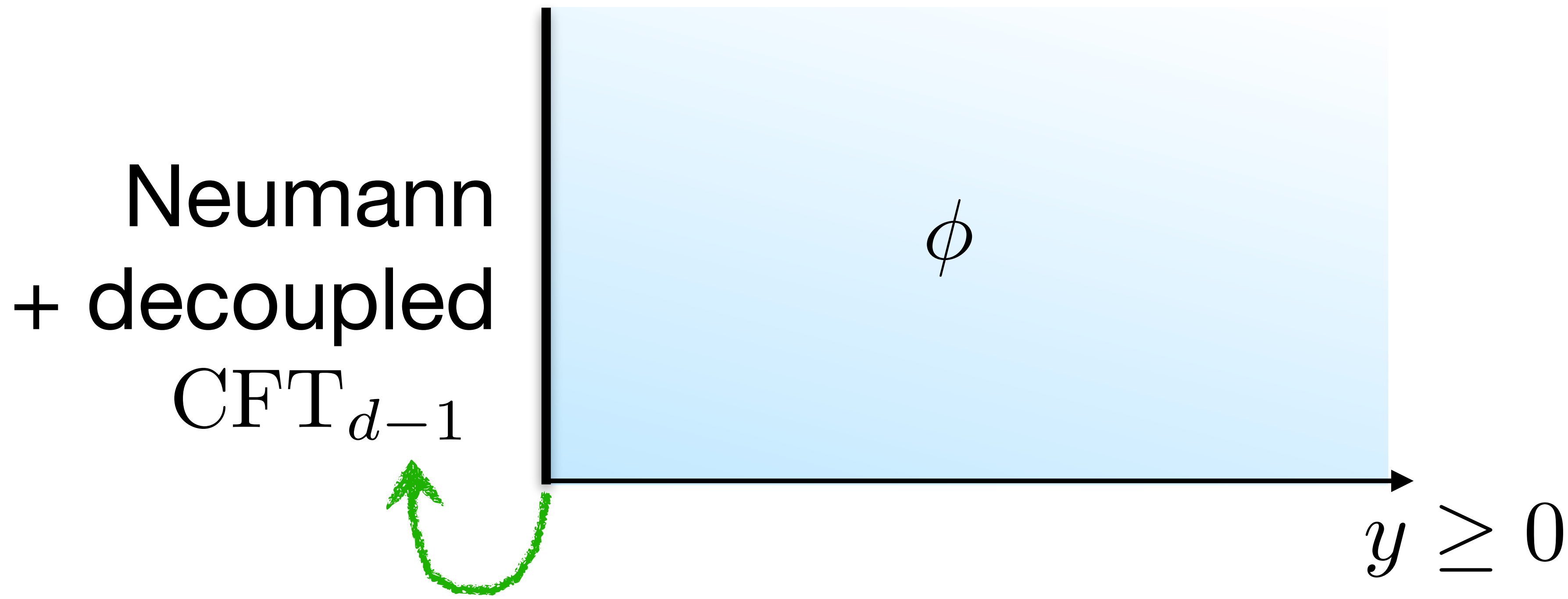
In all these cases: $\widehat{O}_1$ and $\widehat{O}_2$, one set to zero ; only 1 boundary primary in the **boundary OPE**.



Boundary theory: Mean Field Theory for the remaining operator.

All bulk and boundary correlators are computed by Wick contractions.

Adding boundary interactions: Scalar Example



Turn on: $\hat{g} \int d^{d-1}\vec{x}\, \hat{\phi}(\vec{x})\, \mathcal{O}(\vec{x})$, $\mathcal{O} \in \mathrm{CFT}_{d-1}$

