How Scatterings affect the Dark Matter Velocity Distribution in the Laboratory

Timon Emken (Chalmers University of Technology)

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Collaborators

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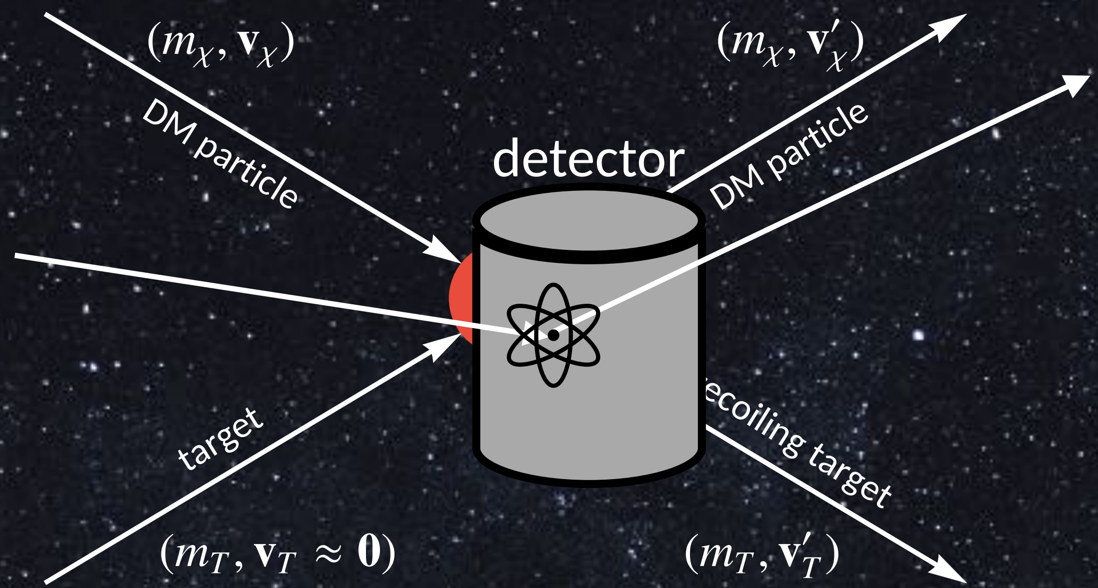
Based on

[arXiv:1905.06348]
[arXiv:1802.04764]
[arXiv:1709.06573]
[arXiv:1706.02249]

Direct Detection of Dark Matter

Basic idea: Look for the aftermath of a DM-atom collision in a detector.



Nuclear recoils as observable for GeV-scale DM searches.

M.W. Goodman and E. Witten, Phys.Rev. D31 (1985) 3059
I. Wasserman, Phys. Rev. D33 (1986) 2071
A.K. Drukier et al., Phys. Rev. D33 (1986) 3495

Event spectrum:

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = N_T \frac{\rho_\chi}{m_\chi} \iiint \mathrm{d}^3\mathbf{v}\; v f_\chi(\mathbf{v}) \frac{\mathrm{d}\sigma_N}{\mathrm{d}E_R} \Theta(v - v_{\min}(E_R)) \,.$$

Detector size

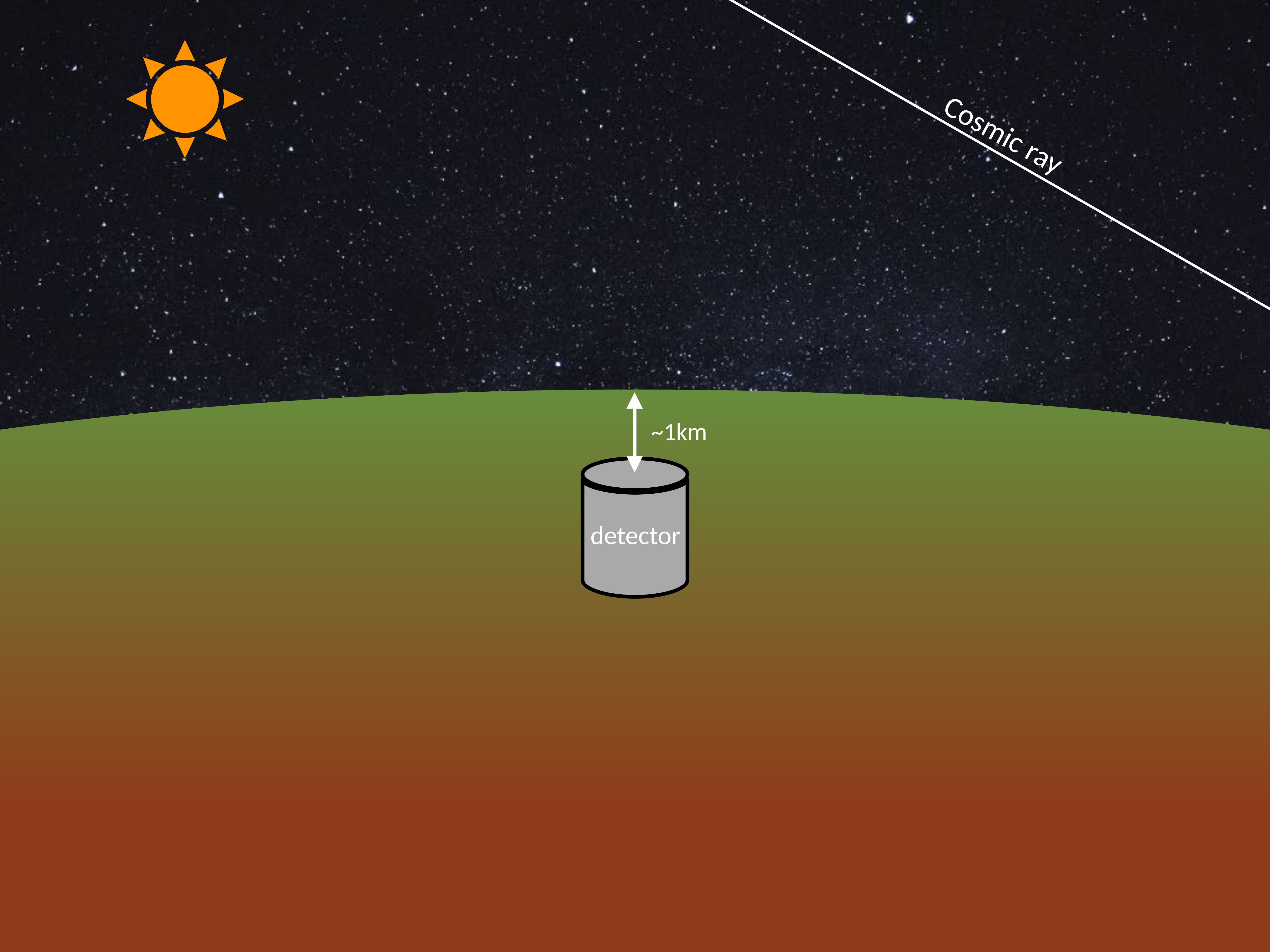
Particle physics

Astrophysics

Kinematics

For sub-GeV DM, one can also look for DM-electron interactions.

R. Essig et al., Phys. Rev. D85 (2012) 076007



Cosmic ray
~1km
detector

How Scatterings affect the Dark Matter Velocity Distribution in the Laboratory

I. Describing DM Scatterings in MC simulations

II. Terrestrial Effects on Direct Detection Experiments

  (a) Diurnal Signal Modulations

  (b) Constraints on strongly interacting DM

III. Solar Reflection of sub-GeV Dark Matter

IV. Cosmic ray up-scattered DM

I.

Describing DM Scatterings in MC simulations

Underground Scatterings of DM Particles

- If DM can scatter in a detector, they scatter anywhere.
- Underground DM scatterings have two consequences:
  1. spatial re-distribution of DM particles,
  2. Change of the DM velocities.
- For stronger DM-nucleus interactions inside the Earth, these two effects could affect DM detection experiments severely.
  1. daily/diurnal modulation of the signal rate,
  2. loss of sensitivity to strongly interacting DM.
- Scattering on energetic targets (in the Sun, or cosmic rays) could also **accelerate** DM.

How to include the effect of 'pre-detection' scatterings into signal rates?

J. Kopp et al., Phys. Rev. D80 (2009) 083502
R. Essig et al., Phys. Rev. D85 (2012) 076007
R. Essig et al., Phys. Rev. Lett. 109 (2012) 021301
R. Essig et al., JHEP 1605 (2016) 046

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = \frac{1}{m_N}\frac{\rho_\chi}{m_\chi}\int_{v>v_{\min}(E_R)} \mathrm{d}v\, v f_\chi(v) \frac{\mathrm{d}\sigma_N}{\mathrm{d}E_R}$$

$$\frac{\mathrm{d}R_{\mathrm{ion}}}{\mathrm{d}E_e} = \frac{1}{m_N}\frac{\rho_\chi}{m_\chi}\sum_{nl}\frac{\sigma_e}{8\mu_{\chi e}^2 E_e}\int \mathrm{d}q\, q \int_{v>v_{\min}(\Delta E_e, q)} \mathrm{d}v \frac{f_\chi(v)}{v} \left|F_{\mathrm{DM}}(q)\right|^2 \left|f_{\mathrm{ion}}^{nl}(k', q)\right|^2$$

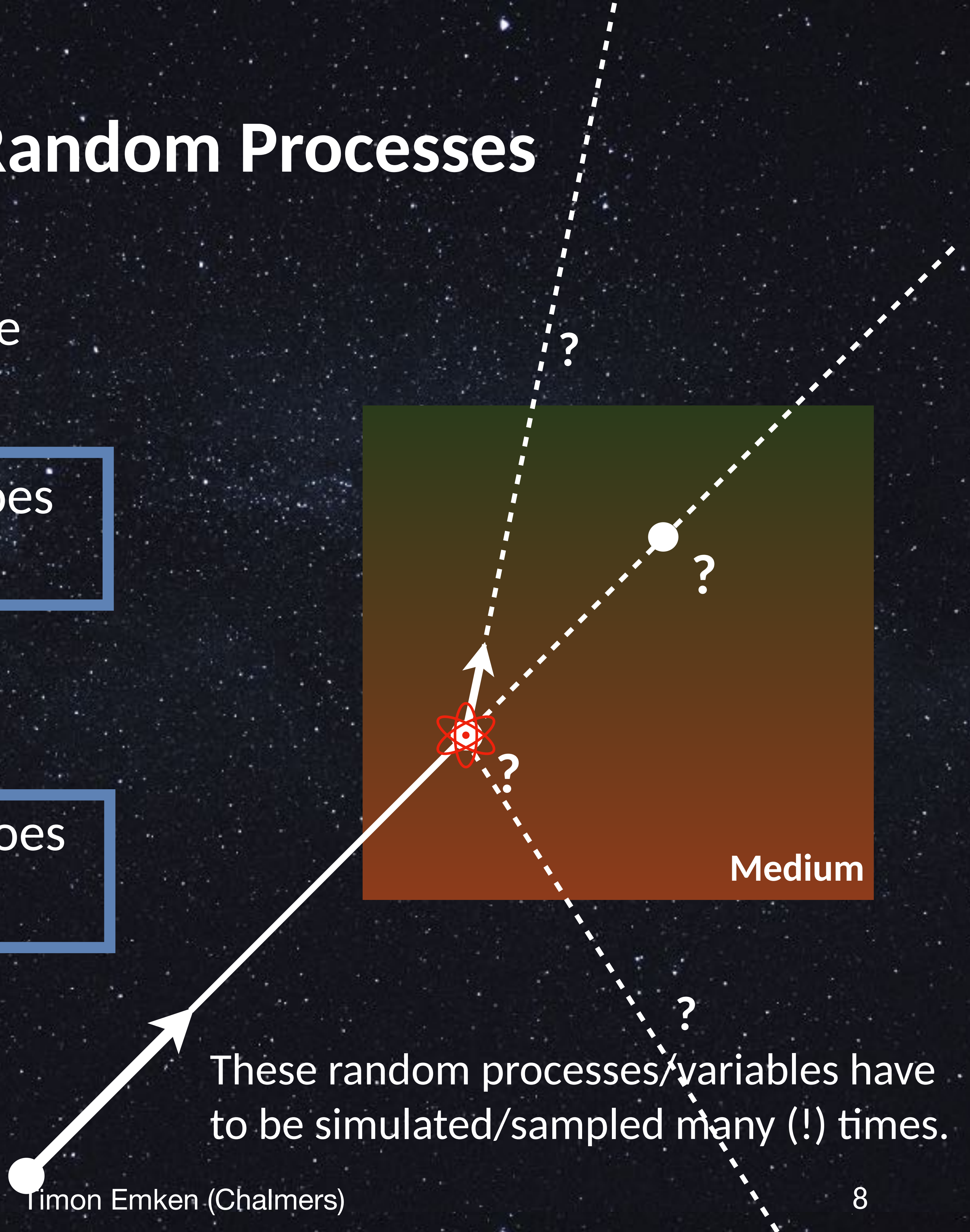
$$\frac{\mathrm{d}R_{\mathrm{crystal}}}{\mathrm{d}E_e} = \frac{\rho_\chi}{m_\chi}\frac{1}{M_{\mathrm{cell}}}\frac{\sigma_e \alpha m_e^2}{\mu_{\chi e}^2}\int \mathrm{d}q\, \frac{1}{q^2}\int_{v>v_{\min}(E_e, q)} \mathrm{d}v \frac{f_\chi(v)}{v} \left|F_{\mathrm{DM}}(q)\right|^2 \left|f_{\mathrm{crystal}}(E_e, q)\right|^2$$

DM scatterings modify the underground DM density and velocity distribution.

