Resummed Massive Spectra in Heavy Quark Decays

work in progress

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Based on the work done in collaboration with:

U. Aglietti L. DiGiustino G. Ferrera A. Renzaglia G. Ricciardi

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Third Workshop on Theory, Phenomenology and Experiments in Heavy Flavour Physics

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Theoretical aspects of resummation in B decays and fragmentation processes

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Second Workshop on Theory, Phenomenology and Experiments in Heavy Flavour Physics

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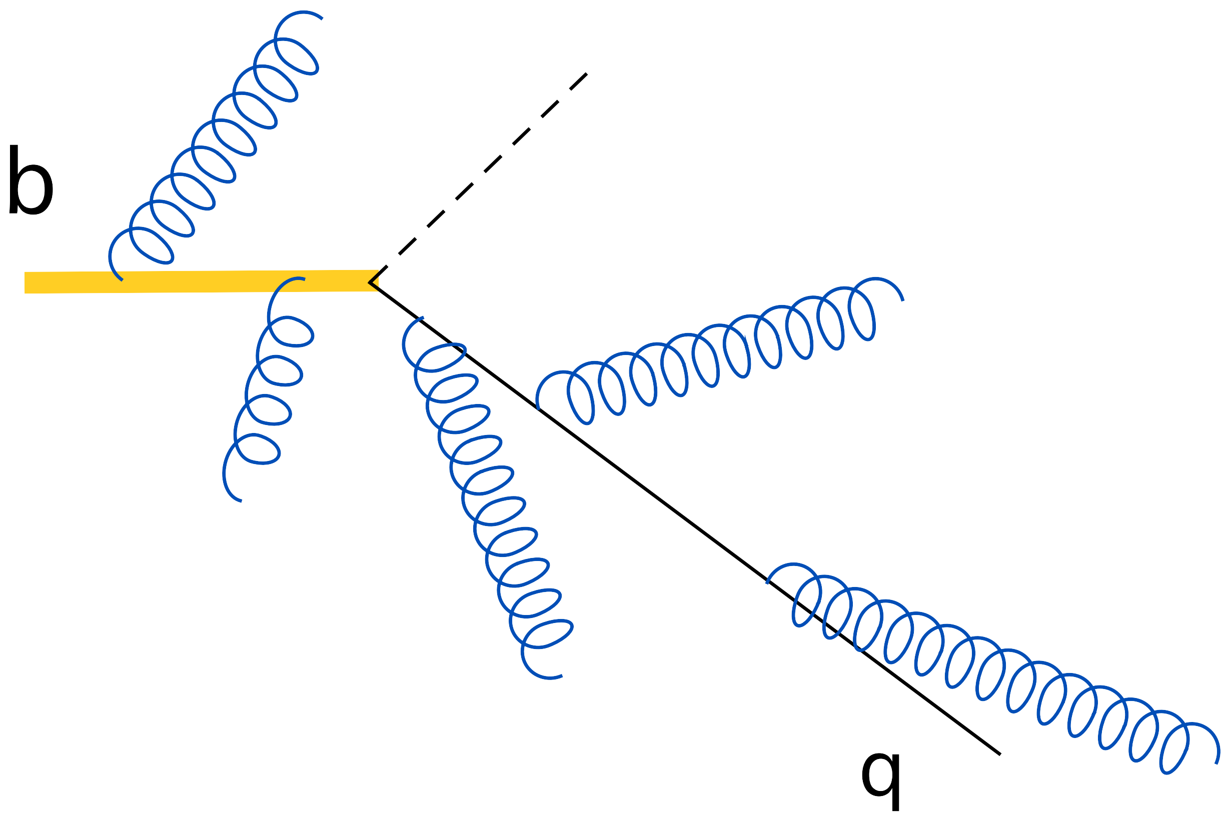
Outlook

- A short introduction to the resummation
- Resummation in b decays
- The massless final parton case
- The massive case
- A generalized “interpolation formula”
- A “not so clever” guess and the real thing
- Conclusions

Radiative and Semi-inclusive b decays



b -> q + non QCD

m_b E_x m_x

$Q=2E_x$ $m_b > E_x$

threshold region $m_x << E_x$ radiation inhibited

b - soft gluons - large angles q emission of soft and collinear gluons

$$\sum_{n=1}^{\infty}\sum_{k=1}^{2n} c_{nk}\,\alpha^n(Q)\,\log^k\frac{Q^2}{m_X^2} = c_{12}\,\alpha(Q)\,\log^2\frac{Q^2}{m_X^2}, + c_{11}\,\alpha(Q)\,\log\frac{Q^2}{m_X^2} + c_{24}\,\alpha^2(Q)\,\log^4\frac{Q^2}{m_X^2} + c_{23}\,\alpha^2(Q)\,\log^3\frac{Q^2}{m_X^2} + \cdots$$

$$\alpha(Q) = \alpha_S(Q)$$

$$2E_X = m_b\left(1 - \frac{q^2}{m_b^2} + \frac{m_X^2}{m_b^2}\right).$$

$$q^2 = 0$$

$$2E_X = m_b\left(1 + \frac{m_X^2}{m_b^2}\right) \simeq m_b$$

$$Q \approx m_b\ (radiative\ decay).$$

$$q^2 \sim O\left(m_b^2\right)$$

$$Q \simeq m_b$$

$$Q \ll m_b\ (semileptonic\ decay)$$

The large logarithms can be factorized into a QCD form factor which is universal: it depends only on the hadronic subprocess the difference between radiative and semileptonic enters in the "short distance" coefficient function

The Form Factor

$$\Sigma(u;\alpha) = 1 - \frac{C_F\,\alpha}{2\pi}L^2 + \frac{7\,C_F\,\alpha}{4\pi}L + O(\alpha^2),$$

$$L = \log\frac{Q^2}{m_X^2}$$

$$\Sigma\left(L,\alpha\right) = 1 + \sum_{n=1}^{\infty}\sum_{k=1}^{2n}\Sigma_{nk}\,\alpha^n\,L^k,$$

The Form Factor

$$\alpha^n L^{2n}$$

double logarithmic approximation

$$\Sigma(u;\alpha) = e^{-C_F\alpha/(2\pi)L^2}$$

$$\Sigma = e^G \qquad G(L;\alpha) = \sum_{n=1}^{\infty}\sum_{k=1}^{n+1} G_{nk}\alpha^n L^k$$

N-moments

$$\sigma_N(\alpha) \equiv \int_0^1 du\,(1-u)^{N-1}\sigma(u;\alpha)$$

$$G_N(\alpha) = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left\{\int_{Q^2(1-z)^2}^{Q^2(1-z)}\frac{dk_t^2}{k_t^2}A\left[\alpha(k_t^2)\right] + B\left[\alpha(Q^2(1-z))\right] + D\left[\alpha(Q^2(1-z)^2)\right]\right\}$$

$$A(\alpha) = \sum_{n=1}^{\infty} A_n\alpha^n = A_1\alpha + A_2\alpha^2 + A_3\alpha^3 + A_4\alpha^4 + \cdots \qquad B(\alpha) = \sum_{n=1}^{\infty} B_n\alpha^n = B_1\alpha + B_2\alpha^2 + B_3\alpha^3 + \cdots$$

$$A_1 = \frac{C_F}{\pi} \qquad B_1 = -\frac{3}{4}\frac{C_F}{\pi} \qquad D(\alpha) = \sum_{n=1}^{\infty} D_n\alpha^n = D_1\alpha + D_2\alpha^2 + D_3\alpha^3 + \cdots$$

$$A_2 = \frac{C_F}{\pi^2}\left[C_A\left(\frac{67}{36} - \frac{z(2)}{2}\right) - \frac{5}{18}n_f\right] \qquad B_2 = \frac{C_F}{\pi^2}\left[C_A\left(-\frac{3155}{864} + \frac{11}{12}z(2) + \frac{5}{2}z(3)\right) - C_F\left(\frac{3}{32} + \frac{3}{2}z(3) - \frac{3}{4}z(2)\right) + n_f\left(\frac{247}{432} - \frac{z(2)}{6}\right)\right]$$

$$A_3 = \frac{C_F}{\pi^3}\left[C_A^2\left(\frac{245}{96} + \frac{11}{24}z(3) - \frac{67}{36}z(2) + \frac{11}{8}z(4)\right) - C_A n_f\left(\frac{209}{432} + \frac{7}{12}z(3) - \frac{5}{18}z(2)\right) + \; - C_F n_f\left(\frac{55}{96} - \frac{z(3)}{2}\right) - \frac{n_f^2}{108}\right]$$

$$D_1 = -\frac{C_F}{\pi} \qquad D_2 = \frac{C_F}{\pi^2}\left[C_A\left(\frac{55}{108} - \frac{9}{4}z(3) + \frac{z(2)}{2}\right) + \frac{n_f}{54}\right]$$

The Form Factor "master formula"

$$\frac{1}{\Gamma}\int_0^u \frac{d^3\Gamma}{dxdwdu'}du' = C[x,w;\alpha(wm_b)]\,\Sigma[u;\alpha(wm_b)] + D[x,u,w;\alpha(wm_b)]$$

$$C(w;\alpha) = C^{(0)}(w) + \alpha C^{(1)}(w) + \alpha^2 C^{(2)}(w) + O(\alpha^3)$$
$$D(w;\alpha) = D^{(0)}(w) + \alpha D^{(1)}(w) + \alpha^2 D^{(2)}(w) + O(\alpha^3)$$

$$w = \frac{2E_X}{m_b}\ (0 \le w \le 2);\ x = \frac{2E_l}{m_b}\ (0 \le x \le 1)$$

Γ

Total width

leptonic semileptonic decays

radiative radiative decays

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process dependent
coefficient function

$$C(w;\alpha) = C^{(0)}(w) + \alpha C^{(1)}(w) + \alpha^2 C^{(2)}(w) + O(\alpha^3)$$
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process dependent coefficient function

universal factor

$$C(w;\alpha) = C^{(0)}(w) + \alpha C^{(1)}(w) + \alpha^2 C^{(2)}(w) + O(\alpha^3)$$
$$D(w;\alpha) = D^{(0)}(w) + \alpha D^{(1)}(w) + \alpha^2 D^{(2)}(w) + O(\alpha^3)$$

$$w = \frac{2E_X}{m_b}\ (0 \le w \le 2);\ x = \frac{2E_l}{m_b}\ (0 \le x \le 1)$$

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Total width

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process dependent coefficient function

universal factor

$$C(w;\alpha) = C^{(0)}(w) + \alpha C^{(1)}(w) + \alpha^2 C^{(2)}(w) + O(\alpha^3)$$
$$D(w;\alpha) = D^{(0)}(w) + \alpha D^{(1)}(w) + \alpha^2 D^{(2)}(w) + O(\alpha^3)$$

$$w = \frac{2E_X}{m_b}\ (0 \le w \le 2);\ x = \frac{2E_l}{m_b}\ (0 \le x \le 1)$$

Γ Total width

leptonic semileptonic decays

radiative radiative decays

The Form Factor "master formula"

$$\frac{1}{\Gamma}\int_0^u \frac{d^3\Gamma}{dxdwdu'}du' = C[x,w;\alpha(wm_b)]\,\Sigma[u;\alpha(wm_b)] + D[x,u,w;\alpha(wm_b)]$$

process dependent coefficient function

universal factor

remainder function

$$C(w;\alpha) = C^{(0)}(w) + \alpha C^{(1)}(w) + \alpha^2 C^{(2)}(w) + O(\alpha^3)$$
$$D(w;\alpha) = D^{(0)}(w) + \alpha D^{(1)}(w) + \alpha^2 D^{(2)}(w) + O(\alpha^3)$$

$$w = \frac{2E_X}{m_b}\ (0 \le w \le 2);\ x = \frac{2E_l}{m_b}\ (0 \le x \le 1)$$

