Electron-Positron Storage Rings

From AdA to LEP

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1. Introduction

Electron-Positron Storage rings have been in operation for high-energy physics since 1967. But even before 1967 much pioneering work had been done on the electron-positron ring AdA at Frascati [1], VEPP I at Novosibirsk [2] and the two intersecting Princeton-Stanford electron rings [3]. It was the 380 MeV ring VEPP II at Novosibirsk where the first e^+e^- annihilation events were observed in 1966 [4] ; soon after that events were equally observed at the ACO storage ring at Orsay [5].

Since then, electron-positron storage rings have played an ever increasing role in elementary particle physics. The simplicity of the initial state of an electron-positron annihilation event, the fact that, given sufficient energy one can produce any particle together with its antiparticle, as long as they have electromagnetic coupling, has made electron-positron storage rings one of the most powerful tools in modern high-energy physics. This is particularly true with storage rings reaching into the energy range where weak neutral currents play a significant role.

The history of electron-positron colliders goes back almost thirty eight years to March 1960 when Bruno Touschek gave a seminar at the Frascati National Laboratory in Italy, in which he pointed out the importance of systematic studies of the electron-positron collisions. He proposed the construction of a single magnetic ring where beams of electrons and positrons at the same energy would rotate in opposite directions and collide head-on at discrete locations around the ring. The study of the products of such a collision has supplied information on the fundamental laws of nature, in the time-like region, and in particular of electroweak interactions and of quantum flavour dynamics.

In all fixed-target accelerators a beam of particles with energy E_0 hits a target at rest. Part of E_0 goes to waste. The percentage of waste increases with E_0. For $E_0 = 10^3$ GeV only 4.5 % is useful energy. It has been obvious that colliders in which two beams of particles with equal and opposite momenta, coming to hit head-on are free of waste.

The beginning of collider physics can be traced back to a study of Kerst and collaborators[2] in 1957 on the efficiency of a system consisting of two tangential or interlaced beams and to the observation by Touschek in 1959 that a single ring of magnets and rf cavities suffice as a collider for counter-rotating particles of opposite charge[7].

It should be said that the idea of constructing accelerators based on the collision of two particle beams, instead of a single particle beam striking a fixed target, was devised for the first time by Ralph Wideröe in the late Summer of 1943, in Norway. As reported by J. Blewett[7], Wideröe himself wrote : "On a nice Summer day I was lying on the grass seeing the clouds drifting by and then I started speculating on what happens when two cars collide. If we have a car moving with velocity v, colliding with a resting car of equal mass, the dissipated energy will be $\frac{1}{4}mv^2$ (inelastic collision), whereas two cars with velocity v having a head-on collision would dissipate four times as much energy (mv^2) in spite of having only twice the energy before the collision. This clearly demonstrated that head-on collisions have to be avoided for cars, but might be very useful for protons".

Following O'Neill's proposal[8] a group of Princeton-Stanford[3] constructed the first two-ring collider which was filled with 300 + 300 MeV electrons from Mark III ; results were reported in 1966. The first single-ring collider, called AdA, was built at Frascati[1]. The first report on e^+e^- interactions at 200 + 200 MeV appeared in 1964.

A person who, at about the same time as Touschek appreciated the constructional advantage of e^+e^- was G.I. Budker at Novosibirsk[9].

The economy of the colliders with respect to useful energy has its price : a low event rate. The target for one collider beam is the other beam which contains a very low density of material, ever so much less than for fixed targets. One of the main problems is therefore to do as well as possible on luminosity, L, which is defined by $\frac{dn}{dt} = L \cdot \sigma$ where σ is the cross section for the reaction at hand, n being the number of events at time t. The luminosity is enhanced by injecting many pulses of particles into the ring(s) before making the two beams interact, whence the other name for colliders : storage rings. This operation of injecting many pulses introduced a crucial new parameter : the beam lifetime. One should be able to keep the beams circulating over long periods of time without an appreciable energy loss. That imposed unprecedented demands on the quality of the vacuum and on the stability of the accelerating rf voltage.