The Fundamental Random Processes

1. **Initial Conditions**: Where does the particle start?
2. **Free distance**: Where does the particle scatter?
3. **Target**: What does the particle scatter on?
4. **Scattering angle**: How does the particle scatter?

Repeat steps 2-4.



These random processes/variables have to be simulated/sampled many (!) times.

Free distance

- The CDF of the free distance is given by

$$P(L) = 1 - \exp\left(-\int_0^L \frac{\mathrm{d}x}{\lambda(x, v)}\right), \quad \text{integrate along the path}$$

where we used the *mean free path*,

$$\lambda(\mathbf{x}, v)^{-1} = \sum_i \lambda_i(\mathbf{x}, v)^{-1} \equiv \sum_i n_i(\mathbf{x})\sigma_{\chi i}\,.$$

- To sample a specific free distance, we need a uniform random number ξ and solve

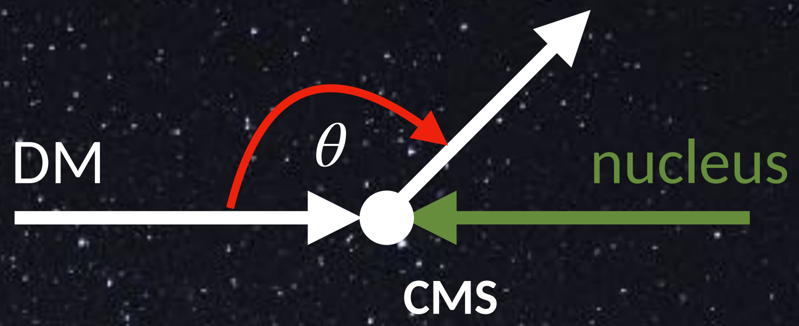
$$P(L) = \xi \in \mathscr{U}_{[0,1]}\,.$$

inverse transform sampling

$$\rightarrow L = P^{-1}(\xi)\,.$$

- Can be complicated for (dis-)continuous changes of the medium.

Scattering angle



- The PDF of the scattering angle $\cos\theta$ is given by

$$f_\theta(\cos\theta) = \frac{1}{\sigma_N}\frac{\mathrm{d}\sigma_N}{\mathrm{d}\cos\theta} = \frac{E_R^{\max}}{2\sigma_N}\frac{\mathrm{d}\sigma_N}{\mathrm{d}E_R}\,.$$

particle physics input

For isotropic contact interactions, this is simply

$$f_\theta(\cos\theta) = \frac{1}{2}\,.$$

inverse transform sampling
or
rejection sampling

For other interaction types, it can be more complicated.

- This angle fixes the new DM velocity after a scattering:

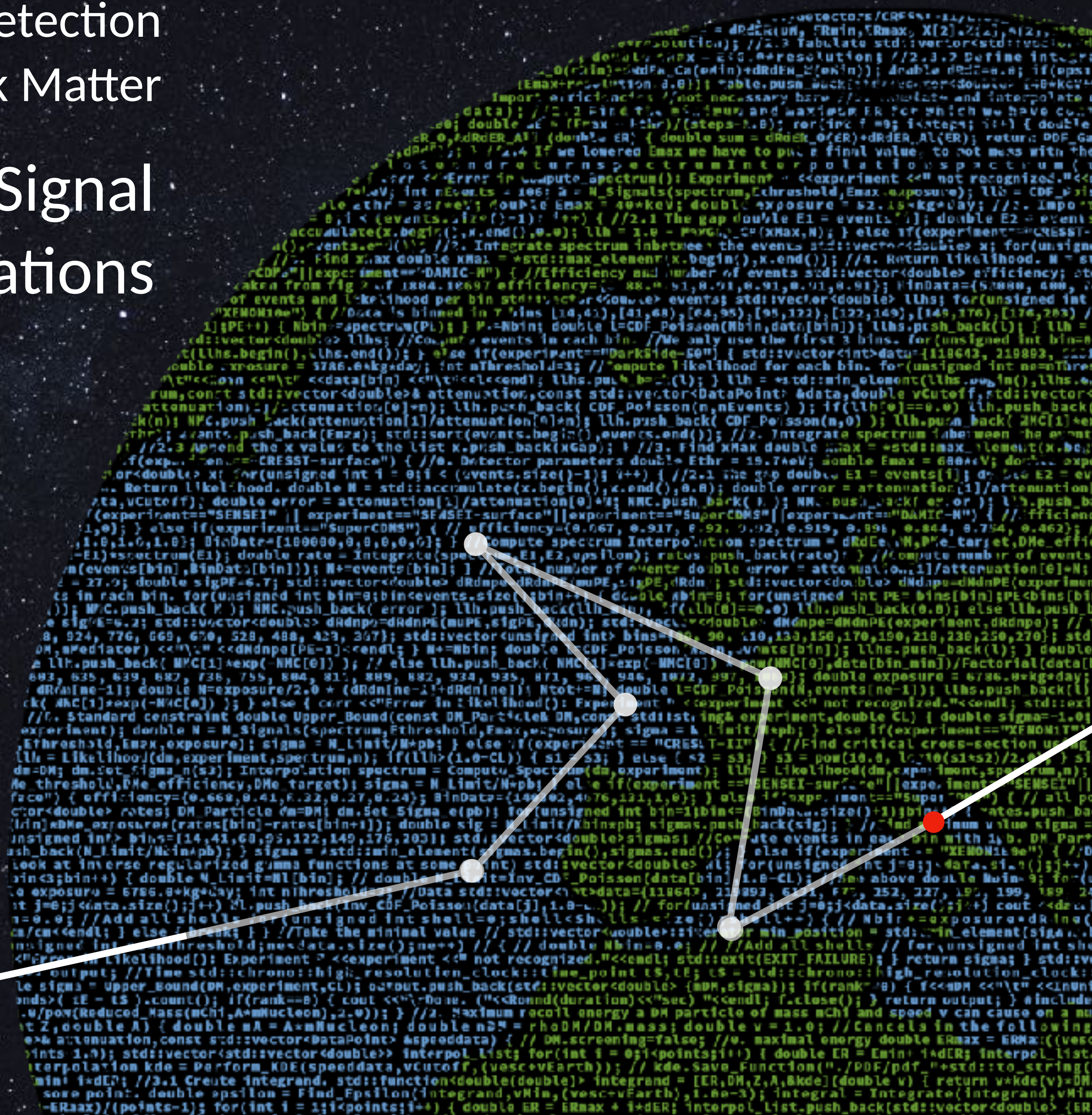
$$\mathbf{v}'_\chi \approx \frac{m_T v_\chi \mathbf{n} + m_\chi \mathbf{v}_\chi}{m_T + m_\chi} \qquad (\mathbf{v}_T \approx \mathbf{0})$$

kinematics

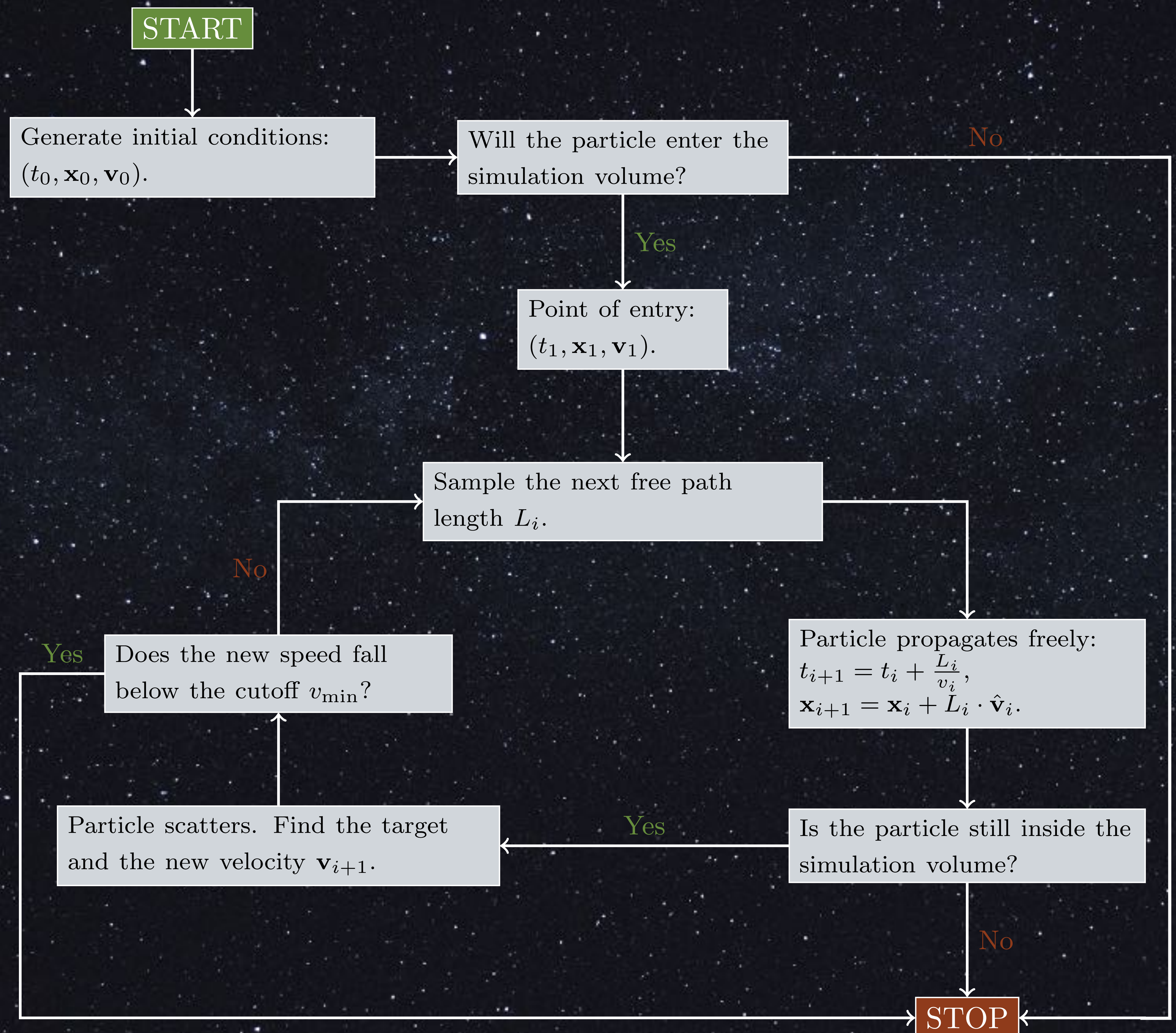
II.