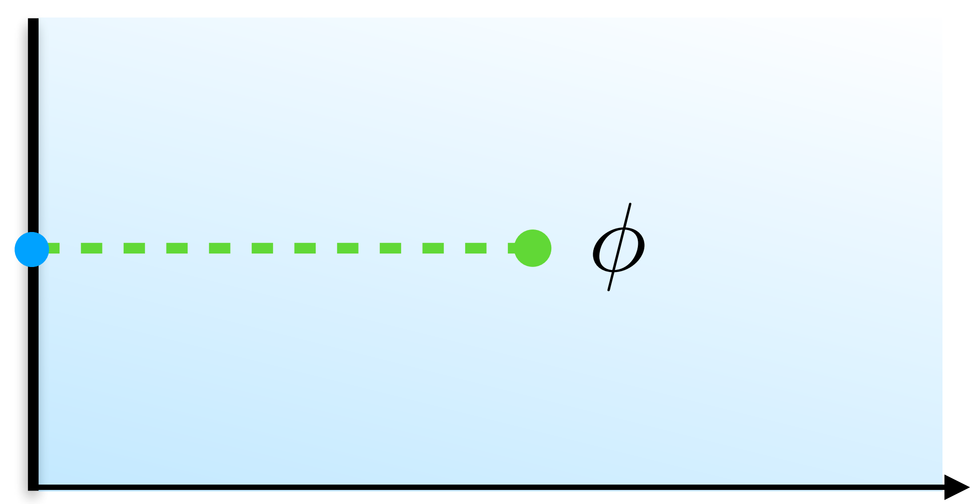
Stationary action: $\widehat{\partial_y \phi} = \hat{g}\mathcal{O}$ **Modified Neumann**

Generically starts an RG, at the endpoint we expect conformal b.c. with both $\hat{\phi} \neq 0$ and $\widehat{\partial_y \phi} \neq 0$.

Interacting boundary condition for free fields:

- Correlation functions are not gaussian
- Both $\widehat{O}_1$ and $\widehat{O}_2$ in the set of boundary operators, and they appear in the boundary OPE of the bulk free field

($\widehat{O}_1$ + desc.)
+
($\widehat{O}_2$ + desc.)



Scaling dimensions $\hat{\Delta}_1$ and $\hat{\Delta}_2$ fixed by e.o.m.

Interesting example: interacting conformal b.c. for 4d Maxwell CFT (1902.09567)

$$S_{\text{bulk}} = -\frac{i}{8\pi}\int \left[\tau(F^-)^2 - \bar{\tau}(F^+)^2\right] \ , \quad \tau \equiv \frac{\theta}{2\pi} + i\frac{2\pi}{g^2}$$

$$\hat{J}_a(\vec{x}) = \frac{1}{g^2}\, F_{ya}(y,\vec{x})\big|_{y=0} \ , \ \hat{I}_a(\vec{x}) = \frac{1}{4\pi i}\epsilon_{abc}\, F^{bc}(y,\vec{x})\Big|_{y=0}$$

Start with Neumann: $\hat{J}_a = 0$, i.e. $A_a|_{y=0}$ free

+ 3d CFT with $U(1)$ global symmetry, and turn on

$$\int d^{d-1}\vec{x}\, A_a(\vec{x})\, J^a_{\text{CFT}}(\vec{x})$$

Stationary action: $\hat{J}_a = J_{\text{CFT}\,a}$ **Modified Neumann**

τ is the coefficient of a local bulk operator

→ boundary interactions cannot renormalize it (locality)

In this setup with interactions localized on the boundary it is **exactly marginal**.
Two-loop check in [Teber], more general argument based on Ward Identities in [Herzog-Huang].

Assuming we can set to zero the beta-function of boundary marginal couplings:

$$\boxed{B(\tau,\bar{\tau})}$$

Continuous family of BCFTs

$\tau \to \infty$: decoupling, $B(\tau, \bar{\tau}) \to \hat{I}_a$ MFT + 3d CFT $T_{0,1}$