Γ Total width

leptonic semileptonic decays

radiative radiative decays

Resummed Mass Distributions

jet initiated by massive partons

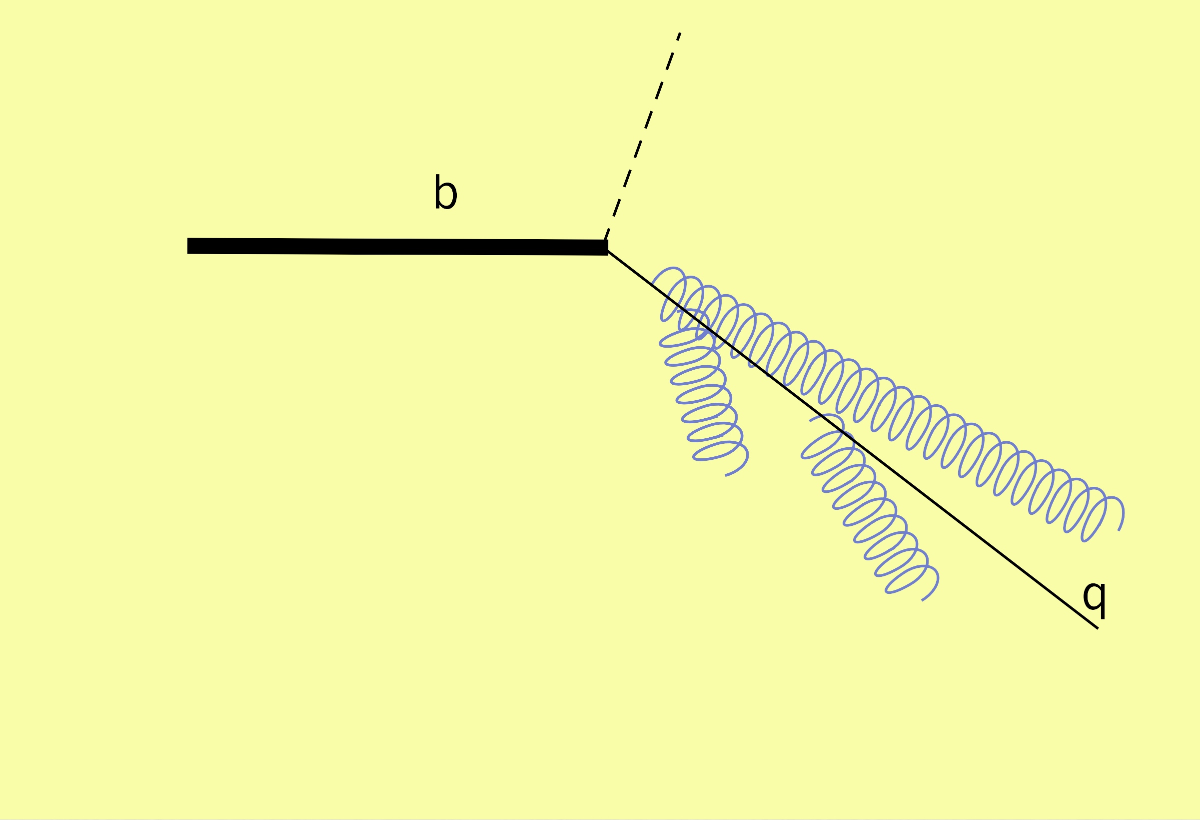
For massless partons
amplitudes contain terms proportional to:

$$\alpha_S \frac{dE}{E}\frac{d\theta^2}{\theta^2}$$

With the inclusion of a (final) massive parton
dead cone effect + soft quanta are radiated
isotropically in its rest frame:

$$\alpha_S \frac{d\theta^2}{\theta^2 + \frac{m^2}{\theta^2}}$$

$r \equiv \frac{m^2}{Q^2} \ll 1$ mass parameter



Let us compare massless and massive cases

Jet of mass m_X^2

$$m_X^2 = (1-z)Q^2$$

$$J_N(Q^2;m^2) = J_N(Q^2)\,\delta_N(Q^2;m^2)$$

$$m \ll Q.$$

$$\delta_N(Q^2;m^2) = \exp\int_0^1 dz \frac{z^{r(N-1)}-1}{1-z}\left\{-\int_{m^2(1-z)^2}^{m^2(1-z)} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] - B\left[\alpha\left(m^2(1-z)\right)\right] + \; + D\left[\alpha\left(m^2(1-z)^2\right)\right]\right\}$$

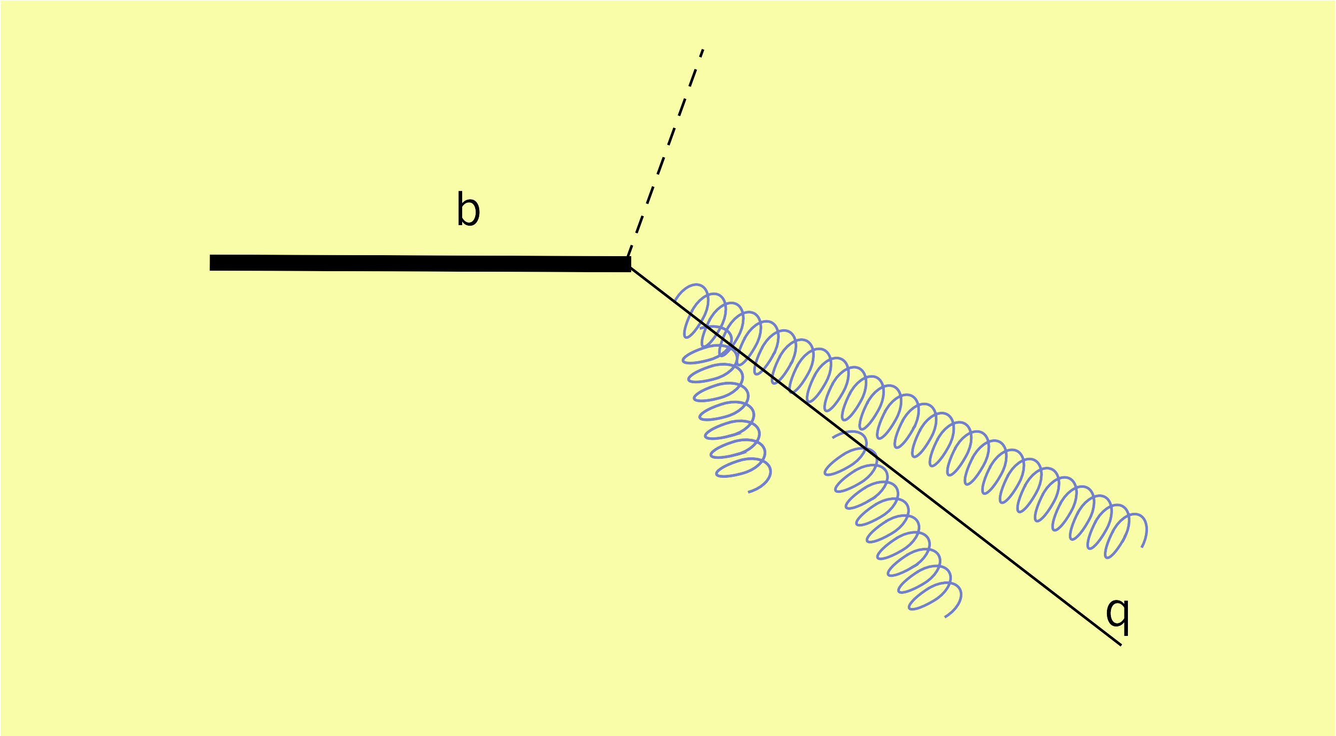
r(N-1) mass effects are "visible" for large $N \geq \dfrac{Q^2}{m^2} \gg 1$

$$J_N(Q^2) = \exp\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left\{\int_{Q^2(1-z)^2}^{Q^2(1-z)} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] + B\left[\alpha\left(Q^2(1-z)\right)\right]\right\}$$

$$A(\alpha) = \sum_{n=1}^{\infty} A_n \alpha^n;\; B(\alpha) = \sum_{n=1}^{\infty} B_n \alpha^n;\; D(\alpha) = \sum_{n=1}^{\infty} D_n \alpha^n.$$

$m_X \gg m$ high jet mass - m neglected

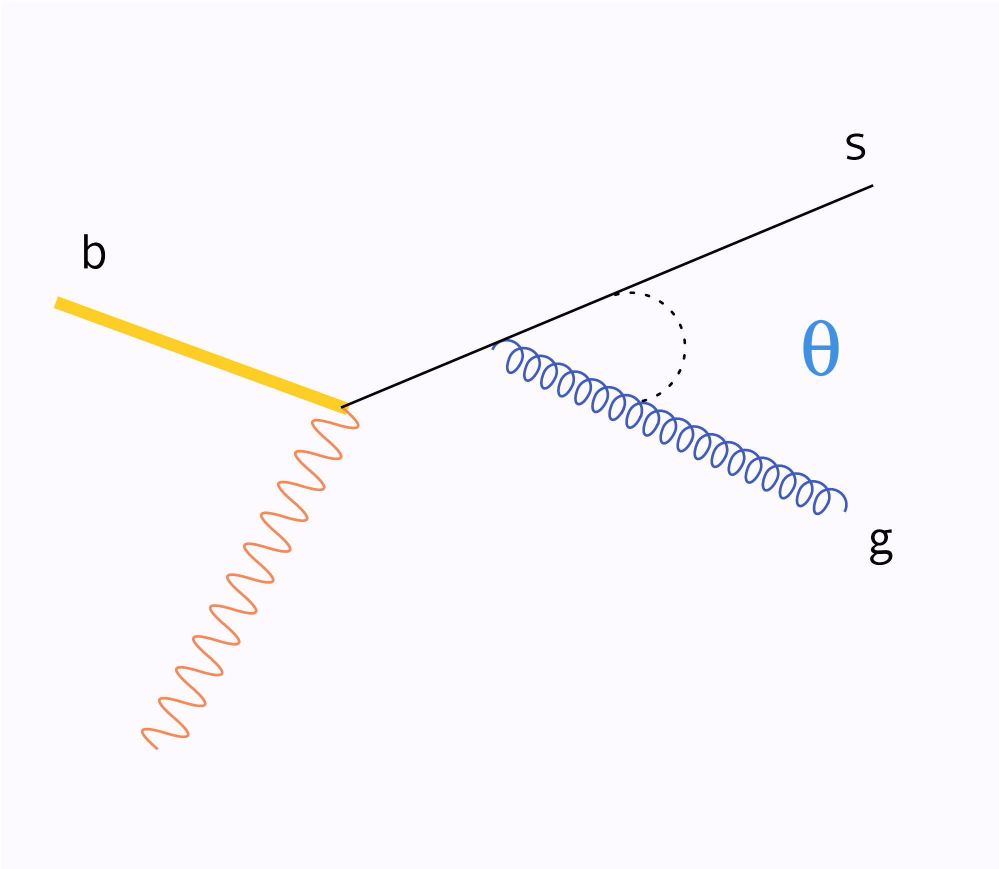
$m_X - m \ll m$ low jet mass and log r appear



$$t \equiv \frac{1-\cos\vartheta}{2}$$

$$b \longrightarrow s + g + \gamma$$

$$r \equiv \frac{m_s^2}{m_b^2}$$



$$\vartheta > \vartheta_{\text{min}} \equiv \frac{m_s}{E_s^{(0)}}$$

$$t > t_{\text{min}} = \frac{1-\cos\vartheta_{\text{min}}}{2} \simeq \left(\frac{\vartheta_{\text{min}}}{2}\right)^2 \simeq r$$

$$E_{Xs} \simeq E_s^{(0)} = \frac{m_b}{2}(1+r)$$

$$y \equiv \frac{m_{Xs}^2 - m_s^2}{m_b^2 - m_s^2}$$

$$y = (1-r)\,\omega\left[t + \frac{r}{1-r}\right] \cong \omega(t+r) \simeq \omega t \ for\ t > r$$

$$\omega \equiv \frac{2E_g}{m_b(1-r)}$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{dy} = \delta(y) + \frac{1}{\Gamma}\frac{d\Gamma^{(1)}}{dy} + \cdots$$

$$\frac{1}{\Gamma}\frac{d\Gamma^{(1)}}{dy} = \int_0^1 d\omega \int_r^1 dt \left[\frac{A_1\alpha}{\omega\, t} + \frac{D_1\alpha}{\omega} + \frac{D_1\alpha\, r}{\omega\, t^2} + \frac{B_1\alpha}{t}\right] \left[\delta(y-\omega t) - \delta(y)\right],$$

$$A_1 = \frac{C_F}{\pi}, \qquad D_1 = -\frac{C_F}{\pi}. \qquad B_1 = -\frac{3}{4}\frac{C_F}{\pi}.$$

$$\alpha \to \alpha(k_\perp^2),$$

$$\frac{\Gamma_N^{(1)}}{\Gamma} = \int_0^1 dy\,(1-y)^{N-1}\,\frac{1}{\Gamma}\frac{d\Gamma^{(1)}}{dy}.$$

$$k_\perp^2 \equiv (1-r)^2\, m_b^2\omega^2 t \simeq E_g^2\,\vartheta^2 \quad \text{for } \vartheta \ll 1.$$

$$\frac{\Gamma_N^{(1)}}{\Gamma} = \int_0^1 dy\left[(1-y)^{N-1} - 1\right]\int_0^1 d\omega\int_r^1 dt\left[\alpha(k_\perp^2)\frac{A_1}{\omega\, t} + \alpha(k_\perp^2)\frac{D_1}{\omega} + \alpha(k_\perp^2)\frac{D_1\, r}{\omega\, t^2} + \right.$$
$$\left. + \alpha(k_\perp^2)\frac{B_1}{t}\right]\delta(y-\omega t).$$

