**Imperial College**
**London**

# The $B_d \rightarrow K^{*0}l^+l^-$ decay

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UE, T. Hurth, J. Matias, M. Ramon, W. Reece (2008, 2010)

# Outline

- Potential of $B \rightarrow K^{*0}\mu^+\mu^-$ exclusive decay
- Kinematic definitions
- Observables
- Symmetries in angular distribution
- Phenomenology
- Moving outside the safe $q^2$ region
- Pending areas

# What is the potential

The $B \to V l^+ l^-$, $V = K^*, \varphi$, $l = e, \mu$ decays have a rich phenomenology

Proceed through penguin loops so puts SM processes and NP on an equal footing

The transversity amplitudes of the decay can be expressed through the Wilson Coefficients in the OPE

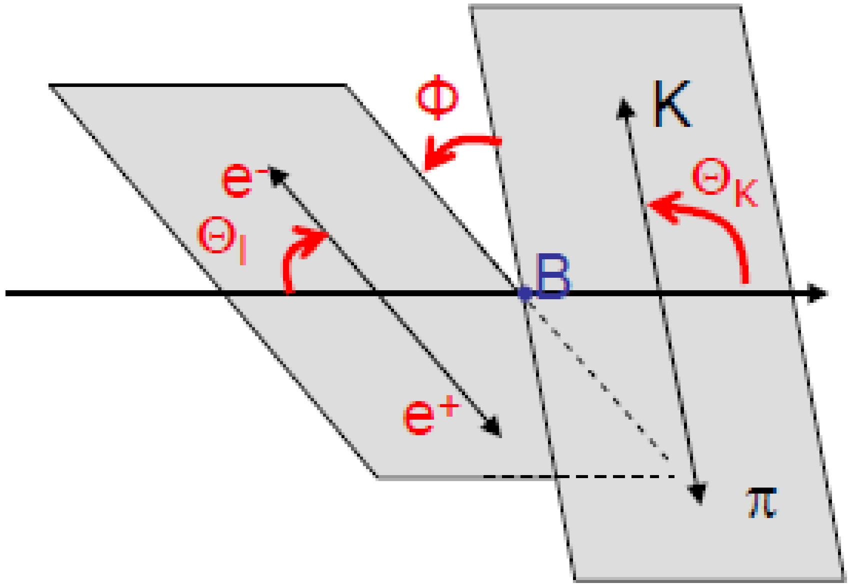
- Sensitivity to $C^{7(\mathrm{eff})}$, $C^9$ and $C^{10(\mathrm{eff})}$ and their right handed counterparts.

# Kinematics

After summing over the polarisation states of the muons, the angular dependence is given as

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$

where



# Kinematics

$$J(q^2,\theta_l,\theta_K,\phi) =$$
$$= J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l + J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$
$$+J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi + (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l$$
$$+J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi + J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi$$

where each of the 12 $J_i$ can (in principle) be experimentally determined (but see later)

Each of the $J_i$ terms can be expressed through the transversity amplitudes

$$J_{1s} \equiv a = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}\right)$$

$$\vdots$$

$$J_9 \equiv m = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L*}A_\perp^L) + (L\to R)\right]$$

# What is the problem

- We are dealing with an exclusive decay
- Multiple problems coming from QCD
  - Form factor calculation from QCDf
  - This leaves us with $\Lambda_{QCD}/m_b$ corrections
  - Mass of charm quark introduce uncertainties
  - Charm loops