Collider physics has brought a new technology to the high-energy physics laboratories and collider experiments are exclusively performed with primary beams and must be executed exactly at the handful of beam intersection regions. The number of experiments that can be accommodated at any given time is therefore quite restricted and further limitations arise because low event rates require long running times for each experiment.

2. An Intermezzo : the Beam Dynamics

Colliders are non-linear dynamic systems whose evolution is governed by electromagnetic forces arising from external fields and from the interaction of the beam particles between themselves and with their environment. These forces among which we must include those caused by external noise and by quantum effects, have a random component so that the time evolution of the beam particles should be properly described by a system of some 10^{12} coupled non-linear stochastic equations.

Solving such a system is, of course, an impossible task. What one usually does is to substitute into the system of coupled equations a single stochastic equation describing the motion on one particle under the combined effect of the external fields and of the forces created by all the other particles, namely an equation of the form :

$$\overset{\circ}{x} = K(x,t;\psi) + \sum_{\alpha} R_{\alpha}(x,t;\psi)\xi_{\alpha}(t) \qquad (1)$$

where $x = \left(\underline{q},\underline{p}\right)$ are the single particle co-ordinates and momenta, while $\psi(x,t)$ is the phase space particle density. The "forces" on the r.h.s. of equation (1) have been split into a deterministic Hamiltonian component K and a random component, depending on the stochastic variable $\xi_\alpha(t)$ which describes both internal and external sources of noise. Since the correlation time of this noise is generally very short, compared to the relaxation time for the distribution ψ, the random variable $\xi_\alpha(t)$ can be assumed to be Gaussian. The stochastic equation (1) becomes equivalent to a Fokker-Planck equation for ψ, having the form of a continuity equation in phase space.

The forces acting on the beam arise from the external electromagnetic fields and from those created by the particles in the beam itself. The external forces, independent of ψ, give rise to the so-called single particle effects, the only ones usually taken into account to determine the equilibrium orbit. The stability of this orbit, with respect to small deviations from the equilibrium values $\underline{q}_o(t)$ and $\underline{p}_o(t)$., can be studied by linearizing the external forces around $\underline{q}_o(t)$ and $\underline{p}_o(t)$. However, a fully non-linear analysis is needed to discuss other important properties of the machine, such as dynamic aperture or the size of the rf bucket.

It is worthwhile noting that the external forces contain a stochastic component arising from scattering on residual gas, fluctuations of the applied fields, field errors, quantum fluctuations of the radiation fields.

The internal forces which depend upon the distribution function ψ, have their origin in collective phenomena such as beam-beam and beam-walls effects. These effects are due to the presence of many charged particles travelling inside a conducting pipe and severely limit the luminosity of particle-antiparticle storage rings. The interaction of the beam with the "environment" is more dangerous for very high-energy machines where the injection energy is relatively low. These storage rings need a very large number of accelerating cavities that, due to the stiffness of the beam at low energy, can easily excite instabilities. Among the collective effects are included stochastic phenomena such as intrabeam scattering and Schottky noise, in fact they depend on the particle density and thus on ψ.

The Lorentz force $\underline{F}$ associated with the average fields can be derived from a Hamiltonian and thus, as a consequence of Liouville's theorem, cannot change the phase space volume (beam emittance). On the contrary, due to the lack of detailed information about the microscopic fields, which describe the collisions, one must describe their effects by means of non-conservative and stochastic forces. These forces give rise to dissipation and diffusion in phase space, i.e. to irreversible phenomena [10].

The main sources of dissipative and stochastic forces in e^+e^- colliders are due to radiation damping and quantum excitation, intrabeam scattering, scattering with other types of particles, density fluctuations. These forces give rise to a particle flux in momentum space described by a vector $\underline{S}$ depending upon ψ : $\underline{S} = \underline{S}(\underline{q}, \underline{p}, t; \psi)$. The distribution function ψ satisfies a Fokker-Planck equation that expresses the local conservation of particle number in phase space. When the irreversible flux $\underline{S}$ is zero, it describes the motion of an incompressible fluid density ψ and is known as the Vlasov equation.