$$\left.\frac{1}{\Gamma}\frac{d\Gamma^{(1)}}{dy}\right|_{\mathrm{fo}} = -A_1\alpha\frac{\log(y+r)}{y} + 2D_1\alpha\frac{1}{y} + (B_1-D_1)\alpha\frac{1}{y+r}.$$

fixed order result
Ali Greub

Resummation to All Orders

$$\frac{\Gamma_N}{\Gamma} \simeq \exp \int_0^1 \frac{dy}{y} \left[(1-y)^{N-1} - 1 \right] \left\{ \int_{m_b^2 y^2}^{m_b^2 y \min[1, y/r]} \frac{dk_\perp^2}{k_\perp^2} A_1 \, \alpha\left(k_\perp^2\right) + \theta(y-r) B_1 \, \alpha\left(m_b^2 y\right) + \right.$$
$$\left. + D_1 \, \alpha\left(m_b^2 y^2\right) + \theta(r-y) D_1 \, \alpha\left(m_b^2 y^2 / r\right) \right\}.$$

$$A_1 \, \alpha \to A(\alpha), \qquad B_1 \, \alpha \to B(\alpha), \qquad D_1 \, \alpha \to D(\alpha),$$

$$\frac{\Gamma_N}{\Gamma} = \exp \int_0^1 \frac{dy}{y} \left[(1-y)^{N-1} - 1 \right] \left\{ \int_{m_b^2 y^2}^{m_b^2 y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] + B\left[\alpha\left(m_b^2 y\right)\right] + D\left[\alpha\left(m_b^2 y^2\right)\right] \right.$$
$$\left. + \theta(r-y) \left[- \int_{m_b^2 y^2 / r}^{m_b^2 y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] - B\left[\alpha\left(m_b^2 y\right)\right] + D\left[\alpha\left(m_b^2 y^2 / r\right)\right] \right] \right\}.$$

$$\delta_N(m_b^2; m_s^2) = \exp \int_0^r \frac{dy}{y} \left[(1-y)^{N-1} - 1 \right] \left\{ - \int_{m_b^2 y^2 / r}^{m_b^2 y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] + \right.$$
$$\left. - B\left[\alpha\left(m_b^2 y\right)\right] + D\left[\alpha\left(m_b^2 y^2 / r\right)\right] \right\}.$$

a specific process

$$B \to X_c + l + \nu_l \qquad\qquad r \gtrsim m_c^2/m_b^2 \approx 0.1.$$

$$\alpha_S^n \log^k\left(\frac{m_{X_c}^2 - m_c^2}{m_b^2 - m_c^2}\right) \log^l\left(\frac{m_{X_c}^2}{m_b^2}\right) \qquad\qquad y \equiv \frac{m_{X_c}^2 - m_c^2}{Q^2 - m_c^2}$$

$$\rho \equiv \frac{m_c^2}{Q^2 - m_c^2}, \qquad\qquad Q \equiv E_{X_c} + |\vec{p}_{X_c}| \,,$$

$$\rho \simeq \frac{1 - u_c}{2u_c}. \qquad\qquad u_c = p_c/E_c$$

$$\sigma_N(\rho, Q^2) = \exp \int_0^1 dy \Big[(1-y)^{N-1} - 1\Big] \Bigg\{ \frac{1}{y} \int_{\frac{Q^2 y^2}{1+\rho}}^{\frac{Q^2 y^2}{y+\rho}} A\left[\rho; \alpha_S(k^2)\right] \frac{dk^2}{k^2} + \frac{1}{y} D\left[\alpha_S\left(\frac{Q^2 y^2}{1+\rho}\right)\right] + $$
$$ + \left(\frac{1}{y} - \frac{1}{y+\rho}\right) \Delta\left[\alpha_S\left(\frac{Q^2 y^2}{y+\rho}\right)\right] + \frac{1}{y+\rho} B\left[\alpha_S\left(\frac{Q^2 y^2}{y+\rho}\right)\right] \Bigg\}$$

$$A(\rho, \alpha_S) = \sum_{n=1}^{\infty} A^{(n)}(\rho)\, \alpha_S^n \,,$$

1. *very slow charm quark:* $u_c \gtrsim 0$ or, equivalently, $\rho \gg 1$.

$$\sigma_N(\rho; Q^2) \to 1 \qquad \text{for} \quad \rho \to +\infty ,$$

2. *non-relativistic charm quark.* $u_c \approx \frac{1}{3}$ or $\rho \approx 1$.

$$\sigma_{S,N}(\rho, Q^2) = \exp \int_0^1 \frac{dy}{y} \left[(1-y)^{N-1} - 1 \right] \left\{ \int_{\frac{Q^2 y^2}{1+\rho}}^{\frac{Q^2 y^2}{\rho}} A\left[\rho; \alpha_S(k^2)\right] \frac{dk^2}{k^2} + D\left[\alpha_S\left(\frac{Q^2 y^2}{1+\rho}\right)\right] + \Delta\left[\alpha_S\left(\frac{Q^2 y^2}{\rho}\right)\right] \right\}$$

3. *fast charm quark:* $u_c \lesssim 1$ or $\rho \ll 1$.

$$\sigma_N(0, Q^2) = \exp \int_0^1 \frac{dy}{y} \left[(1-y)^{N-1} - 1 \right] \left\{ \int_{Q^2 y^2}^{Q^2 y} A\left[\alpha_S(k^2)\right] \frac{dk^2}{k^2} + D\left[\alpha_S\left(Q^2 y^2\right)\right] + B\left[\alpha_S\left(Q^2 y\right)\right] \right\};$$

$$y \to u = \frac{E_X - p_X}{E_X + p_X} \simeq \frac{m_X^2}{4E_X^2}; \qquad Q \to E_X + p_X \simeq 2E_X ,$$

$$\sigma_N(\rho, Q^2) \simeq \sigma_N(0, Q^2)\, \delta_N(\rho, Q^2) \qquad \text{for} \quad \rho \ll 1 ,$$

$$\delta_N(\rho, Q^2) = \exp \int_0^1 dy \frac{(1-y)^{\rho(N-1)} - 1}{y} \left\{ - \int_{\rho Q^2 y^2}^{\rho Q^2 y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] - B\left[\alpha\left(\rho Q^2 y\right)\right] + D\left[\alpha\left(\rho Q^2 y^2\right)\right] \right\}.$$

massless case

"jet function" $J(y;Q^2)$

probability that a massless parton produced in a hard process at the scale Q fragments into an hadronic jet of mass m_X

$$m_X^2 = y\,Q^2$$

$$J(y;\,Q^2) = \delta(y) - A_1\,\alpha\left(\frac{\log y}{y}\right)_+ + B_1\,\alpha\left(\frac{1}{y}\right) + O\left(\alpha^2\right)$$

$y \ll 1$

J(y;Q) expansion coefficients become large

resummation to all orders

$$J_N(Q^2) = \int_0^1 dy\,(1-y)^{N-1}\,J(y;\,Q^2).$$

$$J_N(Q^2) = \exp\int_0^1 dy\,\frac{(1-y)^{N-1}-1}{y}\left\{\int_{Q^2y^2}^{Q^2y}\frac{dk_\perp^2}{k_\perp^2}A\left[\alpha\left(k_\perp^2\right)\right] + B\left[\alpha\left(Q^2y\right)\right]\right\}.$$

massive case

generalized jet function $J(y; Q^2, m^2)$

$$y = \frac{m_X^2 - m^2}{Q^2 - m^2}.$$

$$R(y; Q^2, m^2) \equiv \int_0^y J(y'; Q^2, m^2)\, dy'.$$

1. *high* jet mass: $y \gg r$

$$R(y) = 1 - \int_y^1 J(y'; Q^2, m^2)\, dy' = 1 - \frac{1}{2} A_1 \alpha \log^2 y + B_1 \alpha \log y$$

2. *low* jet mass: $y \ll r$

$$R(y) = 1 - A_1 \alpha \log y \log r + \frac{A_1}{2} \alpha \log^2 r + D_1 \alpha \log y + (B_1 - D_1) \alpha \log r$$

$$J_N(Q^2;m^2) = J_N(Q^2)\,\delta_N(Q^2;m^2)\,,$$

$$y \equiv \frac{m_X^2 - m^2}{Q^2 - m^2}\,; \qquad r \equiv \frac{m^2}{Q^2} \cong \frac{m^2}{Q^2 - m^2} \ll 1\,.$$

$$J_N(Q^2) = \exp \int_0^1 dy \, \frac{(1-y)^{N-1} - 1}{y} \left\{ \int_{Q^2y^2}^{Q^2y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha_S\left(k_\perp^2\right)\right] + B\left[\alpha_S\left(Q^2y\right)\right] \right\},$$

$$\delta_N(Q^2;m^2) = \exp \int_0^1 dy \, \frac{(1-y)^{r\,(N-1)} - 1}{y} \left\{ -\int_{m^2y^2}^{m^2y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] - B[\alpha_S(m^2y)] + D[\alpha_S(m^2y^2)] \right\}.$$

$$A(\alpha_S) = \sum_{n=1}^{\infty} A_n\,\alpha_S^n\,; \qquad B(\alpha_S) = \sum_{n=1}^{\infty} B_n\,\alpha_S^n\,; \qquad D(\alpha_S) = \sum_{n=1}^{\infty} D_n\,\alpha_S^n\,,$$

A,B are related to small angle processes; the first three coefficients are known; giving a NNLLa

D is related to soft emission at large angles w.r. t. the quark; a process-dependent inter-jet quantity; much less accurately known

$r \ll 1$ mass logarithms become large → resummation to all orders

$$J_N(Q^2;\, m^2) \,=\, J_N(Q^2)\,\delta_N(Q^2;\, m^2),$$

$$\delta_N(Q^2;m^2) \;=\; \exp\int_0^1 dy \frac{(1-y)^{r\,(N-1)}-1}{y}\left\{-\int_{m^2y^2}^{m^2y}\frac{dk_\perp^2}{k_\perp^2}A\left[\alpha\left(k_\perp^2\right)\right]-B\left[\alpha\left(m^2y\right)\right]+\right.$$
$$\left.+\,D\left[\alpha\left(m^2y^2\right)\right]\right\}.$$

r (N-1) - mass effects “visible” only for large N

$D(\alpha)$ generalizes D_1 corresponds to soft radiation not collinearly enhanced characteristic of the massive parton

$$\lim_{r\to 0} J_N(Q^2;m^2) = J_N(Q^2)$$

infrared safe

Let us consider the academic
“Frozen Coupling Case”
to define the method to avoid the
Landau poles

The method according to:

ELSEVIER

Nuclear Physics B 478 (1996) 273–310

NUCLEAR PHYSICS B

The resummation of soft gluons in hadronic collisions

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Abstract

We compute the effects of soft-gluon resummation for the production of high mass systems in hadronic collisions. We carefully analyse the growth of the perturbative expansion coefficients of the resummation formula. We propose an expression consistent with the known leading and next-to-leading resummation results, in which the coefficients grow much less than factorially. We apply our formula to Drell–Yan pair production, heavy flavour production, and the production of high invariant mass jet pairs in hadronic collisions. We find that, with our formula, resummation effects become important only fairly close to the threshold region. In the case of heavy flavour production we find that resummation effects are small in the experimental configurations of practical interest.

$$\sigma(\tau, Q^2) = \int_0^1 dx\, dx_1\, dx_2 F(x_1) F(x_2) \delta(x x_1 x_2 - \tau) \Delta(x, Q^2)\,.$$