## From theory to measurements

We start out with a shiny New Physics model

Physics model

# From theory to measurements

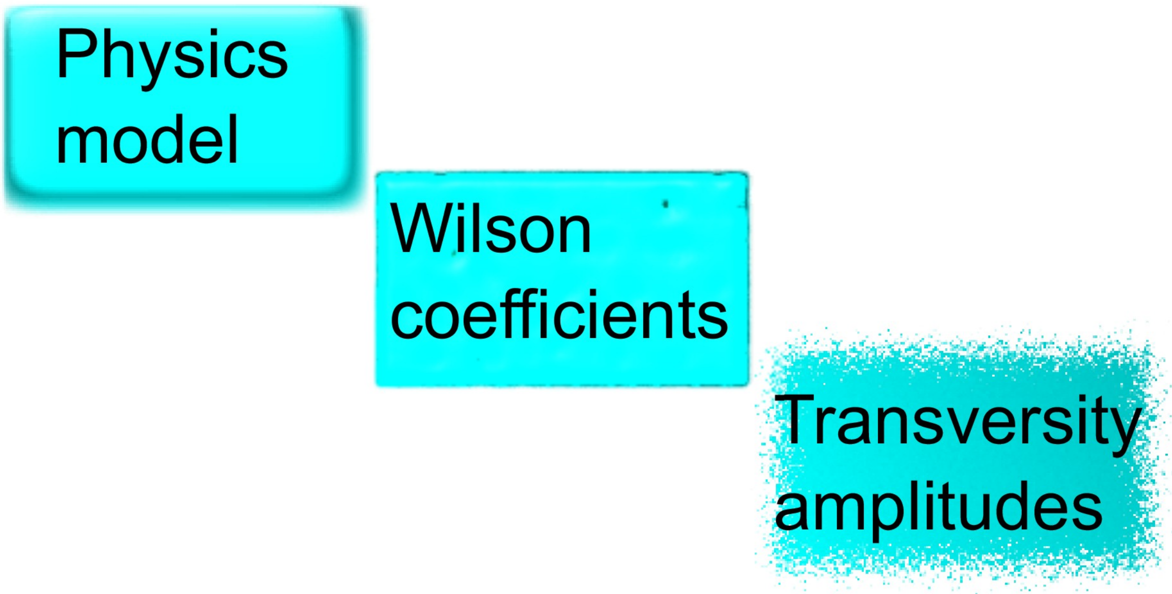
Then calculate the Wilson coefficients

Physics model

Wilson coefficients

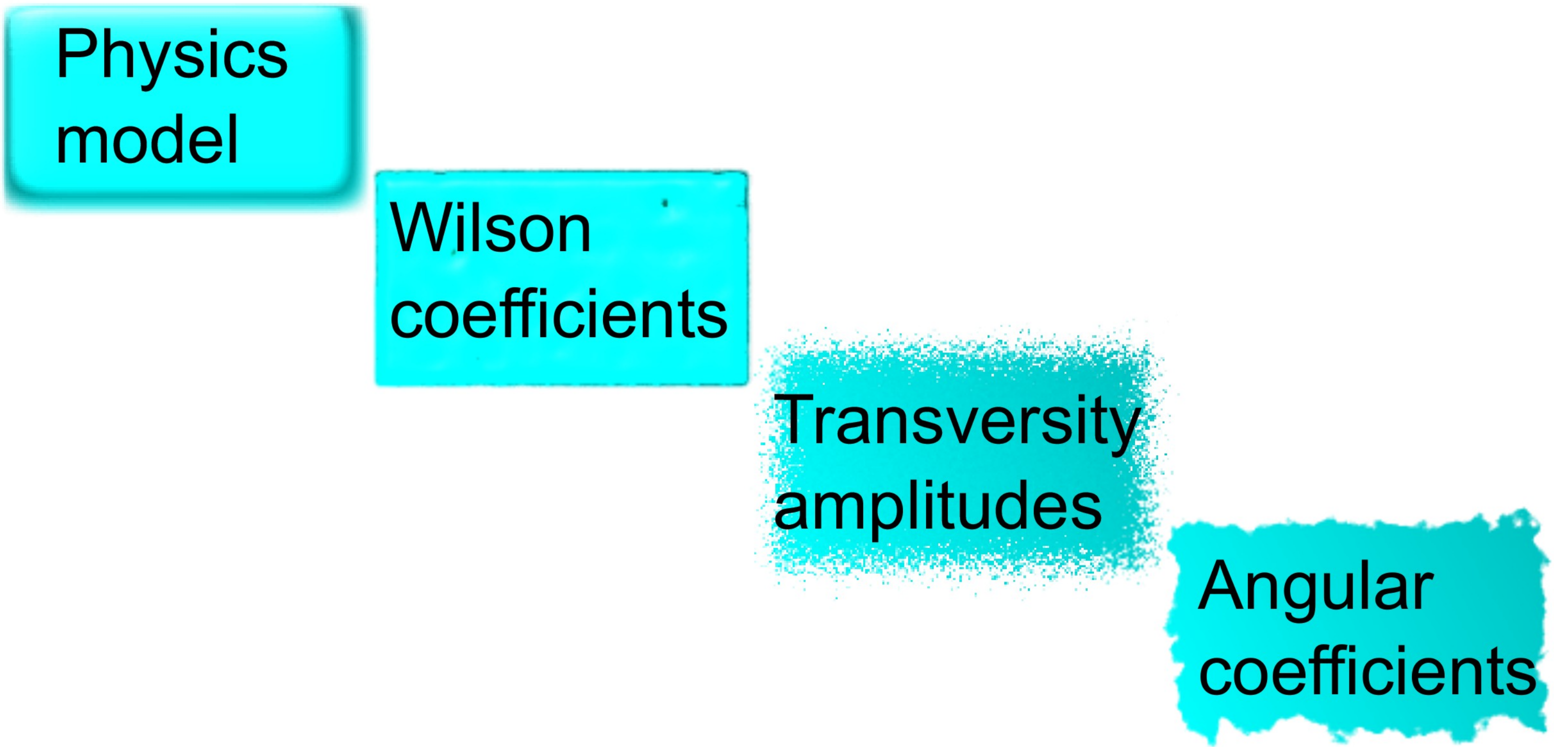
# From theory to measurements

To get to the transversity amplitudes involves form factors and unknown $\Lambda/m_b$ corrections



# From theory to measurements

Finally getting to the angular coefficients involves a loss of information



# From measurement to theory

Now from the experimental side we start with an all shiny set of angular coefficients

Angular coefficients

# From measurement to theory

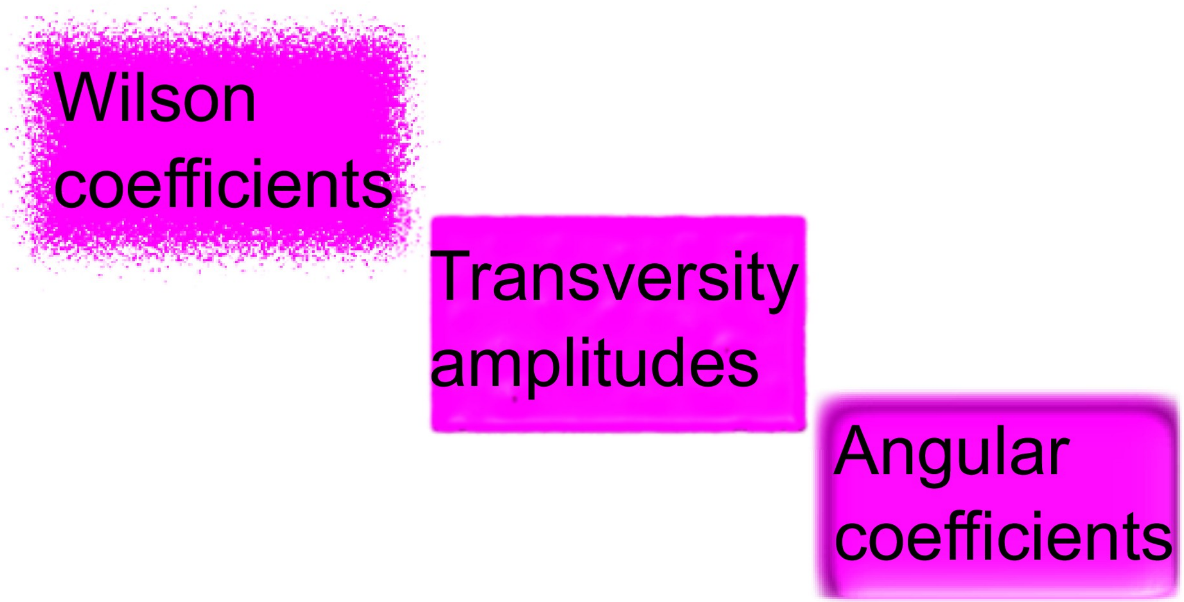
Getting to the transversity amplitudes is not a well defined operation due to symmetries

Transversity amplitudes

Angular coefficients

# From measurement to theory

Getting to the Wilson coefficients introduce the form factor uncertainties



# From measurement to theory

Finally extracting a specific physics model loses model independence.

