Anti-D6-brane singularities and resolution

Johan Blåbäck

Università di Roma
"Tor Vergata"

Primarily based on:

arXiv:
Anti-brane singularities as red herrings
1907.05295

with:
F. F Gautason, A. Ruipérez, T. Van Riet

@ Theories of the Fundamental Interactions
2019-10-21

Introduction – What is brane-flux annihilation?

What is brane-flux annihilation?

If we were to place a localised brane charge in a background of distributed charge, coming from fluxes, of opposite sign

$$\begin{aligned} \mathrm{d}F_5 &= H_3 \wedge F_3 - |Q|\delta(x - x_{\bar{\text{D}}3}) \\ &= e^{-\phi}|H_3|^2 - |Q|\delta(x - x_{\bar{\text{D}}3}) \end{aligned} \tag{1}$$

this solution would generically break supersymmetry. This we call an "*anti-brane*".

The flux-brane combination can decay into a supersymmetric state of pure flux by having the brane consumed by flux.

We sort of have this picture:

$$\begin{aligned} \mathrm{d}F_5 &= e^{-\phi}|H_3|^2 - |Q|\delta(x - x_{\bar{\text{D}}3}) \\ &\to e^{-\phi}|H_3^{\text{leftover}}|^2 \end{aligned} \tag{2}$$

where this arrow is very complicated.

Introduction – What is brane-flux annihilation?

When does this happen? Is the decay perturbative? Why is it interesting?

There is this proposed construction, KKLT[1], which suggests that one would use an anti-brane for uplifting the cosmological constant.

This anti-brane is placed in a background of opposite charge.

KPV[2] showed that brane-flux annihilation can occur in this background as

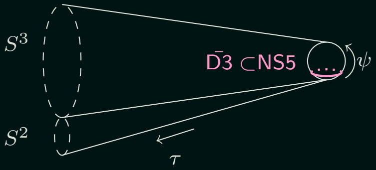
- The anti-D3-brane polarise into an NS5-brane carrying the anti-brane charge
- The NS5-brane is balanced by two forces
  - The background flux pushing the NS5 open
  - The NS5-brane tension (and local geometry) pulling it together
- For small anti-brane–charge/background–flux ratio, these can reach a balance.

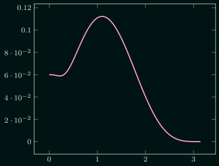
If the background flux pushes too much, the brane would perturbatively decay into a supersymmetric pure flux background.

[1] hep-th/0301240: Kachru, Kallosh, Linde, Trivedi
[2] hep-th/0112197: Kachru, Pearson, Verlinde

Introduction – What is brane-flux annihilation?

KPV performed this calculation at the bottom of the KS[3] conifold, which is relevant for the KKLT scenario.





[3] hep-th/0007191: Klebanov, Strassler

Introduction – Anti-brane singularities

These calculations were done from a probe-brane (worldvolume) perspective.

When the backreaction for the anti-brane was calculated in supergravity a singularity was discovered.[4] The flux surrounding the anti-brane was singular, that is H_3 and F_3, not F_5.

As I mentioned: If the flux push to hard on the anti-brane that has polarised it will decay perturbatively. Interpreting the singular solution as the end-point of an adiabatic time-evolving solution in which flux builds up around the anti-brane, then this means that the anti-brane would decay very fast.[5]

This was the central worry regarding the anti-brane singularity.

Since then, there has been a suggestion as to how this problem is resolved:[6]

The singularity is a signal of instability into polarisation which was not captured by the Ansätze used when computing the backreaction.

---

[4] 0912.3519: Bena, Grana, Halmagyi

[5] 1202.1132: JB, Danielsson, Van Riet

[6] 1507.01022: Cohen-Malonado, Diaz, Van Riet, Vercnocke; 1603.05678: Cohen-Malonado, Diaz, Gautason

Table of Contents

Singular anti-D6 branes

We are using the BPS (but not supersymmetric) background of JMO[7]. From the point of view of the Ansatz this appears to be a generalisation of the Klebanov-Tseytlin[8] singular conifold.

$$\begin{aligned} \mathrm{d}s_{10}^2 &= S^{-1/2}\mathrm{d}s_7^2 + S^{1/2}\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2\right), \quad & e^{\phi} &= g_s S^{-3/4} \\ F_2 &= g_s^{-1}\partial_r S\ r^2\Omega_2\,, & H_3 &= -g_s M r^2 \mathrm{d}r\wedge\Omega_2 \end{aligned} \tag{3}$$

This has the solution

$$S = v^2 - \frac{1}{6}g_s^2 M^2 r^2 + \frac{q}{r} \tag{4}$$

and the similarities to a conifold is that this is BPS and probe anti-D6 branes fall towards $r=0$ (note that $q=\pm|q|$ are O-plane and D-brane, not anti-D-brane and D-brane.).

