The BPS LIMIT of AdS BLACK HOLE THERMODYNAMICS and its MICROSCOPIC COUNTERPART

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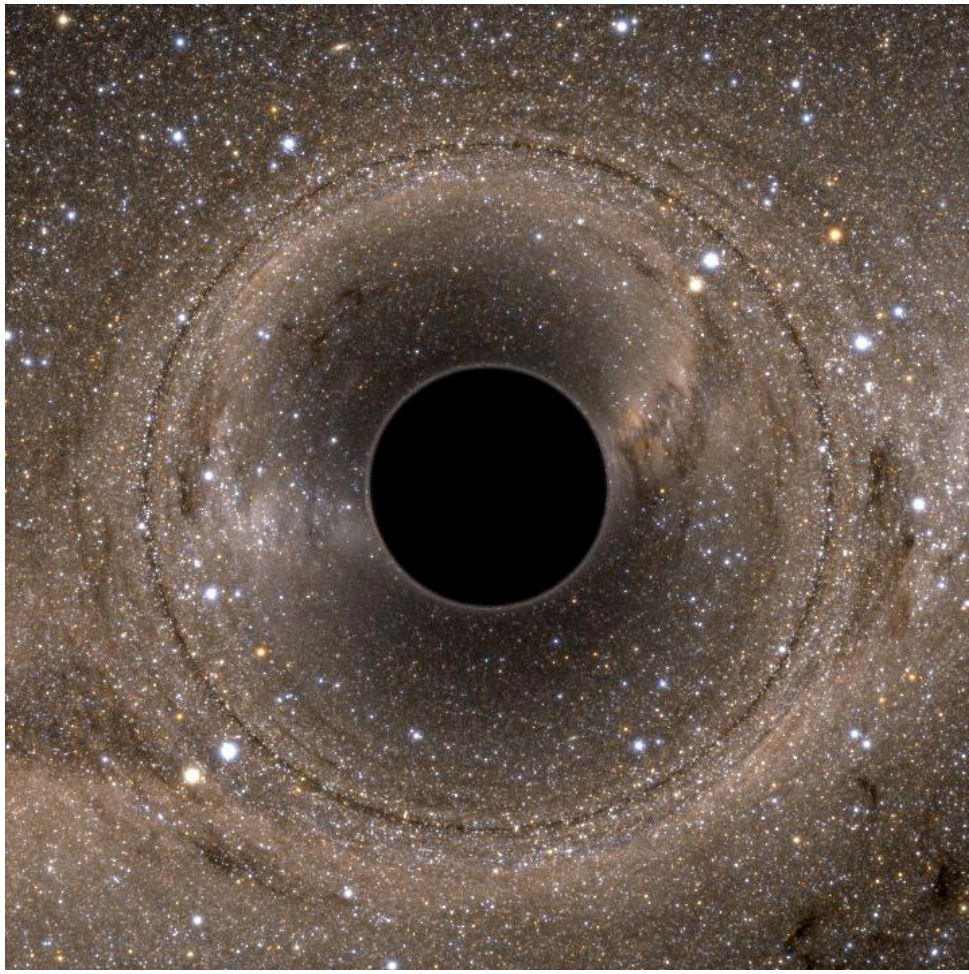
Theories of the Fundamental Interactions

INFN meeting,

Torino, 22 October 2019

Black hole microstate counting

- **Black holes** are "theoretical laboratories" useful to test any theory of quantum gravity

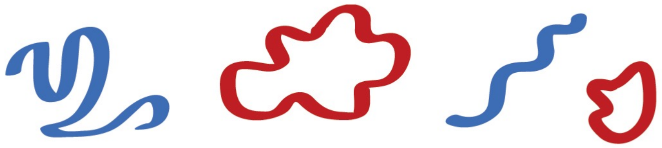


Thermodynamic properties

$$S = \frac{\mathbf{Area}}{\mathbf{4}}$$

Microscopic statistical derivation ?

- Major achievement of string theory:



to provide the microstates for classes of supersymmetric black holes.

asymptotically flat   Strominger, Vafa '96 . . .

asymptotically locally AdS   Benini, Hristov, Zaffaroni '15, . . .

