Nucleons and Deuteron electric dipole moments from Holographic QCD (and few words on Isospin breaking)

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Contents

- θ-term in QCD and EDM state of art
- Top-down Holographic QCD
- Holographic EDMs of nucleons and deuteron
- Isospin breaking: baryons as tops (work in progress!)

Strong $\theta-$term

QCD $\theta-$term Lagrangian

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2}\epsilon^{\mu_1\cdots\mu_4}\int d^4x \mathrm{tr}\left(G_{\mu_1\mu_2}G_{\mu_3\mu_4}\right) = n\theta$$

- "$\vec{E}\cdot\vec{B}$" like term$\Rightarrow \cancel{CP}$
- Boundary term $\Rightarrow$ no effect on classical physics
- Weighs topological classes in path-integral
- $e^{i\theta n}$ dependence $\Rightarrow \theta + 2k\pi = \theta$
- Axial anomaly $\Rightarrow \bar{\theta} = \theta + \arg\det M_q$

Permanent EDMs break CP

$\Downarrow$

good probes for θ

nEDM state of art

An overview on theoretical estimates

Year	Approach/model	$c_n = d_n/(\theta \cdot 10^{-16} e \cdot \text{cm})$
1979	bag model	2.7
1980	ChPT	3.6
1981	ChPT	1
1981	ChPT	5.5
1982	ChPT	20
1984	chiral bag model	3.0
1984	soft pion Skyrme model	1.2
1984	single nucleon contribution	11
1990	Skyrme model $N_f = 3$	2
1991	Skyrme model $N_f = 2$	1.4
1991	ChPT	3.3(1.8)
1991	ChPT	4.8
1992	ChPT	$-7.2, -3.9$
1999	sum rules	2.4(1.0)
2000	heavy baryon ChPT	7.5(3.2)
2004	instanton liquid	10(4)
2007	holographic "hard–wall"	1.08
2015	Lattice QCD	$-3.9(2)(9)$
2016	WSS model	1.8

Experimental upper bound: $|\mathcal{D}_N| \lesssim 3 \times 10^{-26}$e·cm:

$\boxed{\text{Sets the Strong CP-Problem: } \theta \lesssim 10^{-10}}$

DEDM state of art

Much less for the deuteron:

- No existing direct measures
- Hard to evaluate via chiral techniques ($d_n = -d_p$)
- Estimates via sum rules: $\mathcal{D}_D \sim -0.98 \times 10^{-16} \theta e \cdot \text{ cm}$
  [Lebedev et al. [Phys. Rev. D **70** (2004) 016003]]

But it's interesting:

- Storage rings allow the measure of EDMs of charged particles
- JEDI collaboration $\Rightarrow$ potential sensitivity $\sim 10^{-29} e \cdot \text{cm}$

If reached, confrontation with theoretical predictions can push farther the upper bound on θ.

Sakai-Sugimoto basics

Key elements:

- COLOR: Background 10d geometry from SUGRA (stack of N_c D4-Branes)

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

$$e^\phi = g_s\left(\frac{u}{R}\right)^{3/4} \quad , \quad F_4 = dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4 \quad , \quad f(u) = 1 - \frac{u_{KK}^3}{u^3}$$

- FLAVOR: Probe D8-Branes $\Rightarrow$ 5d $U(N_f)$ Yang-Mills/Chern-Simons theory $(\widehat{A}_\alpha, A_\alpha^a)$

$$S = -\kappa \mathrm{tr}\int d^4x dz\left[k(z)\mathcal{F}_{\mu z}^2 + \frac{1}{2}h(z)\mathcal{F}_{\mu\nu}^2\right] + \frac{N_c}{24\pi^2}\int_{\mathcal{M}_4\times R_1}\omega_5$$

	0	1	2	3	(4)	5	6	7	8	9
D4	∘	∘	∘	∘	∘					
D8-$\overline{\text{D8}}$	∘	∘	∘	∘		∘	∘	∘	∘	∘