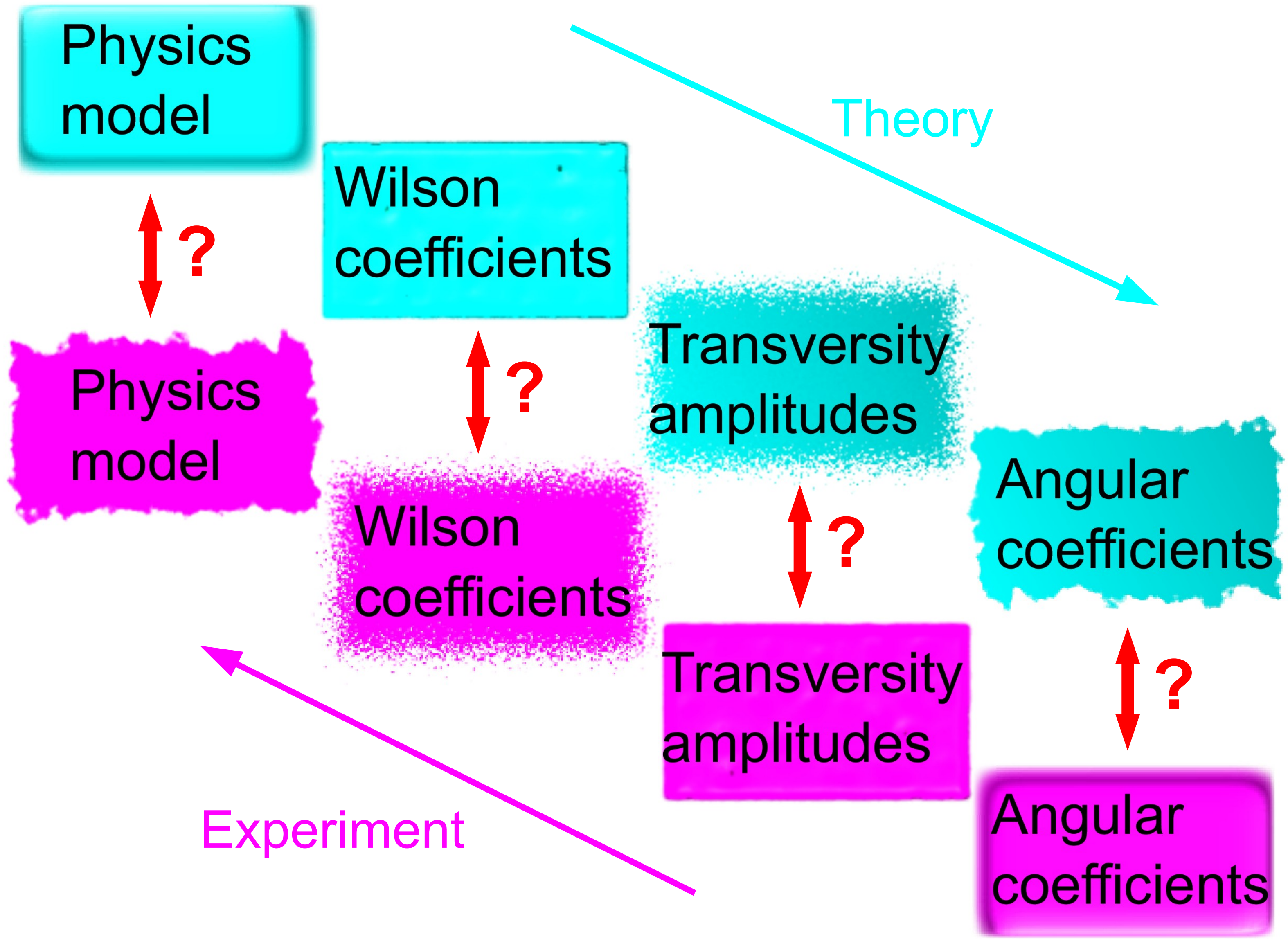
Physics model

Wilson coefficients

Transversity amplitudes

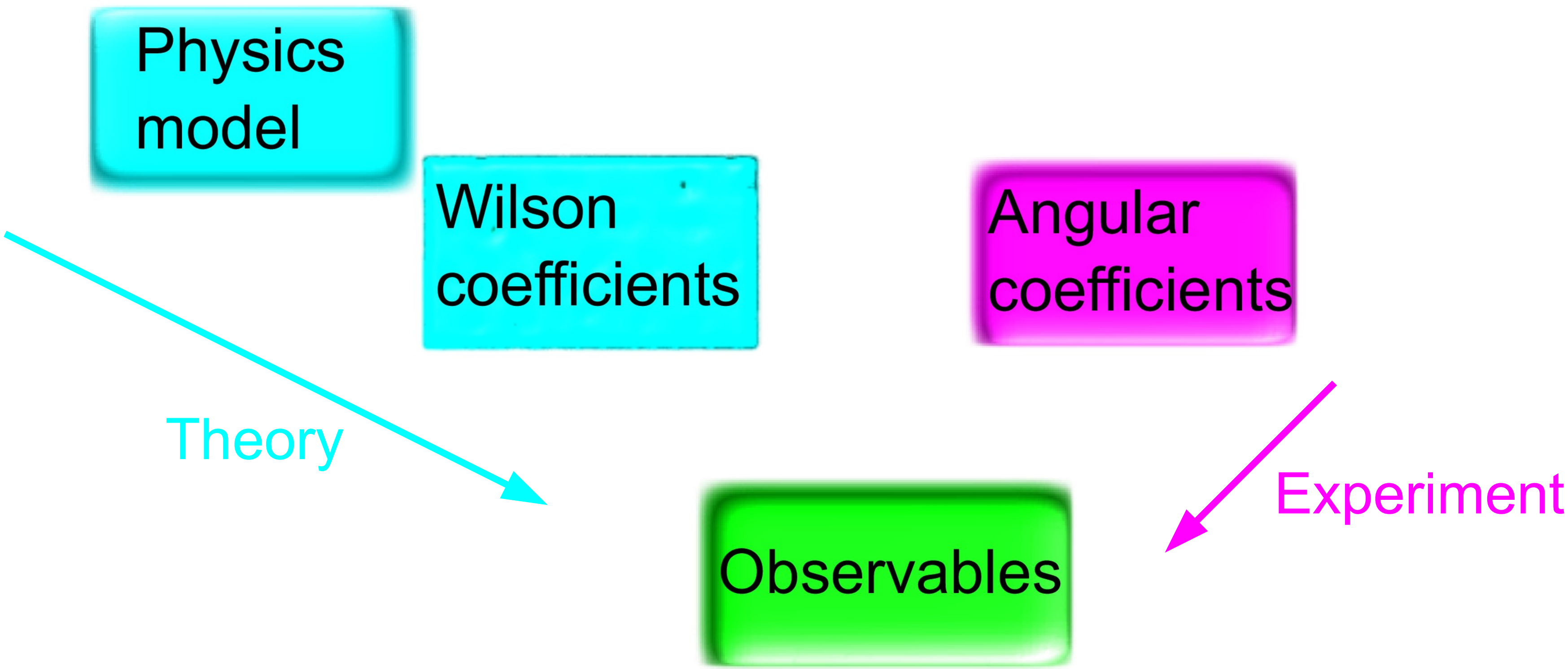
Angular coefficients

# How to compare?



## New observables

Create observables which are made with both theory and experiment in mind



# Constructing observables

New observables are constructed to satisfy multiple criteria

- Sensitivity to a given set of New Physics scenarios
- Form factors should cancel at leading order
- $\Lambda/m_b$ corrections under control
- Respect symmetries of decay
- Have good experimental sensitivity

# Form factor cancellation

In the large recoil region ($q^2$ small) the seven form factors in the decay reduce to the two universal form factors $\xi_\perp$ and $\xi_\parallel$.

Construct observables where they cancel at LO

An example is

Lunghi & Matias 2007

$$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2}$$

$$= \frac{2\left[\mathrm{Re}\left(\mathcal{C}_{10}'\mathcal{C}_{10}^*\right) + F^2\mathrm{Re}\left(\mathcal{C}_7'\mathcal{C}_7^*\right) + F\mathrm{Re}\left(\mathcal{C}_7'\mathcal{C}_9^*\right)\right]}{|\mathcal{C}_{10}|^2 + |\mathcal{C}_{10}'|^2 + F^2(|\mathcal{C}_7|^2 + |\mathcal{C}_7'|^2) + |\mathcal{C}_9|^2 + 2F\mathrm{Re}\left(\mathcal{C}_7\mathcal{C}_9^*\right)} \qquad F \equiv 2m_b m_B/q^2$$

In limit where $C_{10}'=0$, no complex phases, $C_7' \ll C_7$

$$A_T^{(2)} \sim 2\frac{C_7'}{C_7}$$

# Estimate effect of $\Lambda_{QCD}/m_b$ corrections

Write each amplitude as

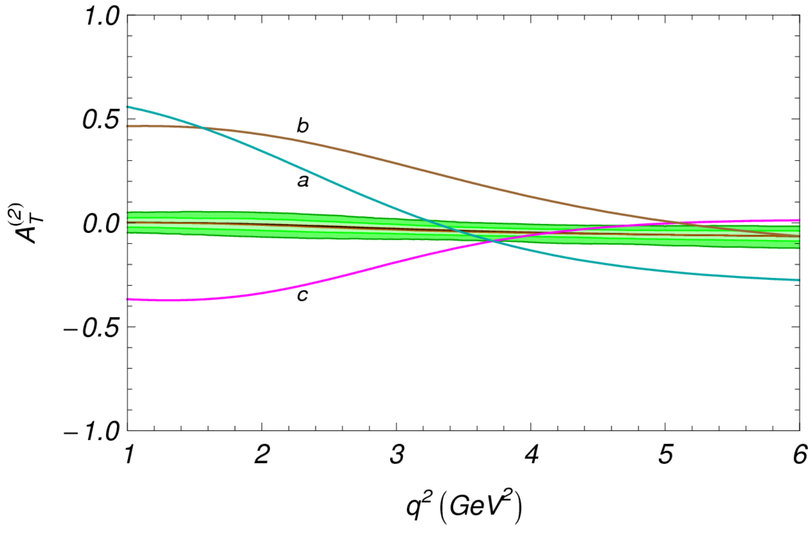
$$A_i' = A_i^{SM}(1 + C_i^{SM} e^{i\theta_i^{SM}}) + A_i^{NP}(1 + C_i^{NP} e^{i\theta_i^{NP}})$$