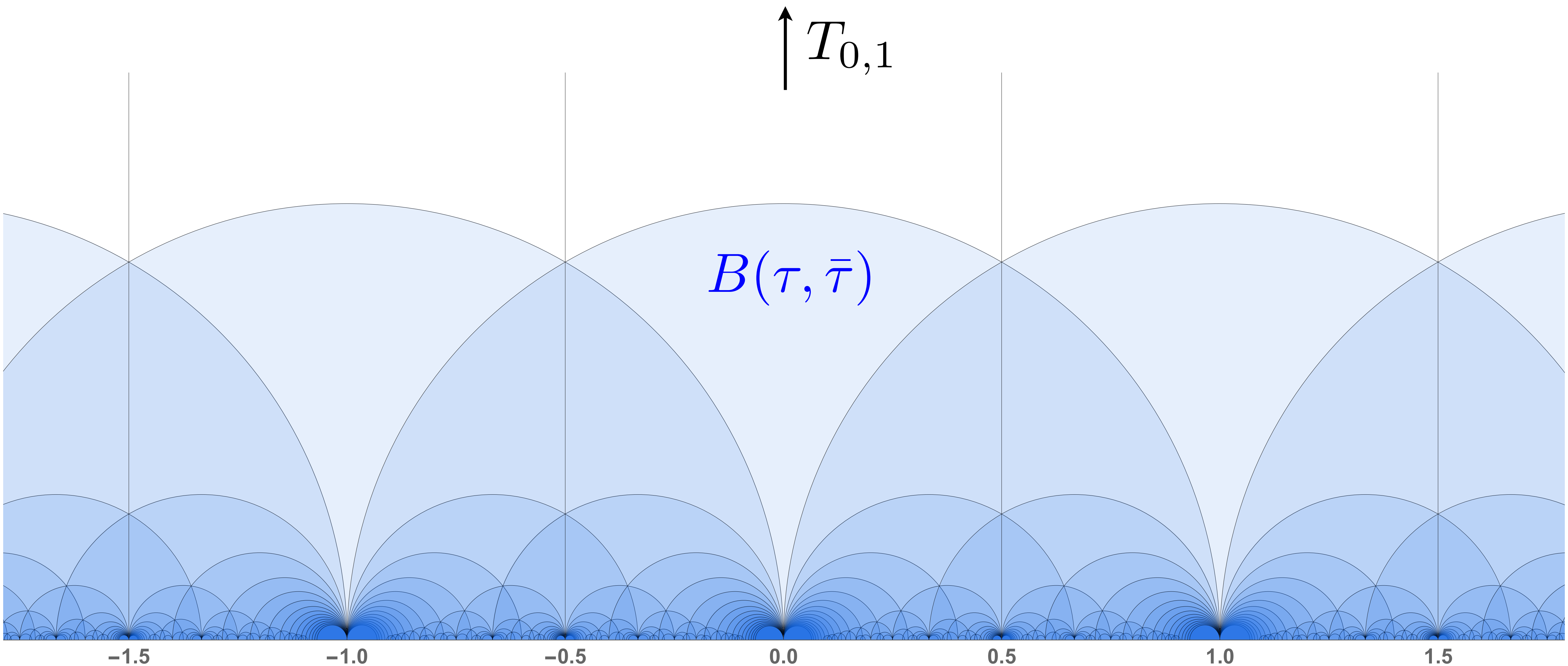
Cusps on the real axis: $\tau = -\frac{q}{p}\ , \quad q, p \in \mathbb{Z}$

Also decoupling, but with (generically) different 3d theory

$$B(\tau, \bar{\tau}) \to \ p\hat{J}_a + q\hat{I}_a \quad \text{MFT + 3d CFT} \ \ T_{p,q}$$

$T_{p,q}$ is obtained from $T_{0,1}$ through the $SL(2, \mathbb{Z})$ action on 3d CFT with $U(1)$ symmetry. [Witten]

[Gaiotto-Witten]