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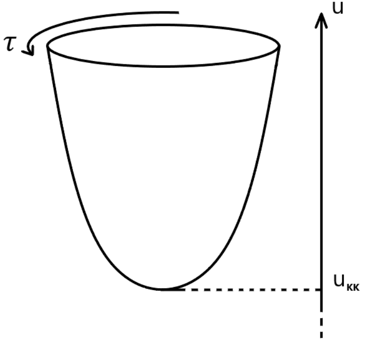
Subspace (u,τ): "Cigar"

$\Downarrow$

$u = u_{KK}$ geometry ends here

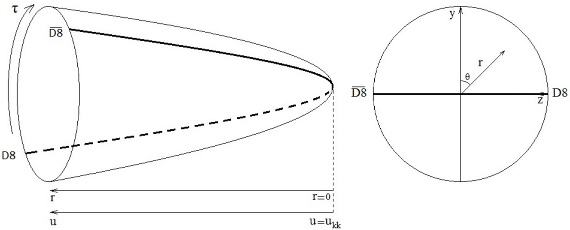


$$R_\tau = \frac{4\pi}{3}\frac{R^{3/2}}{u_{KK}^{1/2}} \Rightarrow M_{KK} = \frac{3}{2}\frac{u_{KK}^{1/2}}{R^{3/2}}$$

Mass scale of the theory (glueballs)

Sakai-Sugimoto basics

The flavor Branes join in the bulk $\Rightarrow \chi-$SSB



The D8-Gauge theory is a theory of mesons: profile in holographic direction encodes the meson content

$$\mathcal{A}_\mu = \sum v_\mu^{2n-1}(x)\psi_{2n-1}(z) + \sum a_\mu^{2n}(x)\psi_{2n}(z) \quad ; \quad \mathcal{A}_z = \Pi(x)\phi_0(z)$$

$$\underbrace{\phi_0(z) = \frac{1}{\sqrt{\pi\kappa}}\frac{1}{k(z)}}_{\text{Massless Pion}} \quad ; \quad \underbrace{-h(z)^{-1}\partial_z\left(k(z)\partial_z\psi_n\right) = \lambda_n\psi_n}_{\text{Vector mesons tower}(m_n)\sim\sqrt{\lambda_n}}$$

Restrict to pions and $S_{YM} \Rightarrow$ Skyrme model as a low energy limit

Sakai-Sugimoto basics

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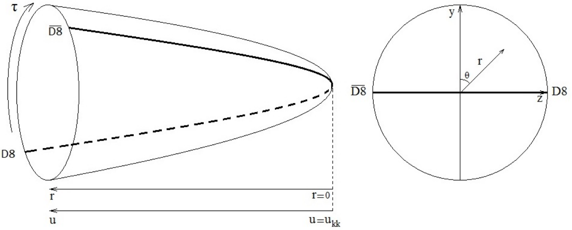

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$$S_{AK} = c\int d^4x \mathrm{tr}\left[Me^{i\varphi}U + h.c.\right]; \quad \varphi \equiv -\int dz\widehat{A}_z; \quad U = e^{-i\int dzA_z}$$

Quark mass action reproduces pion mass potential

Holographic θ-term

RR Form C_1 couples to the color branes ($G_{\mu\nu}$ = D4 field strength):

$$S_{C_1-D4} = \frac{(2\pi\alpha')^2}{2!}\mu_4 \mathrm{tr}\int_{\mathcal{M}_4\times S^1} C_1 \wedge G \wedge G$$

$$\Downarrow$$

$$\boxed{\frac{1}{l_s}\int_{S^1_\tau|_{UV}} C_1 = \theta}$$

F_8 $(\star F_2)$ couples to the flavor D8: nontrivial shift under $\delta_\Lambda \widehat{A}_z$

$$S_{C_7-D8} = \frac{1}{2\pi}\int C_7 \wedge \mathrm{tr}\mathcal{F}\wedge\omega_y \Rightarrow \delta_\Lambda C_1 = \ell_s N_f \Lambda \omega_y$$