General picture for BPS black holes in AdS

Entropy : Legendre transform of a *simple* function of chemical potentials

We focus on rotating BH's that are *asymptotically AdS*

asymptotics	charges	BPS entropy	"log grand-canonical partition function"
M, $AdS_4 \times S^7$	$J,\ Q_1, Q_2, Q_3, Q_4$	$S = S(J_i, Q_I)$	$I = -\frac{i}{2G}\frac{\sqrt{\varphi^1\varphi^2\varphi^3\varphi^4}}{\omega}$
IIB, $AdS_5 \times S^5$	$J_1, J_2,\ Q_1, Q_2, Q_3$		$I = \frac{\pi}{4G}\frac{\varphi^1\varphi^2\varphi^3}{\omega_1\omega_2}$
IIA, $AdS_6 \times_w S^4$	$J_1, J_2,\ \ Q$		$I = \frac{\pi\, i}{3G}\frac{\varphi^3}{\omega_1\,\omega_2}$
M, $AdS_7 \times S^4$	$J_1, J_2, J_3,\ Q_1, Q_2$		$I = -\frac{\pi^3}{128G}\frac{\varphi_1^2\varphi_2^2}{\omega_1\omega_2\omega_3}$

complex constraint $\sum \omega_i - \sum \varphi_I = 2\pi i$

Hosseini, Hristov, Zaffaroni '17, '18;
Choi, Hwang, Kim, Nahmgoong '18

In this talk

- Derivation of **macroscopic** $\mathcal{I}$ in gravity
  - in all cases, $\mathcal{I}$ is a supersymmetric on-shell action
  - complexified solution,

    new BPS limit of black hole thermodynamics

- This will also define the **microscopic** computation via **AdS/CFT**
  - will discuss the AdS_5/CFT_4 case in some detail

1810.11442, 1904.05865 with A. Cabo-Bizet, D. Martelli, S. Murthy

1906.10148 with L. Papini

Supersymmetric black holes in AdS_5

- Supersymmetric black holes in AdS_5 have been known for 15 years

Gutowski, Reall '04, Chong, Cvetic, Lu, Pope '05, Kunduri, Lucietti, Reall '06

1/16 BPS, carry angular momentum & electric charge

start from type IIB on $AdS_5 \times S^5$

↓ ↘

$SO(2,4) \times SO(6)$ symmetry

↓ ↓

break to $\mathbb{R} \times U(1)^2 \times U(1)^3$

↓ ↓

E, J_1, J_2 Q_1, Q_2, Q_3

- replace S^5 with more general M_5 ➜ SO(6) broken to just U(1) ➜ $E,\ J_1,\ J_2,\ Q$

Supersymmetric black holes in AdS_5

Bekenstein-Hawking entropy

$$S = \frac{\mathbf{Area}}{4} = \pi\sqrt{3Q^2 - 8c(J_1 + J_2)}$$

$$c = \frac{\pi\ell^3}{8G_N}$$

microscopic origin ?? use AdS / CFT!

Supersymmetric black holes in AdS_5

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microscopic origin ??   use AdS / CFT!

type IIB on $AdS_5 \times S^5$ $\Longleftrightarrow$ $\mathcal{N} = 4$ SYM ,

replace S^5 with more general M_5 $\Longleftrightarrow$ $\mathcal{N} = 1$ $SCFT_4$, e.g. conifold theory

microstates: 1/16 BPS states with assigned angular momenta and charge

Task: count them at large N and see if there is $O(N^2)$ degeneracy.

Attempts in the past unsuccessful

Difficulties on field theory side

Why failed?

- natural quantity to consider: superconformal index

Romelsberger '05

Kinney, Maldacena, Minwalla, Raju '05

$$\mathcal{I}(\omega_1,\omega_2) = \underbrace{\mathrm{Tr}\,(-1)^F e^{-\beta\{\mathcal{Q},\bar{\mathcal{Q}}\}}}_{\text{Witten index}} {}^{+\omega_1 \underbrace{(J_1+\frac{1}{2}Q)} + \omega_2 \underbrace{(J_2+\frac{1}{2}Q)}}$$

Witten index; commute with supercharge $\mathcal{Q}$

ω_1, ω_2 chemical potentials, taken real

At large N, $\mathcal{I}(\omega_1,\omega_2) \sim \mathcal{O}(1)$ ➜ cannot reproduce $\mathcal{O}(N^2)$ entropy