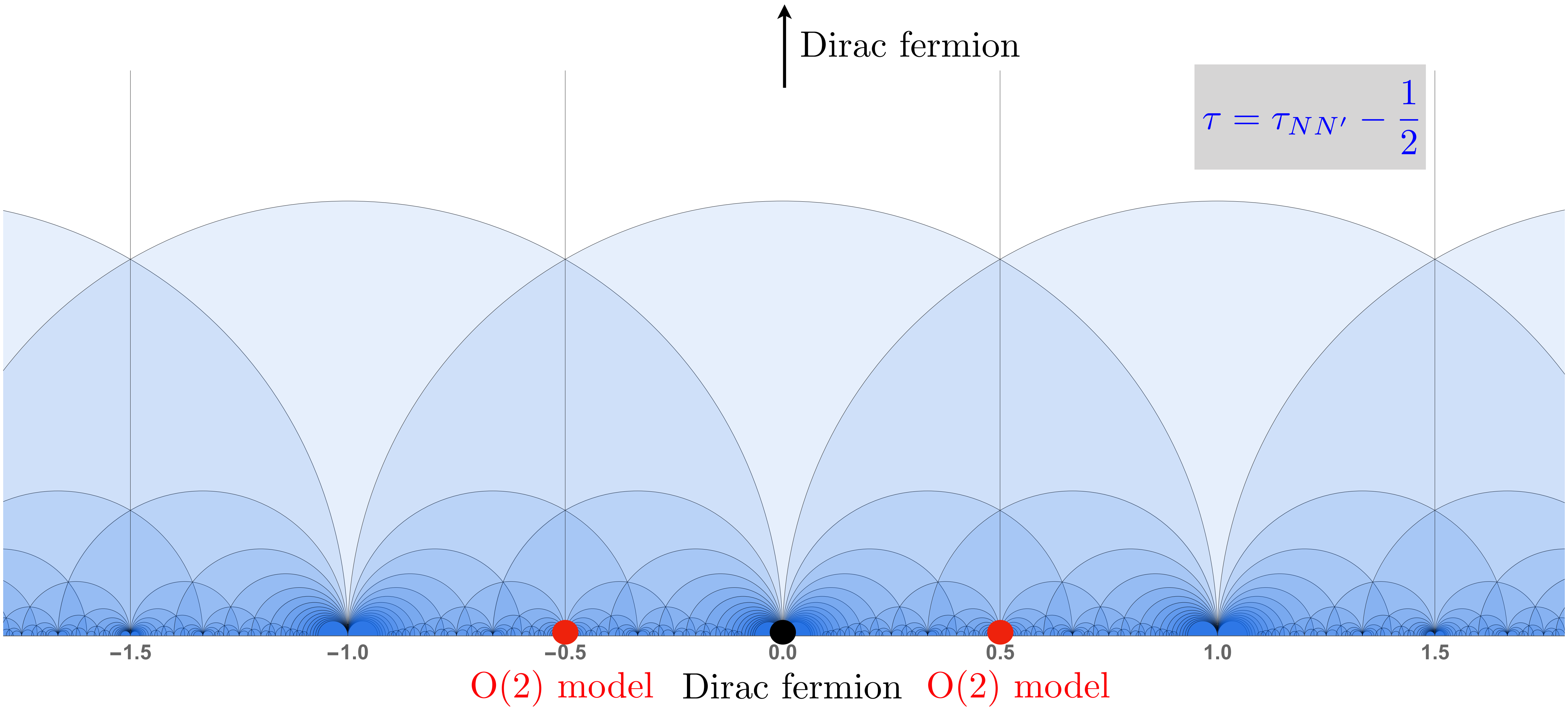
$T_{p,q}$, $(p, q) \neq (0, 1)$

"Gauging of $T_{0,1}$ with fractional CS"

In the favorable situation with no phase transitions the BCFT data of $B(\tau, \bar{\tau})$ smoothly interpolate between the 3d CFT data of the infinite family of theories $T_{p,q}$.

We can pick $T_{0,1}$ to be a free theory, and compute data in perturbation theory in τ^{-1} , $\bar{\tau}^{-1}$. Reaching the gauge-theory cusps on the real axis requires extrapolation.

