Chiral Symmetry and Particle Physics

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9 November 2009

Bruno Touschek – January 1957

IL NUOVO CIMENTO Vol. V, N. 3 1° Marzo 1957

Parity Conservation and the Mass of the Neutrino.

B. F. Touschek

Istituto di Fisica e Scuola di Perfezionamento in Fisica Nucleare dell'Università - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

(ricevuto il 26 Gennaio 1957)

In the frame of the special theory of relativity there are two particles of zero mass: the photon and the neutrino. Whereas the photon mass is fixed to be zero by the principle of gauge invariance no similar principle is known for the field the invariance requirements (1) and (2) ensure that the neutrino mass be zero and that the sum of all Fermi particles minus the sum of all anti-Fermions remains constant. The latter is a property of all known strong inter-

Lee and Yang – June 1956

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

RECENT experimental data indicate closely identical masses[1] and lifetimes[2] of the $\theta^+(\equiv K_{\pi 2}{}^+)$ and the $\tau^+(\equiv K_{\pi 3}{}^+)$ mesons. On the other hand, analyses[3] of the decay products of τ^+ strongly suggest on the grounds of angular momentum and parity conservation that the τ^+ and θ^+ are not the same particle. This poses a rather puzzling situation that has been extensively discussed.[4]

One way out of the difficulty is to assume that parity is not strictly conserved, so that θ^+ and τ^+ are two different decay modes of the same particle, which necessarily has a single mass value and a single lifetime.

PRESENT EXPERIMENTAL LIMIT ON PARITY NONCONSERVATION

If parity is not strictly conserved, all atomic and nuclear states become mixtures consisting mainly of the state they are usually assigned, together with small percentages of states possessing the opposite parity. The fractional weight of the latter will be called $\mathfrak{F}^2$. It is a quantity that characterizes the degree of violation of parity conservation.

The existence of parity selection rules which work well in atomic and nuclear physics is a clear indication that the degree of mixing, $\mathfrak{F}^2$, cannot be large. From

Timely on Time – January 1957

It has recently been suggested by LEE and YANG ([1]) that the evidence on the decay of the K-meson could be interpreted to mean that parity is not conserved in « weak » reactions and this suggestion has recently been brilliantly confirmed by experiments carried out by Miss WU ([2]) at Columbia University.

([1]) C. N. YANG, T. D. LEE: *Phys. Rev.*, **104**, 254 (1956).

([2]) *Time*; Jan. 28 (1957), p. 39.

The Birth of Chiral Symmetry

> Whereas the photon mass is fixed to be zero by the principle of gauge invariance no similar principle is known for the neutrino. It is however, quite clear that the mass of the neutrino can be defined to be 0 by the requirement that the equations of motion—and with them the Lagrangian—of the free neutrino field are invariant under the transformation
>
> $$\nu' = \exp[i\gamma_5\alpha]\nu \,. \tag{1}$$

Chirality as compensation for the loss of Parity

Though it is difficult to see on the basis of the suggested invariance property how the nonconservation of parity in K-decay can be explained it nevertheless seems to suggest a formalism in which parity is not necessarily conserved and in which the loss of one conservation law may be perhaps compensated by the appearance of another.

Abdus Salam – November 1956

IL NUOVO CIMENTO Vol. V, N. 1 1° Gennaio 1957

On Parity Conservation and Neutrino Mass.

ABDUS SALAM
St. John's College - Cambridge

(ricevuto il 15 Novembre 1956)

1. – YANG and LEE(1) have recently suggested that present experimental evidence does not exclude the possibility that parity is not conserved in β-decay. If future experiments confirm this, it may be possible to relate parity-violation in neutrino-decays to the vanishing of neutrino mass and self-mass. The argument is as follows: the free neutrino Lagrangian is invariant for the substitution $\psi_\nu \to \gamma_5 \psi_\nu$ $(\overline{\psi}_\nu \to -\overline{\psi}_\nu \gamma_5)$. If it is further postulated that neutrino interactions produce no self-mass, one way to secure this is to require that the total Lagrangian also remain invariant for the same substitution (2) (so that $\overline{\psi}_\nu \psi_\nu \to -\overline{\psi}_\nu \psi_\nu$) while other fields (barring degeneracies which we consider later) remain unchanged. In so far as ψ_ν and $\gamma_5 \psi_\nu$ have opposite intrinsic parity, most neutrino interactions would then violate parity conservation.