Drell-Yan

$$\sigma(\tau) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} F_N^2\, \Delta_N\, \tau^{-N}\, dN\,,$$

$$\sigma_N(Q^2) = F_N^2(Q^2)\, \Delta_N(Q^2)\,.$$

$$\ln \Delta_N(Q^2) = -\int_0^1 dx\, \frac{x^N - 1}{1-x} \left[2 \int_{(1-x)^2 Q^2}^{(1-x)Q^2} \frac{dq^2}{q^2} A(\alpha_S(q^2)) + B(\alpha_S((1-x)Q^2))\right] + \mathcal{O}(\alpha_S(\alpha_S \ln N)^k)\,, \qquad (2.4)$$

with

$$A(\alpha_S) = \frac{\alpha_S}{\pi} A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)}\,, \qquad B(\alpha_S) = \frac{\alpha_S}{\pi} B^{(1)}$$

where ($C_A = 3\,,\ C_F = 4/3\,,\ T_R = 1/2$ in QCD)

$$A^{(1)} = C_F\,, \quad A^{(2)} = \frac{1}{2} C_F K\,, \quad B^{(1)} = -\frac{3}{2} C_F\,,$$

$$K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9} T_R N_f\,.$$

$$\ln \Delta_N(Q^2) = \ln N \; g_1(b_0 \alpha_S \ln N) + g_2(b_0 \alpha_S \ln N) + \mathcal{O}(\alpha_S^k \ln^{k-1} N)$$

$$g_1(\lambda) = \; + \; \frac{A^{(1)}}{\pi b_0 \lambda}\Big[(1-2\lambda)\ln(1-2\lambda) - 2(1-\lambda)\ln(1-\lambda)\Big] \, ,$$

$$\begin{aligned} g_2(\lambda) = \; & + \; \frac{A^{(2)}}{\pi^2 b_0^2}\Big[2\ln(1-\lambda) - \ln(1-2\lambda)\Big] \\ & - \; \frac{B^{(1)}}{\pi b_0}\ln(1-\lambda) + \frac{2A^{(1)}\gamma_E}{\pi b_0}\Big[\ln(1-\lambda) - \ln(1-2\lambda)\Big] \\ & + \; \frac{A^{(1)} b_1}{\pi b_0^3}\left[\ln(1-2\lambda) - 2\ln(1-\lambda) + \frac{1}{2}\ln^2(1-2\lambda) - \ln^2(1-\lambda)\right] \, . \end{aligned} \tag{2.11}$$

$$\Delta_N = \exp(\, a \log^2 N \,) \, ,$$

$$a = C_F \alpha_S / \pi$$

where only the double logarithmic term has been kept in Δ_N

$$\Delta(x) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} \exp(a\log^2 N)\, x^{-N}\, dN$$
$$= -\frac{d}{dx}\left(\theta(1-\eta-x)\exp[a\log^2(1-x)]\right) \times (1 + \text{NLL terms})$$

neglecting NLL terms, we obtain the following expression for the cross section

$$\sigma(\tau) = \int_0^1 dx\, dx_1\, dx_2\, F(x_1)\, F(x_2)\delta(x\, x_1\, x_2 - \tau)\,\Delta(x)$$
$$= \int_\tau^1 dx\, \exp[a\log^2(1-x)]\frac{d}{dx}\mathcal{L}\left(\frac{\tau}{x}\right), \qquad (3.9)$$

smooth function of x as $x \to 1$, the nature of the divergence is given by the following integral

$$\int_0^1 \exp[a\log^2(1-x)]\, dx = \sum_{k=0}^{\infty}\frac{a^k}{k!}\int_0^1 \log^{2k} z\, dz = \sum_{k=0}^{\infty}\frac{a^k(2k)!}{k!}. \quad (2k)!/k! \approx 4^k\, k!.$$

in eq. (3.10) is irrelevant for this conclusion. It is known that factorially growing terms in the perturbative expansion are associated to power-like ambiguities in the resummed expression. In order to resum the asymptotic expansion, we should in fact truncate the series when the next term is of the same size as the current one, i.e. when $4ak = 1$. The error on the resummed expression is then of the order of the left over term

$$\delta = (4a)^k\, k! \approx (4a)^k\, k^k\, e^{-k} = e^{-\frac{1}{4a}}. \qquad (3.11)$$

smooth function of x as $x \to 1$, the nature of the divergence is given by the following integral

$$\int_0^1 \exp[a \log^2(1-x)]\, dx = \sum_{k=0}^{\infty} \frac{a^k}{k!} \int_0^1 \log^{2k} z\, dz = \sum_{k=0}^{\infty} \frac{a^k (2k)!}{k!}\,. \tag{3.10}$$

when $4ak = 1$. The error on the resummed expression is then of the order of the left over term

$$\delta = (4a)^k\, k! \approx (4a)^k\, k^k\, e^{-k} = e^{-\frac{1}{4a}}\,. \tag{3.11}$$

$$\delta \;=\; \left(\frac{\Lambda}{Q}\right)^{\frac{\pi b_0}{2\,C_F}} \qquad q\bar{q}, \quad \text{DIS},$$

Instead of truncating the perturbative expansion, we may achieve the same goal by putting a cut-off in the integral. In fact, consider the cut-off integral

$$\int_0^{x_0} dx\; \log^{2k} \frac{1}{1-x} = \int_0^{\log \frac{1}{1-x_0}} dt\; t^{2k}\, e^{-t}\,. \tag{3.15}$$

of the expansion. In order to have, as before, a truncation at $k = 1/(4a)$, we need to set the cut-off at $\log 1/(1-x_0) = 2k = 1/(2a)$, corresponding to

$$1 - x_0 = e^{-\frac{1}{2a}}\,. \tag{3.16}$$

In order to include also the subleading single-log terms we consider the full resummation formula

$$\Delta(x) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} \exp(\log N\, g_1(\alpha_S b_0 \log N))\, x^{-N}\, dN$$
$$= -\frac{d}{dx}\left(\theta(1-\eta-x)\exp[l\, g_1(\alpha_S b_0 l)]\right) \times (1 + \text{NLL terms})$$

Using commonly available algebraic programs, it is easy to expand eq. (3.22) up to large orders, and then study numerically the factorial growth. Expanding up to α_S^{32} we have found the behaviour $k! C_{(k)}^k (b_0 \alpha_S)^k$, where $C_{(k)}$ is a slowly increasing function of k. If, for large k, $C_{(k)}$ approaches a limiting value C, this corresponds to a power ambiguity of $(\Lambda/Q)^{2/C}$. For gluon fusion in the $\overline{\text{MS}}$ scheme we get $C_{(32)} = 10.48$,

$$k! C_{(k)}^k \; (b_0\alpha_S)^k \to C^k \; (b_0\alpha_S)^k \; k^k \; e^{-k}$$
$$k \to \infty$$

$$\delta = (C b_0 \alpha_S k)^k \; e^{-k} = e^{-\frac{1}{b_0 \alpha_S C}} = e^{-\frac{2}{C}\log\frac{Q}{\Lambda}} = \left(\frac{\Lambda}{Q}\right)^{\frac{2}{C}}$$

The Minimal Prescription Formula

$$\Delta_N = \sum_{k=0}^{\infty} c_k(\log N)\, \alpha_S^k$$

$$\boxed{\sigma_\tau = \frac{1}{2\pi i} \sum_{k=0}^{\infty} \alpha_S^k \int_{C-i\infty}^{C+i\infty} F_N^2(Q^2)\, c_k(\log N)\, \tau^{-N}\, dN \,. \ *}$$

$$\sigma_{\text{res}}(\tau) = \frac{1}{2\pi i} \int_{C_{\text{MP}}-i\infty}^{C_{\text{MP}}+i\infty} F_N^2(Q^2)\, \Delta_N(Q^2)\, \tau^{-N}\, dN \,, \quad 2 < C_{\text{MP}} < N_L \equiv \exp \frac{1}{2\alpha_S b_0} \,, \tag{4.3}$$

The expansion * converges asymptotically to the MP formula.

• The coefficients of the expansion * do not grow factorially.

• If we truncate the expansion * at the order at which its terms are at a minimum, the difference between the truncated expansion and the full MP formula is suppressed by a factor

$$e^{-\frac{H(1-\tau)}{\lambda}}$$

where H is a slowly varying positive function.

This suppression factor is stronger than any power suppression.

The resummation of logarithmic effects at threshold does not teach us anything about the structure of power corrections.
Resummation formulae should not, therefore, include any power correction.

In our Case

$$J_N(Q^2;\, m^2) \;=\; J_N(Q^2)\; \delta_N(Q^2;\, m^2)$$

$$J_N(Q^2) \;=\; e^{\,f_N\left(Q^2\right)},$$

$$f_N\left(\alpha_S\right) = \exp\left[L\, g_1\left(\lambda\right) + \sum_{n=0}^{\infty} \alpha_S^n\, g_{n+2}\left(\lambda\right)\right] = \exp\left[L\, g_1\left(\lambda\right) + \; g_2\left(\lambda\right) + \alpha_S\, g_3\left(\lambda\right) + \cdots\right]$$

$$\lambda \;=\; \beta_0\; \alpha_S(Q^2)\; L, \qquad\qquad L \;=\; \;\log N$$

$$\delta_N(Q^2;\, m^2) \;=\; e^{F_N\left(Q^2;m^2\right)},$$

$$F_N\left(Q^2; m^2\right) = L_r\; d_1\left(\rho\right) \,+\, \sum_{n=0}^{\infty} \alpha^n\, d_{n+2}\left(\rho\right) = L_r\, d_1\left(\rho\right) \,+\, d_2\left(\rho\right) \,+\, \alpha\, d_3\left(\rho\right) \,+\, \cdots$$

$$\rho \;\equiv\; \beta_0 \alpha(\mu^2)\, L_r, \;\; \text{and} \;\; L_r \;=\; \;\theta\left(N - 1/r\right)\, \log\left(N\, r\right)$$

where
$$g_1\left(\lambda;\frac{\mu^2}{Q^2}\right)=-\frac{A_1}{2\beta_0}\,\frac{1}{\lambda}\left[(1-2\lambda)\log(1-2\lambda)-2(1-\lambda)\log(1-\lambda)\right];$$

$$g_2\left(\lambda;\frac{\mu^2}{Q^2}\right)=+\frac{A_2}{2\beta_0^2}\left[\log(1-2\lambda)-2\log(1-\lambda)\right]+\frac{A_1\gamma_E}{\beta_0}\left[\log(1-2\lambda)-\log(1-\lambda)\right]+$$
$$-\frac{\beta_1 A_1}{4\beta_0^3}\left[\log^2(1-2\lambda)-2\log^2(1-\lambda)+2\log(1-2\lambda)-4\log(1-\lambda)\right]+$$
$$+\frac{D_1}{2\beta_0}\log(1-2\lambda)+\frac{B_1}{\beta_0}\log(1-\lambda)+\frac{A_1}{2\beta_0}\left[\log(1-2\lambda)-2\log(1-\lambda)\right]\log\frac{\mu^2}{Q^2}.$$

$$g_3\left(\lambda;\frac{\mu^2}{Q^2}\right)=-\frac{A_3}{2\beta_0^2}\left[\frac{\lambda}{1-2\lambda}-\frac{\lambda}{1-\lambda}\right]-\frac{A_1\zeta_2}{2}\left[\frac{4\lambda}{1-2\lambda}-\frac{\lambda}{1-\lambda}\right]+-\frac{A_1\beta_2}{4\beta_0^3}\left[\frac{2\lambda}{1-2\lambda}-\frac{2\lambda}{1-\lambda}+\log(1-2\lambda)\text{ -}\right.$$
$$+\frac{A_2\beta_1}{2\beta_0^3}\left[\frac{\log(1-2\lambda)}{1-2\lambda}-\frac{2\log(1-\lambda)}{1-\lambda}+\frac{3\lambda}{1-2\lambda}-\frac{3\lambda}{1-\lambda}\right]+-\frac{A_1\beta_1^2}{2\beta_0^4}\left[\frac{1}{2}\frac{\log^2(1-2\lambda)}{1-2\lambda}-\frac{\log^2(1-\lambda)}{1-\lambda}+\frac{\log(1-2\lambda}{1-2\lambda}\right.$$
$$+\frac{D_1\beta_1}{2\beta_0^2}\left[\frac{\log(1-2\lambda)}{1-2\lambda}+\frac{2\lambda}{1-2\lambda}\right]+\frac{B_1\beta_1}{\beta_0^2}\left[\frac{\log(1-\lambda)}{1-\lambda}+\frac{\lambda}{1-\lambda}\right]+-\frac{D_2}{\beta_0}\frac{\lambda}{1-2\lambda}-$$
$$\frac{B_2}{\beta_0}\frac{\lambda}{1-\lambda}-\frac{A_1\gamma_E^2}{2}\left[\frac{4\lambda}{1-2\lambda}-\frac{\lambda}{1-\lambda}\right]++\frac{A_1\beta_1\gamma_E}{\beta_0^2}\left[\frac{\log(1-2\lambda)}{1-2\lambda}-\frac{\log(1-\lambda)}{1-\lambda}+\frac{1}{1-2\lambda}-\frac{1}{1-\lambda}\right]+$$
$$-\frac{A_2\gamma_E}{\beta_0}\left[\frac{1}{1-2\lambda}-\frac{1}{1-\lambda}\right]-\frac{D_1\gamma_E 2\lambda}{1-2\lambda}-\frac{B_1\gamma_E\lambda}{1-\lambda}+-\frac{A_1}{2\beta_0}\left[\frac{2\lambda^2}{1-2\lambda}-\frac{\lambda^2}{1-\lambda}\right]\log^2\frac{\mu^2}{Q^2}-$$
$$\frac{A_2}{\beta_0^2}\left[\frac{\lambda}{1-2\lambda}-\frac{\lambda}{1-\lambda}\right]\log\frac{\mu^2}{Q^2}+-\frac{A_1\gamma_E}{\beta_0}\left[\frac{2\lambda}{1-2\lambda}-\frac{\lambda}{1-\lambda}\right]\log\frac{\mu^2}{Q^2}-\frac{D_1}{\beta_0}\frac{\lambda}{1-2\lambda}\log\frac{\mu^2}{Q^2}-$$
$$\frac{B_1}{\beta_0}\frac{\lambda}{1-\lambda}\log\frac{\mu^2}{Q^2}++\frac{A_1\beta_1}{\beta_0^3}\left[\frac{\lambda\log(1-2\lambda)}{1-2\lambda}-\frac{\lambda\log(1-\lambda)}{1-\lambda}+\frac{\lambda}{1-2\lambda}+-\frac{\lambda}{1-\lambda}+\frac{1}{2}\log(1-2\lambda)-\log(1-\lambda)\right]\log$$