Sample using flat distribution for each $C_i$ and $\theta_i$

- Use 5% or 10% variation for $C_i$, $-\pi < \theta_i < \pi$
- Illustrates effect without making assumption about level

Mark error band as variation at given $q^2$ containing 68% of ensembles

$A_T^{(2)}$



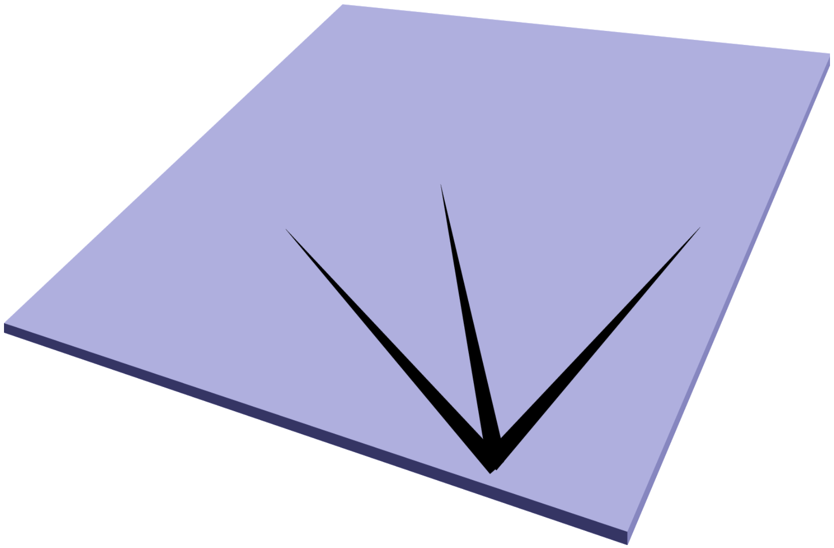

Green 5% and 10% $\Lambda/m_b$ uncertainty bands

# Symmetries

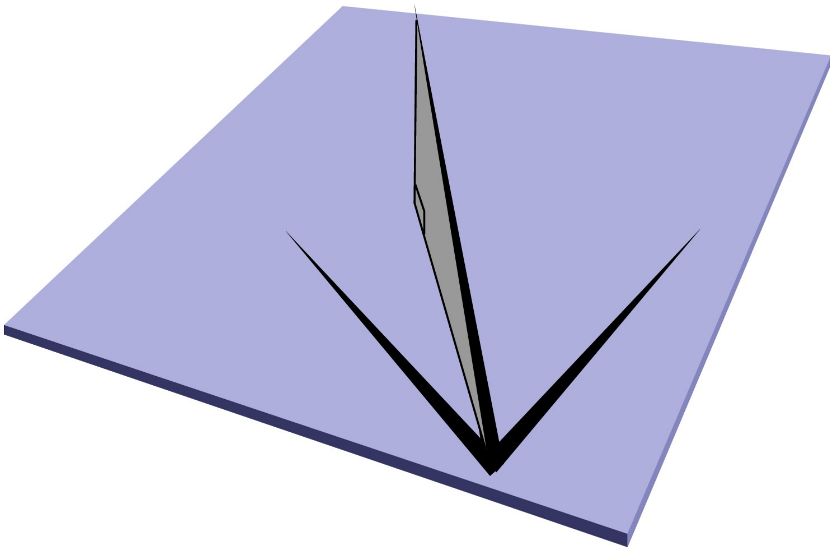
Look at differential of coefficients w.r.t. amplitudes

Example: $\vec{\nabla}(J_{1c}) = (0,0,0,0,2\mathrm{Re}(A_0^L),2\mathrm{Im}(A_0^L),0,0,0,0,2\mathrm{Re}(A_0^R),2\mathrm{Im}(A_0^R))$

If gradients span a lower number of dimensions than hyperspace, the amplitudes can't all be determined.



3rd gradient in plane



3rd gradient out of plane

# Symmetries

The number of symmetries depend on assumptions

| Case | Coefficients | Dependencies | Amplitudes | Symmetries |
|---|---|---|---|---|
| $m_\ell = 0,\ A_S = 0$ | 11 | 3 | 6 | 4 |
| $m_\ell = 0$ | 11 | 2 | 7 | 5 |
| $m_\ell > 0,\ A_S = 0$ | 11 | 1 | 7 | 4 |
| $m_\ell > 0$ | 12 | 0 | 8 | 4 |

Of course $m_l$=0 is never true, but if $q^2$>>$m_l$, it will be an approximate symmetry that experimentally is equivalent

In massless case there are 3 dependencies between 11 non-zero coefficients.

$$J_{1s} = 3J_{2s} \qquad J_{1c} = -J_{2c} \qquad J_{1c} = 6\frac{(2J_{1s}+3J_3)\left(4J_4^2+J_7^2\right)+(2J_{1s}-3J_3)\left(J_5^2+4J_8^2\right)}{16J_1^{s\,2}-9\left(4J_3^2+J_6^{s\,2}+4J_9^2\right)} - 36\frac{J_{6s}(J_4J_5+J_7J_8)+J_9(J_5J_7-4J_4J_8)}{16J_{1s}^2-9\left(4J_3^2+J_{6s}^2+4J_9^2\right)}.$$

In total 8 meaningful observables

If ignored by experiments they will reduce their sensitivity

# Symmetries

For the massless case we know the explicit form of the symmetries

$$n_1 = (A_\|^L, A_\|^{R*}),$$
$$n_2 = (A_\perp^L, -A_\perp^{R*}),$$
$$n_3 = (A_0^L, A_0^{R*}),$$

$$n_i' = \begin{bmatrix} e^{i\phi_L} & 0 \\ 0 & e^{-i\phi_R} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cosh i\tilde{\theta} & -\sinh i\tilde{\theta} \\ -\sinh i\tilde{\theta} & \cosh i\tilde{\theta} \end{bmatrix} n_i$$

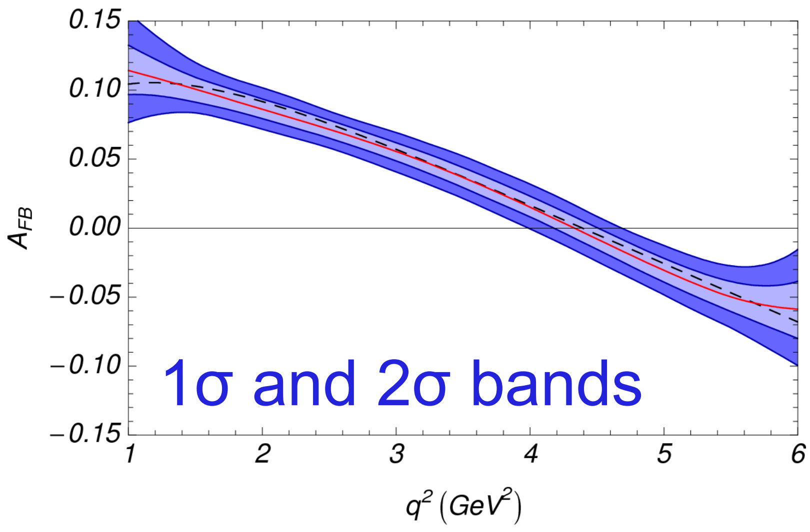
Consequence of symmetries are that some “observables” can't be observed!