This mechanism reproduces ($\omega_y = \delta(y)dy$ for antipodal branes)

Axial shift

$$\theta \to \theta + N_f \int \mathrm{d}\Lambda\wedge\omega_y = \theta + 2\alpha N_f$$

Quark mass phase

Choose Λ to cancel C_1:

$$Me^{-\frac{i}{2}\int dz\widehat{A}_z} \to Me^{i\frac{\theta}{N_f}} e^{-\frac{i}{2}\int dz\widehat{A}_z}$$

Baryons as Solitons

$\boxed{\text{Leading } (\lambda, N_c) \text{ order: BPST Instanton+Electric field}}$

$$A_M = -if(\xi)g\partial_M g^{-1} \quad ; \quad \widehat{A}_0 = \frac{N_c}{8\pi^2\kappa}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\xi^2+\rho^2)^2}\right]$$

$$M = i, z \quad ; \quad \xi^2 = |\vec{x} - \vec{X}|^2 + (z - Z)^2$$

Real moduli

- $\vec{X}$ ⇒Position in flat "ordinary" R^3
- $\mathbf{a} \in SU(2)$ ⇒ Orientation in (Iso-)space

Pseudo-moduli

- Z ⇒Position in bulk (broken by curvature)
- ρ ⇒Size (broken by curvature and electric field)

Turn on time derivatives:

$$\widehat{A}_z = -\frac{N_c}{16\pi^2\kappa}\frac{\rho^2}{(\xi^2+\rho^2)^2}\vec{\chi}\cdot\vec{x} \quad ; \quad \widehat{A}_i = -\frac{N_c}{16\pi^2\kappa}\frac{\rho^2}{(\xi^2+\rho^2)^2}\epsilon^{iab}\chi^a x^b$$

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Baryons as Solitons

Promote moduli to operators: moduli space quantization

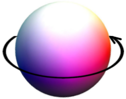
Single set $\mathbf{a}(t)$ to describe (iso-)orientation

$$\Downarrow$$

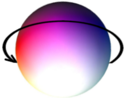
only $J = I$ states in spectrum

$\boxed{\text{"Skyrme-like" picture}}$

Proton

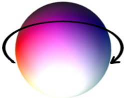


$|p\uparrow\rangle \propto (a_1 + ia_2)$

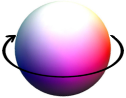


$|p\downarrow\rangle \propto (a_4 - ia_3)$

Neutron



$|n\uparrow\rangle \propto (a_4 + ia_3)$



$|n\downarrow\rangle \propto (a_1 - ia_2)$

[soliton picture taken from C. J. Halcrow, N. S. Manton and J. I. Rawlinson, Phys. Rev. C **97** (2018) no.3, 034307 doi:10.1103/PhysRevC.97.034307 [arXiv:1712.07786 [hep-th]].

Holographic nucleon EDM

Holographic EM current:

$$J_{EM} = \mathrm{tr}\left(J_V^\mu \tau^3\right) + \frac{1}{N_c}\widehat{J}_V^\mu \quad ; \quad \mathcal{J}_V^\mu = \kappa\left[k(z)\mathcal{F}^{z\mu}\right]_{z=-\infty}^{z=+\infty}$$

Electric dipole moment of a baryon state $|B\rangle$:

$$\mathcal{D}_B^i \equiv \langle B| \int d^3x x^i J_{EM}^0 |B\rangle = \mathcal{D}_B \langle B|\sigma^i|B\rangle$$

Charge operator breaks Isospin symmetry, different EDMs for p, n:

$$\mathcal{D}_N = d_N + \Delta_N \quad ; \quad d_n = -d_p \quad ; \quad \Delta_n = \Delta_p$$