Functions $d_i(\rho)$ represent mass effects, can be obtained from the standard ones $g_i(\lambda)$ of the massless case by means of the replacements:

$$A(\alpha) \to -A(\alpha); \quad B(\alpha) \to -B(\alpha); \quad D(\alpha) \to D(\alpha); \quad \log\frac{\mu^2}{Q^2} \to \log\frac{\mu^2}{m^2}; \quad \lambda \to \rho.$$

U. Aglietti G. Ricciardi G. Ferrera, Phys. Rev. D 74 (2006) 034004

The general inverse Mellin transform is defined as:

$$M^{-1}[f^\star(N);x] = f(x) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN\, x^{-N} f^\star(N)$$

C represents the integration path in the complex N-plane

The inverse Mellin transform of the product is:

$$M^{-1}[f^\star(N)g^\star(N);x] = \int_x^1 f\left(\frac{x}{u}\right) g(u)\frac{du}{u}$$

In the frozen coupling limit $\beta_0 \to 0$

$$g_1 = -\frac{A_1}{2\,\beta_0}\,\lambda$$

$$g_2 = \left(-\frac{B_1}{\beta_0} - \frac{D_1}{\beta_0} - \frac{A_1 \gamma_E}{\beta_0} \right) \lambda$$

$$g_3 = \left(-\frac{B_2}{\beta_0} - \frac{D_2}{\beta_0} - \frac{A_2 \gamma_E}{\beta_0} \right) \lambda.$$

and similarly for the $d_i(\rho)$

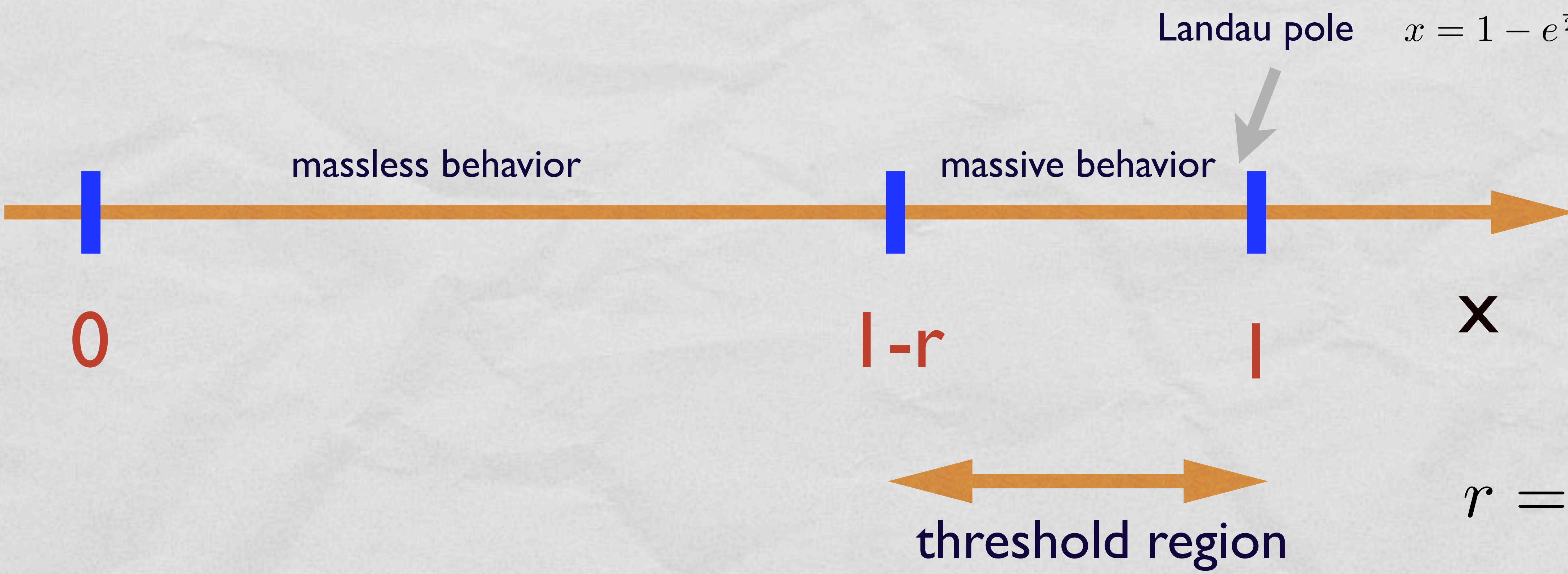
$$\boxed{\log J_N(Q^2, m^2) = L\; f_N(Q^2) + L_r\; F_N(Q^2, m^2) \simeq -\,\frac{A_1}{2\beta_0}\lambda\, L + \frac{A_1}{2\beta_0}\rho\, L_r}$$

$$\lambda \;=\; \beta_0\;\alpha_S(Q^2)\;L, \qquad\qquad L \;=\; \log N$$

$$\rho \equiv \beta_0 \alpha(\mu^2)\, L_r, \;\text{ and }\; L_r = \;\theta\,(N - 1/r)\,\log\,(N\,r)$$

$$\log J_N(Q^2, m^2) \simeq \frac{A_1}{2\beta_0}\;\log N \;\cdot\; \log r \quad , \qquad r = \frac{m^2}{Q^2}$$

scales and kinematics



$$x = 1 - e^{\frac{1}{2\alpha_S\ \beta_0}}$$

$$r = \frac{m^2}{Q^2}$$

$$x = 1 - y \qquad\qquad y = \frac{m_X^2 - m^2}{Q^2 - m^2}$$

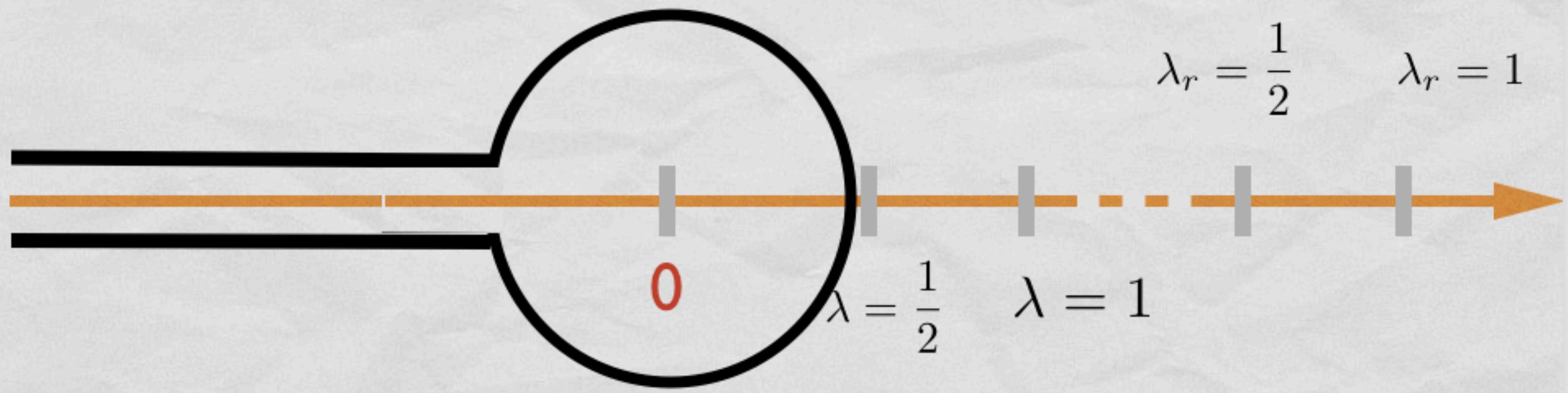
x likewise the Bjorken variable represents the inelasticity of the final state

$$x = \frac{2E_w}{m_b} = 1 - y \qquad\qquad x = \frac{Q^2}{2Q \cdot p}$$

$$\frac{\Gamma_N(x)}{\Gamma} = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN\; x^{-N}\; J_N(Q^2)\;\cdot\delta_N(Q^2;m^2)$$

$$J_N(Q^2;m^2) = J_N(Q^2)\;\cdot\delta_N(Q^2;m^2)$$

in our massive case the Minimal Prescription Formula consists in taking the contour as in the massless case



$$\frac{\Gamma_N(x)}{\Gamma} = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN\; x^{-N}\; J_N(Q^2;m^2)$$

$$J_N(Q^2;m^2) = e^{\frac{A_1}{2\beta_0}\log N\cdot\log r}$$

Finally

$$a = \frac{A_1}{2\beta_0}$$

$$\frac{d\Gamma(x)}{dx} = \frac{1}{2\pi i}\int_{C_{MP}-i\infty}^{C_{MP}+i\infty} dN\ x^{-N}\ e^{a\log N\cdot\log r}$$

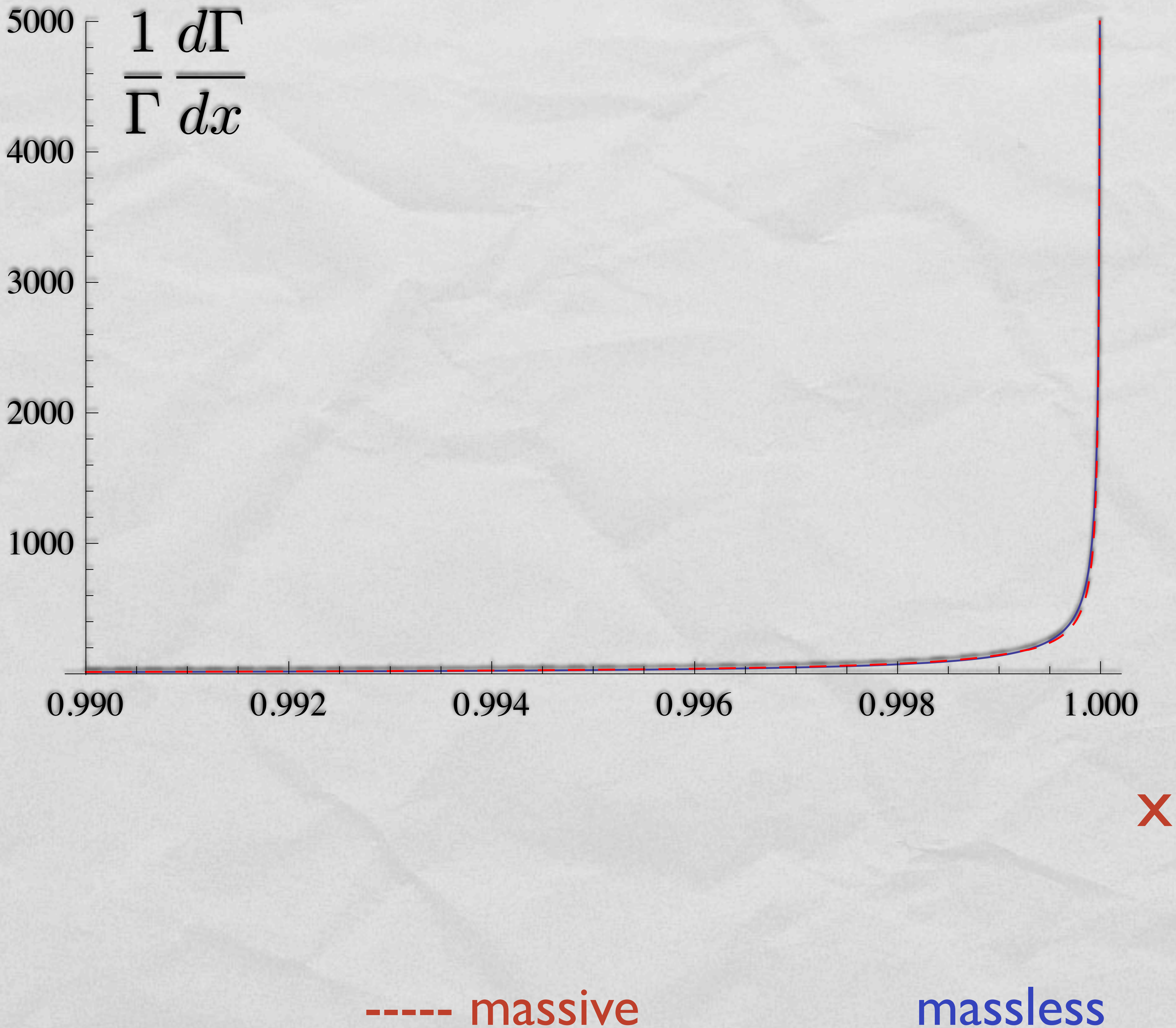
$$= -\frac{d}{dx}\theta(1-x)e^{a\log r\cdot\log\frac{1}{1-x}}\cdot(1+NLL\ \ terms)$$