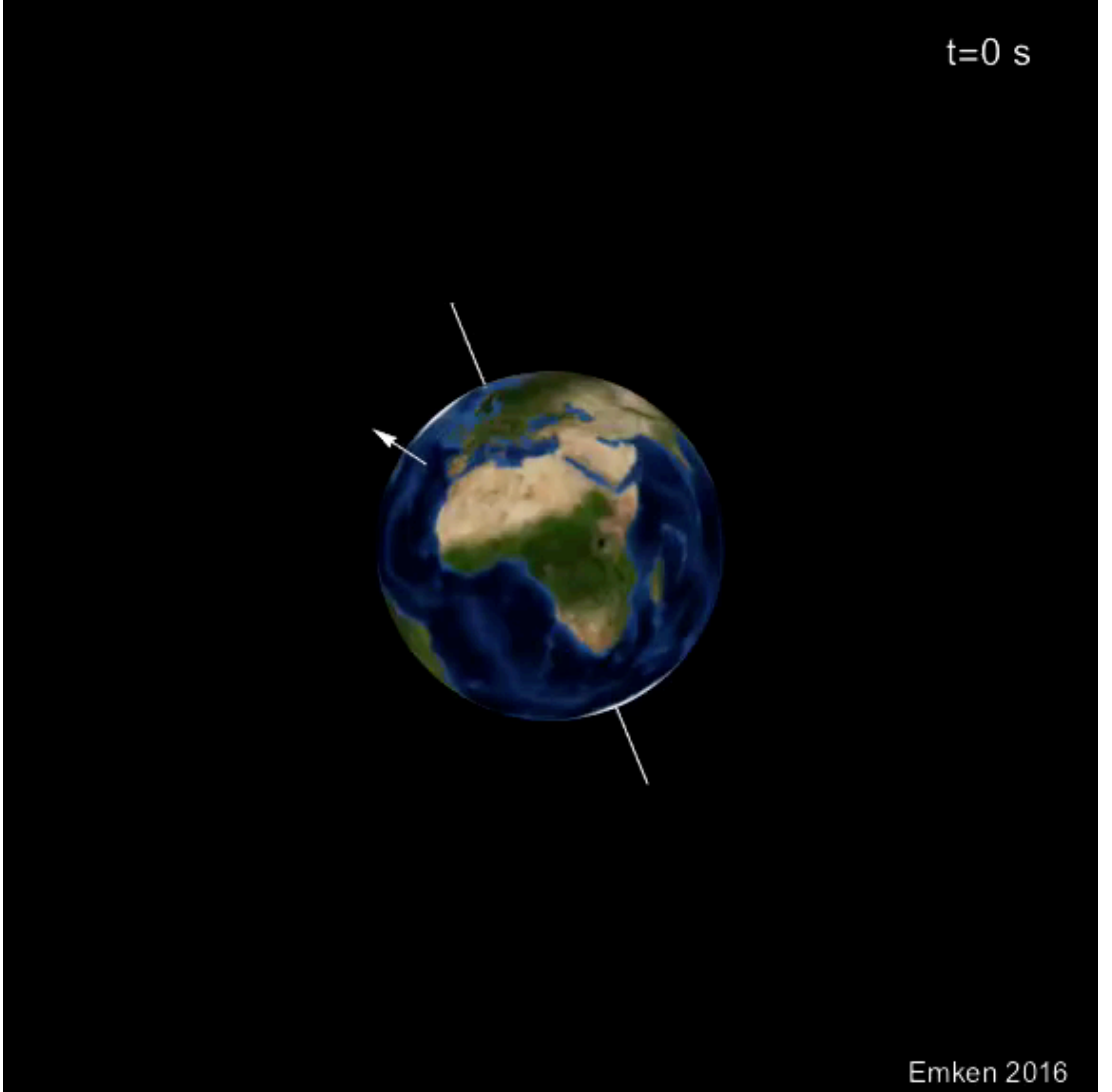
Terrestrial Effects on Direct Detection of Dark Matter

Diurnal Signal Modulations

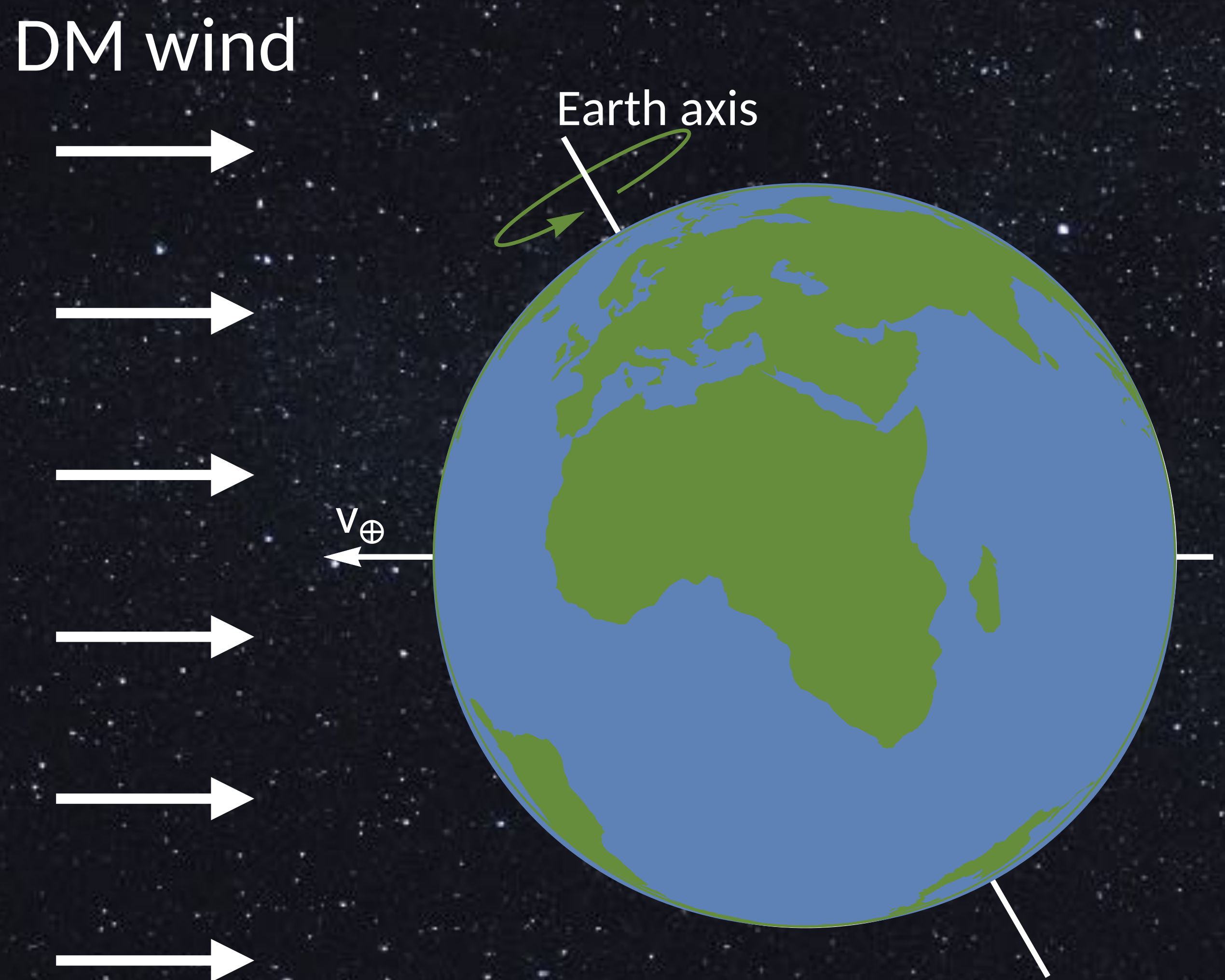


Monte Carlo Simulation Algorithm




t=0 s
Emken 2016

Isodetection angle



How deep is a laboratory in Earth's 'shadow'?

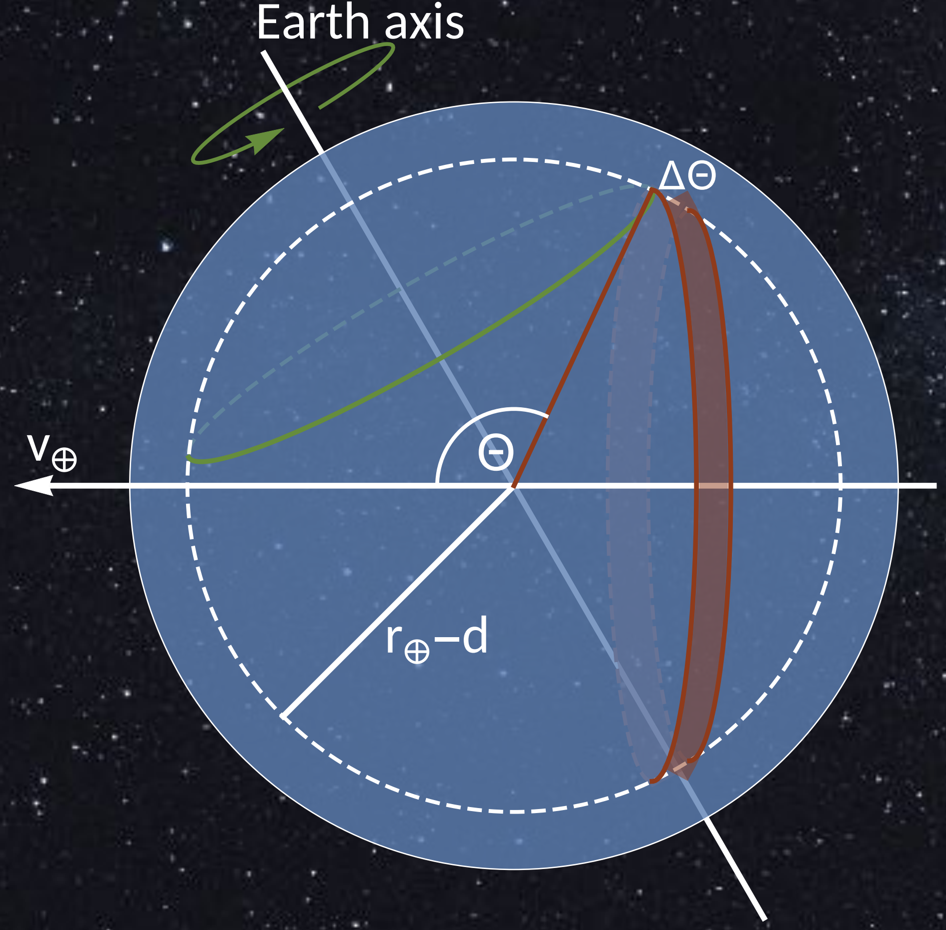
$$\Theta(t) = \arccos\left[\frac{\mathbf{v}_\oplus(t)\cdot\mathbf{x}^{\mathbf{(gal)}}_{\mathbf{lab}}(t)}{v_\oplus(t)(r_\oplus - d)}\right]$$