- reason: many cancellations between bosonic and fermionic states

Difficulties on gravity side

Black hole thermodynamics:

Gibbons, Hawking

entropy S and on-shell gravity action are related as

$$I(\beta, \Omega_i, \Phi) = \beta E - S - \beta \Omega_i J_i - \beta \Phi Q$$

Quantum Statistical Relation

$$E = \frac{\partial I}{\partial \beta}\,, \qquad J_i = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_i}\,, \qquad Q = -\frac{1}{\beta}\frac{\partial I}{\partial \Phi} \qquad \beta = T^{-1}$$

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- Thermodynamics for supersymmetric black holes is subtle : $\beta \to \infty$
  - what are the relevant chemical potentials for $\beta \to \infty$ **?** $\Omega_i \to 1,\ \Phi \to 3/2$
    frozen!
  - How do these match ω_1, ω_2 on the field theory side?

Let us clarify the issues on the gravity side first

The non-BPS solution

- Five-dimensional minimal gauged supergravity

$$\mathcal{L} = (R + 12) *1 - \frac{2}{3} F \wedge *F + \frac{8}{27} F \wedge F \wedge A$$

The non-BPS solution

- Five-dimensional minimal gauged supergravity

$$\mathcal{L} = (R+12)*1 - \tfrac{2}{3}F\wedge *F + \tfrac{8}{27}F\wedge F\wedge A$$

- *Non*-supersymmetric, *non*-extremal black hole solution

Chong, Cvetic, Lu, Pope

4 parameters $r_+ \; , \; a \; , \; b \; , \; q$

$\updownarrow$

4 independent charges $E \, , \, J_1 \, , \, J_2 \, , \, Q$

$\updownarrow$

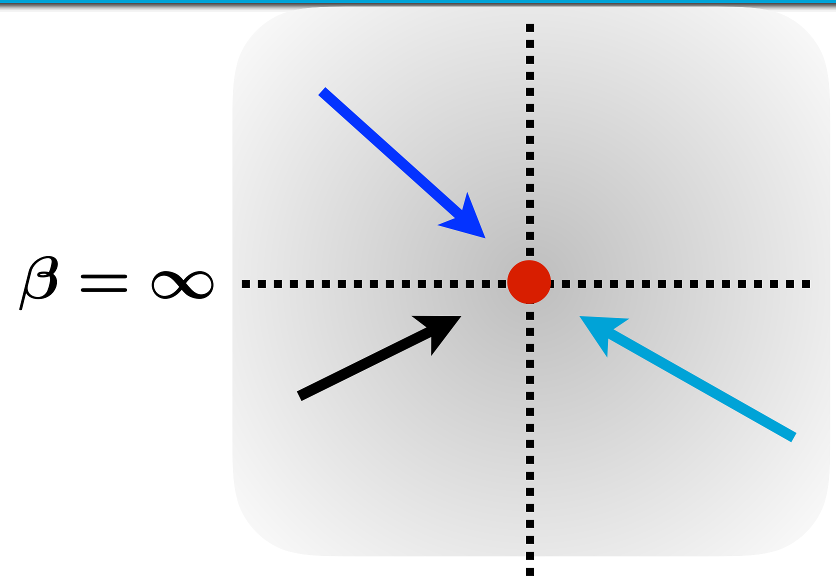
4 independent chemical pot. $\beta \, , \, \Omega_1 \, , \, \Omega_2 \, , \, \Phi$

$$I(\beta, \Omega_i, \Phi) = \beta E - S - \beta\Omega_i J_i - \beta\Phi Q$$ ✔

We want to take susy & extremal limit $\beta \to \infty$

The BPS limit

- many possible limits towards susy & extremal BH



- supersymmetry is :

$$q = -ab + (1+a+b)\, r_+^2 \pm \sqrt{-r_+^2 (r_+^2 - r_*^2)^2}$$

reality requires $r_+ = r_*$

$$r_* = \sqrt{a+b+ab}$$

susy & extremal horizon radius

➜ tune two parameters

➜ in the Lorentzian causally meaningful solution, susy implies extremality.