Some Results in frozen coupling

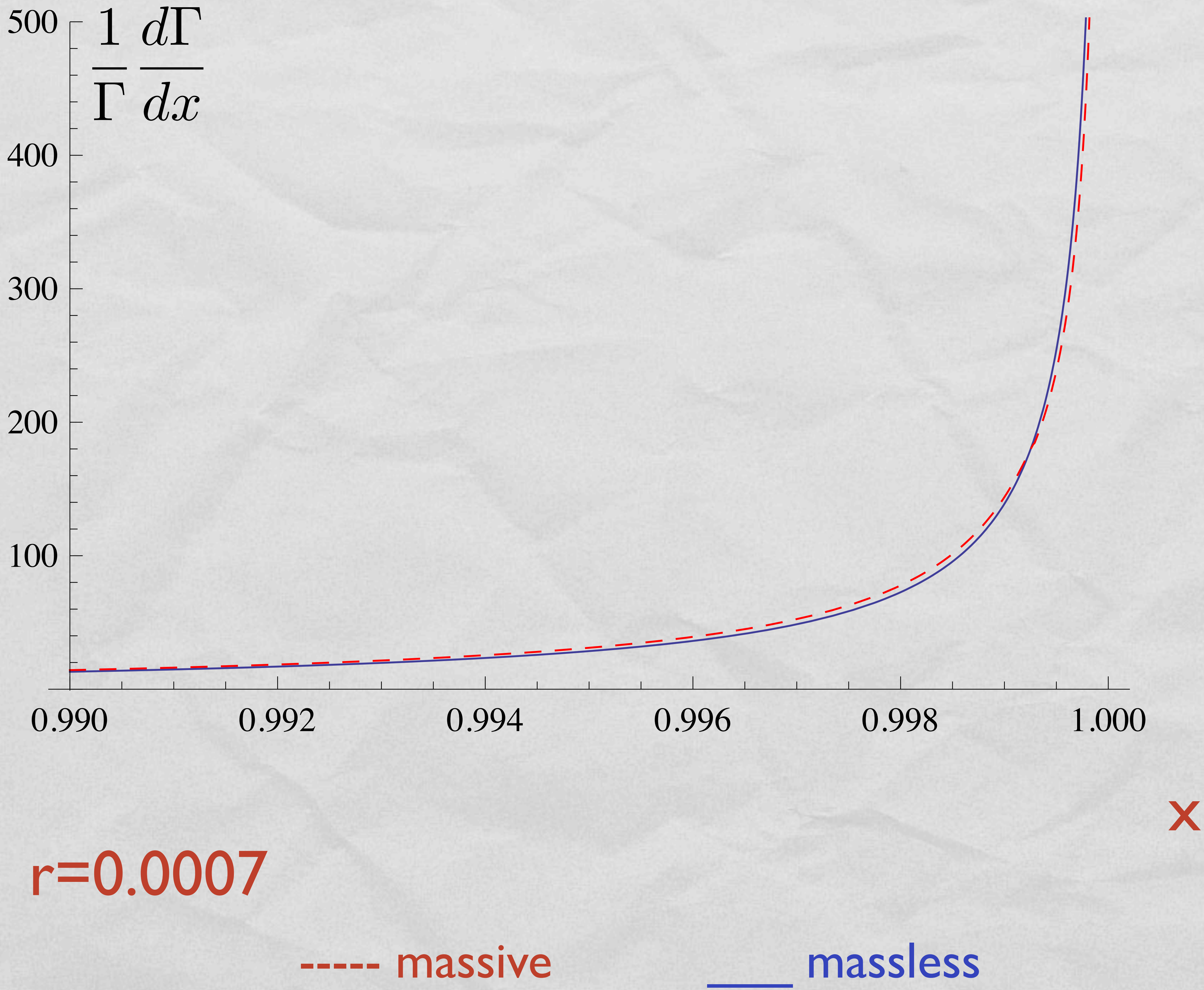
(no regularization)

