Emitted radiation and geometry

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Theories of Fundamental Interactions 2019

Based on [1910.06332] with L. Bianchi, M. Billò, A. Lerda

Emitted energy by a charged particle

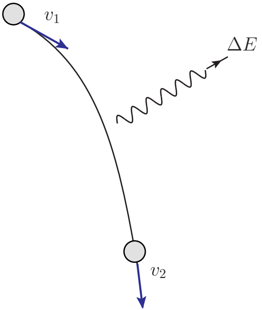
- Larmor formula for electrodynamics

$$\Delta E = 2\pi B \int dt(\dot{v})^2$$

- B = Bremsstrahlung function
- Problem modeled by a **Wilson line operator**

$$W(C) = e^{\int_C A}$$

- Bremsstrahlung = Wilson line deformations



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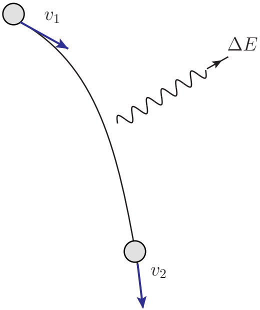
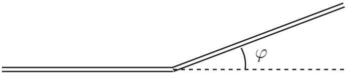


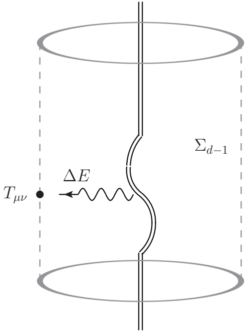
Example: Cusped Wilson loop



$$\langle W_{\text{Cusp}} \rangle \sim e^{-\Gamma_{\text{Cusp}} \log \frac{\Lambda}{\epsilon}}, \qquad \Gamma_{\text{Cusp}} = -B\,\varphi^2 + O(\varphi^4)$$

Measuring the radiated energy

- Idea[1]: ΔE proportional to the integrated energy flux h_W through a $(d-1)$–dim surface around the line.
- h_W is the coefficient defining the stress tensor $T_{\mu\nu}$.
- Knowing h_W and computing the energy flux integral, one obtains an expression for the Bremsstrahlung B.



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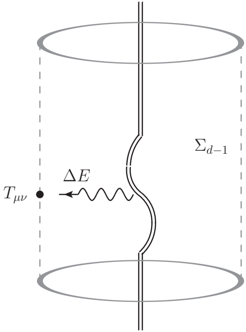

Example (Maxwell):

$$h_W = \frac{e^2}{32\pi^2}\,, \qquad B = \frac{e^2}{12\pi^2}\,, \qquad B = \frac{8}{3}h_W$$

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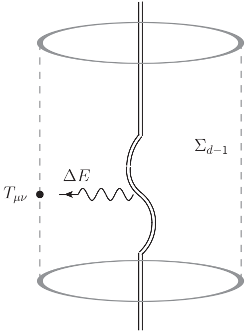

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Generalizations?

- B and h_W have **no universal relation**.
- Difficulties in **computing** h_W for non abelian cases
- Super Conformal Field Theories (SCFTs)

[1] [Boulware,1980], [Lewkowycz, Maldacena, 2013]

Action of Conformal symmetry

Residual Conformal symmetry

$$SO(1, d+1) \to \; SO(1,2) \times SO(d-1)$$

fixes the 1-pt function of local operators

$$\langle T_{00} \rangle_W \equiv \frac{\langle T_{00}\, W \rangle}{\langle W \rangle} = \frac{h_W}{r^4}\,, \quad \langle T_{0i} \rangle_W = 0$$

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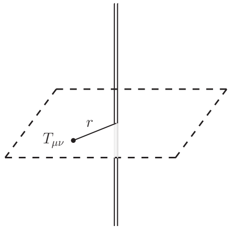

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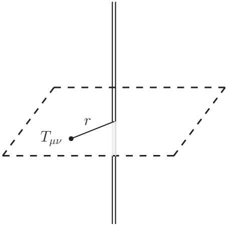


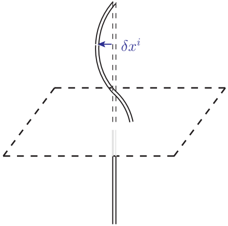
Conformal preserving deformations of the defect

$$\delta \langle W \rangle = -\int d\tau\, \delta x^i(\tau) \langle D_i(\tau)\rangle_W$$

have non trivial 2-pt function

$$\left\langle D_i(\tau) D_j(0) \right\rangle_W = 12\, B\, \frac{\delta_{ij}}{\tau^4}$$

and define the Bremsstrahlung function[2]



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4-dim maximally supersymmetric case $\mathcal{N} = 4$

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$$B = 3h_W \ .$$

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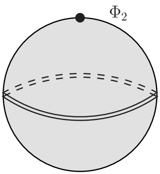
$$\langle \Phi_2 \rangle_W \Big|_{S^4} = h_W = \frac{1}{12\pi^2}\, g \frac{\partial}{\partial g} \log \langle W \rangle_{S^4}$$



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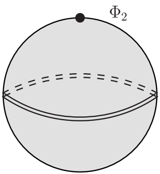

- Hence in $\mathcal{N} = 4$: • Well defined **relation between** B **and** h_W
  - h_W is an **exact function** in g.