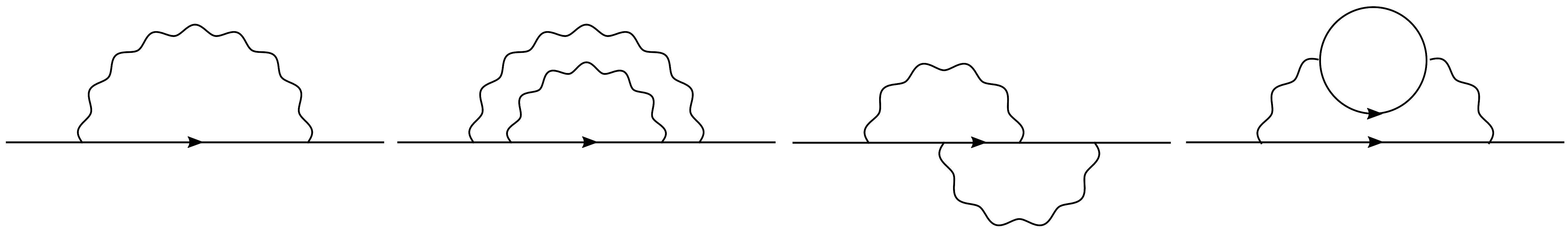
Best case: some gauge theory $T_{p,q}$ might be known (or conjectured) to be equivalent to $T_{0,1}$. Then we can perform a **duality-improved extrapolation**, matching also with the cusp on the real axis.



$$U(1)_{-\frac{1}{2}} + \psi \longleftrightarrow \phi$$

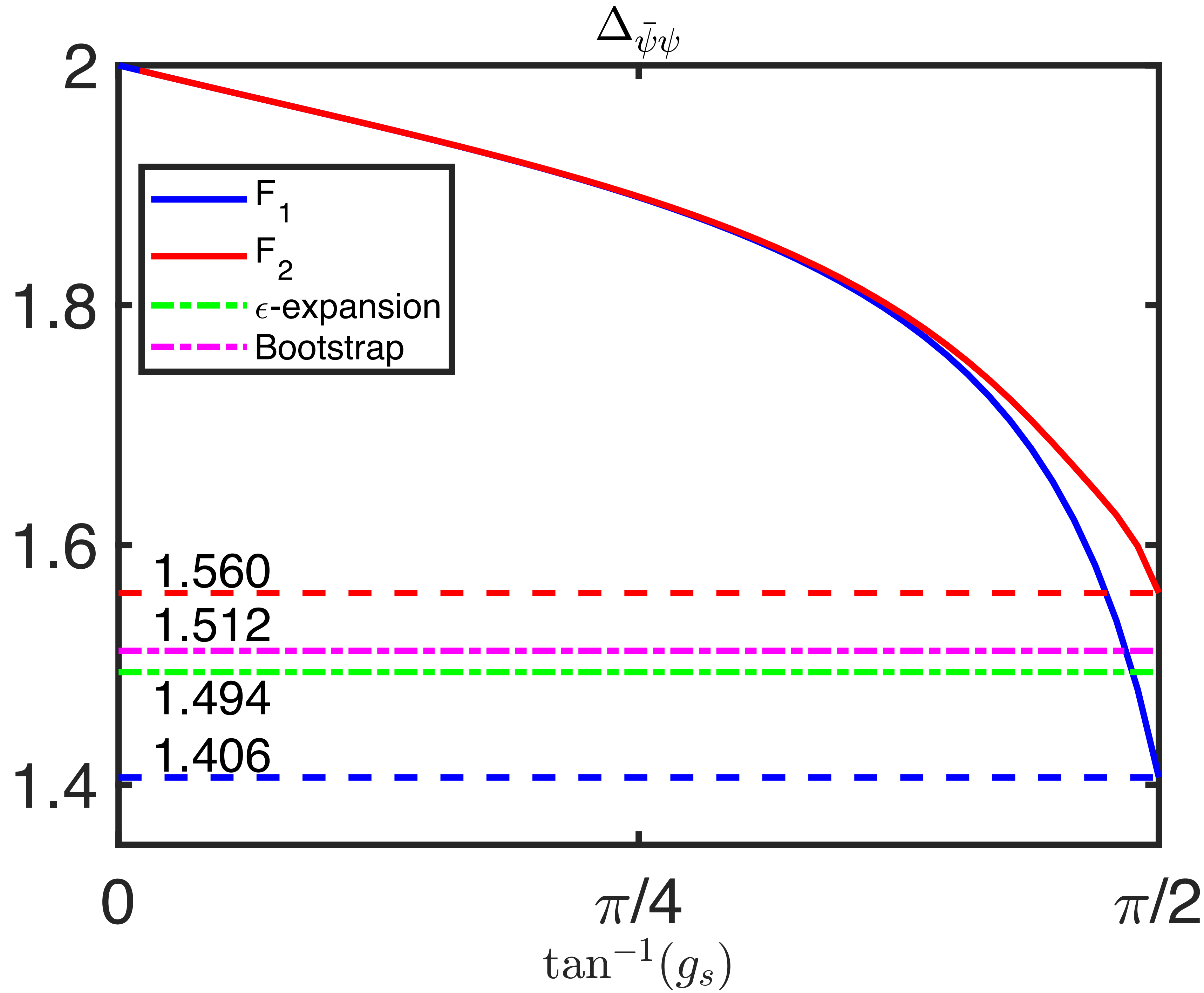
$$U(1)_{-2} + (U(1)_{-\frac{1}{2}} + \psi) \longleftrightarrow \psi$$