Bruno Touschek – March 1957

IL NUOVO CIMENTO   Vol. V, N. 5   1° Maggio 1957

The Mass of the Neutrino and the Non-Conservation of Parity.

B. F. Touschek

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Istituto Nazionale di Fisica Nucleare - Sezione di Roma

(ricevuto il 5 Marzo 1957)

Summary. — A special form of the theory of Lee and Yang (1) is obtained by imposing an invariance principle, which insures that the mass as well as the magnetic moment (2) of the neutrino is identically zero. This invariance property leads to the conservation of a quantum number n. The non-conservation of parity is discussed for the chain of decays $\pi \to \mu + \nu$, $\mu \to e + \bar{\nu} + \nu$. It is shown that the present theory contains as a special case the « screwon » theory recently proposed by Lee and Yang (3) and a similar theory proposed by Salam (4).

Bruno Touschek – March 1957

In a recent letter (5) it has been proposed to connect the observed non-conservation of parity in processes involving the emission and absorption of neutrinos to an invariance principle which insures that the mass (and also the magnetic moment) of the neutrino is identically zero. The invariance principle may be stated in the following form:

All observable quantities of field theory are invariant under the transformation

$$\nu' = \exp\,[i\gamma_5\alpha]\nu\,, \qquad \bar{\nu}' = \bar{\nu}\,\exp\,[i\gamma_5\alpha]\,, \tag{1}$$

of the neutrino field and the simultaneous transformation

$$\psi_i' = \exp\,[in_i\alpha]\psi_i\,, \tag{2}$$

of all the other fields.

In Touschek's theory — but also in Lee and Yang, and in Salam — only the neutrino is chiral.

Feynmann and Gell-Mann – September 1957

PHYSICAL REVIEW VOLUME 109, NUMBER 1 JANUARY 1, 1958

Theory of the Fermi Interaction

R. P. FEYNMAN AND M. GELL-MANN
California Institute of Technology, Pasadena, California

(Received September 16, 1957)

The representation of Fermi particles by two-component Pauli spinors satisfying a second order differential equation and the suggestion that in β decay these spinors act without gradient couplings leads to an essentially unique weak four-fermion coupling. It is equivalent to equal amounts of vector and axial vector coupling with two-component neutrinos and conservation of leptons. (The relative sign is not determined theoretically.) It is taken to be "universal"; the lifetime of the μ agrees to within the experimental errors of 2%. The vector part of the coupling is, by analogy with electric charge, assumed to be not renormalized by virtual mesons. This requires, for example, that pions are also "charged" in the sense that there is a direct interaction in which, say, a π^0 goes to π^- and an electron goes to a neutrino. The weak decays of strange particles will result qualitatively if the universality is extended to include a coupling involving a Λ or Σ fermion. Parity is then not conserved even for those decays like $K\to 2\pi$ or 3π which involve no neutrinos. The theory is at variance with the measured angular correlation of electron and neutrino in He^6, and with the fact that fewer than 10^{-4} pion decay into electron and neutrino.

But now we go further, and suppose that the same rule applies to the wave functions of all the particles entering the interaction. We take for the β-decay interaction the form

$$\sum C_i(\overline{a\psi_n}O_i a\psi_p)(\overline{a\psi_\nu}O_i a\psi_e),$$

and we should like to discuss the consequences of this hypothesis.

Feynmann and Gell-Mann: learn to distrust experiments!