[7] hep-th/9901078: Janssen, Meessen, Ortín

[8] hep-th/0002159: Klebanov, Tseytlin

Singular anti-D6 branes

To make it possible to add an anti-brane: generalise the Ansatz.

$$\begin{aligned} &\mathrm{d}s^2_{10} = e^{2A}\mathrm{d}s^2_7 + e^{2B}\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2\right) , \\ &F_2 = -e^{-7A} \star_3 \mathrm{d}\alpha \, , \qquad H_3 = -\alpha e^{-7A+2\phi} \star_3 F_0 \end{aligned} \tag{5}$$

Four functions: $F_2 \sim \alpha$, dilaton $\sim \phi$, warping $\sim A$, and conformal factor $\sim B$.

This system has been studied since the early days of the anti-brane singularity. It was the first that showed the presence of a singularity beyond perturbation theory.[9]

The proof goes like this.[10] Equations of motion tells us:

1. The derivative near the source is determined by the charge: $\operatorname{sgn}\alpha'|_{r=0} = \operatorname{sgn} Q$.
2. If H_3 is to be regular, non-singular, then $\alpha|_{r=0} = 0$.
3. Solution should asymptotically be BPS, in the sense of $\alpha > 0$ for large r.
4. Equations of motion shows that: $\operatorname{sgn}\alpha = \operatorname{sgn}\alpha''$ at any regular point.

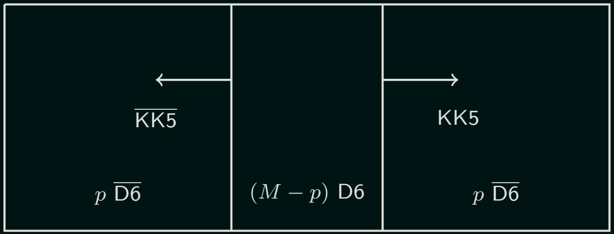
Can this be avoided by polarisation somehow? Brane-flux decay for an anti-D6?

[9] 1111.2605: JB, Danielsson, Junghans, Van Riet, Wrase, Zagermann
[10] 1105.4879: JB, Danielsson, Junghans, Van Riet, Wrase, Zagermann

The polarisation

For D5 and D6, the topology of the state is different: a brane–anti-brane pair.[11]

$$\begin{array}{lcl} \text{D5} & \to & \text{NS5} + \overline{\text{NS5}} \\ \text{D6} & \to & \text{KK5} + \overline{\text{KK5}} \end{array}$$



[11] 1609.06529: Danielsson, Gautason, Van Riet

The polarisation

We perform the calculation from the point of view of the anti-D5-brane polarisation.

We need a background to place this brane-configuration in

$$\begin{aligned} \mathrm{d}s^2 &= S^{-1/2}\mathrm{d}s_6^2 + S^{1/2}\left(\mathrm{d}\psi^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2\right) \,, \\ F_1 &= M\mathrm{d}\psi \,, \qquad H_3 = g_s M r^2 \mathrm{d}r \wedge \Omega_2 \,, \\ e^{2\phi} &= g_s^2 S^{-1} \,, \qquad F_3 = g_s^{-1}\tilde{\star}_4 \mathrm{d}S \,. \end{aligned} \tag{6}$$

With the same S as before. This has a $H_3 \wedge F_1$ charge distribution in it, opposite that of the brane charges we will add.

The full worldvolume action for this process would consist of

$$\begin{aligned} S &= S_+ + S_- + S_{\text{int.}} \\ &= (S_+^{(\text{DBI})} + S_+^{(\text{WZ})}) + (S_-^{(\text{DBI})} + S_-^{(\text{WZ})}) + S_{\text{int.}} \end{aligned} \tag{7}$$

with $S_{\text{int.}}$ some interaction action capturing gravitational and Coulomb interactions between the pair.

The polarisation

The explicit expression for the individual brane actions are

$$S_\pm = -\mu_5 \left\{ \int_{W_\pm^6} \mathrm{d}^6 x e^{-2\phi} \sqrt{-\det g_6} \sqrt{1 + e^{2\phi} \mathcal{G}_\pm^2} \mp \int_{W_\pm^6} (B_6 - \mathcal{G}_\pm C_6) \right\} , \quad (8)$$

where $\mathcal{G}_\pm$ carries the D5-brane charge $\mathcal{G}_\pm = \pm \frac{p}{2} - P[C_0]$ derived via T-duality.[12]

This action is then evaluated in this background: C_0, ϕ, B_6, C_6, and S are evaluated to their supergravity values.