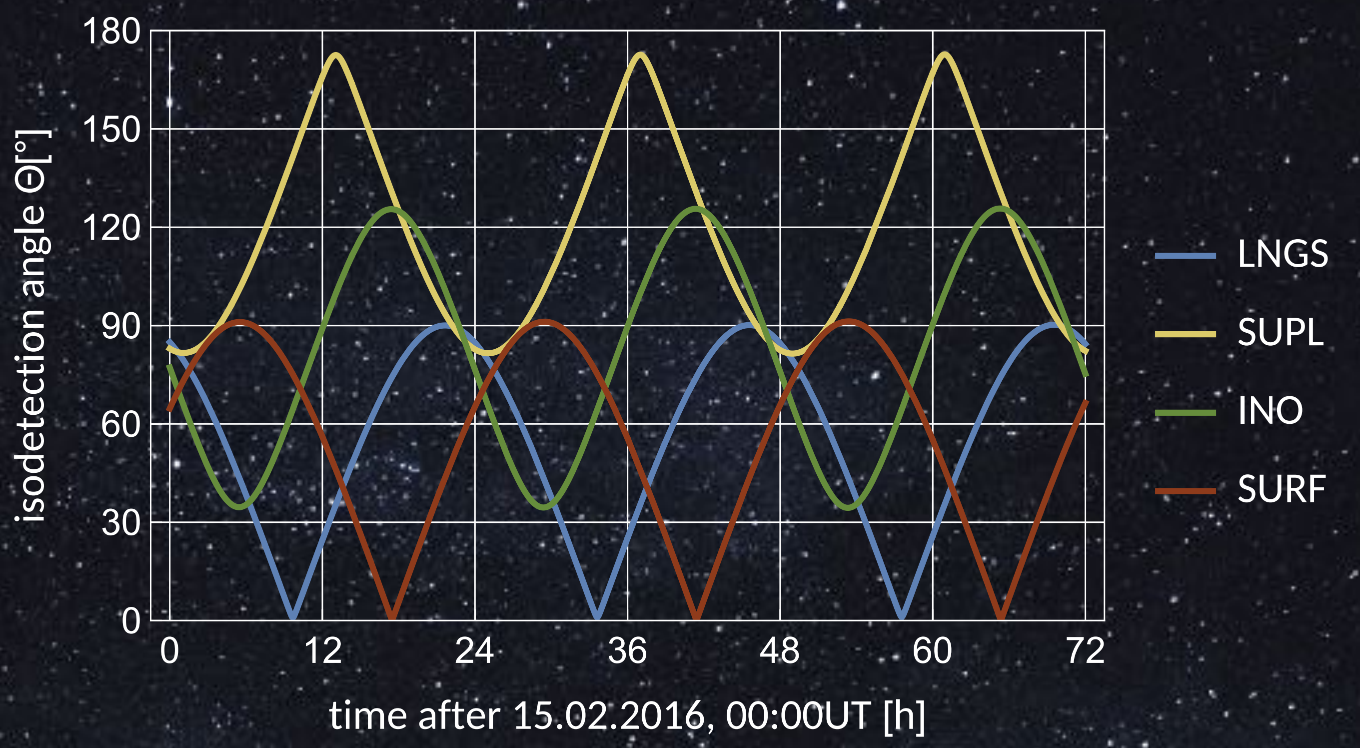
$\Theta = 0^\circ$ ➡ DM wind from above

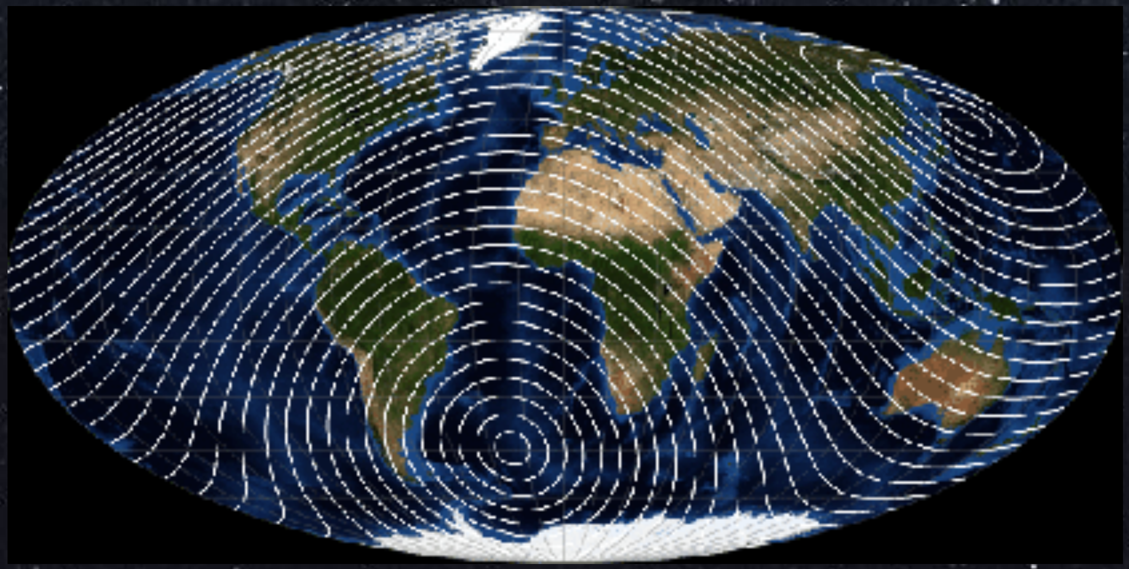
$\Theta = 180^\circ$ ➡ Shadow

J.I. Collar, F.T. Avignone, Phys. Lett. B275 (1992), 181-185
J.I. Collar, F.T. Avignone, PRD 47 (1993), 5238-5246
Hasenbalg et al., PRD 55 (1997), 7350-7355