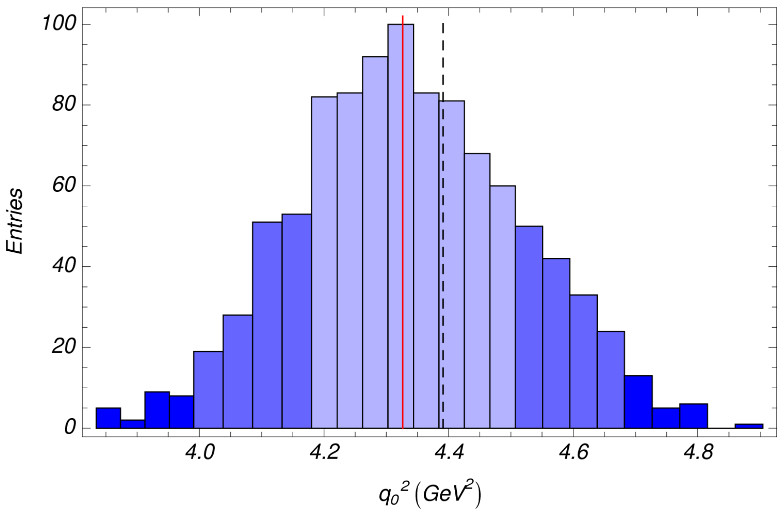
- $A_T^{(1)}$, polarisation asymmetry is not measurable!
- Introduce instead

$$A_T^{(5)} = \frac{|A_\perp^L A_\|^{R*} + A_\perp^{R*} A_\|^L|}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\|^L|^2 + |A_\|^R|^2}$$

# The experimental sensitivity

- From public LHCb information take
  - Signal yield, background rate
  - Assume flat angular acceptance
- Run an ensemble of toy MC studies to judge statistical sensitivity
  - Illustrated with 10 $fb^{-1}$ data for LHCb on $A_{FB}$ and zero point

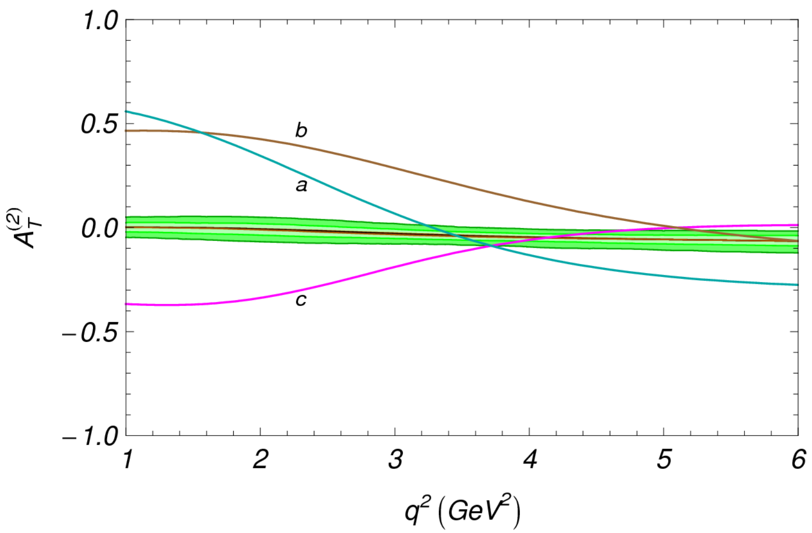


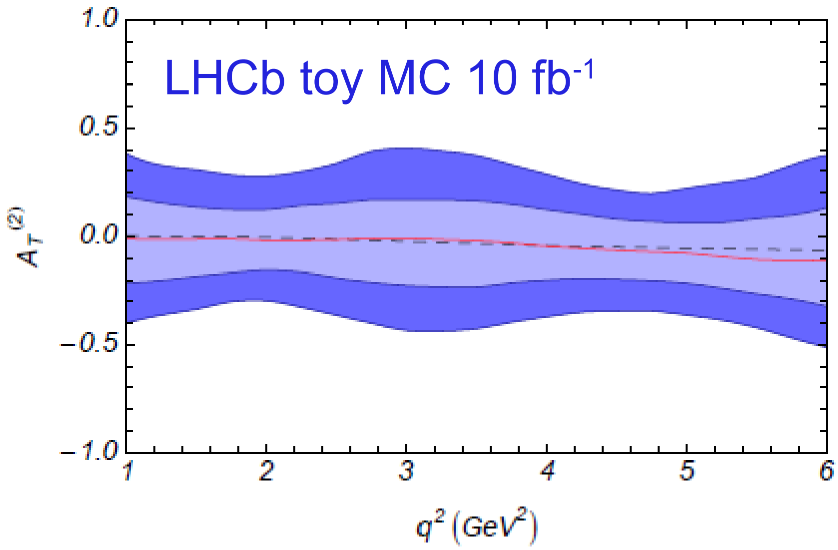



# $A_T^{(2)}$

Consider how AT(2) is changing when looking at different scenarios

- All currently experimentally allowed
- Good sensitivity to complex phases



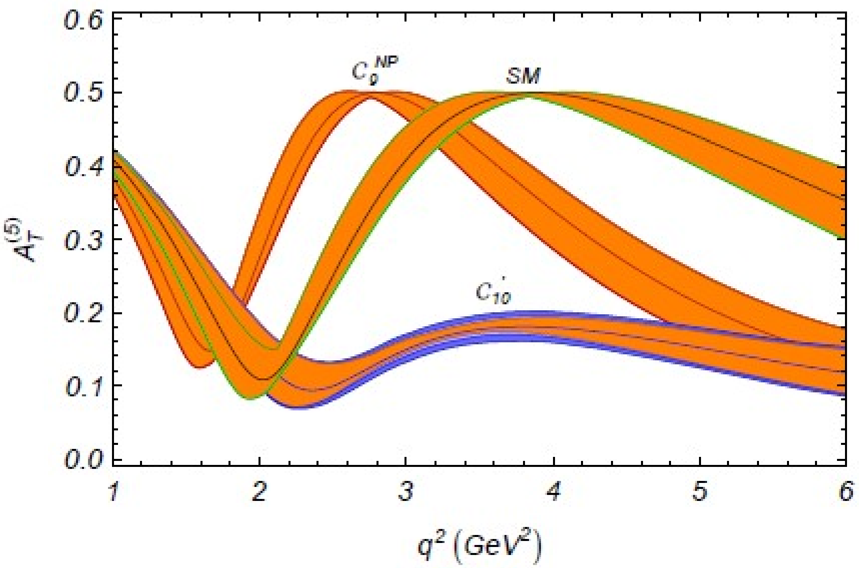



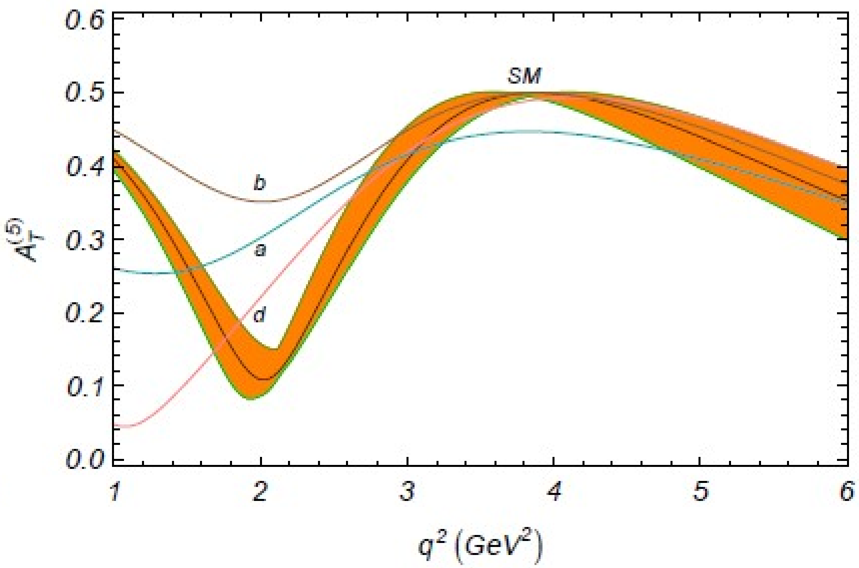

$$(\mathcal{C}_7^{\rm NP}, \mathcal{C}_7') = (0.26e^{-i\frac{7\pi}{16}}, 0.2e^{i\pi})$$
$$(0.07e^{i\frac{3\pi}{5}}, 0.3e^{i\frac{3\pi}{5}})$$
$$(0.03e^{i\pi}, 0.07)$$