Anomalous dimension of $\bar{\psi}\psi$



$$\gamma_{\bar{\psi}\psi} = \frac{4i}{3\pi}\left(\frac{1}{\bar{\tau}} - \frac{1}{\tau}\right) + \left(\frac{8}{27\pi^2} - 1\right)\left(\frac{1}{\bar{\tau}^2} + \frac{1}{\tau^2}\right) + \left(-\frac{16}{27\pi^2} - \frac{2}{3}\right)\frac{1}{\tau\bar{\tau}} + \mathcal{O}(|\tau|^{-3})$$

$$\Rightarrow \quad \gamma_{\bar{\psi}\psi} = -\frac{8}{3k^2} + \mathcal{O}(k^{-4}) \quad \text{at large k}$$



Comparison with energy-operator of O(2) model

[Kos, Poland, Simmons-Duffin, Vichi]

[Kleinert, Neu, Schulte-Frohlinde, Chetyrkin, Larin]

Work in progress: **Conformal Bootstrap** to look for interacting boundary conditions.

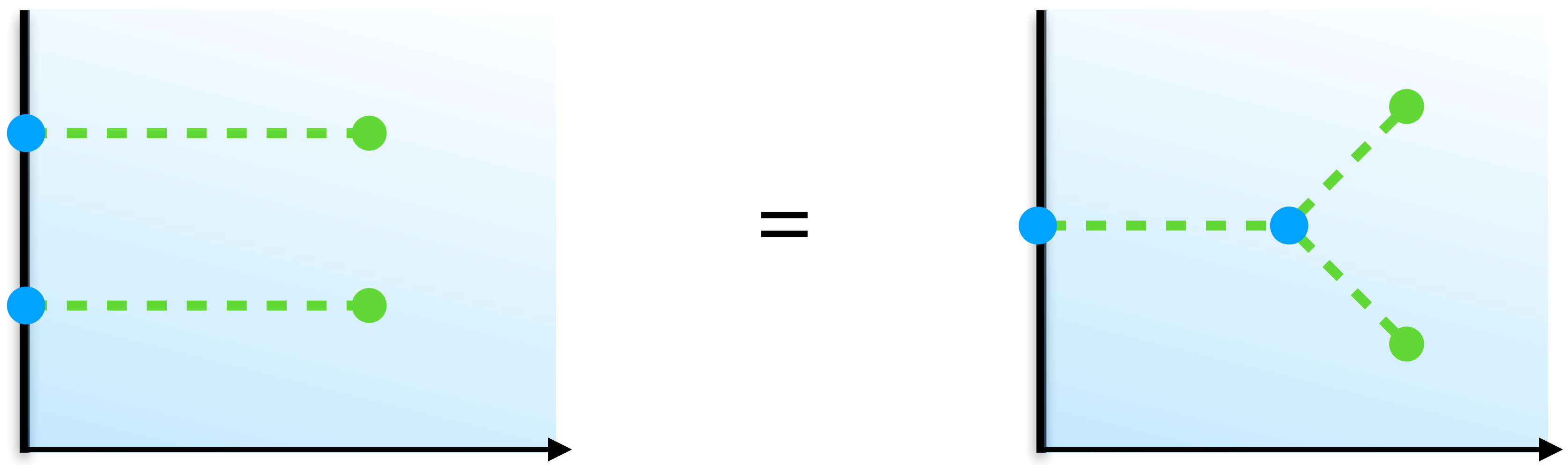
Long-term goal: non-perturbative approach to study the conformal manifold $B(\tau, \bar{\tau})$ and learn about 3d CFTs with $U(1)$ global symmetry.