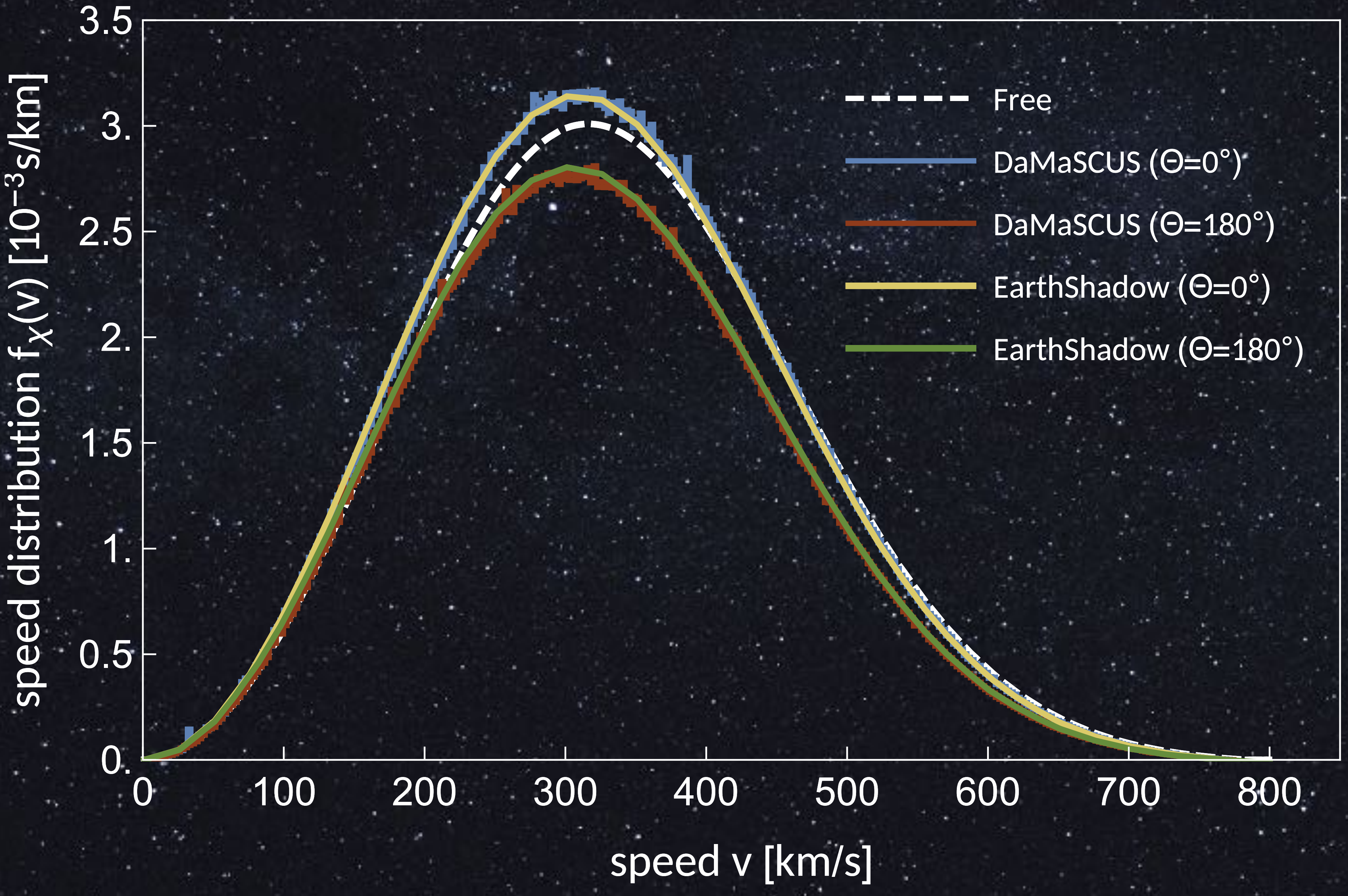
Isodetection angle





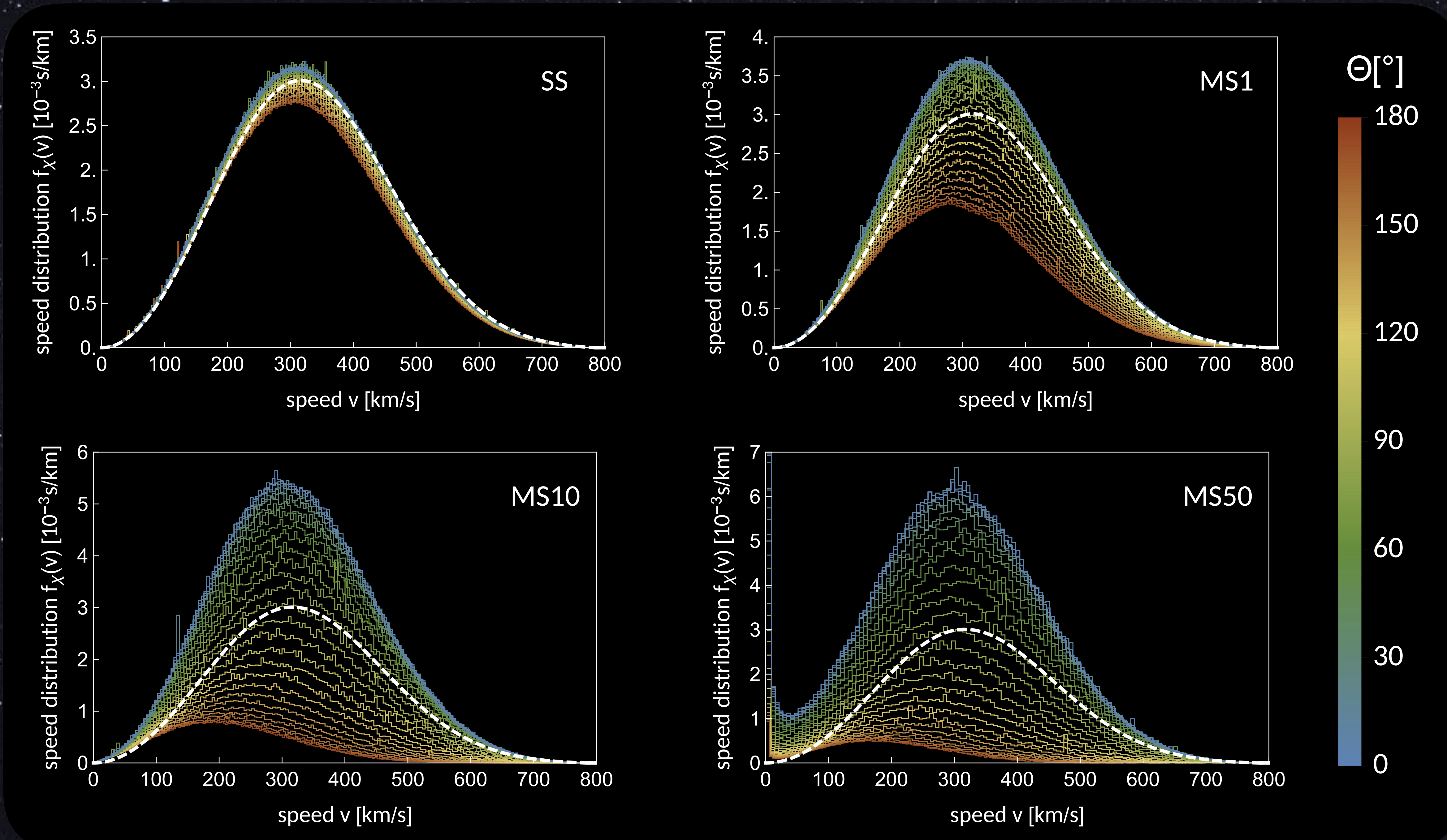


DM distribution inside the Earth



B.J. Kavanagh, R. Catena, C. Kouvaris, JCAP 1701 (2017) no 01, 012

DM distribution inside the Earth



$m_\chi = 500\text{MeV}$

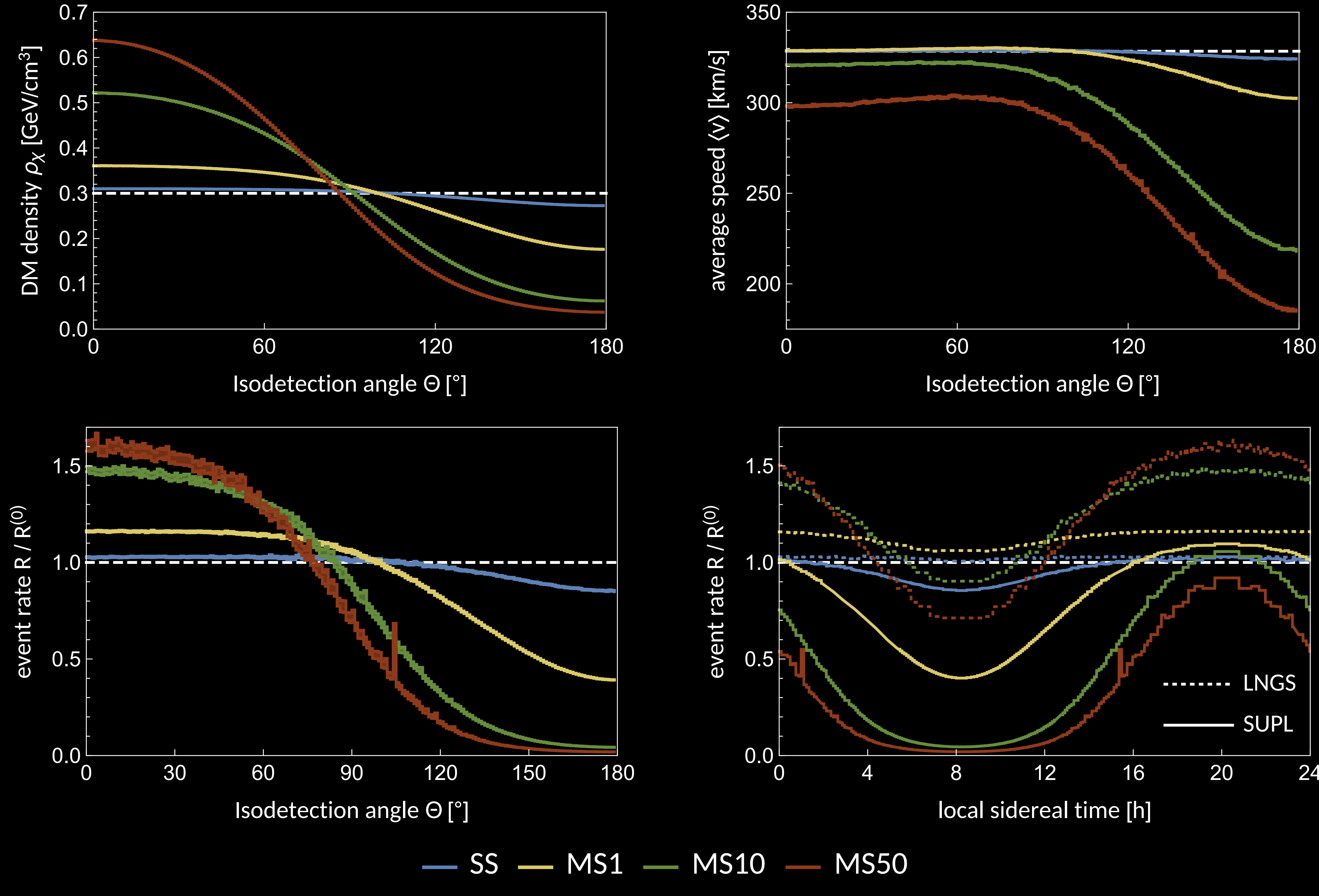
SS: $\sigma_p \approx 0.5\text{pb}$

MS1: $\sigma_p \approx 4.3\text{pb}$

MS10: $\sigma_p \approx 42.5\text{pb}$

MS50: $\sigma_p \approx 300\text{pb}$

Diurnal Modulation Results



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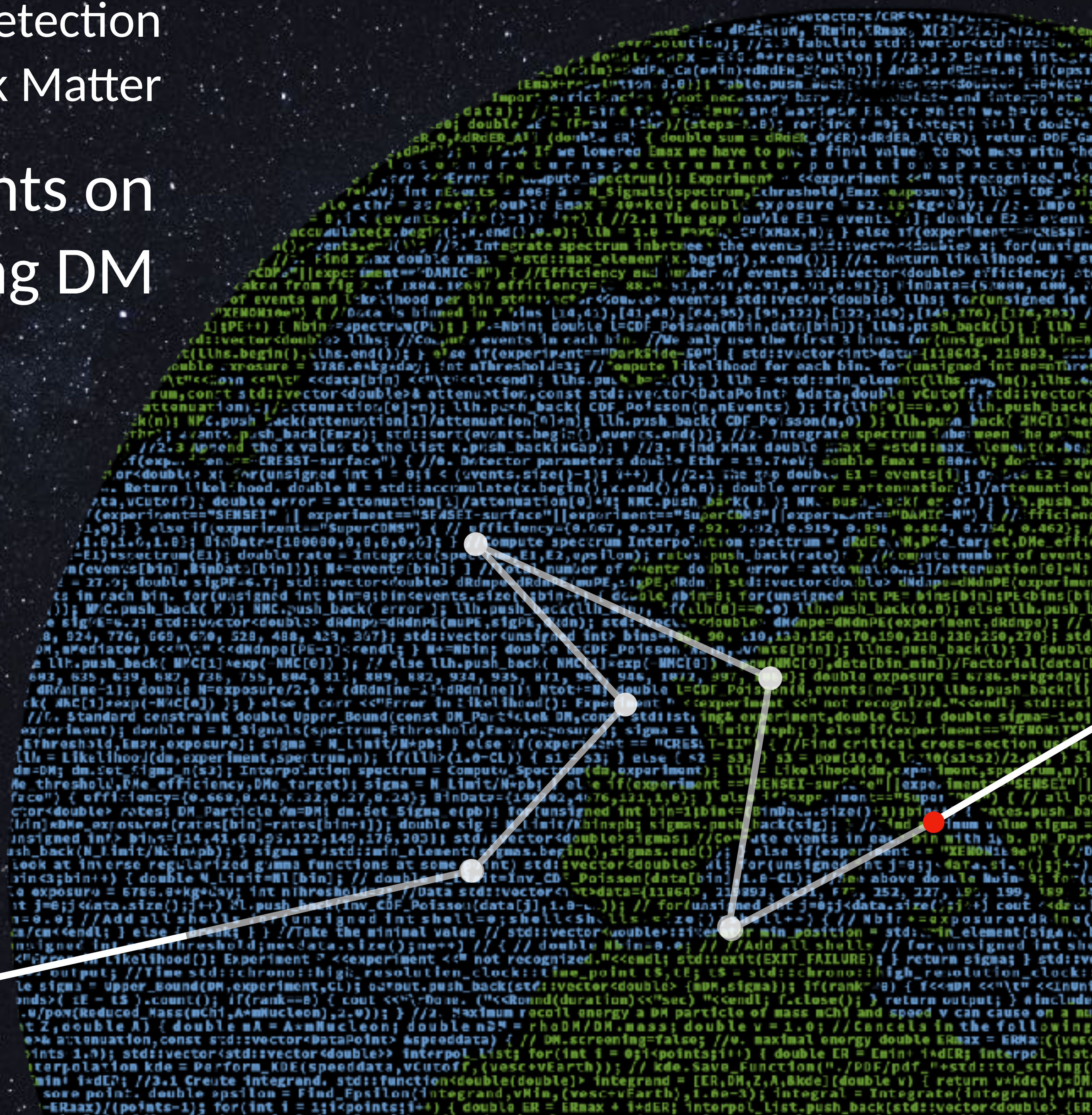
II.

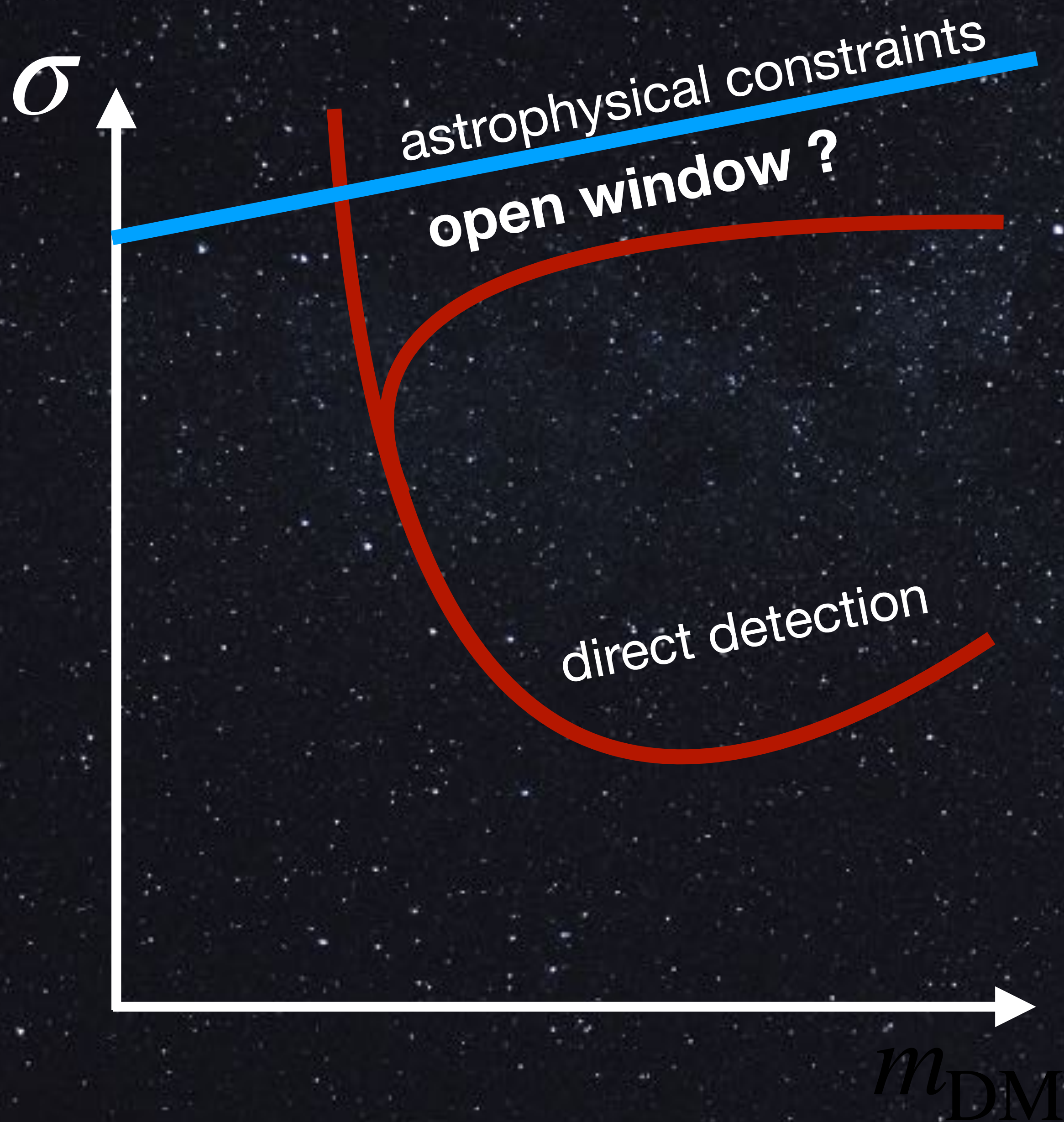
Terrestrial Effects on Direct Detection of Dark Matter

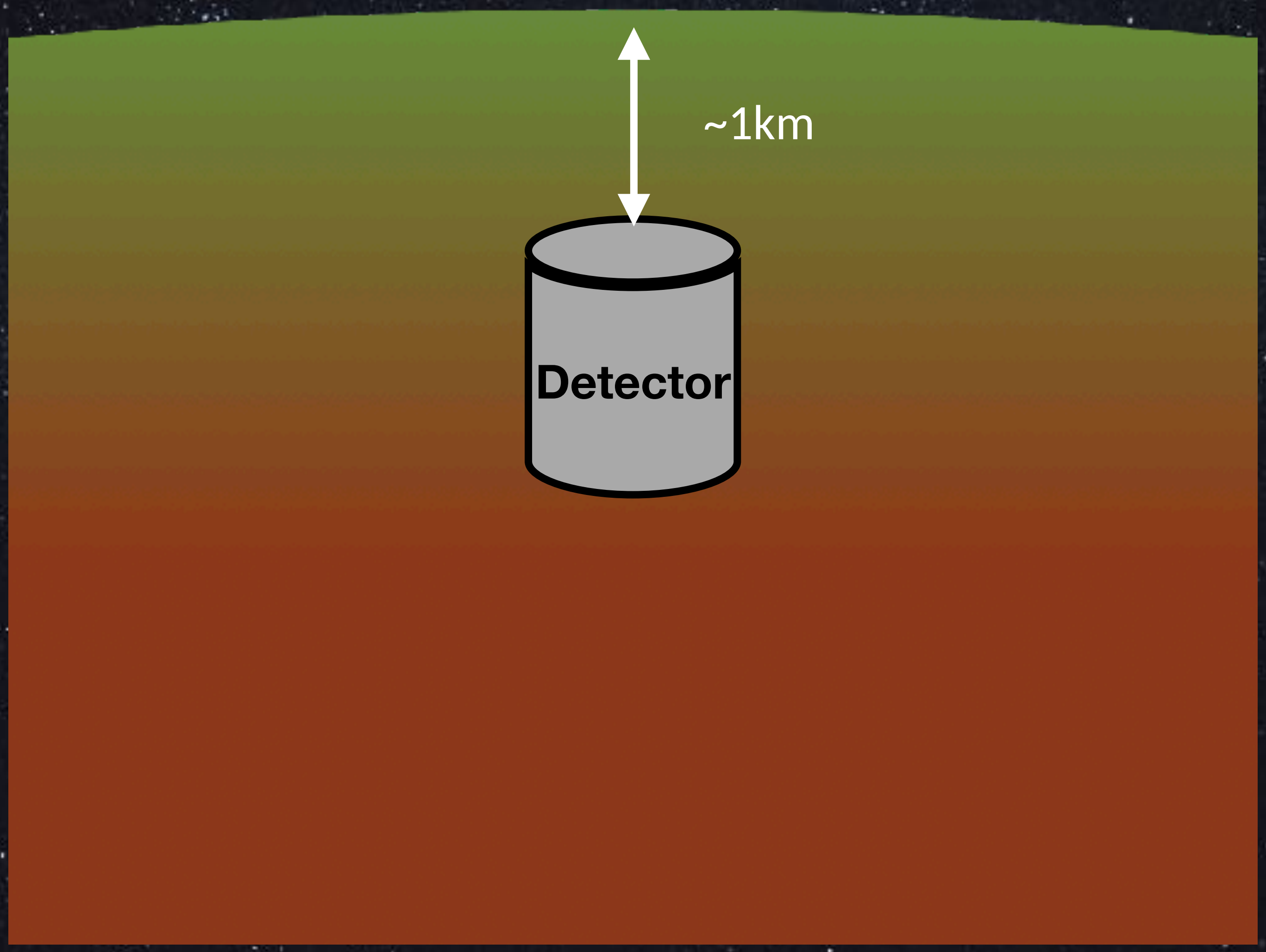
Constraints on strongly interacting DM



Direct detection of strongly interacting DM

Goodman and Witten, Phys.Rev. D31 (1985) 3059
Starkman et al, Phys.Rev. D41 (1990) 3594





PHYSICAL REVIEW D VOLUME 41, NUMBER 12 15 JUNE 1990

Opening the window on strongly interacting dark matter

Glenn D. Starkman and Andrew Gould
Institute for Advanced Study, Princeton, New Jersey 08540

Rahim Esmailzadeh
Center for Particle Astrophysics, University of California, Berkeley, California 94720

Savas Dimopoulos*
CERN TH-Division, 1211 Gen...