# $A_T^{(5)}$

Measure $A_T^{(5)}$ as a function of $q^2$

Very different behaviours for different NP contributions




$$\mathcal{C}_{10}' = 3e^{i\frac{\pi}{8}}$$
$$\mathcal{C}_9^{\rm NP} = 2e^{i\frac{\pi}{8}}$$

$$(\mathcal{C}_7^{\rm NP}, \mathcal{C}_7') = (0.26e^{-i\frac{7\pi}{16}}, 0.2e^{i\pi})$$
$$(0.07e^{i\frac{3\pi}{5}}, 0.3e^{i\frac{3\pi}{5}})$$
$$(0.18e^{-i\frac{\pi}{2}}, 0)$$

# CP violation

The data can be considered separately for $B_d$ ($J_i$) and $B_d$ ($J_i$)

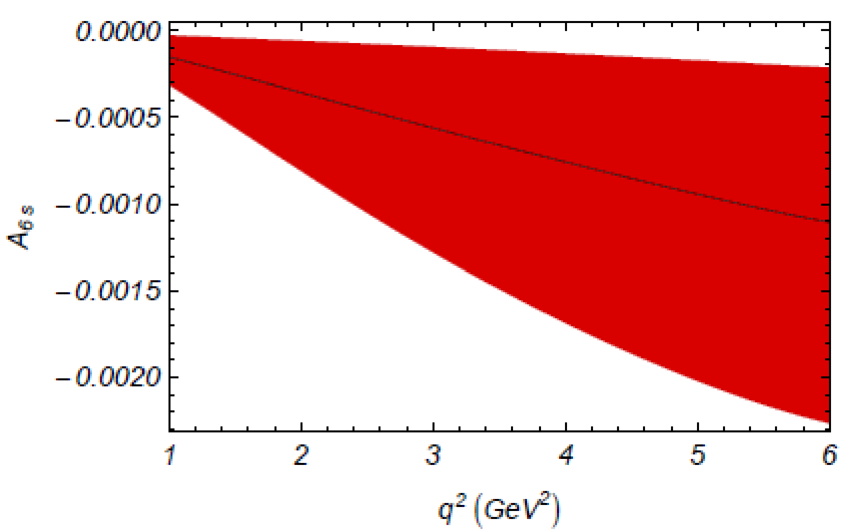
- Differences are a measure of CP violation
- In Altmannshofer et. al. (2009) a comprehensive study of CPV in angular coeficients

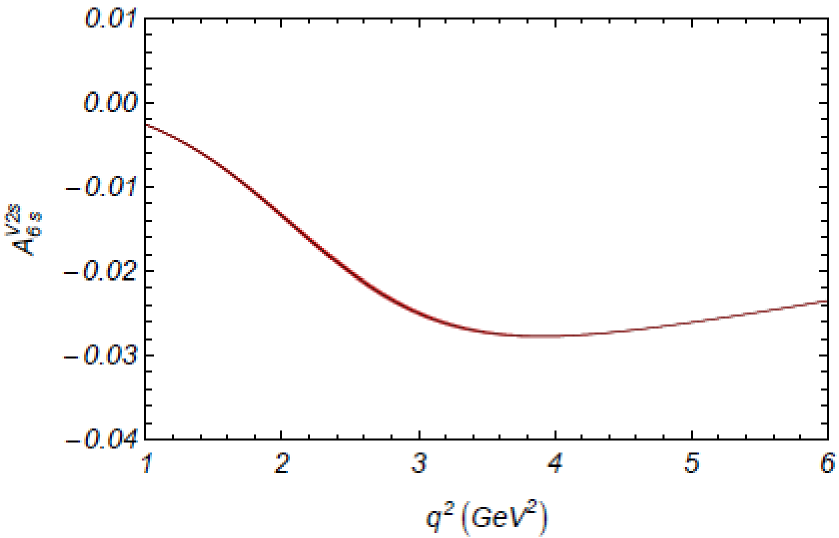
We have again taken approach of making normalisation that cancels FF's at LO

$$A_{6s}^{V2s} = \frac{J_{6s} - \bar{J}_{6s}}{J_{2s} + \bar{J}_{2s}} \qquad A_8^V = \frac{J_8 - \bar{J}_8}{J_8 + \bar{J}_8}$$

# CP violation

Example of use of explicit normalisation to cancels FF's




$$A_{6s} = \frac{J_{6s} - J_{6s}}{d(\Gamma + \overline{\Gamma})/dq^2}$$

$$A_{6s}^{V2s} = \frac{J_{6s} - J_{6s}}{J_{2s} + \bar{J}_{2s}}$$

Only theoretical error from FF's plotted

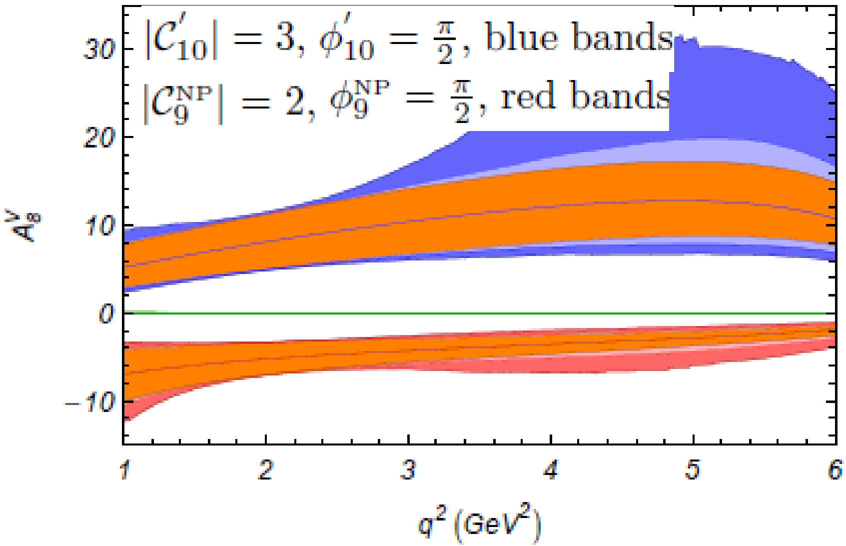
- Relative error drops dramatically

# CP violation

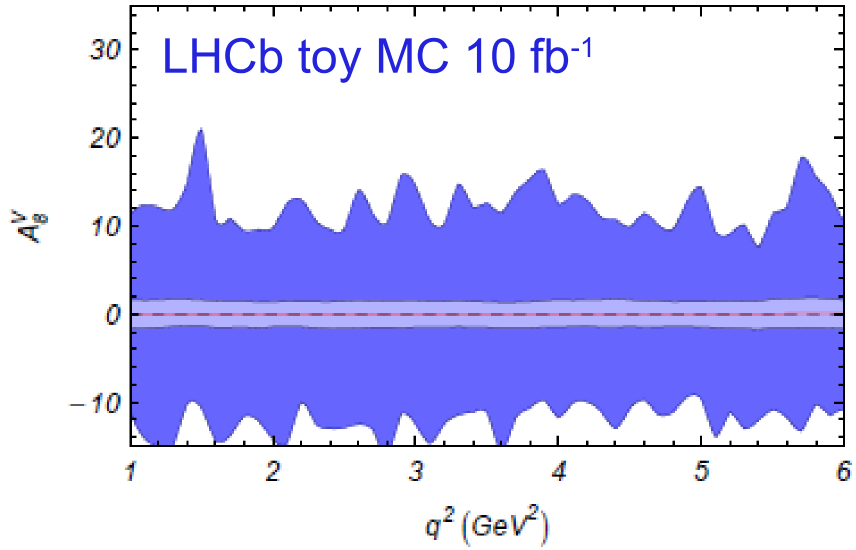
Λ/mb corrections are insignificant in SM, but very sizeable if NP CPV effects are large

In addition poor experimental uncertainty.