One of the most important irreversible processes in e^+e^- colliders is the incoherent synchrotron radiation. In the extreme relativistic limit ($\gamma \gg 1$) the radiation reaction force $\underline{R}$ acting on electrons with velocity $\underline{v}$ in a magnetic field $\underline{B}$ orthogonal to $\underline{v}$ is given by :

$$\underline{R} = -\frac{W(t)}{c^2}\underline{v} \tag{2}$$

where W(t) is the instantaneous radiated power which fluctuates due to quantum effects. In writing equation (2) the angular deviations of the emitted photons have been neglected, this is of the order $1/\gamma$ from the direction of $\underline{v}$. From the stochastic force (2) the irreversible flux $\underline{S}$ can be deduced [10]:

$$\underline{S} = -\left(p^2 + m^2c^2\right)\underline{p}\left\{\frac{\overline{W}}{c}\psi + \frac{1}{2c^2}\left(k\overline{W}\varepsilon_{ph}\right)^{\frac{1}{2}}\frac{\partial}{\partial p}\left[\left(\frac{k\overline{W}\varepsilon_{ph}}{p^2 + m^2c^2}\right)^{\frac{1}{2}}\psi\right]\right\} \tag{3}$$

where $\overline{W}$ and ε_{ph} depend upon $\underline{p}$, through the Lorentz factor $\gamma(p)$ and upon $\underline{q}$ and t through $\underline{B}(\underline{q}, t)$. The quantity ε_{ph} is the critical photon energy and the constant k is of the order one.

The first term in curly brackets represents the dissipation associated with classical synchrotron radiation, the second one is a diffusion term arising from the discrete nature of quantum emission, similar to that appearing in Brownian movement.

The Fokker-Planck equation with irreversible flux (3) is far too complicated to be solved exactly. The usual way is to linearize it with respect to co-ordinate and momentum and then to introduce a dispersion function of radial co-ordinate, thus decoupling the radial betatron oscillations from the energy oscillations. A further averaging over one turn allows an appropriate solution from the longitudinal part of the Fokker-Planck equation (10).

An alternative procedure to investigate the time evolution of ψ is to track a given sample of particles, typically of the order of 10^3, by means of stochastic equation (1) over subsequent small intervals Δt, thus relating the new co-ordinates and momenta of each particle $x_i(t+\Delta t)$ to previous values $x_i(t)$ and to $\psi(x,t)$.

3. From AdA to LEP

The e^+e^- physics was born in 60's with AdA. The first two machines fruitful for many years in high-energy physics were constructed at Novosibirsk VEPP II (2 x 700 MeV) and at Orsay ACO (2 x 550 MeV). They allowed the clarification of a well-defined domain of particle physics : the electromagnetic interactions of hadrons, and settled the detailed properties of the vector mesons ρ, ω, ϕ and their importance in the hadron-like behaviour of the photon described by the Vector Dominance Model.

A first surprise was observed in 1970 with Adone (2 x 1.5 GeV), in fact it revealed a high intensity of e^+e^- annihilations into multihadrons above 1.2 GeV and the existence of two bumps in the cross section interpreted as higher vector mesons ρ' and ρ'' excited states of the ρ meson. Between 2 and 3 GeV the ratio $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ appeared to be roughly constant and equal to 2. This is exactly the value predicted by the quark parton model with the three light quarks u, d, and s.

In 1973 the e^+e^- by pass at Cambridge (USA) announced that the $\sigma(e^+e^- \rightarrow \text{hadrons})$ at 4 and 5 GeV was nearly equal to the value measured below the 3 GeV at Adone. In November 1974 at SPEAR (2 x 4.2 GeV) the discovery of the ψ particle was announced, interpreted as a $c\bar{c}$ vector meson bound state of charmed quarks. During a few months discoveries were obtained both at SPEAR and at DORIS (2 x 5 GeV) : a complete charmonium spectroscopy ; a heavy lepton $\tau^{\pm}$ (1.8 GeV) produced in $e^+e^- \rightarrow \tau^+\tau^-$; charmed D, D^*, F, F^*. In December 1979 CESR (2 x 8 GeV) confirmed the two narrow states (observed at DORIS) at 9.4 GeV and 10 GeV, interpreted as $b\bar{b}$ bound states, and added a new one at 10.4 GeV. PETRA (2 x 19 GeV) began operating in 1978 and produced interesting results, such as $e^+e^- \rightarrow q\bar{q}g \rightarrow \text{hadrons}$.