The decay of the π^- into a μ^- and $\bar{\nu}$ might be understood as a result of a virtual process in which the π becomes a nucleon loop which decays into the $\mu+\bar{\nu}$. In any event one would expect a decay into $e+\bar{\nu}$ also. The ratio of the rates of the two processes can be calculated without knowledge of the character of the closed loops. It is $(m_e/m_\mu)^2(1-m_\mu^2/m_\pi^2)^{-2}=13.6\times10^{-5}$. Experimentally[16] no $\pi\rightarrow e+\nu$ have been found, indicating that the ratio is less than 10^{-5}. This is a very serious discrepancy. The authors have no idea on how it can be resolved.

At the present time several β-decay experiments seem to be in disagreement with one another. Limiting ourselves to those that are well established, we find that the most serious disagreement with our theory is the recoil experiment in He^6 of Rustad and Ruby[10] indicating that the T interaction is more likely than the A. Further check on this is obviously very desirable.

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These theoretical arguments seem to the authors to be strong enough to suggest that the disagreement with the He^6 recoil experiment and with some other less accurate experiments indicates that these experiments are wrong. The $\pi\rightarrow e+\bar{\nu}$ problem may have a more subtle solution.

Bravo!

Nambu: from Chiral Symmetry to PCAC

VOLUME 4, NUMBER 7 PHYSICAL REVIEW LETTERS APRIL 1, 1960

AXIAL VECTOR CURRENT CONSERVATION IN WEAK INTERACTIONS*

Yoichiro Nambu
Enrico Fermi Institute for Nuclear Studies and Department of Physics
University of Chicago, Chicago, Illinois
(Received February 23, 1960)

In analogy to the conserved vector current interaction in the beta decay suggested by Feynman and Gell-Mann, some speculations have been made about a possible conserved axial vector current.[1-3] One can formally construct an axial vector nucleon current, which satisfies a continuity equation,

$$\Gamma_\mu^A(p',p) = i\gamma_5\gamma_\mu - 2M\gamma_5 q_\mu/q^2, \quad q = p'-p, \qquad (1)$$

where p and p' are the initial and final nucleon momenta. Such an attempt has some appeal in view of the apparently modest renormalization effect on the axial vector beta decay constant ($g_A/g_V \approx 1.25$), although the second appealing point,[1] namely, the possible forbidding of $\pi \rightarrow e + \nu$, has now lost its relevance.

The expression (1), unfortunately, can be easily ruled out experimentally, as was pointed out by Goldberger and Treiman,[3] since it introduces a large admixture of pseudoscalar interaction.

Our final remark concerns the theoretical basis for the assumptions made here. If the baryons are derived from some fundamental field ψ which possesses an invariance under a transformation of the type $\psi \rightarrow \exp(i\vec{\alpha}\cdot\vec{\tau}\gamma_5)\psi$,[8] then there will be a conservation of the pseudovector charge-current.

See also Gell-Mann and Levy, Nuovo Cimento, 16,705(1960).

At this point Bruno was deeply engaged with AdA!

Nambu: Broken Chiral Symmetry and "Goldstone" Bosons

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

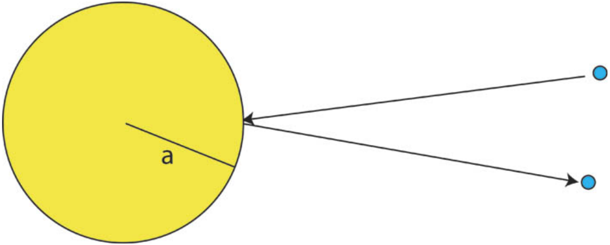
It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.

The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

From Chiral Symmetry to Chiral Dynamics

- Chiral SU(3) $\times$ SU(3)
- Current Algebra and sum rules.
- Anomalies
- Quarks
- Soft Pions
- Pion-pion scattering

The Scattering Length.