- Hard to see these will ever be useful observables
- The $A_T^{(i)}$ observables are more sensitive to CPV!




Theoretical uncertainties

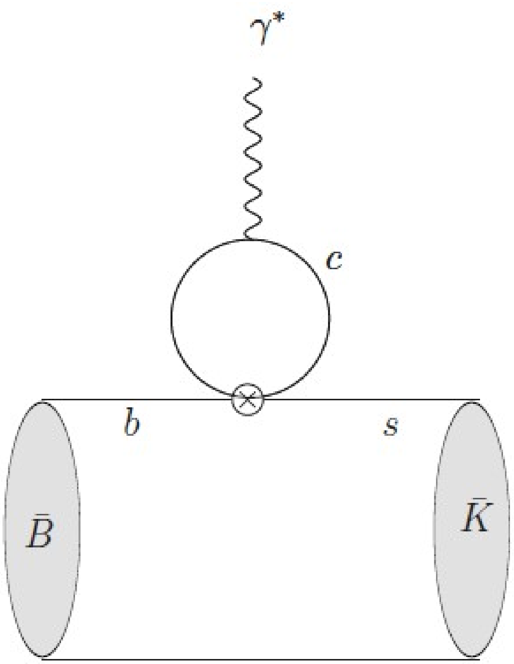



Experimental uncertainty

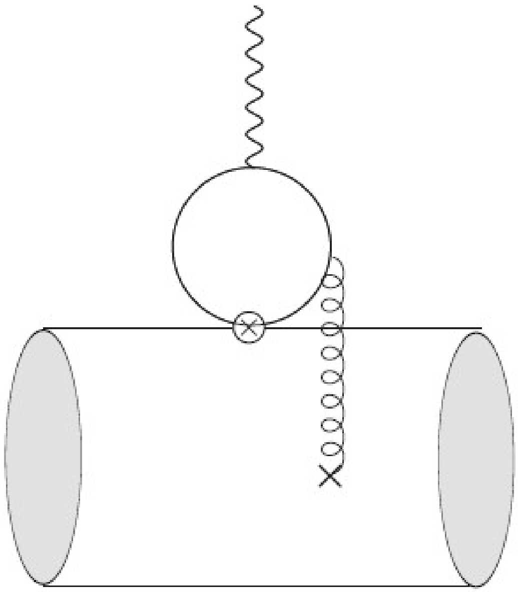
# Charm loops

Khodjamirian *et. al.* 2010

To go for region with $q^2$>6 $GeV^2$ require a better understanding of charm loops