Short-term: start with free scalar, explore the space of interacting boundary conditions. A priori unclear whether to expect that this space is empty, or huge, or something in between.

??? → Empty: simplicity of the Hilbert space

??? → Huge: lower-dim. CFTs; analogy with Maxwell

Usual approach to bootstrap boundary CFTs:

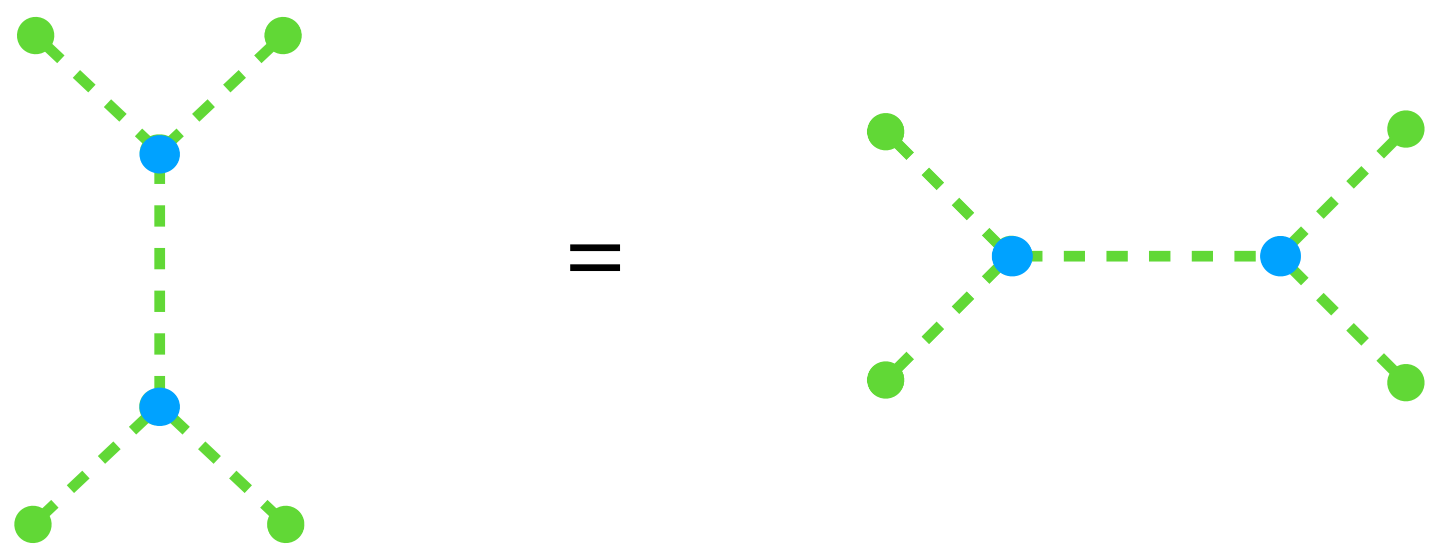


Crossing symmetry of the bulk two-point functions.

[Liendo, Rastelli, Van Rees]

In our setup, the simplicity of the boundary OPE allows us to skip this step and concentrate on the boundary correlators. Akin to bootstrapping a non-local conformal theory. [Behan]

Idea: numerical bootstrap applied to boundary 4pt functions of $\widehat{O}_1$ and $\widehat{O}_2$.