Cracking Open the Window for Strongly Interacting Massive Particles as the Halo Dark Matter

Patrick C. McGuire, Paul J. Steinhardt

(Submitted on 31 May 2001)

A Window in the Dark Matter Exclusion Lim...

Gabrijela Zaharijas, Glennys R. Farrar

(Submitted on 24 Jun 2004 ...)

...wards Closing the Window on Strongly Interacting Dark Matter: Far-Reaching Constraints from

... Gianfranco Bertone (IAP)

Closing the Window on Strongly Interacting Dark Matter with IceCube

Ivone F. M. Albuquerque (1 and 2), Carlos Pérez de los Heros (3) ((1) Center for Particle Astrophysics FERMILAB ... Universidade de São Paulo, São Paulo, Brazil, (3) Department of Physics and Astronomy, Upp...

(Submitted on 8 Jan 2010 (v1), last revised 16 Feb 2010 (this version, v2))

Closing the Window on Strongly Interacting Dark Matter

Closing the window on $\sim$GeV Dark Matter with moderate ($\sim$$μ$b) interaction with nucleons

M. Shafi Mahdawi, Glennys R. Farrar

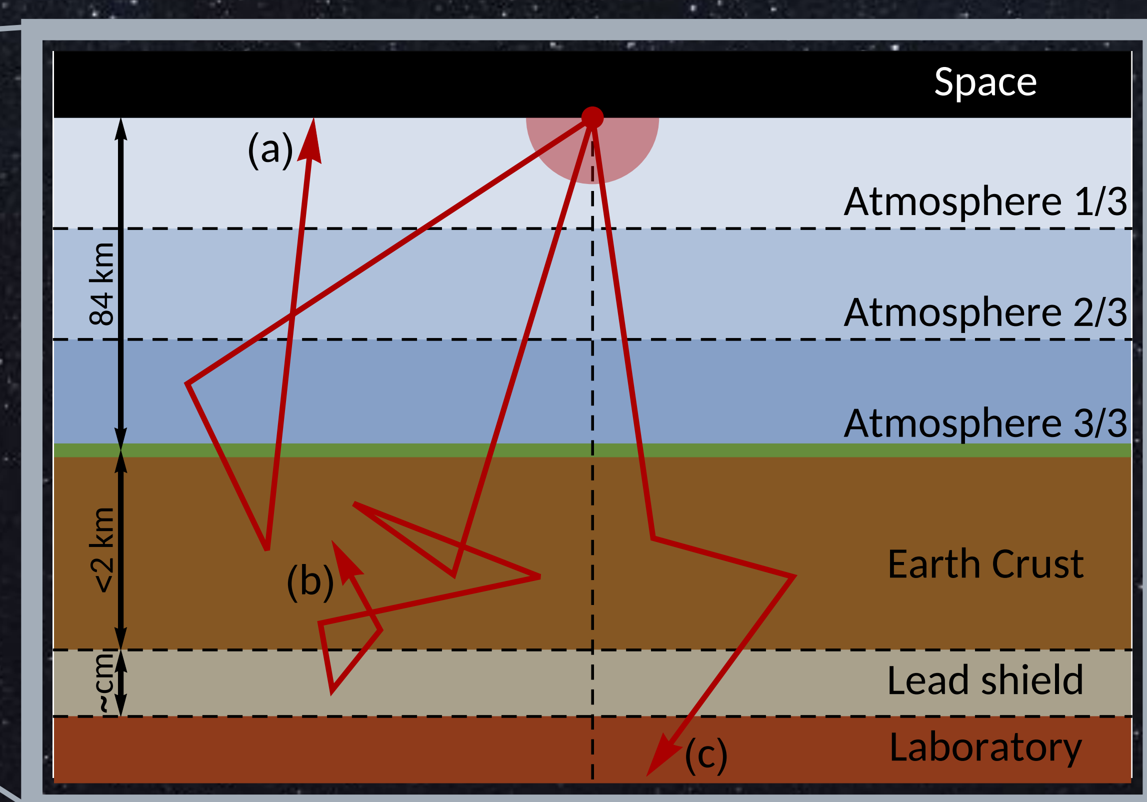
(Submitted on 1 Sep 2017 (v1), last revised 20 Dec 2017 (this version, v3))

Direct Detection Constraints on Strongly Interacting DM

- DM detectors are typically underground to *shield off* background sources from the detector.
- Above some high critical cross section, the overburden (Earth crust/ atmosphere) shields off DM particles itself.

Goodman and Witten, Phys.Rev. D31 (1985) 3059
Starkman et al, Phys.Rev. D41 (1990) 3594

→ Terrestrial experiments lose sensitivity to DM above this critical value.



Overburden Shielding vs. Detection I

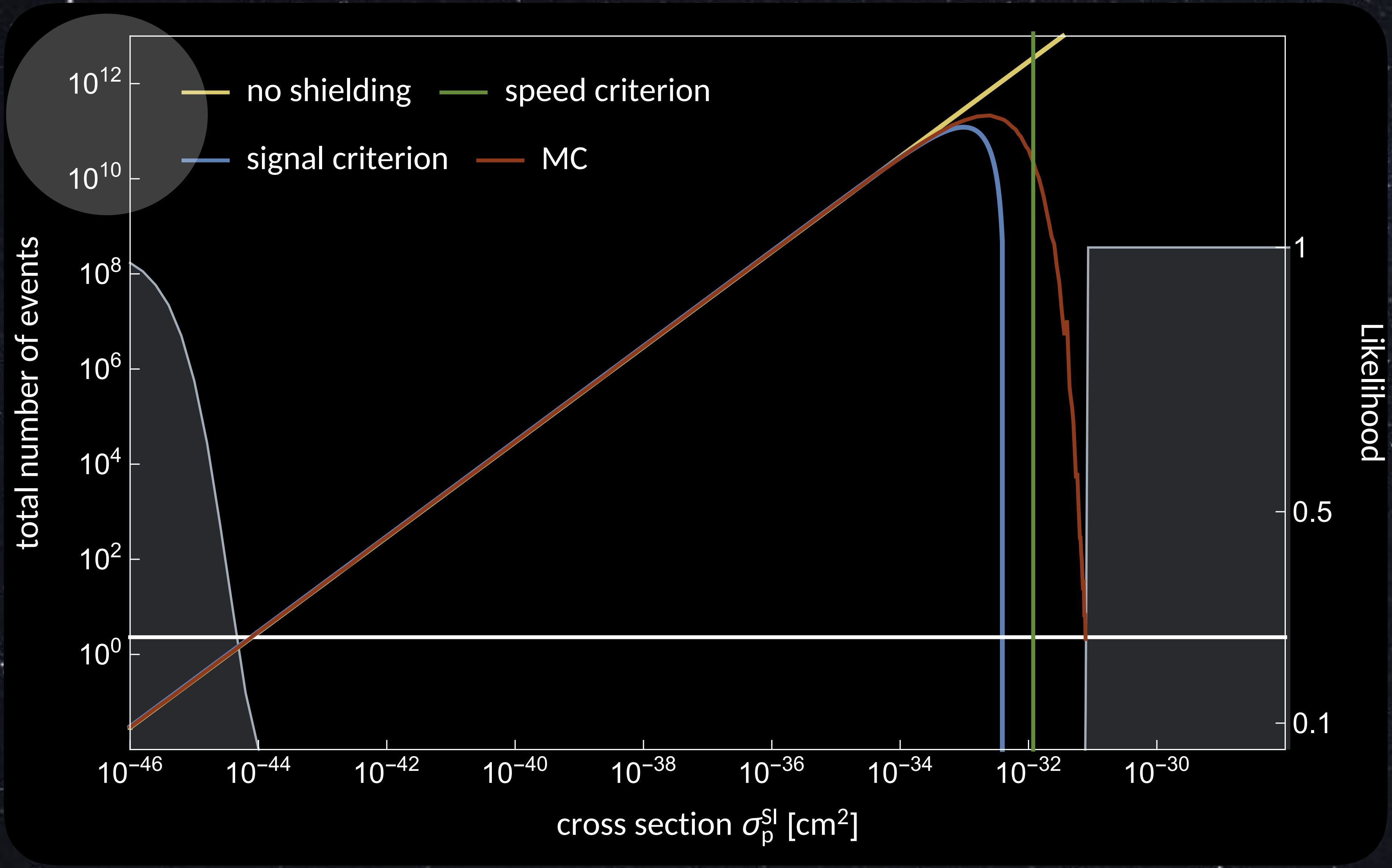
larger cross section

increases shielding

decreases event rate

increases event rate

Overburden Shielding vs. Detection II



Rare-event techniques are absolutely crucial!

- Importance Sampling
- Importance Splitting

Importance Sampling (IS)

- Rare event technique, which modifies the PDFs of the simulation.

$$\langle Y \rangle_I = \int_I \mathrm{d}x\ Y(x) f(x) = \int_I \mathrm{d}x\ Y(x) \frac{f(x)}{\hat{g}(x)} \hat{g}(x)$$

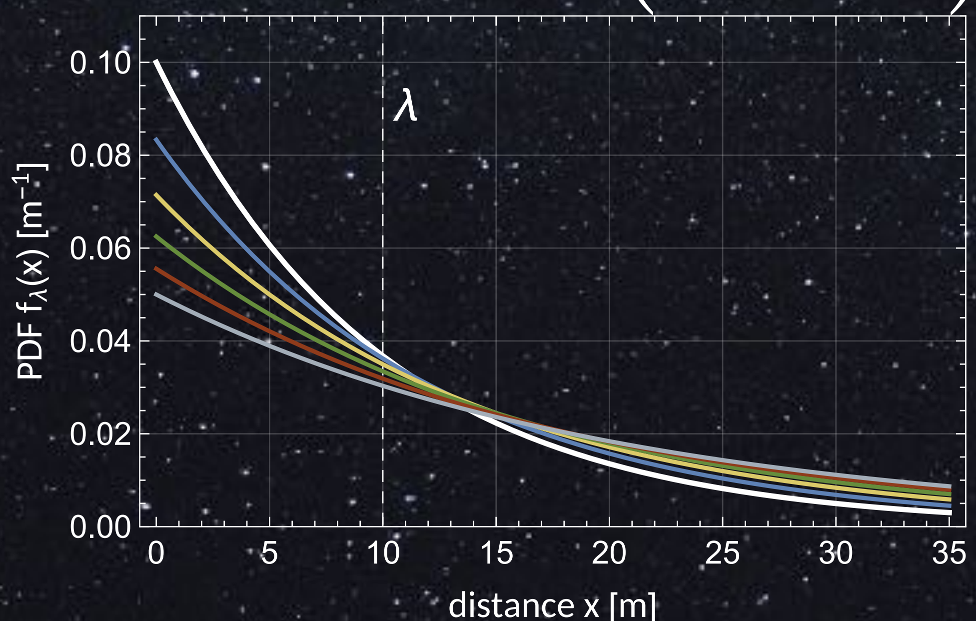
- Try to "mimic" the successful runs by introducing a bias into the simulations.