PEP, VEPP 4, DCI pursued studies of quarkonia spectroscopy and the new class of reactions, such as $e^+e^- \rightarrow e^+e^- + X$, that are the photon-photon collisions.

TRISTAN, HERA, SLC and LEP are the new colliders. Hera is an ep collider. They have performed as expected. From LEP we obtained evidence for three neutrinos and therefore the confirmation of three families of quarks and leptons, and the clear evidence that the Electroweak Standard Model is a good model up to 1 ‰ accuracy. The search for new physics is still being pursued at LEP 200.

4. <u>Selected considerations on LEP</u>[11]

In the preparation of the LEP project, various factors had to be kept in mind, these dealt with the setting of the parameters defined by the physicists, first and foremost the energy and the luminosity of the machine, with investments and operating costs kept at a minimum.

In colliding rings one extremely important parameter is the luminosity. This is proportional to the number of particles (N) in each beam and it increases with the decrease of the transverse section (A) of the particle beam $\left(L \cong Ne^+Ne^-/A\right)$. For any physical process (such as for example Z°), the rate at which the events are produced is given by

$$\frac{dn}{dt} = \sigma L \tag{4}$$

where σ is the cross section of the process. For beams colliding "head-on" the luminosity at any collision point is given by :

$$L = \frac{N_e N_p k_b f_{rev}}{4\pi\sigma_x^*\sigma_y^*} \tag{5}$$

with N_e, N_p the number of electrons, positrons per bunch and $\sigma_x^*\sigma_y^*$ the cross section of the bunches in the transverse plane ; f_{rev} is the rotation frequency of the particles, k_b the number of electron and positron bunches.

A fundamental limitation to all electron-positron colliders results from the influence of the electromagnetic field associated with each beam on the motion of the particles in the 'other' beam. This 'beam-beam effect' is quantified by the beam-beam strength parameter (ξ) or often referred to as beam-beam tune shift. For $\sigma_x^* >> \sigma_y^*$ one can find [11]

$$L = \frac{\gamma f_{rev} k_b}{2r_e} \cdot \frac{N_b \xi_y}{\beta_y^*} \tag{6}$$

where γ is the relativistic energy factor, ξ_y is the vertical tune shift, β_y^* is the vertical betatron amplitude function at the collision points, r_e is the classical electron radius equal to 2.8 x 10^{-15} m.

The second important performance parameter is the beam energy. In the case of LEP the energy was clearly defined by the energy of the Z° and the $W^\pm$ particles and resulted in the range of 40 $\rightarrow$ 100 GeV per beam. Having decided upon the energy, the next question in the design procedure is to optimise the capital and the running costs. The crucial component in the definition of the circumference of any high energy e^+e^- storage ring is the mechanism of synchrotron radiation. The radiation loss per revolution (U_0) of a particle of energy E_b being bent in a circular path of radius ρ is given by

$$U_o = c_\gamma \frac{E_b^4}{\rho} \quad (7)$$

where $$c_\gamma = \frac{4\pi}{3} \frac{r_o}{E_o^3} = 8.85 \times 10^{-5} \mathrm{m\ GeV}^{-3} \quad (8)$$

with r_e = 2.8 x 10^{-15} m (the classical electron radius) and E_0 = 0.511 MeV, the rest energy of the electron.

It is the function of the acceleration system which consists of radio frequency resonant cavities to replenish the energy lost by the particles of both beams on each revolution. Therefore the power and the cost of the r.f. system is defined to a large extent by the radiated power due to synchrotron radiation.