Perturbative approach: $\theta \longrightarrow \delta\mathcal{A}$

$$\mathcal{A}^{full} = \mathcal{A}^{bar} + \delta\mathcal{A}(m\theta)$$

$$\Downarrow$$

Turn on $\delta J_{EM}^0 \Rightarrow \mathcal{D}_N \neq 0$

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Holographic nucleon EDM

Equations of motion for perturbations:

$$\boxed{\hat{z}} \quad \kappa k(z)\partial_\mu \delta \widehat{F}^{z\mu} = -cm\theta(\cos\alpha + 1)\cos\varphi$$

$$\boxed{\hat{i}} \quad \kappa\left[h(z)\partial_j \delta\widehat{F}^{ij} + \partial_z\left(k(z)\delta\widehat{F}^{iz}\right)\right] = 0$$

$$\boxed{\hat{0}} \quad \kappa\left[h(z)\partial_\nu \delta\widehat{F}^{0\nu} + \partial_z\left(k(z)\delta\widehat{F}^{0z}\right)\right] +$$

$$+\frac{N_c}{32\pi^2}\epsilon^{ijk}\left(\delta F^a_{ij}F^a_{kz} + F^a_{ij}\delta F^a_{kz} + \delta\widehat{F}_{ij}\widehat{F}_{kz} + \widehat{F}_{ij}\delta\widehat{F}_{kz}\right) = 0$$

$$\boxed{z} \quad \kappa k(z)\delta\left[D_\nu F^{z\nu}\right] = \frac{cm\theta}{2}\sin\alpha\sin\varphi\frac{x^k}{r}\mathbf{a}\tau^k\mathbf{a}^{-1}$$

$$\boxed{i} \quad \kappa\left[h(z)\delta\left(D_\nu F^{i\nu}\right) + \delta\left(D_z\left(k(z)F^{iz}\right)\right)\right] = 0$$

$$\boxed{0} \quad \kappa\left[h(z)\delta\left(D_i F^{0i}\right) + \delta\left(D_z\left(k(z)D_z F^{0z}\right)\right)\right]^a$$

$$+\frac{N_c}{32\pi^2}\epsilon^{ijk}F^a_{ij}\delta\widehat{F}_{kz} = 0$$

Holographic nucleon EDM

Neglect time derivatives of moduli:

$$\boxed{\hat{z}} \quad \kappa k(z)\partial_\mu \delta \widehat{F}^{z\mu} = -cm\theta(\cos\alpha + 1)\cancelto{1}{\cos\varphi}$$

$$\boxed{\hat{i}} \quad \kappa \left[h(z)\partial_j \delta \widehat{F}^{ij} + \partial_z \left(k(z)\delta \widehat{F}^{iz} \right) \right] = 0$$

$$\boxed{\hat{0}} \quad \kappa \left[h(z)\partial_\nu \delta \widehat{F}^{0\nu} + \partial_z \left(k(z)\delta \widehat{F}^{0z} \right) \right] + \\ + \frac{N_c}{32\pi^2}\epsilon^{ijk} \left(\delta F^a_{ij} F^a_{kz} + F^a_{ij}\delta F^a_{kz} + \delta \widehat{F}_{ij}\widehat{F}_{kz} + \widehat{F}_{ij}\delta \widehat{F}_{kz} \right) = 0$$

$$\boxed{z} \quad \kappa k(z)\delta \left[D_\nu F^{z\nu} \right] = \frac{cm\theta}{2} \sin\alpha \cancelto{0}{\sin\varphi} \frac{x^k}{r} \mathbf{a}\tau^k \mathbf{a}^{-1}$$

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$$\boxed{0} \quad \kappa \left[h(z)\delta \left(D_i F^{0i} \right) + \delta \left(D_z \left(k(z) D_z F^{0z} \right) \right) \right]^a \\ + \frac{N_c}{32\pi^2}\epsilon^{ijk} F^a_{ij}\delta \widehat{F}_{kz} = 0$$

Holographic nucleon EDM

Turn on linear order in angular velocity $\vec{\chi} = -i\,\mathrm{tr}(\mathbf{a}^{-1}\dot{\mathbf{a}}\vec{\tau})$:

$$\boxed{\hat{z}} \quad \kappa k(z)\partial_\mu \delta\widehat{F}^{z\mu} = -cm\theta(\cos\alpha + 1)\cos\varphi$$