The BPS limit

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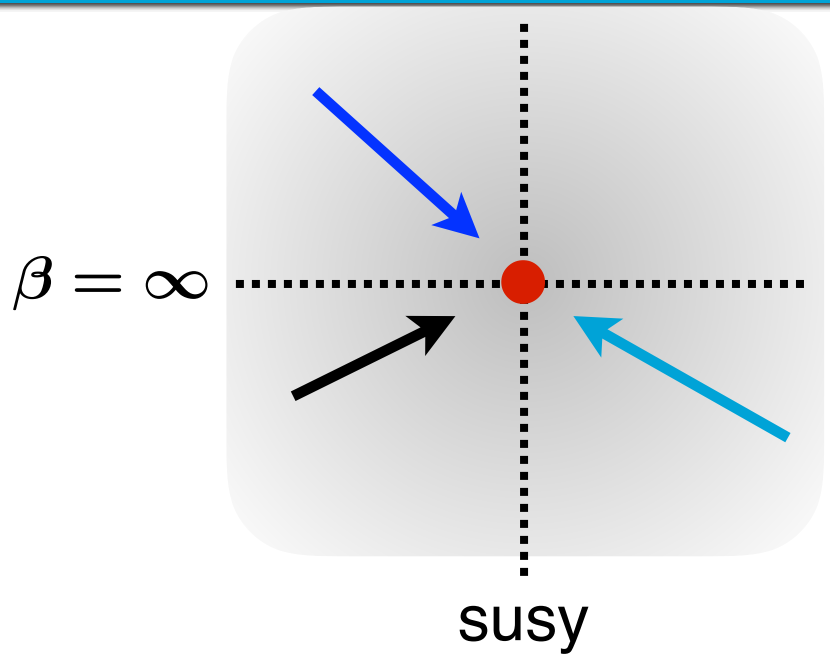

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susy & extremal horizon radius

➜ tune two parameters

➜ in the Lorentzian causally meaningful solution, susy implies extremality.

- impose susy and only later $\beta \to \infty$.

➜ allow q to be complex ➜ 3-param family of complexified, susy solutions at finite β

BPS limit of BH thermodynamics

$a\ ,\ b\ ,\ r_+$

$\updownarrow$

$J_1\ ,\ J_2\ ,\ Q$ $\qquad$ $E = J_1 + J_2 + \frac{3}{2}Q$

follows from superalgebra

$\{\mathcal{Q}, \overline{\mathcal{Q}}\} = E - J_1 - J_2 - \frac{3}{2}Q$

$\updownarrow$

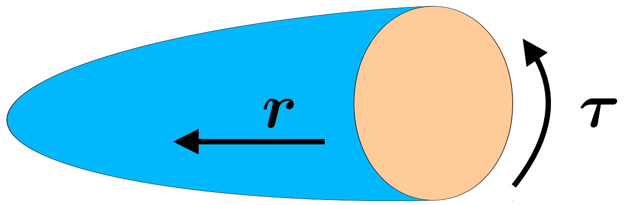
$\beta\ ,\ \Omega_1\ ,\ \Omega_2\ ,\ \Phi$ $\qquad$ $\beta\,(1 + \Omega_1 + \Omega_2 - 2\Phi) = 2\pi i$

constraint on chemical potentials

- chemical potentials are complex!

- physical meaning?

  regularity condition ensuring the Killing spinor is antiperiodic along the shrinking thermal circle



crucial that we have not taken $\beta \to \infty$ yet $\qquad$ $\infty \cdot 0 = ?$

BPS limit of BH thermodynamics

Define difference between the chemical potentials and their BPS values

$$\omega_1 = \beta(\Omega_1 - 1)\ , \qquad \omega_2 = \beta(\Omega_2 - 1)\ , \qquad \varphi = \beta(\Phi - \tfrac{3}{2})$$

Silva

These are conjugate to $J_1\ ,\ J_2\ ,\ Q$ if one takes time translations

to be generated by the susy Hamiltonian $\{\mathcal{Q}, \overline{\mathcal{Q}}\} = E - J_1 - J_2 - \frac{3}{2}Q$

(as in the index)

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(as in the index)

The constraint $\beta\,(1 + \Omega_1 + \Omega_2 - 2\Phi) = 2\pi i$ becomes:

$$\omega_1 + \omega_2 - 2\varphi = 2\pi i$$

on-shell action $I = \dfrac{16}{27}\dfrac{\varphi^3}{\omega_1\omega_2}\,c$ ➜ matches the entropy function!