For the collision of a point particle with a hard sphere of radius a, the outgoing wave is advanced with respect to the ingoing wave:

Incoming wave	:	$\exp(-ikr)$
Outgoing wave	:	$\exp\left(ik(r+2a)\right) = \exp(2i\delta)\exp(ikr)$
With a phaseshift	:	$\delta = ka$

For S-wave $\pi\pi$ scattering we have two scattering lengths, a_0, a_2, corresponding to the possible I-spin values, I=0, 2.

Pions and QCD

The QCD Lagrangian	$\mathcal{L} = (\bar{\psi}_L i\gamma{\cdot}\mathcal{D}\, \psi_L) + (\bar{\psi}_R i\gamma{\cdot}\mathcal{D}\, \psi_R)$
(mass terms)	$+ (\bar{\psi}_R M\, \psi_L) + (\bar{\psi}_L M\, \psi_R)$
(e.m. interactions)	$+ e\,(\bar{\psi}_L Q\, \gamma{\cdot}A\, \psi_L) + e\,(\bar{\psi}_R Q\, \gamma{\cdot}A\, \psi_R)$

M is the mass matrix and Q the charge matrix. Neglecting strange and heavier quarks,

$$M = \begin{vmatrix} m_u & 0 \\ 0 & m_d \end{vmatrix}, \qquad Q = \begin{vmatrix} 2/3 & 0 \\ 0 & -1/3 \end{vmatrix}$$

In the limit $M = 0, e = 0$ there is exact SU(2)×SU(2) chiral symmetry — separate isospin for left handed and right handed quarks. In the real world this symmetry is broken to SU(2) — isospin, with pions acting as Nambu-Goldstone bosons. This leads to accurate predictions for the low energy pion interactions, both in π-Nucleon and in $\pi - \pi$ scattering.

Pion scattering lengths and chiral dynamics

In the limit of exact I-spin we have accurate predictions for the $\pi - \pi$ scattering lenghts.

Weinberg 1966 :

$$a_0 m_{\pi^+} = \frac{7 m_{\pi^+}^2}{16\pi f_\pi^2} = 0.159$$

$$a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_\pi^2} = -0.045$$

Colangelo et al. 2001 :

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$

$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

That the scattering lengths are small is a general consequence of the fact that pions act as "pseudo Goldstone Bosons" for chiral symmetry breaking.

How to measure $\pi\pi$ scattering lengths.

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$
$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$
$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

Note the precision
of the theoretical prediction.
Can it be matched by
experiment?

Correlations in K$\to \pi\pi\, e\nu$ (K_{e4}) decays. The recent result from BNL (hep-ex/0301040):

$$a_0 m_{\pi^+} = 0.216 \pm 0.013\,(\mathrm{stat.}) \pm 0.002\,(\mathrm{syst.}) \pm 0.002\,(\mathrm{theor.})$$

A new result of comparable accuracy should emerge from the NA48 experiment at CERN.

Lifetime of the pionium — the $\pi^+\pi^-$ atom. The decay rate of the pionium atom into $\pi^0\pi^0$ is proportional to $(a_0 - a_2)^2$. The preliminary result from the DIRAC experiment at CERN (hep-ex/00504044) is:

$$|a_0 - a_2| m_{\pi^+} = 0.264^{+0.033}_{-0.020}$$

K_{e4}: Experiment or theory?

High statistics measurement of K_{e4} decay properties

S. Pislak[7,6], R. Appel[6,3], G.S. Atoyan[4], B. Bassalleck[2], D.R. Bergman[6][*], N. Cheung[3], S. Dhawan[6], H. Do[6], J. Egger[5], S. Eilerts[2][‡], H. Fischer[2][§], W. Herold[5], V.V. Issakov[4], H. Kaspar[5,6], D.E. Kraus[3], D.M. Lazarus[1], P. Lichard[3], J. Lowe[2], J. Lozano[6][‖], H. Ma[1], W. Majid[6][**], A.A. Poblaguev[4], P. Rehak[1], A. Sher[3][‡‡], Aleksey Sher[7], J.A. Thompson[3], P. Truöl[7,6], and M.E. Zeller[6]