The system for the acceleration of particles makes use of high-frequency resonant cavities ; these must supply very high power in the case of electrons. Electrons are accelerated in the cavity containing electrodes across which is applied the electric field. This field between two electrodes continually changes in polarity. At a given time (t = 0), a bunch of electrons is accelerated in cell number 1. When the electrons enter cell (number 2) at time t = 1.4 ns, the field changes direction so that the electrons are always submitted to an accelerating filed and so on. The positron bunches, which have a positive charge, go in the opposite direction to that of the electrons and are accelerated with the same principle. As the speed of the electrons and positrons in LEP approaches that of light and the distance between the electrodes is of 42.5 cm, the field must change direction every 1.4 ns, corresponding to a frequency of 352 MHz.

Colliders are systems where stable conditions are obtained for quite long intervals of time with respect to the time it takes for the particles to go round, in fact in LEP, electrons and positron go round about 10^8 to 10^9 times.

The magnetic functions are very clearly defined : the magnetic dipoles give the curvature to the electron and the positron trajectories ; the quadrupoles give the beam its stability, the sextupoles correct the inevitable chromaticity of the machine. This system of electromagnets (dipoles, quadrupoles and sextupole) is distributed all along the LEP tunnel in an even and repetitive manner. The fundamental structure of the magnetic system – the half-cell – is made up of three coupled dipoles, one quadrupole and one sextupole, the total length is of the order of 40 metres. In the LEP tunnel there are 1'700 coupled dipoles installed, each of 11.7 metres ; 760 quadrupoles, some of them 1.6 metres long, others 5 metres ; 560 sextupoles of 50 cm each, plus a few hundred smaller dipoles for the fine correction of the trajectory position. Superconducting quadrupoles have been installed near the experimental magnetic spectrometer to reduce the betatron function at the points of collision to enhance the luminosity there.

Other important achievements have been obtained in the vacuum system and in the superconducting rf cavities to reach the energy of 100 GeV per beam. LEP has been described in detail in many articles and we suggest to consult the references [11] for more technical details.

LEP is an octagonal polygon with eight straight sections of about 500 m each, and eight circular arcs with a radius of curvature of 3'300 metres. LEP has a circumference of 26.7 km, with a circumference known better than $1 : 10^6$.

LEP components are situated in an underground tunnel with an average diameter of about 4 metres and at an average depth of 100 metres. Eighteen access points join the surface areas to the underground areas by means of shafts of varying depths, from a minimum of 50 to a maximum of 150 metres.

LEP is a powerful and precise scientific instrument, put at the disposal of the high-energy physics community so as to verify the predictions made by the electroweak model. It has been conceived to produce a great quantity of Z° vector bosons every day. Up to now more than 20 million Z° have been produced and analysed. Nowadays LEP runs at about 94.5 GeV per beam exploring the $W^\pm$ region ; in the near future (1999) LEP will reach an energy of 100 GeV per beam.

Synchrotron radiation [12], [13] emitted by the electrons and the positrons induces a well-defined polarisation in the beam. This polarisation mechanism is due to the fact that there is a non-zero probability that the radiation emitted is accompanied by a change of the spin state (spin-flip). This important quantum phenomenon allows that a beam of electrons and positrons initially non-polarised, gradually acquires a polarisation in the direction perpendicular to the trajectory described by them. Transverse polarised electrons are depolarised in a controlled way by applying an oscillating horizontal magnetic field. Under the influence of such a weak field, the spin is slightly rotated away from the vertical axis on each turn, and a depolarising resonance occurs if the depolarising field is in phase with the spin precession. The number of spin precessions per revolution, the spin tune ν_S, is related to the beam energy via

$$E_{beam} = 2\left(\nu_s m_e c^2 \Big/ g_e - 2 \right) = 0.4406486\ (1)\ (\mathrm{GeV} x \nu_s) \quad (9)$$

The depolarising field occurs at a frequency which is independent of the integer part of the spin tune :

$$f_{dep} = (\nu_s - \mathrm{int}(\nu_s)) \ x \ f_{rev} \quad (10)$$

where f_{rev} = 11245.50 (4) Hz is the revolution frequency of the beam particles. The integer part of ν_S is 105 at 46.5 GeV ; it is well-known from the other calibration techniques, since a unit tune change corresponds to a ~ 440 MeV change in beam energy. A beam energy definition of about 2 x 10^{-5} has been reached with this technique.