BPS limit of BH thermodynamics

on-shell action $I(\omega_i, \varphi) = \frac{16}{27} \frac{\varphi^3}{\omega_1 \omega_2} c$

constraint $\omega_1 + \omega_2 - 2\varphi = 2\pi i$

using $E = J_1 + J_2 + \frac{3}{2} Q$ Quantum Statistical Relation becomes :

$$I = -S - \omega_1 J_1 - \omega_2 J_2 - \varphi Q$$

Now take extremal limit $r_+ \to r_*$

$\beta \to \infty$ but $\omega_1 , \omega_2 , \varphi$ remain finite ➜ the limit is smooth

➜ these relations define a **BPS black hole thermodynamics**

Entropy: $S = \pi \sqrt{3Q^2 - 8c(J_1 + J_2)} = \frac{\text{Area}}{4}$ upon requiring reality of Legendre transform

Other dimensions

A similar derivation holds for the other cases

DC, Papini '19, for AdS_7 also Kantor, Papageorgakis, Richmond

(explicitly proven for restricted sets of charges)

asymptotics	charges	BPS entropy	"log grand-canonical partition function"
M, AdS_4 x S^7	$J,\ Q_1, Q_2, Q_3, Q_4$	$S = S(J_i, Q_I)$	$I = -\frac{i}{2G}\frac{\sqrt{\varphi^1\varphi^2\varphi^3\varphi^4}}{\omega}$
IIB, AdS_5 x S^5	$J_1, J_2,\ Q_1, Q_2, Q_3$		$I = \frac{\pi}{4G}\frac{\varphi^1\varphi^2\varphi^3}{\omega_1\omega_2}$
IIA, AdS_6 x_w S^4	$J_1, J_2,\ \ Q$		$I = \frac{\pi\, i}{3G}\frac{\varphi^3}{\omega_1\,\omega_2}$
M, AdS_7 x S^4	$J_1, J_2, J_3,\ Q_1, Q_2$		$I = -\frac{\pi^3}{128G}\frac{\varphi_1^2\varphi_2^2}{\omega_1\omega_2\omega_3}$

complex constraint $\sum \omega_i - \sum \varphi_I = 2\pi i$

From gravity to field theory

Now that we have gained insight on the gravity side

let's see how the dual field theory computation is defined.

From gravity to field theory

AdS/CFT master equation (at large N)

$$Z_{\mathbf{gravity}} \simeq \mathrm{e}^{-I_{\mathrm{on\,shell}}} = Z_{\mathbf{CFT}}$$

AdS_5 gravity boundary conditions $\Longleftrightarrow$ CFT_4 background fields

Take $\mathcal{N} = 4$ SYM, or a Lagrangian $\mathcal{N} = 1$ SCFT, such as the conifold theory

The partition function, *Z*, can be computed as a path integral.

➜ need to specify

- background fields (ds^2 and A)
- boundary conditions on the dynamical fields

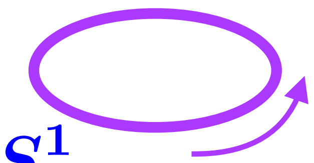
Localization computation

- in the regular Euclidean section, boundary fields are:

S^3

$$ds^2 = d\tau^2 + d\theta^2 + \sin^2\theta\,(d\phi_1 - i\,\Omega_1 d\tau)^2 + \cos^2\theta\,(d\phi_2 - i\,\Omega_2 d\tau)^2$$

→ S^3 fibered over S^1



S^1_β

$$A = i\,\Phi\,d\tau$$

complexify chemical pot, $\beta(1 + \Omega_1 + \Omega_2 - 2\Phi) = 2\pi i n\,, \quad n \in \mathbb{Z}$