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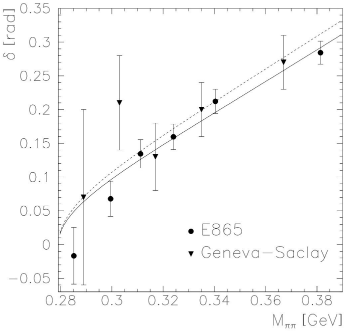
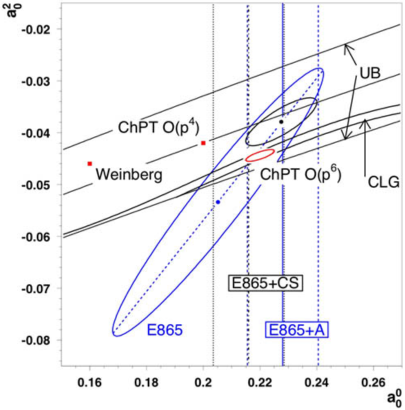
[2] *Department of Physics and Astronomy, University of New Mexico, Albuquerque, NM 87131, USA*
[3] *Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA*
[4] *Institute for Nuclear Research of Russian Academy of Sciences, Moscow 117 312, Russia*
[5] *Paul Scherrer Institut, CH-5232 Villigen, Switzerland*
[6] *Physics Department, Yale University, New Haven, CT 06511, USA*
[7] *Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland*

(Dated: January 31, 2003)

We report experimental details and results of a new measurement of the decay $K^+ \to \pi^+\pi^- e^+\nu_e$ (K_{e4}). A sample of more than 400,000 K_{e4} events with low background has been collected by Experiment 865 at the Brookhaven Alternate Gradient Synchrotron. From these data, the branching ratio $(4.11 \pm 0.01 \pm 0.11) \cdot 10^{-5}$ and the $\pi\pi$ invariant mass dependence of the form factors F, G, and H of the weak hadronic current as well as the phase shift difference $\delta_0^0 - \delta_1^1$ for $\pi\pi$-scattering were extracted. Using constraints based on analyticity and chiral symmetry, a new value with considerably improved accuracy for the s-wave $\pi\pi$-scattering length a_0^0 has been obtained also: $a_0^0 = 0.216 \pm 0.013\,(\text{stat.}) \pm 0.002\,(\text{syst.}) \pm 0.002\,(\text{theor.})$.

Extracting Scattering Lengths from K_{e4}.

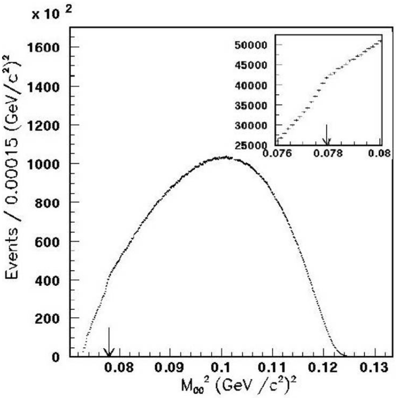




- One measures directly $\delta_0 - \delta_1$, the difference of $\pi\pi$ phase-shifts in I=J=0 and I=J=1 (N. C., and A. Maksymowicz, Phys. Rev. **137**, B438 (1965)) over a 100 MeV range. Theory is used to extrapolate to threshold.
- Chiral perturbation theory is used to restrict the result to the "universal band" (UB) of a_0 vs. a_2.

The $K^+ \to \pi^+\pi^0\pi^0$ anomaly.

In early 2004 Italo Mannelli showed me an early histogram of the $\pi^0\pi^0$ spectrum in $K^+ \to \pi^+\pi^0\pi^0$ from NA48 data ($\approx 25 \times 10^6$ events), with a funny anomaly near the $\pi^+\pi^-$ threshold. Could it be some sort of resonance? some sort of super-pionium?