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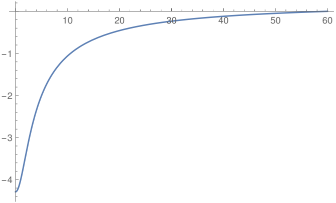
$$+\frac{N_c}{32\pi^2}\epsilon^{ijk}F^a_{ij}\delta\widehat{F}_{kz} = 0$$

Isovector nucleon EDM

Neglect time derivatives ($\sim N_c^{-1}$)$\Rightarrow$ Consistent to set $\delta A^a_\alpha = 0$



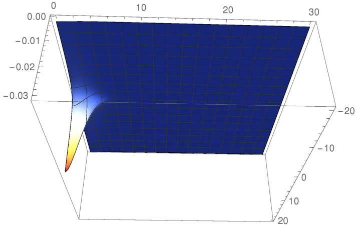
How perturbations arise:

- Quark mass term sources

$$\delta\widehat{A}_z = \frac{cm\theta}{\kappa}\frac{u(r)}{k(z)} \quad ; \delta\widehat{A}_i = 0$$

- $\delta\widehat{A}_z$ via Chern-Simons sources

$$\delta A^0 = 27\pi\frac{cm\theta}{\lambda\kappa}W(r,z)\mathbf{a}(\vec{x}\cdot\vec{\tau})\mathbf{a}^{-1}$$



$$\mathcal{D}^i_N = -\frac{4\pi}{3}\int drr^4\kappa\left[k(z)\partial_z W\right]^{+\infty}_{-\infty}\langle \mathrm{tr}\left(\tau^3\mathbf{a}\tau^i\mathbf{a}^{-1}\right)\rangle_N = -d_n\langle\tau^3\rangle_N\langle\sigma^i\rangle$$

Isovector nucleon EDM

Neglect time derivatives ($\sim N_c^{-1}$)$\Rightarrow$ Consistent to set $\delta A_\alpha^a = 0$

How perturbations arise:

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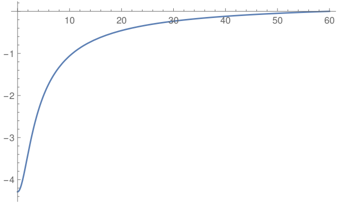
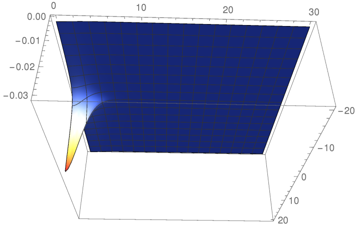




Extrapolation to phenomenological values of $\kappa, N_c, M_{KK}, m_\pi$

$$d_n = 1.8 \times 10^{-16}\theta e \cdot \text{ cm}$$

Isoscalar nucleon EDM

Turn on time derivatives$\Rightarrow$ $\vec{\chi}$ appears in $\int dz \widehat{A}_z^{bg}$
δA_0 in YM and $\widehat{A}_z^{bg}$ in quark mass provide sources for δA_M
Non abelian fields are wildly coupled: no easy ansatz this time...

Same mechanism, but Abelian$\leftrightarrow$ Non Abelian:

1. Find all possible structures $\propto \vec{\chi}$
   - 3 scalar structures for δA_z
   - 8 vector structures for δA_i
2. Write down and solve the set of 11 coupled equations
3. Plug solutions in Chern-Simons term to source $\delta \widehat{A}^0$
   - Only 1 possible scalar structure $(\hat{r} \cdot \vec{\chi})$
4. Solve and plug in $\widehat{J}_V^0 \Rightarrow$ profit