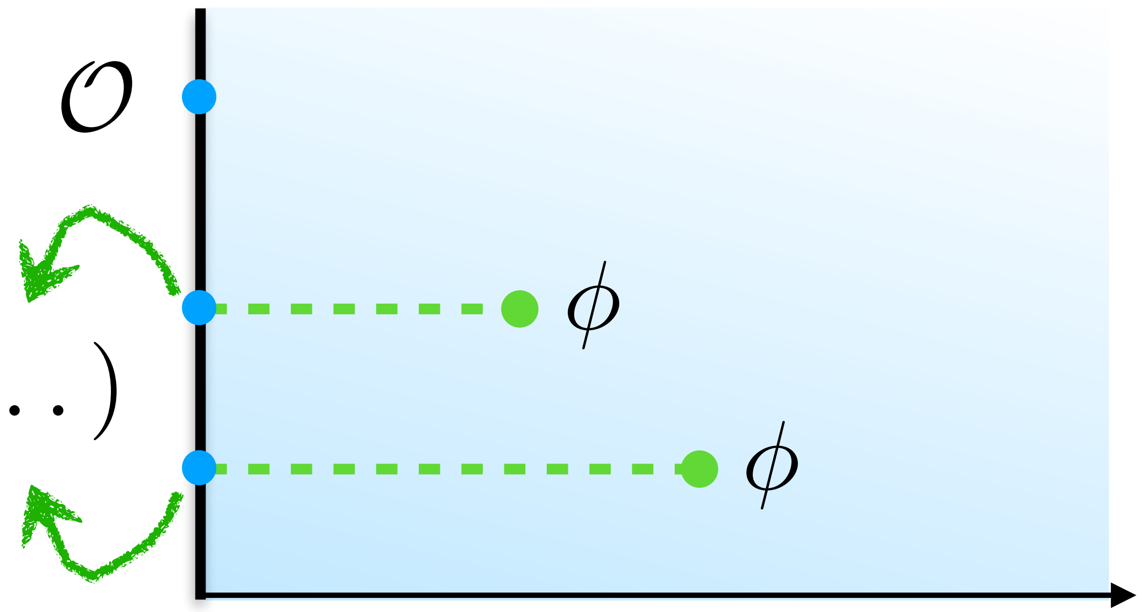


Conformal Data that enters: $\Delta_{\mathcal{O}}, \lambda_{11\mathcal{O}}, \lambda_{22\mathcal{O}}, \lambda_{12\mathcal{O}}$

Input from the free bulk theory: **constraints among the OPE coefficients**. They allow to organize exchanges of $\mathcal{O}$ in different channels as a unique "superblock".

E.g. free scalar theory: $\widehat{O}_1 \equiv \hat{\phi}$, $\widehat{O}_2 \equiv \widehat{\partial_y \phi}$

$$b_1(\widehat{O}_1 + \ldots) + y\, b_2(\widehat{O}_2 + \ldots)$$



3pt function computed resumming the boundary OPE.

Compatibility with bulk OPE limit:

$$\begin{aligned} \lambda_{11\mathcal{O}} &= F_1(\lambda_{12\mathcal{O}}, \Delta_{\mathcal{O}}, b_1/b_2) \\ \lambda_{22\mathcal{O}} &= F_2(\lambda_{12\mathcal{O}}, \Delta_{\mathcal{O}}, b_1/b_2) \end{aligned}$$

b_1/b_2 constrained by unitarity in a known interval.

$\Delta_T \geq d-1$: Gap on spin 2 exchange (non-locality)

$\Delta_\varepsilon \geq \dfrac{d-1}{2} - 1$: Gap on spin 0 exchange

$(\Delta_T,\, \Delta_\varepsilon,\, b_1/b_2)$

NO or MAYBE

Further assumption: the additional boundary d.o.f. must be local.

Preliminary result: in $d=4$ the answer is NO for most of the allowed parameter space.

Indicates that in this case there is nothing beyond Neumann and Dirichlet.

Summary:

- Interacting boundary conditions for free fields;
- Example of Maxwell: conformal manifold, action of EM duality, interpolates conformal data of interesting 3d CFTs;
- Bootstrap approach: first results for free scalar in 4d;

To do list:

- Scalars in 3d: compare to perturbative constructions;
- Apply bootstrap approach to Maxwell case;
- Compare with boundary effects in experiments/ simulations of systems described by free scalars (e.g. superfluids)

Thank You