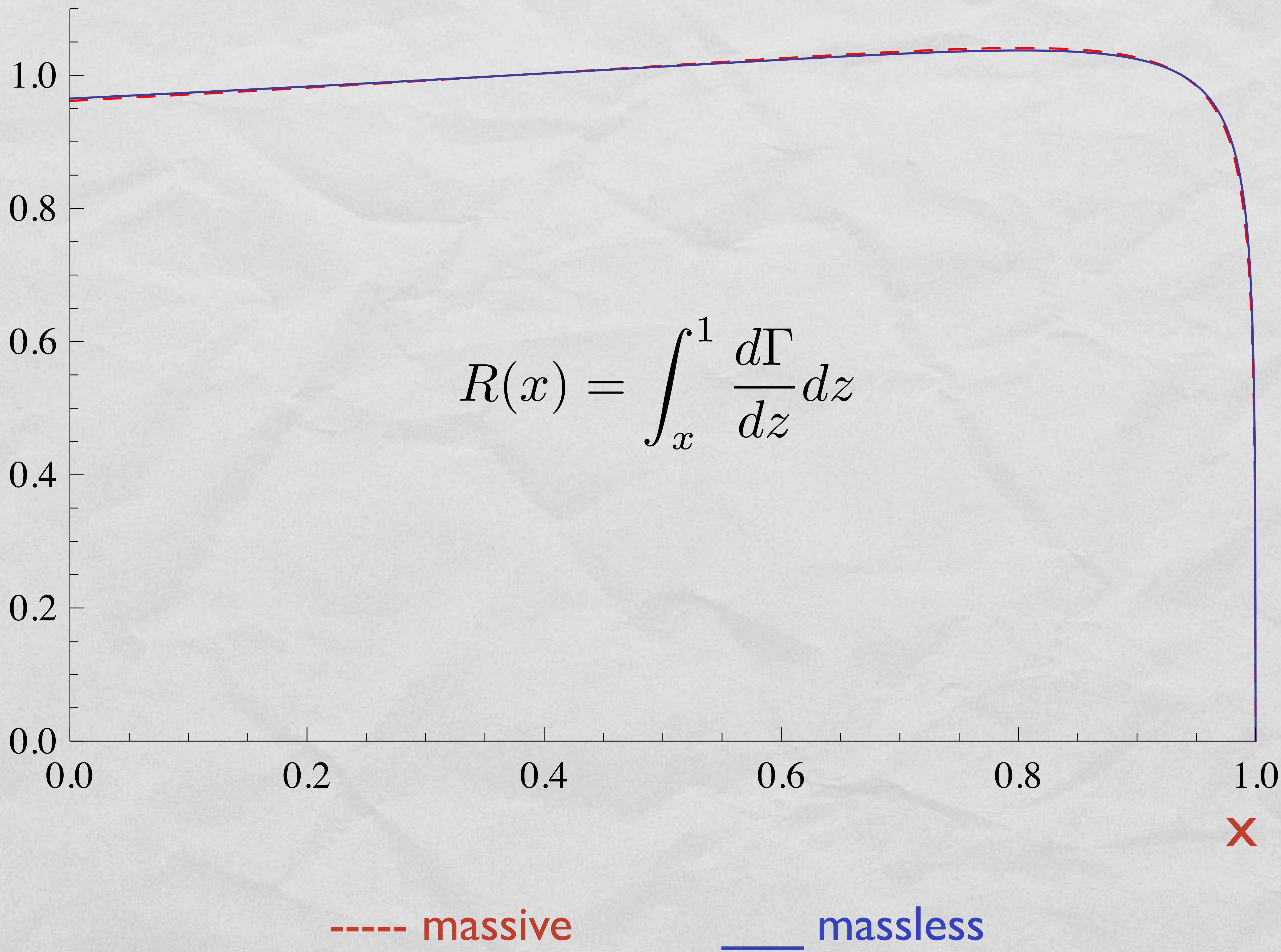
in order to estimate the mass effects

top to b

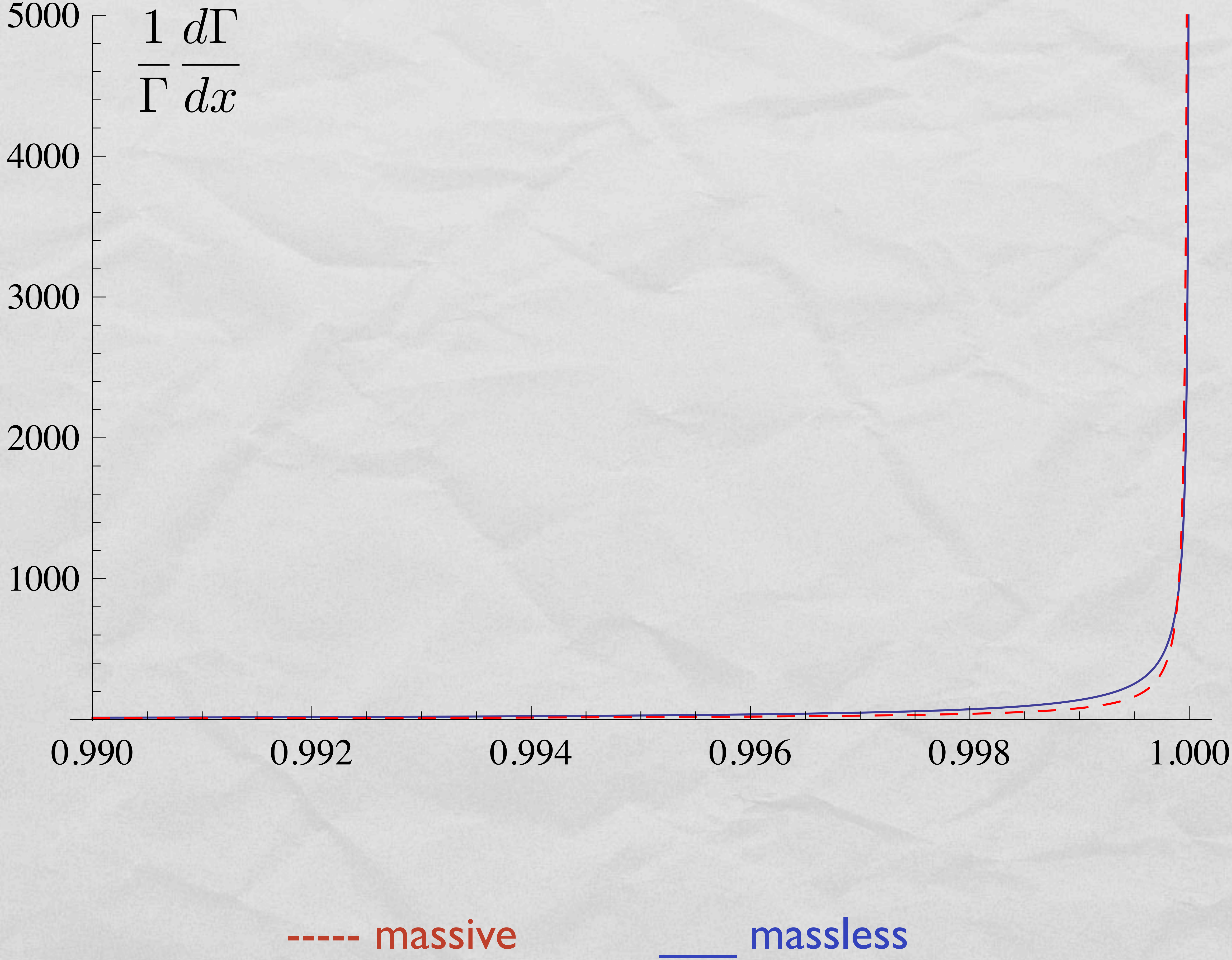


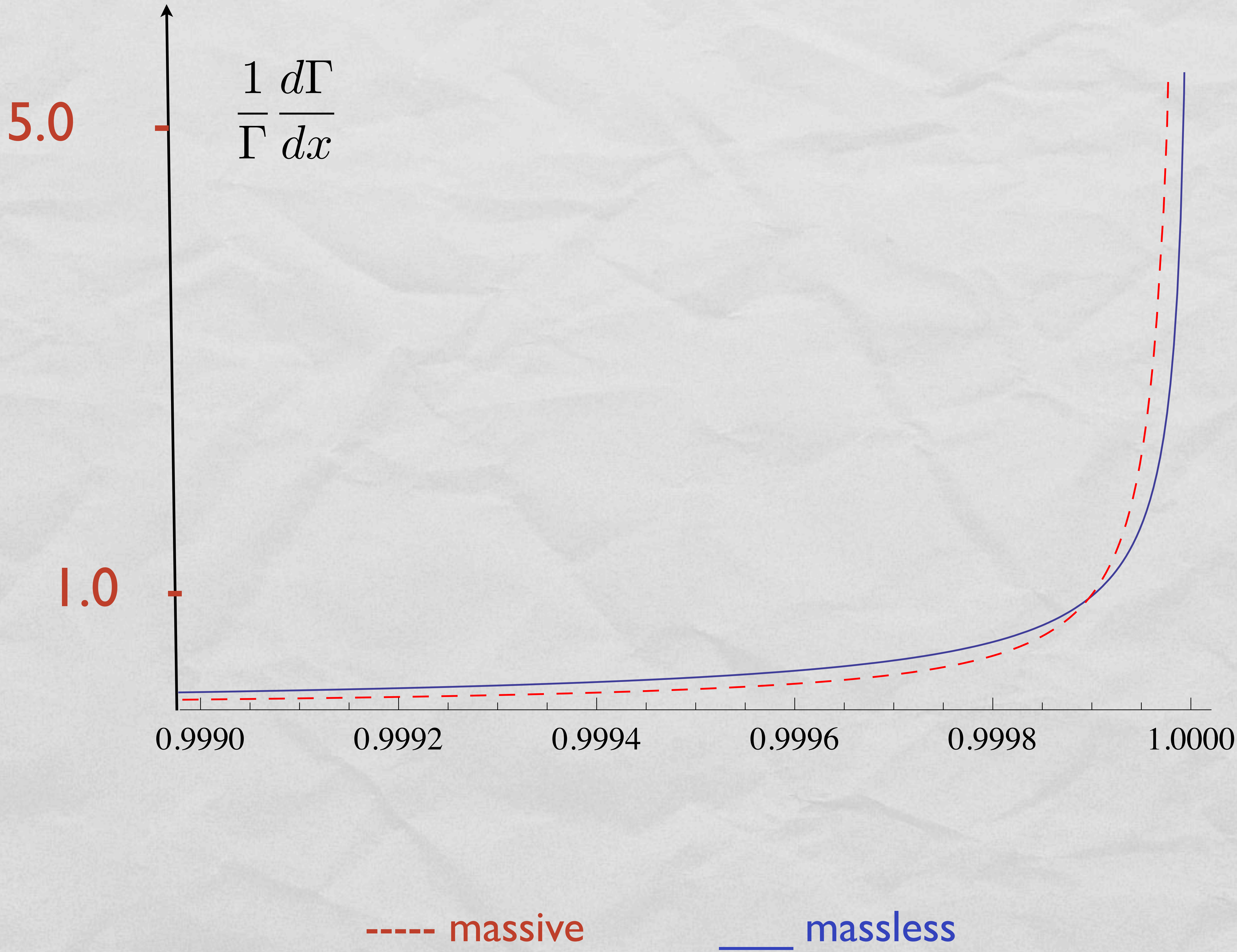
$\frac{1}{\Gamma}\frac{d\Gamma}{dx}$
5000
4000
3000
2000
1000
0.990
0.992
0.994
0.996
0.998
1.000
x
----- massive
___ massless

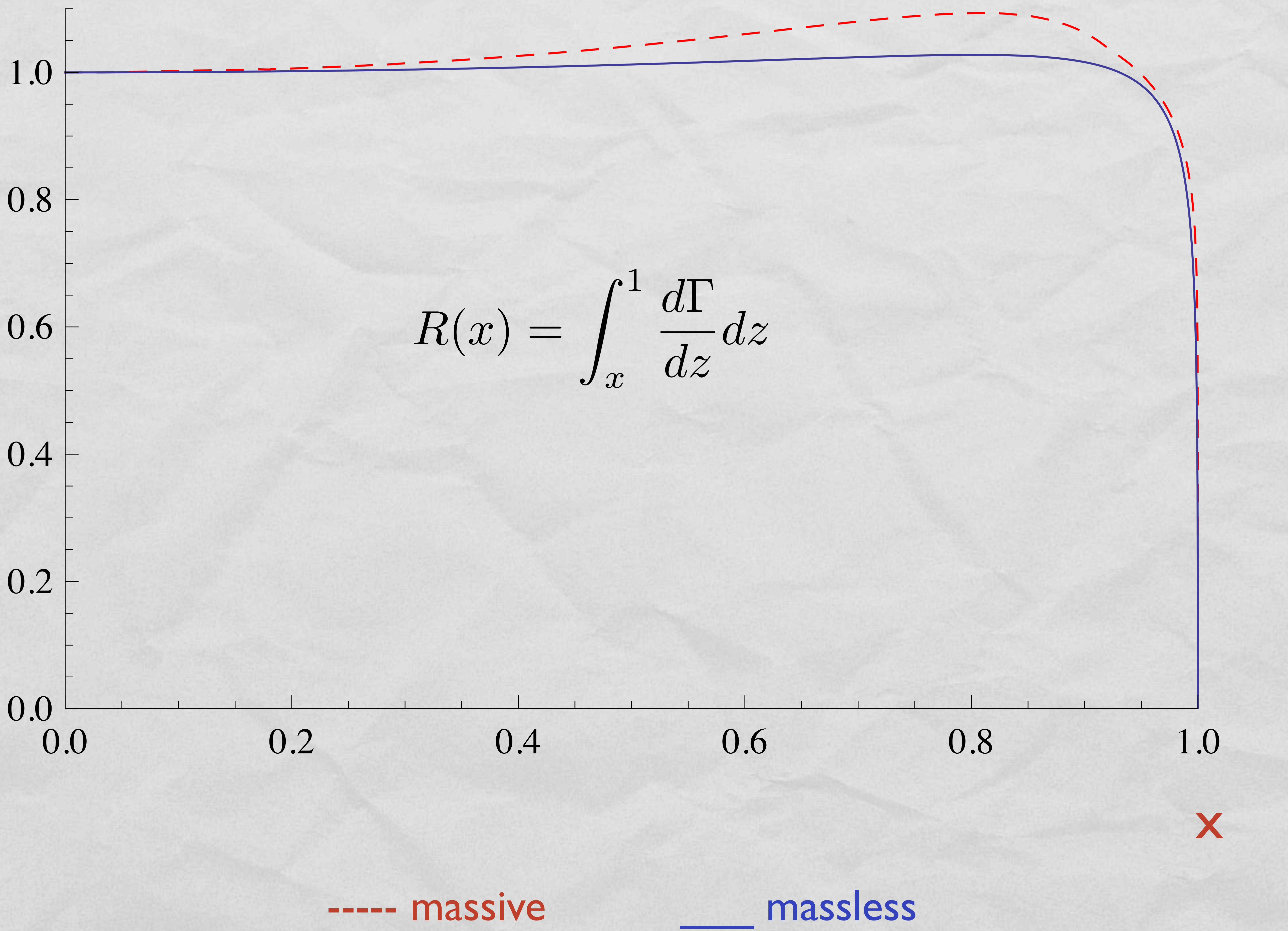

$\frac{1}{\Gamma}\frac{d\Gamma}{dx}$
500
400
300
200
100
0.990
0.992
0.994
0.996
0.998
1.000
x
r=0.0007
----- massive
____ massless


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
$R(x) = \int_x^1 \frac{d\Gamma}{dz} dz$
----- massive
____ massless

b to c


$\frac{1}{\Gamma}\frac{d\Gamma}{dx}$
5000
4000
3000
2000
1000
0.990
0.992
0.994
0.996
0.998
1.000
----- massive
___ massless


$\frac{1}{\Gamma}\frac{d\Gamma}{dx}$
5.0
1.0
0.9990
0.9992
0.9994
0.9996
0.9998
1.0000
----- massive
___ massless



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$$R(x) = \int_x^1 \frac{d\Gamma}{dz} dz$$
x
----- massive
____ massless

CONCLUSIONS AND A WISHFUL OUTLOOK

- The regularization of a generalized resummed approach to mass corrections in HQ decays in the threshold region up to NNLLa terms

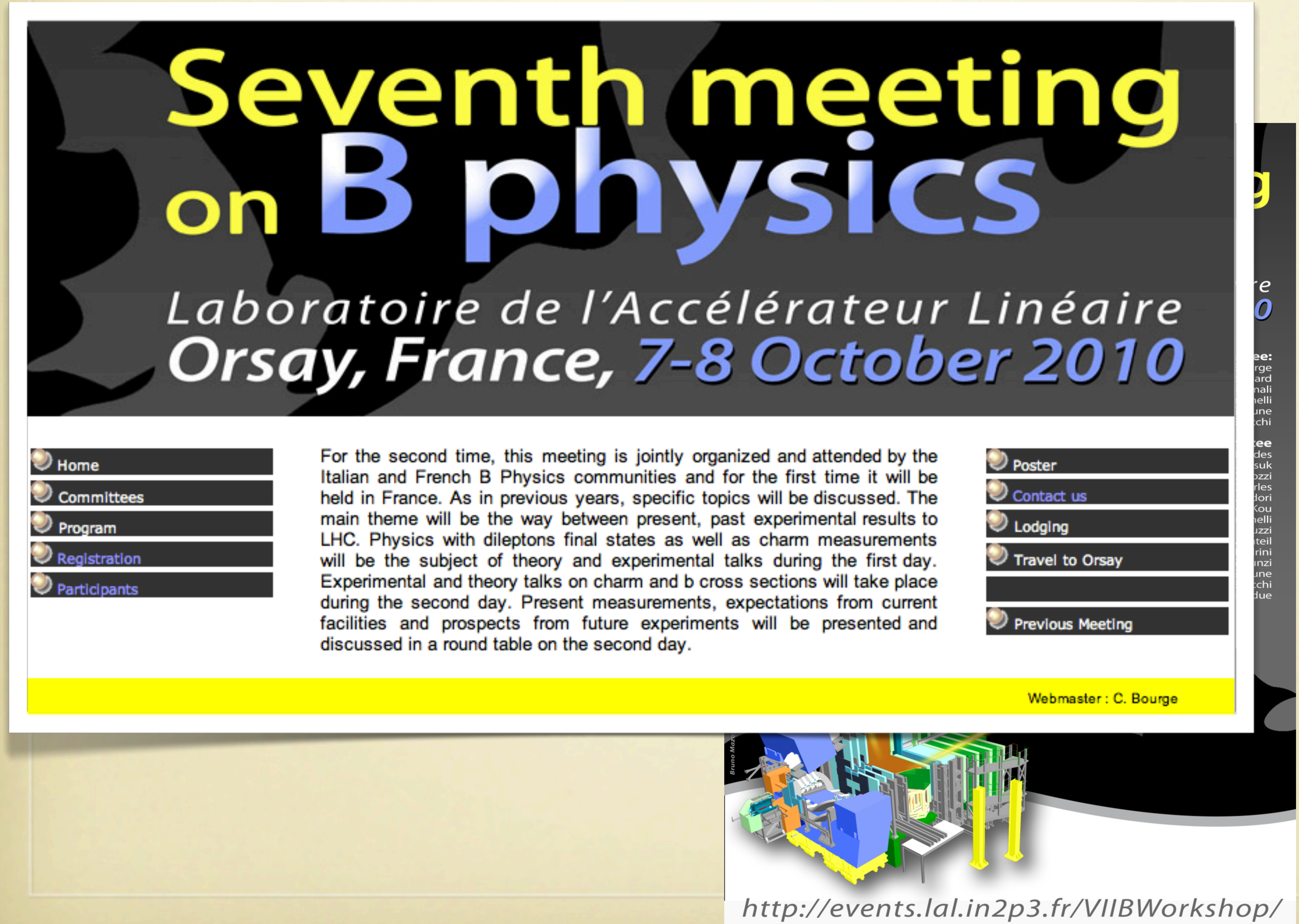
- In the frozen coupling constant case the massive Minimal Prescription Formula regularization, for the leading terms, shows a milder divergent behaviour (shadow of the singularity in the massless case) with respect to the massless one.