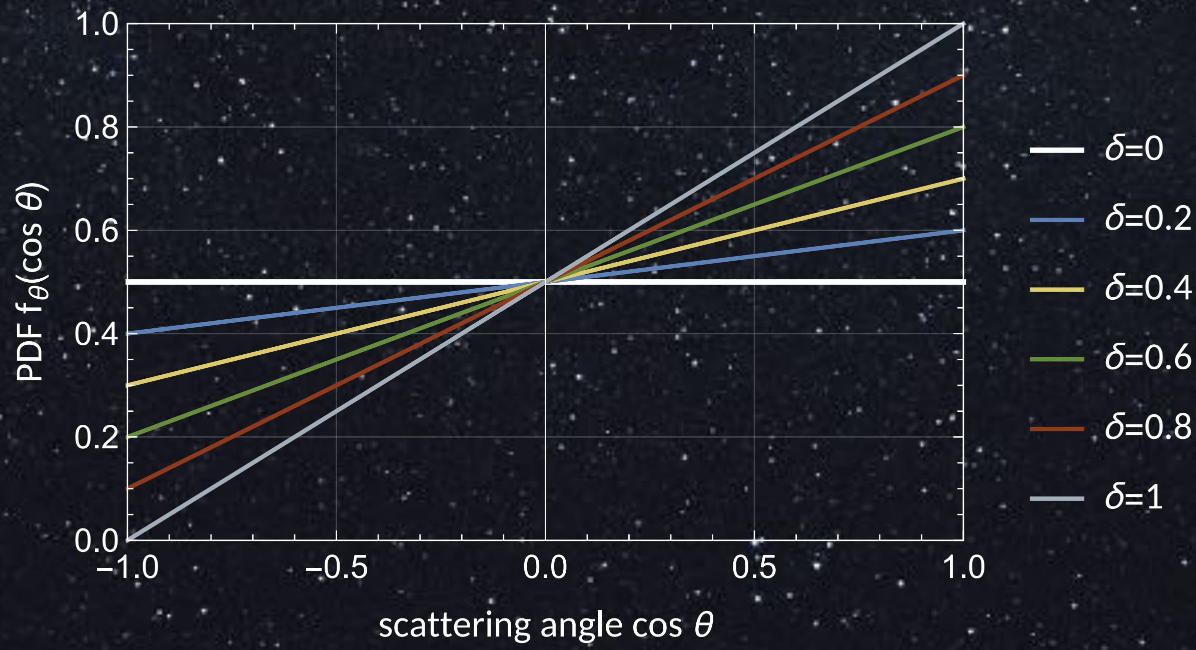
$$f_\lambda(x) = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right)$$

$$g_\lambda(x) = \frac{1}{(1+\delta_\lambda)\lambda} \exp\left(-\frac{x}{(1+\delta_\lambda)\lambda}\right)$$

$$f_\theta(\cos\theta) = \frac{1}{2}$$

$$g_\theta(\cos\theta) = \frac{1+\delta_\theta \cos\theta}{2}$$



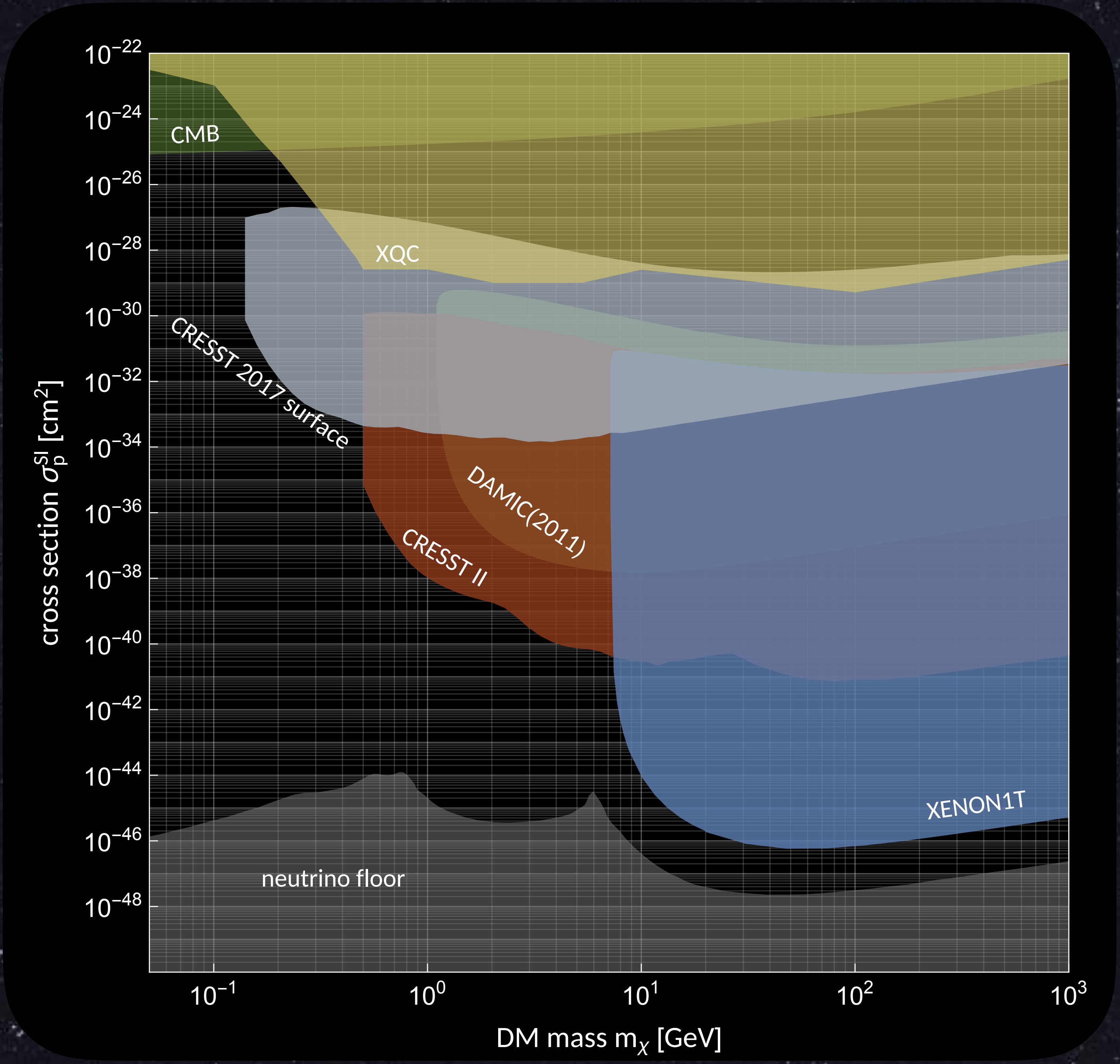


- Compensate by a statistical weight factor.

$$w_{\lambda,i} = \frac{f_\lambda(l_i)}{g_\lambda(l_i)}$$

M.S. Mahdawi, G.R. Farrar, JCAP 1712 (2017) 004

Constraints from Nuclear Recoil Experiments



Including DM-electron scatterings

The incoming DM flux gets attenuated by

1. Elastic nuclear scatterings.
2. Elastic DM-electron scatterings.
3. Inelastic DM-electron scatterings (ionizations/excitations).

detection process $\neq$ attenuation/stopping process

↓

We need a model.

The Dark Photon Model

- Extend the SM by a DM particle and a U(1) gauge group with kinetic mixing.

$$\mathscr{L}_D = \bar{\chi}(i\gamma^\mu D_\mu - m_\chi)\chi + \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + m^2_{A'}A'_\mu A'^\mu + \varepsilon F_{\mu\nu}F'^{\mu\nu}$$

- For kinetic mixing with the photon, the DM couples to electric charge.

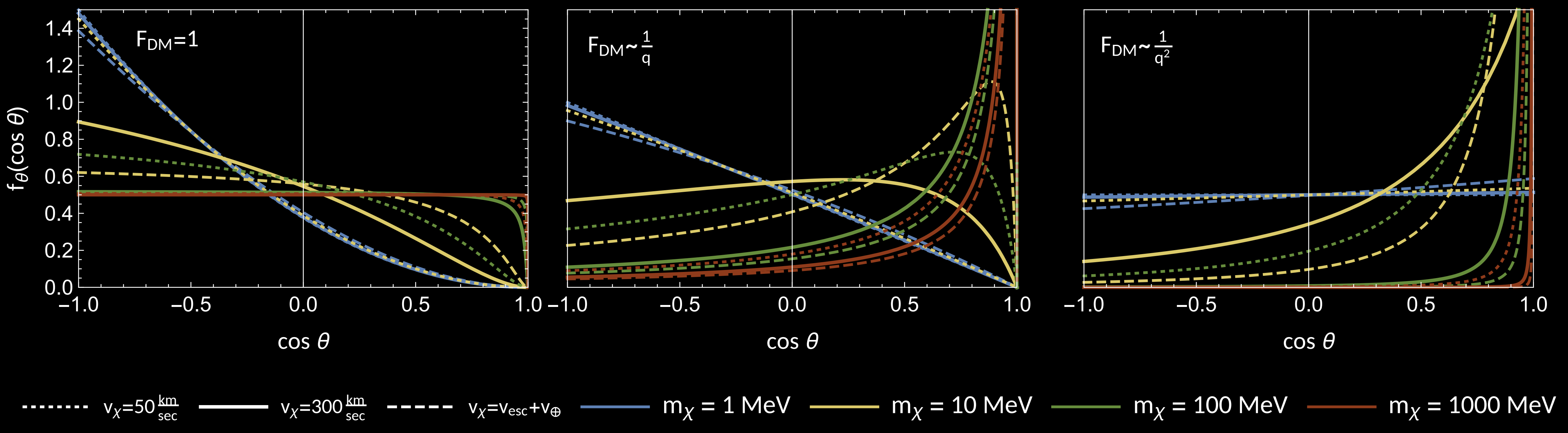
$$\frac{\mathrm{d}\sigma_N}{\mathrm{d}q^2} = \frac{\sigma_p}{4\mu^2_{\chi p}v^2_\chi}F_{\mathrm{DM}}(q)^2\,F_N(q)^2\,Z^2$$

- Hierarchy between the DM-proton and DM-electron cross section:

$$\frac{\sigma_p}{\sigma_e} = \left(\frac{\mu_{\chi p}}{\mu_{\chi e}}\right)^2$$

S.K. Lee et al, PRD92 (2015) 083517

New scattering kinematics



DM form factor **vs** Charge screening

$$F_{\rm DM}(q) = \begin{cases} 1\,, & \text{for heavy mediator}\,, \\ \frac{q_{\rm ref}}{q}\,, & \text{for ED interaction}\,, \\ \left(\frac{q_{\rm ref}}{q}\right)^2\,, & \text{for light mediator}\,. \end{cases}$$

$$F_A(q) = \frac{a^2q^2}{1+a^2q^2}$$

Geometric Importance Splitting (GIS)

- Rare event method.
- "More interesting" particles get split into copies.
- Requires the definition of an importance function,

$$I : \mathbb{R}^3 \to \mathbb{R}$$

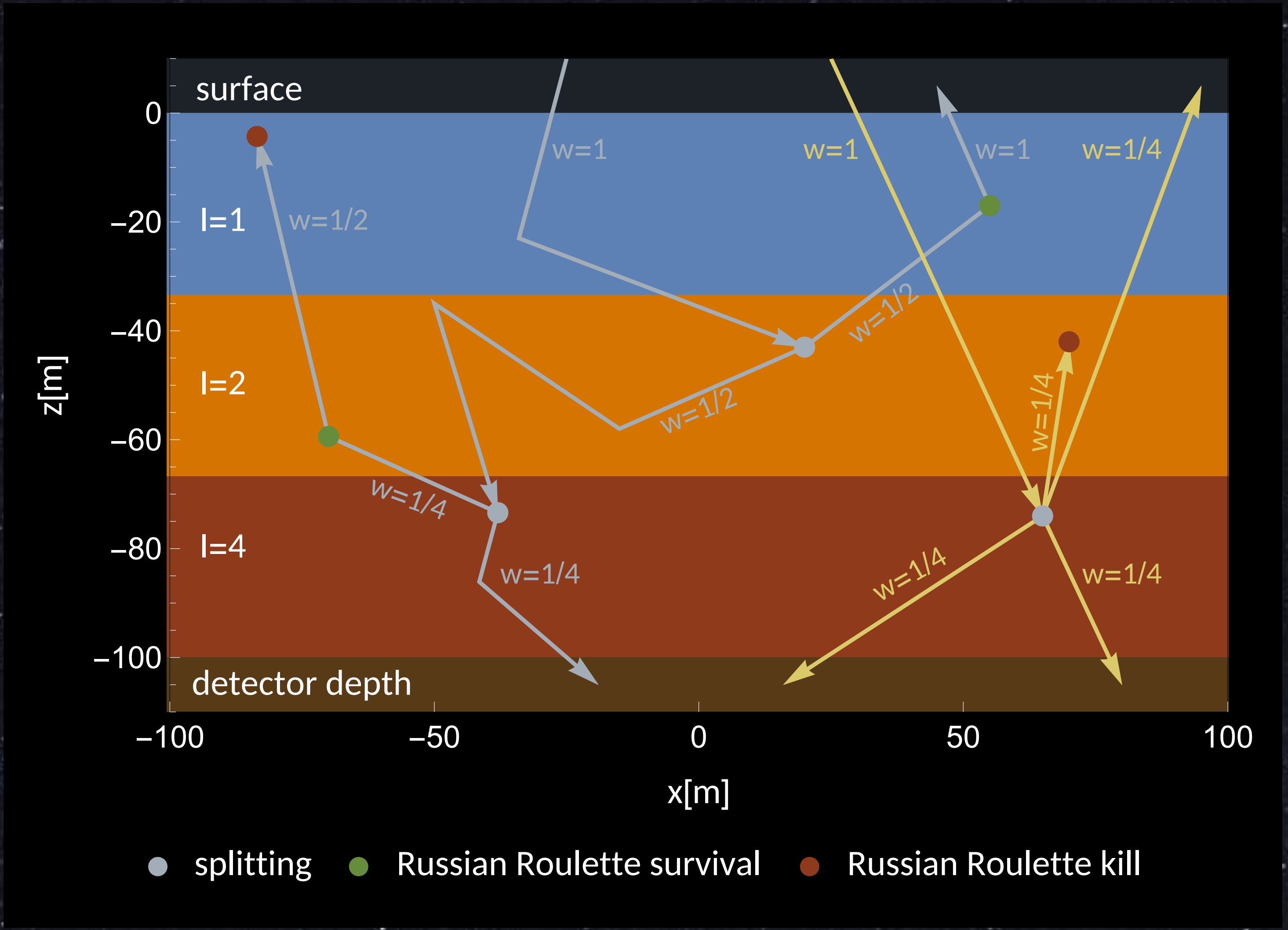
- If the importance increases,

$$\nu \equiv \frac{I_{i+1}}{I_i} > 1$$

the particle gets split into

$$n = \begin{cases} \nu\,, & \textbf{if } \nu \in \mathbb{N}\,, \\ \lfloor \nu \rfloor\,, & \textbf{if } \nu \notin \mathbb{N} \wedge \xi \geq \Delta\,, \\ \lfloor \nu \rfloor + 1\,, & \textbf{if } \nu \notin \mathbb{N} \wedge \xi < \Delta\,, \end{cases}$$

copies.