Isoscalar nucleon EDM

Consistent ansatze that factorize away $\mathbf{a}(t)$ dependence:

$$\begin{aligned}\delta A_z = K\mathbf{a}\,[\beta(r,z)\,(\hat{r}\cdot\vec{\chi})\,(\hat{r}\cdot\vec{\tau}) + \\ + \gamma(r,z)\,(\vec{\chi}\cdot\vec{\tau}) + \\ + \delta(r,z)\epsilon^{abc}\chi^a\hat{r}^b\tau^c +]\,\mathbf{a}^{-1}\end{aligned}$$

$$\begin{aligned}\delta A_i = K\mathbf{a}\,\big[B(r,z)\chi^i\,(\hat{r}\cdot\vec{\tau}) + \\ + C(r,z)\,(\hat{r}\cdot\vec{\chi})\,\tau^i + \\ + D(r,z)\hat{r}^i\,(\hat{r}\cdot\vec{\tau}) + \\ + E(r,z)\epsilon^{iab}\chi^a\tau^b + \\ + F(r,z)\hat{r}^i\,(\hat{r}\cdot\vec{\chi})\,(\hat{r}\cdot\vec{\tau}) + \\ + G(r,z)\hat{r}^i\epsilon^{abc}\chi^a\hat{r}^b\tau^c + \\ + H(r,z)\epsilon^{iab}\chi^a\hat{r}^b\,(\hat{r}\cdot\vec{\tau}) + \\ + I(r,z)\,(\hat{r}\cdot\vec{\chi})\,\epsilon^{iab}\hat{r}^a\tau^b\big]\,\mathbf{a}^{-1}\end{aligned}$$

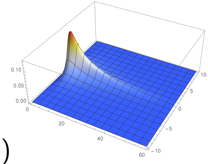
$$\delta\widehat{A}^0 = \Upsilon\mathcal{M}(r,z)\,(\hat{r}\cdot\vec{\chi})$$

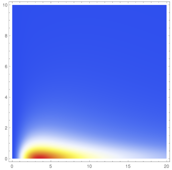
Preliminary results

Approximation: only turn on δA_z (hint from numerical results)

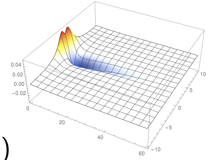
$$\Downarrow$$

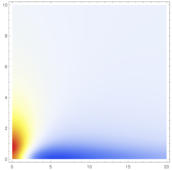
Set is differential in r, algebraic in z: easy numerical task



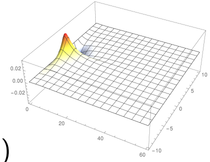


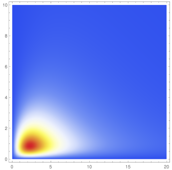
$\beta(r,z)$





$\gamma(r,z)$





$\delta(r,z)$

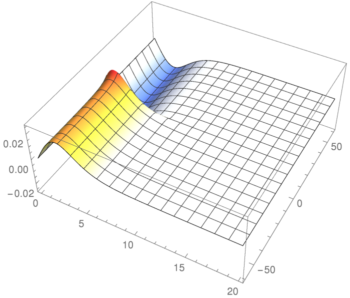
Preliminary results

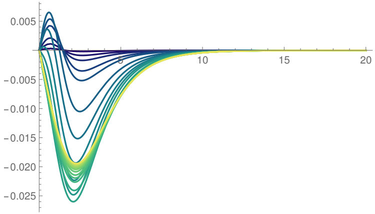
EDM formula:

$$\Delta_N{}^i = \frac{cm\theta}{192\pi^3\kappa^2}\sqrt{\frac{5}{6}}\int drr^3\left[k(z)\partial_z\mathcal{M}\right]_{-\infty}^{+\infty}\langle\sigma^i\rangle$$

Integrand function:

$$\left[k(z)\partial_z\mathcal{M}(r,z)\right]_{-\infty}^{+\infty}$$



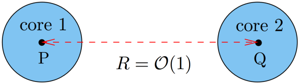


Extrapolation to phenomenological values of $\kappa, N_c, M_{KK}, m_\pi$

$$\boxed{\Delta_N = -0.4\times 10^{-16}\theta\mathrm{e}\cdot\mathrm{cm}}$$

Holographic Deuteron

Holographic approach to nuclear physics
[arXiv:1703.08695 [hep-th]] (Baldino, Bolognesi, Gudnason, Koksal)

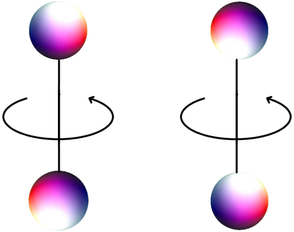


Tail of each soliton "feels" the core of the other.