The important thing I want to illustrate is the effective charge:

$$S_+^{(\mathrm{WZ})} + S_-^{(\mathrm{WZ})} = -\mu M \int \left(\frac{p}{M} - (\Phi_+ - \Phi_-) \right) C_6 \, , \quad (9)$$

The effective charge is the coefficient to C_6. At $\Phi_+ - \Phi_- =: \Delta\Phi = 0$, this charge is $-p$ for the number of anti-D5-branes that it carries. At the other side, where $\Delta\Phi = 1$ the charge is instead $M - p$, BPS with the background.

[12] 1505.00159: Gautason, Truijen, Van Riet; 1609.06529: Danielsson, Gautason, Van Riet

The polarisation

First we need to need to take a detour though.

This system is not a pure NS5–anti-NS5 pair, but a system carrying D5 charge. If we annihilate the two NS5-brane pairs at $\Delta\Phi = 0$ they should condense into the D5 brane. However:

$$S_+ + S_- = -\mu_5 p \left\{ \frac{2}{g_s^2 v p} \int \mathrm{d}^6 x \sqrt{1 + \frac{g_s^2 p^2}{4v^2}} \sqrt{-\det \eta_{\mu\nu}} + \int C_6 \right\} \qquad (10)$$

which is the action for a $-p$ charged D5-action except for the "1" in the square root.

This is because we have not accounted for the tachyon dynamics.[13]

[13] hep-th/9805170: Sen; hep-th/0204203: Hashimoto

The polarisation

The tachyon field is governed by a potential

$$V(T) \sim e^{-|T|^2}(1 + L^2|T|^2)\,, \tag{11}$$

and has two behaviours depending the distance between the branes, L.

- $L < 1$: Branes are close, and $V \to 0$ as $|T| \to \infty$.
- $L > 1$: Branes are far, meta-stable at $|T| = 0$ and $V = 1$.

The resolution to our problem: The branes are close, the tachyon falls into its true vacuum: $V \to 0$, and the combined action is

$$\begin{aligned} S_+ + S_- &= -\mu_5 p \left\{ \frac{2}{g_s^2 v p} \int \mathrm{d}^6 x \sqrt{V(T) + \frac{g_s^2 p^2}{4v^2}} \sqrt{-\det \eta_{\mu\nu}} + \int C_6 \right\} \\ &\to -\mu_5 p g_s^{-1} \left\{ \int \mathrm{d}^6 x \sqrt{-\det \eta_{\mu\nu}} + \int C_6 \right\} \end{aligned} \tag{12}$$

which is a D5-brane with charge $-p$.

We will therefore keep the tachyon in the true vacuum.

The polarisation

At the end of the day, we arrive at the effective potential

$$V_{\text{eff.}} = \frac{\mu_5 M}{g_s v^2} \left\{ \left| \frac{p}{M} - \Delta\Psi \right| + \left(\frac{p}{M} - \Delta\Psi \right) + V_{\text{int.}}(\Delta\Psi) \right\} . \tag{13}$$

without knowing much about $V_{\text{int.}}$ we can say:

There is a meta-stable minima at $\Delta\Psi = p/M$ as long as $\partial_{\Delta\Psi} V_{\text{int.}}|_{\Delta\Psi = p/M} < 2$.

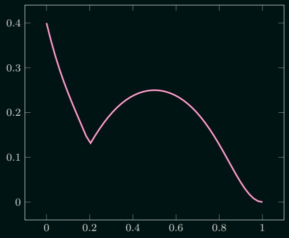
Furthermore, since the effective charge carried by the NS5 pair is given by:

$$Q_5 = -\mu_5 (p - M\Delta\Psi) , \tag{14}$$

it is *zero* at the meta-stable state.

The polarisation

Just as a toy-example. Set $V_{\text{int.}} = k\sin(\pi\Delta\Psi)^2/(1+\sin(\pi\Delta\Psi)^2)$, as long as k is not too large we get a meta-stable minimum



This leads us to believe that the same is true for our D6-brane: A meta-stable minima at zero charge.

Supergravity solution of this?

Regular supergravity solution – General statements

What was the big deal with zero charge?

The known results regarding the anti-D6-brane background was:

If there is (anti-brane) charge at $r = 0$, there is a singularity there in H_3.

But if we are looking for something at $r = 0$ with zero net-charge, what would we be looking for?

This would be our wish-list:

- A probe D6-brane should feel a force towards the $r = 0$ is there is a anti-D6-brane remnant there.
- Since the flux is non-BPS w.r.t. the anti-brane, there should be some (finite) flux clumping at small r.
- A probe D6-brane should have zero force *at* the point in UV.
- The UV should be preserved.