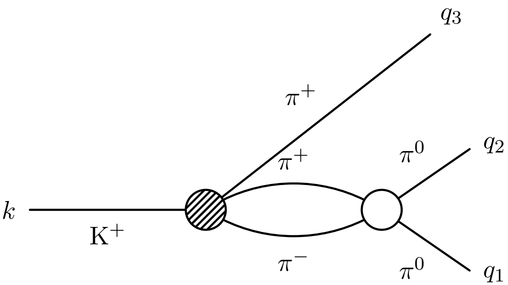


Distribution of M_{00}^2, the square of the $\pi^0\pi^0$ invariant mass. The insert is an enlargement of a narrow region centred at $M_{00}^2 = (2m_+)^2$ (this point is indicated by the arrow). The statistical error bars are also shown in these plots.

The diagram on the right is more recent, but similar to the one Italo had at the time.

The anomaly is in fact a cusp due to the interference of the $K^+ \to \pi^+\pi^0\pi^0$ decay with $K^+ \to \pi^+\pi^+\pi^-$ followed by $\pi^+\pi^- \to \pi^0\pi^0$.

The Cusp in $K^+ \to \pi^+\pi^0\pi^0$



$\pi^+\pi^- \to \pi^0\pi^0$ re-scattering from $K^+ \to \pi^+\pi^+\pi^-$ causes
a cusp singularity in the $\pi^0\pi^0$ spectrum at the $\pi^+\pi^-$ threshold.
The cusp amplitude is proportional to $(a_0 - a_2)$

N. Cabibbo, Phys. Rev. Lett. **93** (2004) 121801.

Time reversal, Unitarity and Analyticity

The structure of the cusp is determined by very general principles.

1. Time reversal: In the sectors of interest the **S**-matrix is symmetric,

$$\langle B|\mathbf{S}|A\rangle = \langle A|\mathbf{S}|B\rangle \qquad K \to 3\pi \text{ and } \pi\pi \to \pi\pi$$

2. Unitarity:

$$\mathbf{S} = \mathbf{1} + i(\mathbf{R} + i\mathbf{I}) \qquad \text{where } \mathbf{R} \text{ and } \mathbf{I} \text{ are hermitian.}$$

   From time reversal: **R** and **I** are symmetric, their matrix elements are the real and imaginary parts of the matrix elements of **S**.

$$2\mathbf{I} = \mathbf{R}^2 + \mathbf{I}^2 \qquad \text{or, solving for } \mathbf{I},$$

$$\mathbf{I} = \mathbf{1} - \sqrt{\mathbf{1} - \mathbf{R}^2} \quad = \quad \frac{1}{2}\mathbf{R}^2 + \frac{1}{8}\mathbf{R}^4 + \frac{1}{16}\mathbf{R}^6 + \frac{5}{128}\mathbf{R}^8 \ldots$$

3. Analyticity.

How does the cusp arise

Let us write:

$$\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = \mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1$$

where $\mathcal{M}_1$ is the contribution of the re-scattering graph.
We must consider two cases, with $s_{\pi\pi}$ above or below the $\pi^+\pi^-$ threshold.

$$s_{\pi\pi} > 4m_{\pi^+}^2 \quad : \quad \mathcal{M}_1 = i2a_x m_{\pi^+}\mathcal{M}_{+,\text{thr}} \sqrt{(s_{\pi\pi} - 4m_{\pi^+}^2)/s_{\pi\pi}}$$