- Separation $R \sim \mathcal{O}(1)$
- Single soliton size $R \sim \mathcal{O}(\lambda^{-1/2})$

Relevant properties

- Phase locked configuration
- Moduli can be expressed through the single-soliton ($\mathbf{b}$,$\mathbf{c}$)
- Only S-wave component (D-wave suppressed with λ due to inertia)



$$\psi_D = \frac{1}{\pi^2}(b_4 c_3 - c_3 b_4 + b_1 c_2 - b_2 c_1)$$

Deuteron EDM

Configuration with which to build current:

$$\delta\mathcal{A} = B\delta\mathcal{A}^{cl}_{(p)}(\vec{x} + \frac{\vec{R}}{2}, z)B^\dagger + C\delta\mathcal{A}^{cl}_{(q)}(\vec{x} - \frac{\vec{R}}{2}, z)C^\dagger$$

Plug in EDM formula

$$\mathcal{D}^i_D = \int d^3x x^i \langle D|\mathrm{tr}\left(\delta J^0_V \tau^3\right) + \frac{1}{N_c}\delta\widehat{J}^0_V|D\rangle = \left(d_D + \Delta_D\right)\langle D|\sigma^i|D\rangle$$

Mixed terms vanish because of symmetry, others because of $|D\rangle$

$$\langle D|\delta J_{(p,q)}|D\rangle = 0$$

$$\mathrm{tr}\left(\delta J^0_V \tau^3\right) = \cancel{\delta J^0_{(p)}} + \cancel{\delta J^0_{(q)}} + iK\kappa\Big[\cancel{k(z)\left[\delta A^0_{(p)}, A^z_{(q)}\right] + (p \leftrightarrow q)}\Big]^{+\infty}_{-\infty} \to 0$$

Abelian part is trivially

$$\frac{1}{N_c}\delta\widehat{J}^0_V = \Upsilon\frac{\kappa}{N_c}\left[k(z)\partial_z\mathcal{M}^{(1)}(\hat{r}_1\cdot\vec{\chi}^{(1)}) + k(z)\partial_z\mathcal{M}^{(2)}(\hat{r}_2\cdot\vec{\chi}^{(2)})\right]^{+\infty}_{-\infty}$$

We end up with $\mathcal{D}_D = 2\Delta_N \simeq -0.8\times 10^{-16}\theta\mathrm{e}\cdot\mathrm{cm}$

What are we missing?

We found $\mathcal{D}_D = \mathcal{D}_p + \mathcal{D}_n$
Expected both $1-$ and $2-$bodies contributions:

$$\mathcal{D}_D = \mathcal{D}_p + \mathcal{D}_n + \mathcal{D}_D^{(2)}$$

About $\mathcal{D}_D^{(2)}$ [Liu et al.[nucl-th/0408060]]:

$$\mathcal{L}_{\pi NN} = \bar{g}_{\pi NN}^{(0)} \bar{N} \vec{\pi} \cdot \vec{\tau} N + \bar{g}_{\pi NN}^{(1)} \bar{N} \pi^0 N$$

- Can be decomposed in $\mathcal{D}_D^{pol} + \mathcal{D}_D^{exc}$
- $\mathcal{D}_D^{pol}$ from $\bar{g}_{\pi NN}^{(1)}$ (expected to dominate)
- $\mathcal{D}_D^{exc}$ from both $\bar{g}_{\pi NN}^{(0,1)}$

Why we miss it:

- P-wave component $|\widetilde{D}\rangle = |L=1\rangle$ needed for $\mathcal{D}_D^{pol}$
- $\mathcal{D}_D^{exc} \propto \bar{g}_{\pi NN}^{(0)}$ should arise from $\theta-$perturbation of linear tails: investigate soon