→ black hole requires $n = \pm 1$

for n odd, supercharge is antiperiodic

➜ dynamical fields are: periodic bosons, antiperiodic spinors

Localization computation

- A localization computation gives the exact partition function:

$$Z(\omega_1, \omega_2, \varphi) = \mathrm{e}^{-\mathcal{F}(\omega_1, \omega_2, \varphi)} \mathcal{I}(\omega_1, \omega_2, \varphi)$$

where again $\omega_1 = \beta(\Omega_1 - 1)\ ,\qquad \omega_2 = \beta(\Omega_2 - 1)\ ,\qquad \varphi = \beta(\Phi - \frac{3}{2})$

with $\omega_1 + \omega_2 - 2\varphi = 2\pi i\, n$

The index

$$\mathcal{I}(\omega_1, \omega_2, \varphi)$$

- Can be expressed as

$$\mathcal{I}(\omega_1, \omega_2, \varphi) = \mathrm{Tr}\,(-1)^F\, \mathrm{e}^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+(\omega_1-2\pi i n)(J_1+\frac{1}{2}Q)+\omega_2(J_2+\frac{1}{2}Q)}$$

$$= \mathcal{I}(\omega_1 - 2\pi i\, n\,,\, \omega_2)$$

➜ superconformal index with a shifted chemical potential

Non-trivial as the shift is *not* an invariance of the index! Introduces extra phases

Cardy limit of the index

- limit of large charges (at finite N) : $\omega_1, \omega_2 \to 0$

  $\varphi = \frac{1}{2}(\omega_1 + \omega_2 - 2\pi i\, n)$ remains finite when $n = \pm 1$

  Choi, J. Kim, S. Kim, Nahmgoong; Honda; Arabi Ardehali;
  CCMM '19; J. Kim, S. Kim, Song; Amariti, Garozzo, Lo Monaco

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- universal saddle point controlled by anomalies.

  Dominant under some assumptions

$$-\log \mathcal{I} \sim \frac{8\varphi^3}{27\omega_1\omega_2}(5a - 3c) + \frac{8\pi^2\varphi}{3\omega_1\omega_2}(a - c)$$

$n = \pm 1$ version of
Di Pietro, Komargodski

at large N : $a = c$ ➜ matches grand can. fct. I ➜ Bekenstein-Hawking entropy

holds at finite N ➜ prediction for quantum black hole entropy !

Entropy from the index

- large N limit of the $SCFT_4$ index Benini, Milan Cabo-Bizet, Murthy

  - There is evidence that the $SCFT_4$ index counts $O(N^2)$ states for suitable complexified chemical potentials

  - the same appears to be true in other dimensions

    - e.g. $SCFT_3$

      Choi, Hwang, S. Kim;
      Bobev, Crichigno;
      Nian, Pando Zayas;
      Benini, Gang, Pando Zayas 2019.

The prefactor

localization gives $$\mathcal{F} = -\frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}\,c$$ at large N

$$\varphi = \tfrac{1}{2}(\omega_1 + \omega_2 - 2\pi i\, n)$$

- $n = 0$ ➜ $$\mathcal{F} = -\frac{2}{27}\frac{(\omega_1+\omega_2)^3}{\omega_1\omega_2}\,c$$ susy Casimir energy

  Assel, DC, Martelli '14

  ➜ Legendre transform = 0 ➜ no entropy

- $n = 1$ ➜ $$\mathcal{F} = -\frac{2}{27}\frac{(\omega_1+\omega_2-2\pi i)^3}{\omega_1\omega_2}\,c$$ ➜ matches *minus* I

➜ Legendre transform of $-\mathcal{F}$ is the Bekenstein-Hawking entropy

$$S = \pi\sqrt{3Q^2 - 8c(J_1+J_2)} = \frac{\text{Area}}{4}$$

Summary

entropy

$$S(J_i, Q) = \pi\sqrt{3Q^2 - 8c(J_1 + J_2)}$$

Legendre transform

$$I(\omega_i, \varphi) = \frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}c \qquad \omega_1 + \omega_2 - 2\varphi = 2\pi i$$

Hosseini, Hristov, Zaffaroni

BPS limit

supergravity on-shell action (at finite β)

$$I(\beta, \Omega_i, \Phi) = \beta E - S - \beta\Omega_i J_i - \beta\Phi Q$$

CCMM

SCFT partition function

$$\mathcal{Z}(\omega_1, \omega_2, n) = \mathrm{e}^{-\mathcal{F}(\omega_1,\omega_2,n)}\mathcal{I}(\omega_1, \omega_2, n)$$

variant of Casimir energy

modified index

Two open questions

- Why is I encoded both in prefactor and index?

  4d Cardy formula relating degeneracy of states to vacuum energy?

- subleading corrections to Bekenstein-Hawking entropy

  localization in supergravity?

  → New window into **quantum gravity**

thanks for your attention !