Regular supergravity solution – General statements

How would we start looking for this?

We write down the Ansatz presented before for the anti-D6 backreaction

$$ds^2 = S_a^{-1/2} ds_7^2 + S_b^{1/2}(dr^2 + r^2 d\Omega_2^2)\,, \qquad e^\phi = g_s S_f^{-3/4}$$

$$F_2 = -g_s^{-1}\left(\frac{S_a^7 S_b}{S_f^8}\right)^{1/4} \star_3 dS_l\,, \quad H_3 = -g_s M \left(\frac{S_a^7 S_b^3}{S_f^6 S_l^4}\right)^{1/4} (1 - S_l^{(0)} S_l) r^2 dr \wedge \Omega_2 \qquad (15)$$

which is the BPS solution when $S_a = S_b = S_f = S_l = S$.

The only candidate initial condition ($\sim$ boundary condition at $r = 0$) that exists[14] the ones having

$$S_x|_{r=0} = \text{positive constants}\,, \quad S'_x|_{r=0} = 0\,. \qquad (16)$$

[14] 1111.2605: JB, Danielsson, Junghans, Van Riet, Wrase, Zagermann

Regular supergravity solution – General statements

The full non-BPS equations are: F_2 Bianchi, dilaton, and three components of the Einstein's equations. These are very complicated, and for the present work we aim to find a solution only numerically.

Numerically we have some "problems"

- Equations of motion are singular at $r = 0$
- Equations of motion are singular at $r = r_{\text{UV}}$
- We have four functions and five differential equations

These are not serious problems, we simply have to

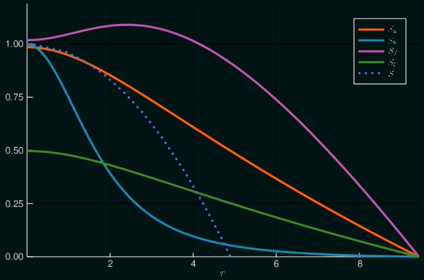
- Find consistent approximate boundary conditions at $r = \epsilon \ll 1$
- Solve towards $r = r_{\text{UV}}$ but be careful with interpreting behaviour there
- Make sure that a numerical solution satisfies the remaining equation

Regular supergravity solution – Numerical solutions

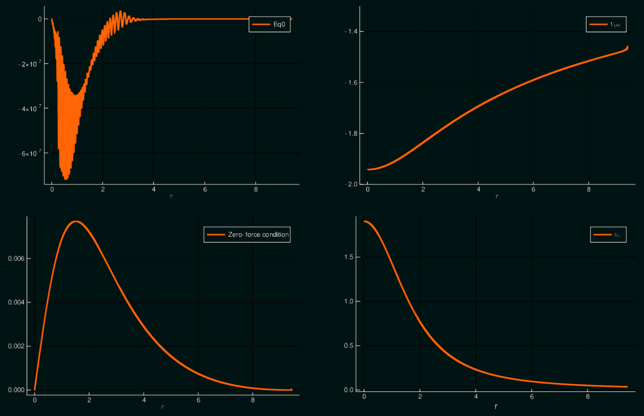
Our numerical solution is given by the boundary conditions

$$S_x|_{r=\epsilon} \approx \begin{cases} 0.9849456962929038 \\ 0.9981866313616979 \\ 1.018395334382631 \\ 0.49703367584074376 \end{cases}, \quad S'_x|_{r=\epsilon} \approx \begin{cases} -2.214565823966585 \cdot 10^{-8} \\ -1.917892841125644 \cdot 10^{-7} \\ 6.231007744084667 \cdot 10^{-8} \\ -2.192092217351898 \cdot 10^{-8} \end{cases}. \tag{17}$$

where $\epsilon = 10.0^{-6}$, for $m = 1.0$, $g_s = 0.5$, and $S_l^{(0)} = 0$.



Regular supergravity solution – Numerical solutions



All code is available on

`https://gitlab.com/johanbluecreek/non-bps_polarised_anti-d6_notebooks`

Concluding remarks

- Using a T-dual version of KPV, we argue for the existence of a meta-stable KK5-monopole–pair polarisation of anti-D6-branes.
- This meta-stable polarised state is such that it carries zero net charge.
- This would imply a supergravity solution of this polarised state, without localised charge.
- Given a list of requirements for such a solution we are able to show there exists numerical solutions that appear to obey all requirements.

Going forward it would be interesting to find the supergravity solution analytically.

- Can one analytically find parameters corresponding to remaining tension of polarised source?
- Is the zero-force at UV condition actually true?

Thank you for your attention.