- In the running coupling case we expect that, by including the subleading terms, a finite order truncated expansion can be obtained

- We expect that resummed formulae will not be plagued by renormalon power corrections in perturbative treatment of HQ decays

- The MP formula for the massive case is potentially extensible to any other massive perturbative evaluation

7-8 October 2010



spare transparencies

Theoretical aspects of resummation in B decays and fragmentation processes

Based on the work done in collaboration with:
U. Aglietti L. Di Giustino G. Ferrera A. Renzaglia G. Ricciardi
Phys.Lett. B651:275-292,2007 B653:38-52 2007
e-Print: arXiv:0804.3922 [hep-ph]

Second Workshop on Theory, Phenomenology and Experiments in Heavy Flavour Physics

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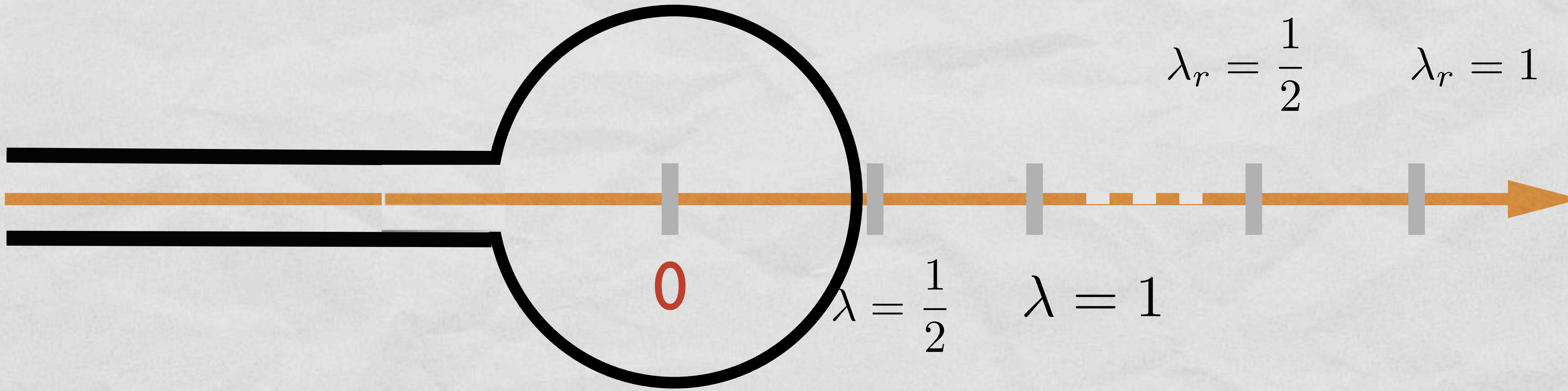
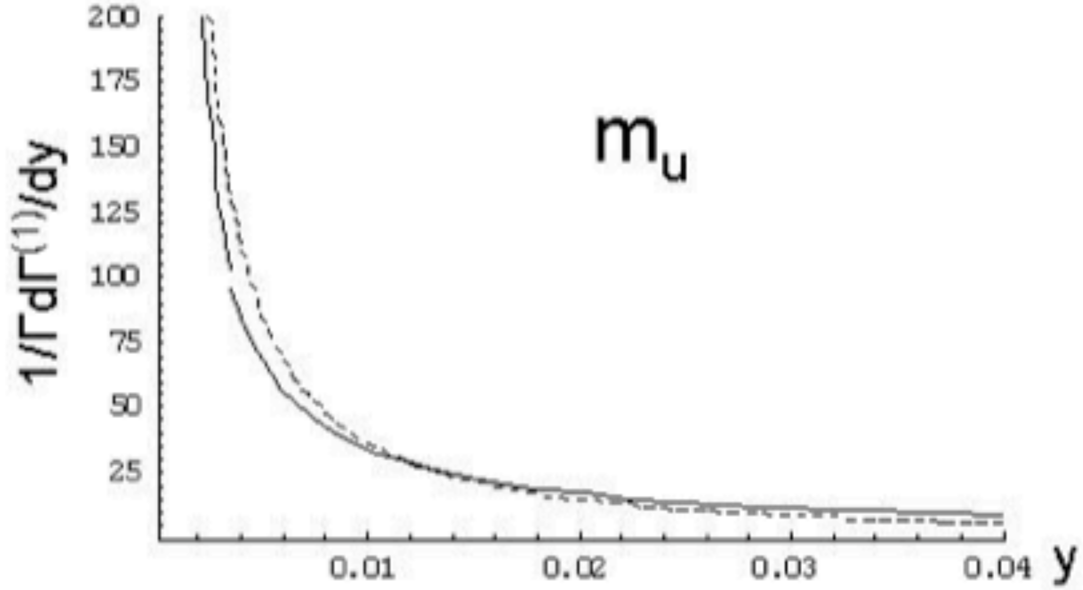
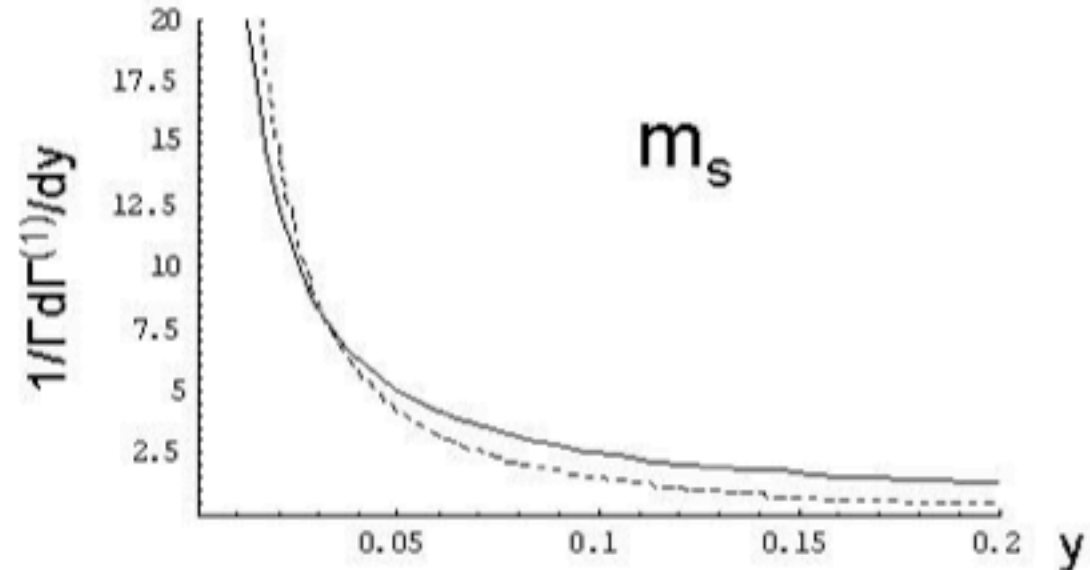
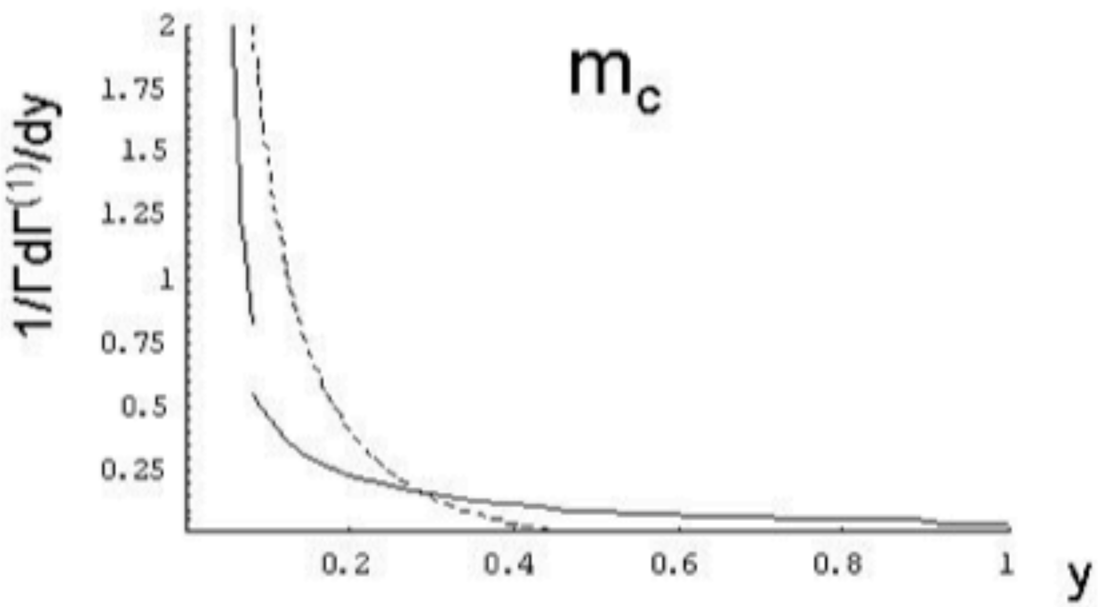

$\lambda_r = \frac{1}{2}$
$\lambda_r = 1$
0
$\lambda = \frac{1}{2}$
$\lambda = 1$

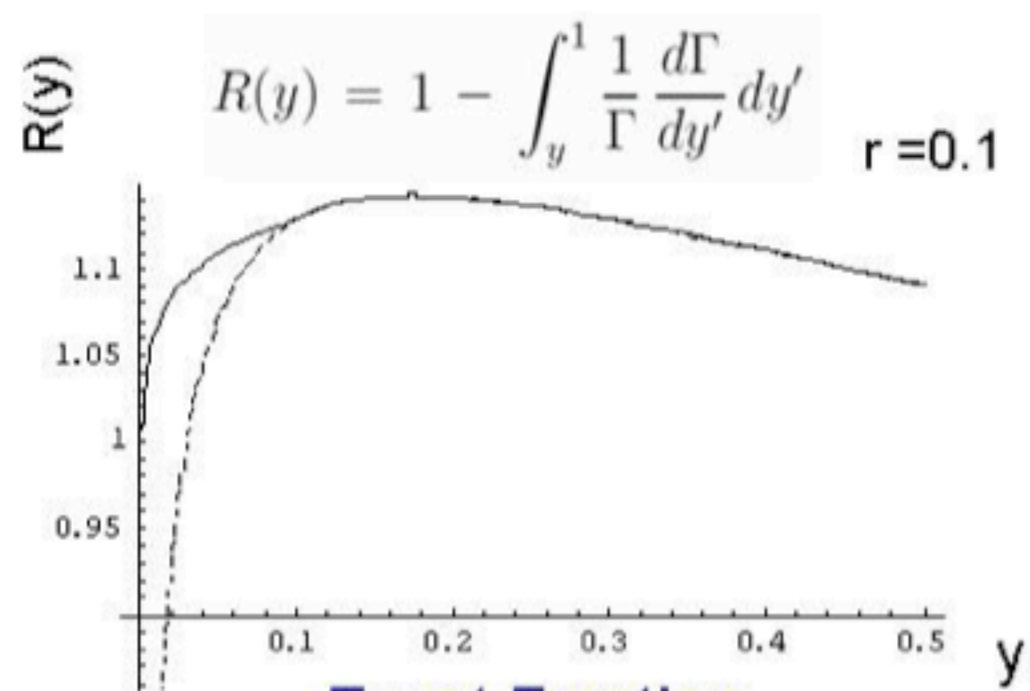
Plot of the Order (α) Distribution





Dashed: massless
Plane : massive





Event Fraction

Interpolation Formula

$$\sigma_N(\rho;m_b^2) = \exp\int_0^1 dy\left[(1-y)^{N-1}-1\right]\left\{\frac{1}{y}\int_{\frac{m_b^2y^2}{1+\rho}}^{\frac{m_b^2y^2}{y+\rho}} A\left[\rho;\,\alpha_S(k^2)\right]\frac{dk^2}{k^2}+\frac{1}{y}D\left[\alpha_S\left(\frac{m_b^2y^2}{1+\rho}\right)\right]+\right.$$
$$\left.+\left(\frac{1}{y}-\frac{1}{y+\rho}\right)\Delta\left[\alpha_S\left(\frac{m_b^2y^2}{y+\rho}\right)\right]+\frac{1}{y+\rho}B\left[\alpha_S\left(\frac{m_b^2y^2}{y+\rho}\right)\right]\right\}$$

Interpolation of soft and soft+collinear radiation

Resummation is valid for any ratio of $\frac{m}{m_b}$; reduces to the massless formula for m->0

Coherence effects are included i.e. no-radiation for no-recoil of the “light” quark u_c=0
(most of the phase space of the b->c transitions is for a “not too fast” charm)

Derivation comes from the universal properties of the QCD radiation
Resummed formula can be combined with a first order differential distribution

Phenomenological Outputs:

A more precise determination of m_c
b->c is dominated by few hadronic states (unlike the charmless channels)
Insights on the parton-hadron duality and hadron dynamics at the scale of few GeV

Improved inclusive extraction of the CKM lV_cbl and a better subtraction of backgrounds
to b->u transitions due to larger windows of data analysis (see D