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4-dim $\mathcal{N} = 2$ SCFTs

- **Relation between B and** h_W:
  Using superconformal Ward identities, ([5]) were able to prove that

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- **Conjecture** ([6]):
  Insertion of a stress tensor $\leftrightarrow$ small variation of the action w.r.t. the geometry
  $$h_W = \frac{1}{12\pi^2}\partial_b \log \langle W \rangle_b \Big|_{b=1} \tag{1}$$
  where W is placed on a squashed sphere with squashing parameter b.

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- **Our goal**: proof of (1).

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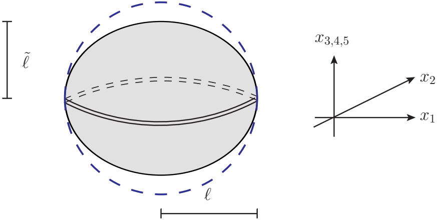
Ellipsoid geometry[7]

$$\frac{x_1^2 + x_2^2}{\ell^2} + \frac{x_3^2 + x_4^2}{\tilde{\ell}^2} + \frac{x_5^2}{r^2} = 1 .$$



$$\begin{aligned} x_1 &= \ell \sin\rho \cos\theta \cos\varphi , \\ x_2 &= \ell \sin\rho \cos\theta \sin\varphi , \\ x_3 &= \tilde{\ell} \sin\rho \sin\theta \cos\chi , \\ x_4 &= \tilde{\ell} \sin\rho \sin\theta \sin\chi , \\ x_5 &= r \cos\rho , \end{aligned}$$

- When $\ell = \tilde{\ell} = r$, the ellipsoid becomes a round sphere S^4.
- Squashing parameter $b = \sqrt{\ell/\tilde{\ell}}$
- Circular Wilson loop inserted on the maximal circle along (12) direction.
- For simplicity, in this talk we choose $\ell = r$ such that $\partial_b W = 0$ (in general our derivation does not depend on the parametrization)

[7] [Hama, Hosomichi, 2012]

Relating h_W to ellipsoid deformation: naive idea

If we try to perform the direct computation

$$\left\langle W \right\rangle_b = \frac{1}{Z_b} \int [\mathcal{D}A] \, \mathrm{e}^{-S_b} \, W \, ,$$

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We evaluate $\partial_b S_b$ on the ellipsoid ($\xi^\mu = (\rho, \theta, \varphi, \chi)$):

$$\partial_b S_b = \int d^4\xi \, \partial_b(\sqrt{g}\mathcal{L}) = \int d^4\xi \frac{\partial(\sqrt{g}\mathcal{L})}{\partial g^{\mu\nu}} \partial_b g^{\mu\nu} = \int d^4\xi \left(-\frac{\sqrt{g}}{2} T_{\mu\nu} \right) \partial_b g^{\mu\nu}$$

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- $\left\langle T_{\mu\nu} \right\rangle_W$ is fixed by conformal symmetry
- $\partial_b g^{\mu\nu}$ is known from the ellipsoid embedding in $\mathbb{R}^5$
- **But!** The dependence on the deformation is not only through the metric: we need a well defined supersymmetric theory on a curved space.

$\mathcal{N} = 2$ supersymmetry on the ellipsoid

- We exploit previous results to build rigid supersymmetry on a curved space, following the approach of ([8]).
- $\mathcal{N} = 2$ Super Yang Mills theory is coupled to the full gravity supermultiplet

$$g_{\mu\nu}\,, \quad \psi_\mu^{\mathcal{I}}\,, \quad k_{\mu\nu}\,, \quad \bar{k}_{\mu\nu}\,, \quad V_\mu^0\,, \quad (V_\mu)^{\mathcal{I}}_{\mathcal{J}}\,, \quad \eta^{\mathcal{I}}\,, \quad M\,,$$

metric	gravitino	(anti) self-dual tensors	R-symmetry gauge fields	dilatino	scalar