Factorisable
LO effect



Non-Factorisable
soft gluon emission

Khodjamirian *et. al.* 2010

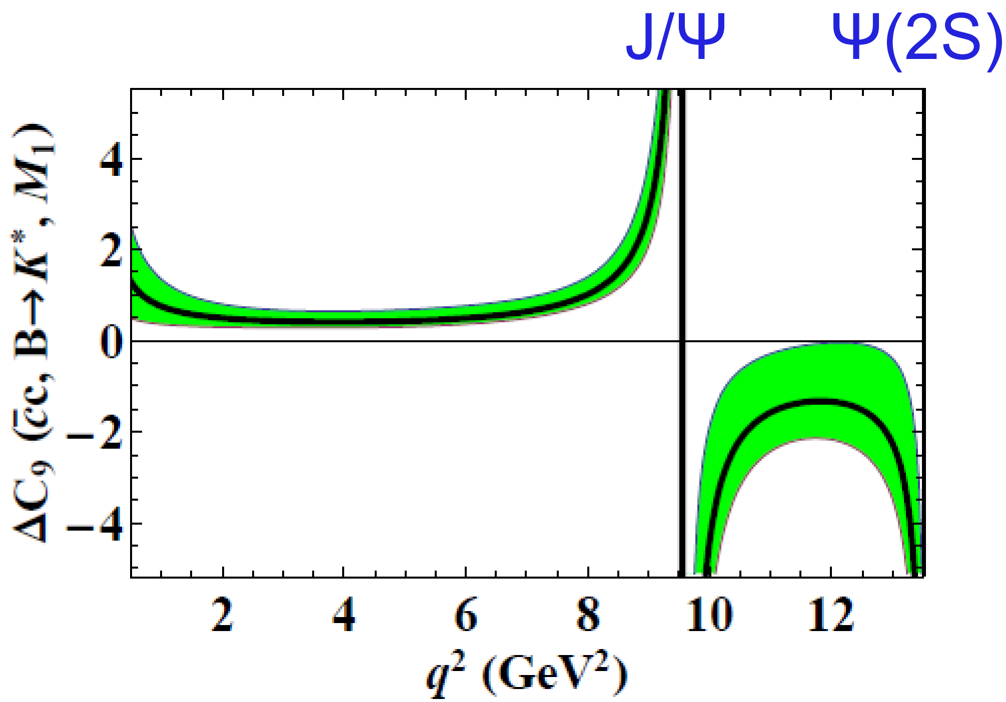
# Charm loops

Use of $J/\Psi K^{*0}$ and $\Psi(2S)K^{*0}$ measurements from data

Higher charmonium resonances and open charm treated as a single effective pole

Calculations considered valid upto $\Psi(2S)$ but not beyond

NLO parts not treated

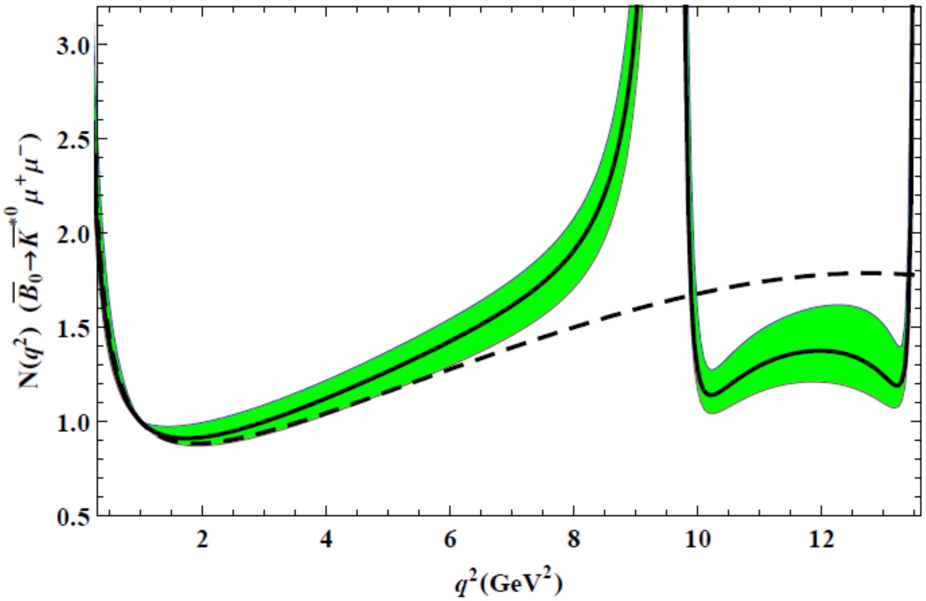


Khodjamirian *et. al.* 2010

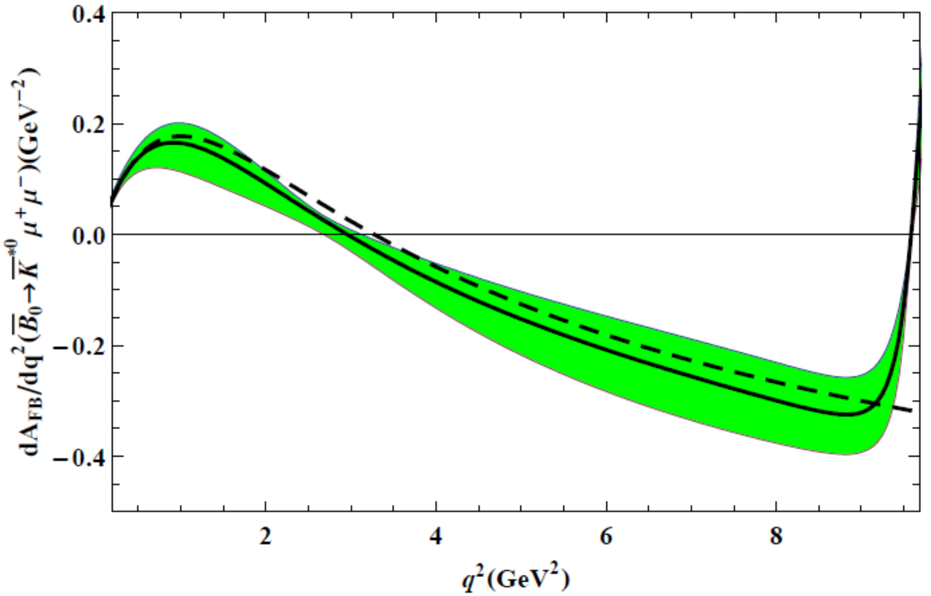
# Charm loops

Negative interference predicted between Ψ's

- Hump shape in differential cross section




For $A_{FB}$ the effect on absolute value is quite significant everywhere



Bobeth *et. al.* 2010

# Soft recoil region (large $q^2$)

Use HQET framework as applied by Grinstein and Pirjol (2004)

- Valid specificly in soft recoil region

Observables constructed in similar way to us

- System is very constrained at LO

$$\rho_1 \equiv \left| \mathcal{C}_9^{\text{eff}} + \kappa \frac{2\hat{m}_b}{\hat{s}} \mathcal{C}_7^{\text{eff}} \right|^2 + |\mathcal{C}_{10}|^2 ,$$

$$\rho_2 \equiv \text{Re}\left\{ \left( \mathcal{C}_9^{\text{eff}} + \kappa \frac{2\hat{m}_b}{\hat{s}} \mathcal{C}_7^{\text{eff}} \right) \mathcal{C}_{10}^* \right\}$$

$$A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}$$

$$H_T^{(1)} = \frac{\text{Re}(A_0^L A_\parallel^{L*} + A_0^{R*} A_\parallel^R)}{\sqrt{(|A_0^L|^2 + |A_0^R|^2)(|A_\parallel^L|^2 + |A_\parallel^R|^2)}} = \frac{\sqrt{2} J_4}{\sqrt{-J_2^c(2J_2^s - J_3)}} = 1$$

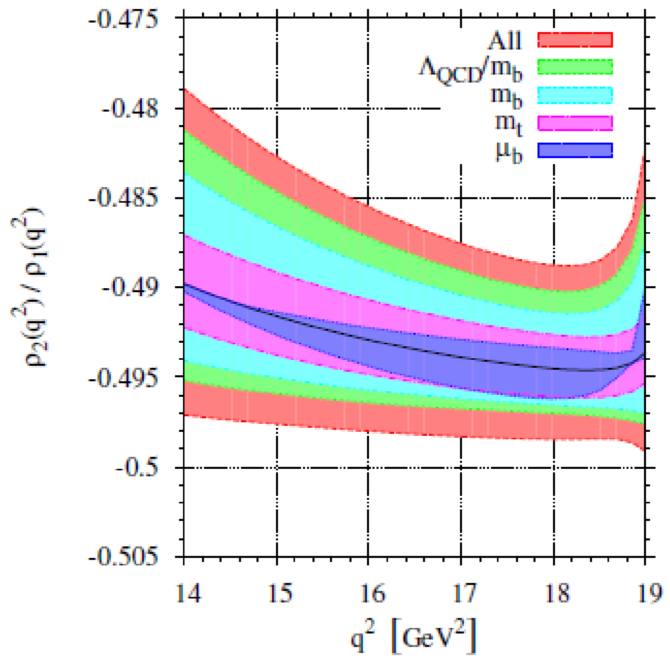
$$H_T^{(2)} = \frac{\text{Re}(A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R)}{\sqrt{(|A_0^L|^2 + |A_0^R|^2)(|A_\perp^L|^2 + |A_\perp^R|^2)}} = \frac{\beta_l J_5}{\sqrt{-2J_2^c(2J_2^s + J_3)}} = 2\frac{\rho_2}{\rho_1}$$

Bobeth *et. al.* 2010

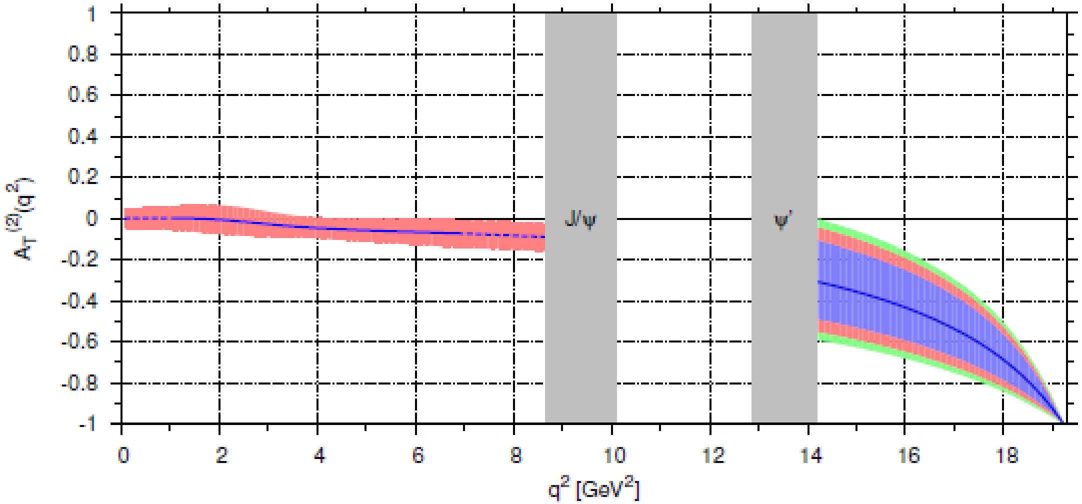
# Soft recoil region (large $q^2$)

Error on several observables estimated

New $H_T^{(1)}$ and $H_T^{(2)}$ has very small relative uncertainty




$A_T^{(2)}$ at high $q^2$ acts as cross check



# Conclusion

- When making measurements in $B_d \to K^{*0}\mu^+\mu^-$ great care has to be taken to
  - Minimise theoretical errors
  - Make observables that satisfy symmetries
- Framework developed for how to get such observables
  - Theoretical and experimental errors estimated
  - CPV observables have no experimental sensitivity
- Pending areas
  - The contribution of S-waves
  - Use of higher K* resonances
  - What will $B_s \to J/\psi\ \varphi$ add?