The most spectacular source of energy variations comes from ground motion. Strong focussing amplifies the movements by a factor of nearly 10^4, so small expansions by $\pm 10^{-8}$ lead to a practical error on the Z° mass and the width of 10 MeV. Terrestrial tides are one strong cause of such variations and these have indeed been observed. Energy jumps are correlated with orbit movements. This effect has been cured by modulating the main field at the level of 10^{-4} and the present knowledge of the beam energy leads to a systematic error of 4 MeV on the Z° mass and 3 MeV on the Z° width before correction of the orbit.

5. LEP performance at 94.5 GeV [14]

The operation of LEP at 94.5 GeV is very smooth. The 102°/90° optics is performing very well. The peak performances are summarised in the following Table 1.

Year	$L(cm^{-2}s^{-1})$	$I_b(\mu A)$	Max ξ_y
1996	3.4×10^{31}	520	0.040
1997	5.0×10^{31}	650	0.055
1998	7.4×10^{31}	740	0.062

The beam-beam tune shift $\xi_y \sim L/I_b$.

The efficiency (physics/total time) : 47%. The major contributions to the down-time are due to SPS & CPS : 50%, RF : 13%. The time needed to refill LEP (turn-around time) is about 70 minutes. The best integrated luminosity is 2.9 pb^{-1} over 24 hours. The maximum efficiency obtained is of 80%. The beam lifetime τ in collisions is dominated by beam-beam bremsstrahlung :

$$1/\tau \sim L/I_b \sim \xi_y \qquad (11)$$

from which one can deduce that the better the LEP performance, the faster it decays. The luminosity lifetime is about $\tau/2 \approx$ 3 hours, and the typical fill length is 3.5 hours.

6. Conclusions

In 1975 particle physics entered the jet age when at SPEAR they found that $e^+e^- \rightarrow q\bar{q}$ was followed by the fragmentation of q and $\bar{q}$ into hadrons. Jet studies have been carried out on PETRA at DESY which began operating in late 1978 and which has reached energies of 2E over 20 GeV ; and on PEP at SLAC, which began operating in 1980 at 14.5 GeV. The first three-jet events were found at PETRA in 1979 and later on thousands of them at PETRA and at PEP ;

these events are caused by gluon bremsstrahlung of either q or $\bar{q}$ followed by gluon $\rightarrow$ hadron.

New dedicated colliders will soon be entering into operation to study the physics of the beauty-antibeauty system and the ϕ physics, here at Frascati with DAΦNE.

Electron storage rings are used to produce synchrotron light in many places and at different energies for applications in various scientific areas. Electron storage rings have been developed and constructed for the sole use as colliding beam facilities for high energy physics. Very soon the synchrotron radiation emanating from these storage rings became a powerful research tool in a variety of disciplines other than high energy physics. Recently additional applications of storage ring radiation have been proposed for industrial use to produce miniature computer circuits through an X-ray lithographic process and for medical use to diagnose symptomatic cardio-vascular diseases. Recently, as the area of storage rings for high energy physics seemed to reach an end by the development of linear collider facilities, it has become quite clear that storage rings are still an essential part of such a facility as dumping rings to prepare particle beams of extremely small emittance.

In contrast to the requirement of a large beam emittance for colliding beams, the smallest possible beam emittances are desired, for example, in synchrotron light sources, in low energy storage rings used as drivers for free electron lasers or in damping rings for linear colliders.

LEP II is now exploring a new range : we are looking forward to either further confirmation of the Standard Model or deviations from it. This, if ever it happens, will be the sign of a new physics.

A question remains : What will the future of e^+e^- colliders be ?

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