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- For example in this parametrization:

$$\partial_b M\Big|_{b=1} = \frac{3\cos(2(\theta-\rho)) + 3\cos(2(\theta+\rho)) + 2\cos(2\theta) - 6\cos(2\rho) - 10}{2r^2}$$

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Stress tensor supermultiplet one-point functions

- We build the stress tensor multiplet consistently with SUSY transformations

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$$\downarrow \frac{\delta \mathcal{L}}{\delta \bullet}$$

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stress tensor — SUSY current — (anti) self-dual operators — R-symmetry currents — fermion — scalar superprimary

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- Defect CFT fixes the one-point functions in presence of a line defect ([10]).
  - Fixed kynematics (distance in terms of ellipsoid coordinates).
  - Unique coefficient h_W for the whole multiplet (Superconformal Ward identities).
  - Non vanishing 1 pt functions: $\left\langle T_{\mu\nu}\right\rangle_W\,,\ \left\langle H_{\mu\nu}\right\rangle_W\,,\ \left\langle \bar{H}_{\mu\nu}\right\rangle_W\,,\ \langle O_2\rangle_W\,.$

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- For example:

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Relating h_W to ellipsoid deformation: full computation

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Relating h_W to ellipsoid deformation: full computation

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- The conjecture is now fully proven.
- For any $\mathcal{N} = 2$ SCFT the Bremsstrahlung function ($B = 3h_W$)

$$B = \frac{1}{4\pi^2} \partial_b \log \Big\langle W \Big\rangle \Big|_{b=1}$$

Perturbation theory computations

- **Change of perspective**:
  - Proven conjecture $\Rightarrow$ predictions in field theory.
  - $\mathcal{N} = 2$ $\langle W \rangle_b$ still localizes ([11]) on a matrix model.
  - Ellipsoid matrix model provides a nice tool to infer diagrammatic behaviours.

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- After a rescaling, we write (in 0-instanton sector)

$$\left\langle \mathcal{W} \right\rangle_b = \frac{1}{Z_b} \int da \, \mathrm{e}^{-\mathrm{tr}\, a^2} \left| Z_b^{\text{1-loop}} \right|^2 \left(\frac{1}{N} \mathrm{tr}\, e^{\frac{b\, g}{\sqrt{2}}\, a} \right) .$$

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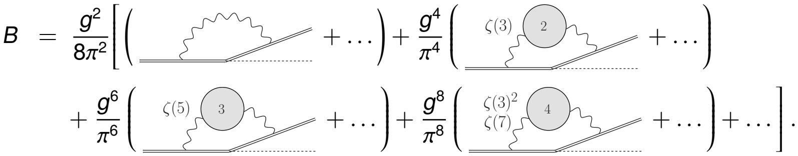
- $\mathcal{N}=2$ SCFT:

$$\left| Z_b^{\text{1-loop}} \right|^2 = e^{-S_{\mathrm{int}}(a,g)}$$

no longer exact results, but we can obtain expansions in transcendentality

[11] [Hama, Hosomichi, 2012]

Perturbative computation the Bremsstrahlung function ([12])



[12] [Andree, Young, 2010], [Gomez, Mauri, Penati, 2018]

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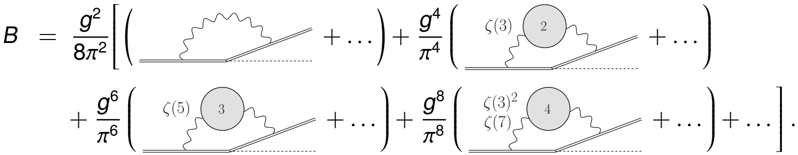
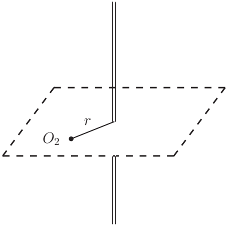


What about h_W?

- h_W is related to the 1-pt function of a **non-chiral operator**
  $$O_2 \sim \mathrm{Tr}\,\phi\bar{\phi} + \text{hypers}$$
- the expansion of B provides non-trivial suggestions for higher loop diagrams of $\langle O_2 \rangle_W$



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Resume and final comments

- We proved a conjecture which relates the emitted radiation of an accelerated particle to a deformation of the geometry, for any $\mathcal{N} = 2$ Lagrangian SCFT on a 4-dim squashed sphere.
- Derivation uses general properties of background geometry and Defect CFTs data.

Future directions:

- Insights on non-protected observables.
- Integrability structure of transcendentality expansion ([14]).
- In principle we have a recipe to extract $\left\langle T_{\mu\nu} \right\rangle_D$ for any superconformal (line) defect by perturbing the geometry.

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THANK YOU!

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