$$|\mathcal{M}|^2 = (\mathcal{M}_0)^2 + |\mathcal{M}_1|^2$$

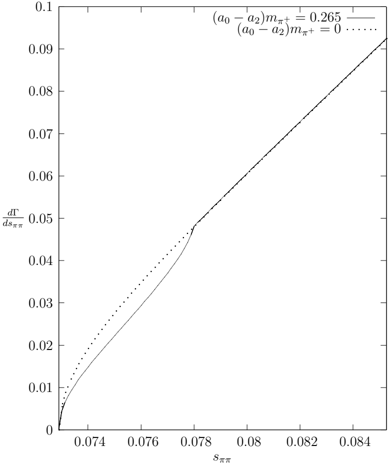
$$s_{\pi\pi} < 4m_{\pi^+}^2 \quad : \quad \mathcal{M}_1 = -2a_x m_{\pi^+}\mathcal{M}_{+,\text{thr}} \sqrt{(4m_{\pi^+}^2 - s_{\pi\pi})/s_{\pi\pi}}$$

$$|\mathcal{M}|^2 = (\mathcal{M}_0)^2 + (\mathcal{M}_1)^2 + 2\mathcal{M}_0\mathcal{M}_1$$

where $\mathcal{M}_{+,\text{thr}}$ is the value of the $K^+ \to \pi^+\pi^+\pi^-$ amplitude at the $\pi^+\pi^-$ threshold, and $a_x = (a_0 - a_2)/3$

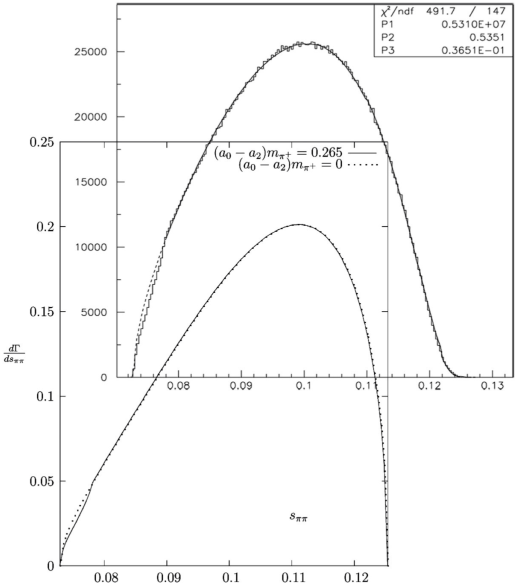
- Above threshold: $\mathcal{M}_1$ imaginary — No interference
- Below threshold: $\mathcal{M}_1$ real and negative — Interferes destructively with $\mathcal{M}_0$

How the Cusp looks



The $\pi^0\pi^0$ spectrum in $K^+ \longrightarrow \pi^+\pi^0\pi^0$ in the $\pi^+\pi^-$ threshold region, at the first order in an expansion in powers of a_i

A first comparison



Promising!

NA48 data and the cusp

With 10^8 $K^+ \to \pi^+\pi^0\pi^0$ events, a $1 \div 2$ % measurement of $(a_0 - a_2)$ seems possible, but since the cusp is a 10% effect, this requires a theory good to $1 \div 2$ parts in 10^{-3}, and implicates higher order rescattering effects, and radiative corrections.

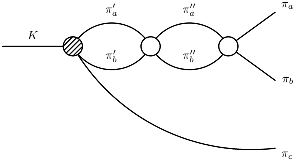
It is possible to set up a systematic computation of the singular parts of an amplitude in terms of its non-singular parts. This leads to an expansion of the $K \to 3\pi$ amplitudes in powers of the $\pi\pi$ scattering lengths a_0, a_2.

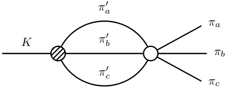
$3\pi \to 3\pi$ scattering amplitudes are also implicated but give negligible correction at our target precision

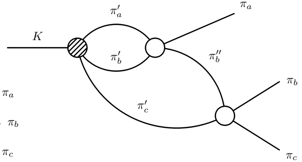
The development is useful because the scattering lengths are small, a general consequence of the "Goldstone Boson" nature of the pions.