Anyway θ enters only $\bar{g}_{\pi NN}^{(0)}$: miss only $\mathcal{D}_D^{exc} \propto \bar{g}_{\pi NN}^{(0)}$

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Isospin breaking and spinning tops

Introduce quark mass difference (Isospin explicitly broken)

$$M = m\mathbb{I} + \epsilon m \tau^3$$

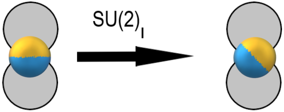
Baryon is perturbed

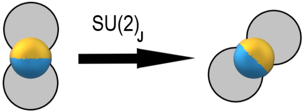
[Bigazzi, Niro [arXiv:1803.05202 [hep-th]]]:

- Mass splitting of nucleons $\Delta M \propto \epsilon m I_3$
- $\delta \widehat{A}_z = cm\epsilon \frac{b(r)}{k(z)} \frac{x^j}{2r} \mathrm{tr}\left(\tau^3 \mathbf{a} \tau^j \mathbf{a}^{-1}\right)$

  $\delta \widehat{A}_z$ (and others δA_z^a) makes Iso-rotations not allowed
- Iso-rotation: does not send a solution in another
- Rotation: still a zero mode: sends solution into another

$\delta \widehat{A}_z = \eta'$ cloud with cylindrical symmetry locked in the τ^3-oriented direction ζ





Isospin breaking and spinning tops

$\delta\widehat{A}_z, \delta A_z$ in $S_{YM} \Rightarrow$ splitting of moment of inertia

$$L_{YM} = \frac{1}{2} I_A \left(\chi_\xi^2 + \chi_\eta^2\right) + \frac{1}{2} I_C \chi_\zeta^2 - M_0 \quad ; \quad I_{A,C} = I_{(0)} + \epsilon^2 \Delta I_{A,C}$$

Mass term in effective Lagrangian produces linear term:

$$L_{AK} = \frac{\epsilon m c N_c}{12\kappa} \rho^3 R_{3i} \chi^i \mathcal{J}_2 \quad ; \quad R_{3i} \equiv \frac{1}{2} \mathrm{tr}\left(\tau^3 \mathbf{a} \tau^j \mathbf{a}^{-1}\right)$$

this modifies momentum $J_i = I_{ij}\chi_j + \frac{\epsilon m c N_c}{12\kappa} \rho^3 R_{3i} \mathcal{J}_2 \equiv I_{ij}\chi_j - K_i$

Modified Hamiltonian: symmetric top with linear term

$$H = \frac{1}{2I_A}\left(J_\xi^2 + J_\eta^2\right) + \frac{1}{2I_C}\left(J_\zeta^2 + K_\zeta^2 + 2J_\zeta K_\zeta\right) + M_0$$

J_ζ identified with $I_3 \Rightarrow$ shifts and splittings in Isospin multiplets

$$E = \underbrace{M_0 + \frac{1}{2I_A} j(j+1)}_{\text{unperturbed}+\epsilon^2} + \underbrace{\frac{1}{2}\left(\frac{1}{I_C} - \frac{1}{I_A}\right) i_3^2}_{\sim\epsilon^2} - \underbrace{\frac{1}{I_C} K_\zeta i_3}_{\sim\epsilon} + \underbrace{\frac{1}{2I_C} K_\zeta^2}_{\sim\epsilon^2}$$

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Conclusions and what's next

Wrapping up our results at this point

- We derived full equations for baryon $\theta-$induced perturbations.
- We used a simpler set to succesfully compute the missing isoscalar part of the nucleon EDMs and estimate the deuteron EDM.
- We extracted an effective Hamiltonian that accounts for strong splittings in masses isospin multiplets.

What's next:

- Find full $\theta-$induced perturbations and refine estimate for the EDMs.
- Investigate two-bodies contributions.
- Make the "symmetric top" model quantitative including non-Abelian perturbations.

Thanks for your attention