Two loop contributions

a)







b) c)

$K \to 3\pi$ rescattering topologies at the two-loop level:

a) single-channel $\pi\pi$ scattering;

b) irreducible $3\pi \to 3\pi$ contributions;

c) $3\pi \to 3\pi$ amplitude due to $\pi\pi$ scattering.

The fine points

Second and higher order rescattering effects With Gino Isidori we have fully evaluated $O(a_i^2)$ effects, but it is worthwhile to consider $O(a_i^3)$ effects, that could contribute $\approx 3-5\%$ corrections.

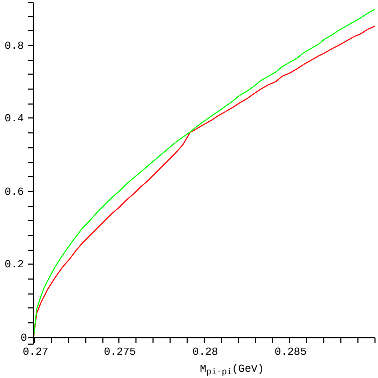
I-spin breaking I-spin is maximally violated around the $\pi^+\pi^-$ threshold. The cusp itself is an I-spin breaking effect. Are there subtler I-spin breaking effects?

Radiative corrections These are relevant close to the $\pi^+\pi^-$ threshold.

At present the Bonn-Berne group is providing valuable contributions to the analysis, including radiative corrections.

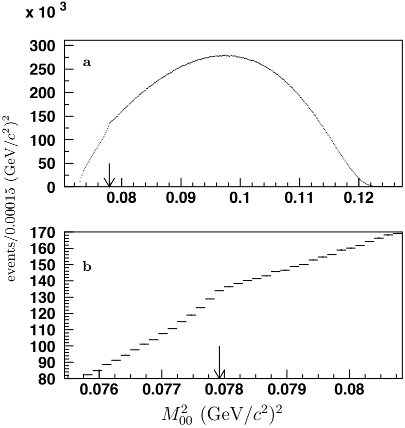
N. Cabibbo and G.Isidori – J. High Energy Phys. JHEP03(2005)021
G.Colangelo,J.Gasser,B.Kubis,A.Rusetsky – Phys.Lett. B638, 187 (2006)
M. Bissegger, et al. – Nucl. Phys. B806, 178 (2009), 0807.0515

The cusp region



Theoretical $\pi^0\pi^0$ spectrum in $K^+ \to \pi^+\pi^0\pi^0$ at $O(a_i{}^2)$
compared to the "unperturbed" amplitude.
The picture will be complicated by the existence of a $\pi^+\pi^-$ pionium atom signal at the $\pi^+\pi^-$ threshold!.

NA48 results (6×10^7 $K^+ \to \pi^+\pi^0\pi^0$ events)



$$(a_0 - a_2)m_+ = 0.2571 \pm 0.0048(stat.) \pm 0.0025(syst.) \pm 0.0014(ext.);$$

$$a_2 m_+ = -0.024 \pm 0.013(stat.) \pm 0.009(syst.) \pm 0.002(ext.).$$

Batley et al., European Phys. Jour. C, 64, 589-608 (2009).

Where the story started

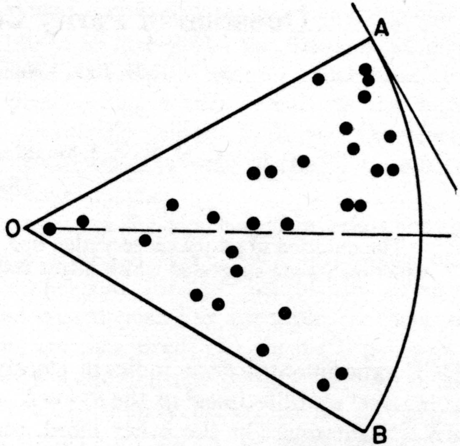


FIG. 4. The data on twenty-nine τ-meson decay events.

R. H. Dalitz, Phys. Rev. 94, 1046 (1954)

The End