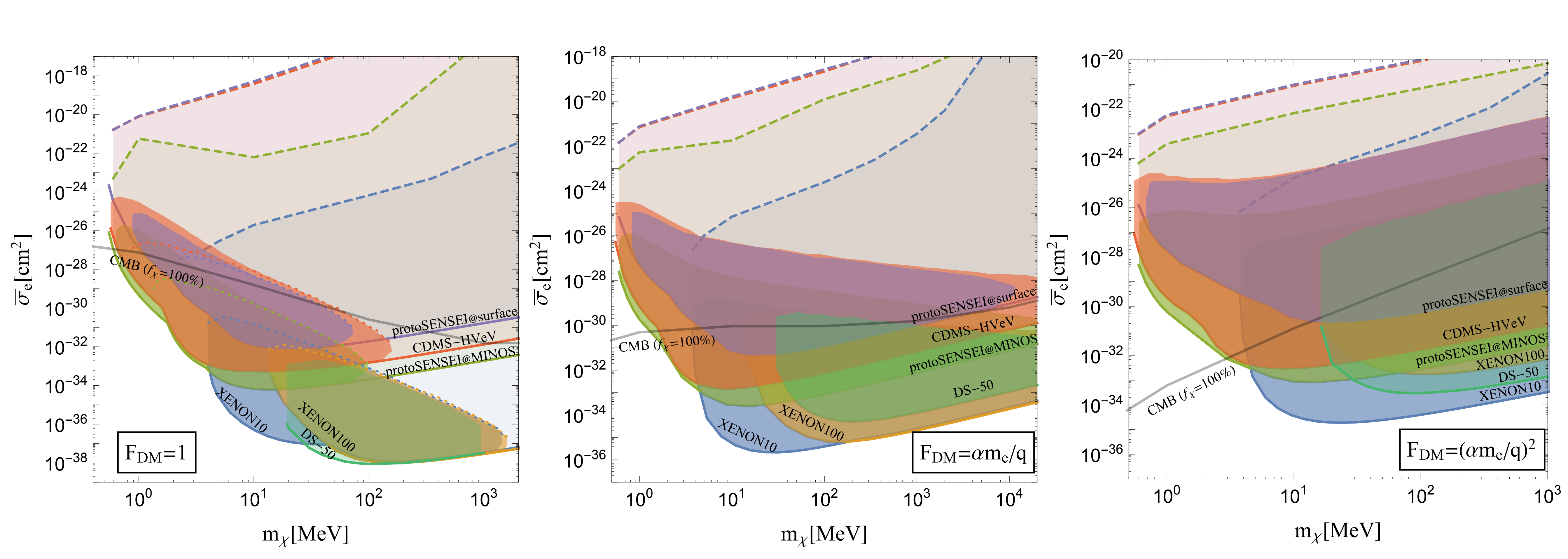


New statistical weight

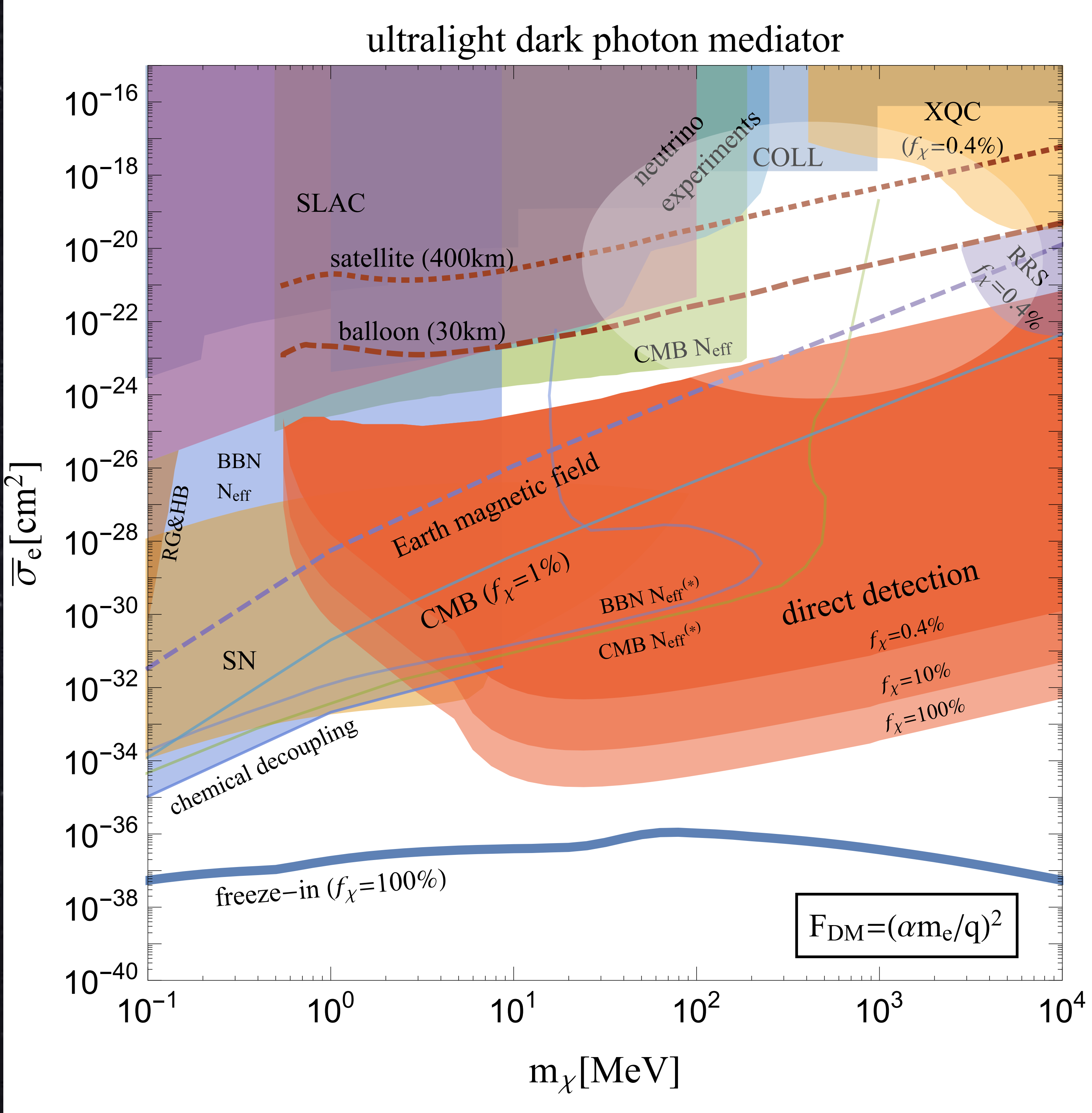
$$w_{i+1} \equiv \frac{w_i}{n}$$

- Otherwise: Russian Roulette

Constraints on DM-Electron Scatterings



Constraints on a sub-dominant component of strongly interacting DM

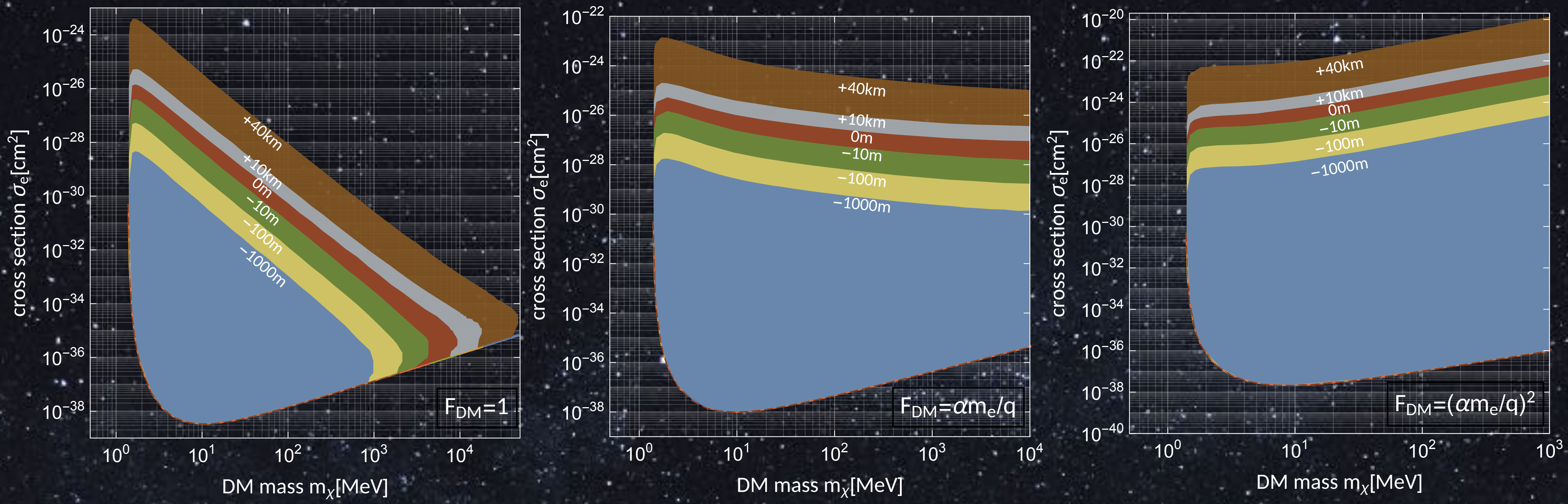


- So is there an open window in parameters space?
- Probably not for milli-charged DM.
- Definitely not for

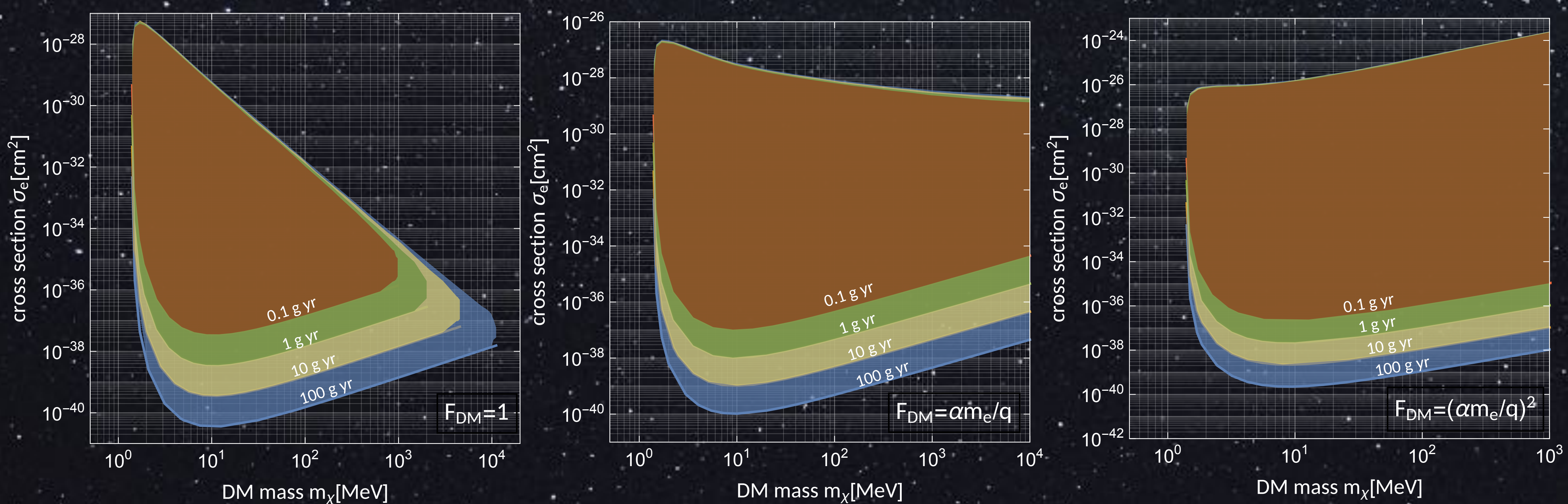
$$f_\chi = 100\,\%$$

- Yes, under certain conditions:
  - Sub-dominant component.
    $$f_\chi < 0.4\,\%$$
  - Ultralight, but not massless mediator.
  - Small dark gauge coupling.

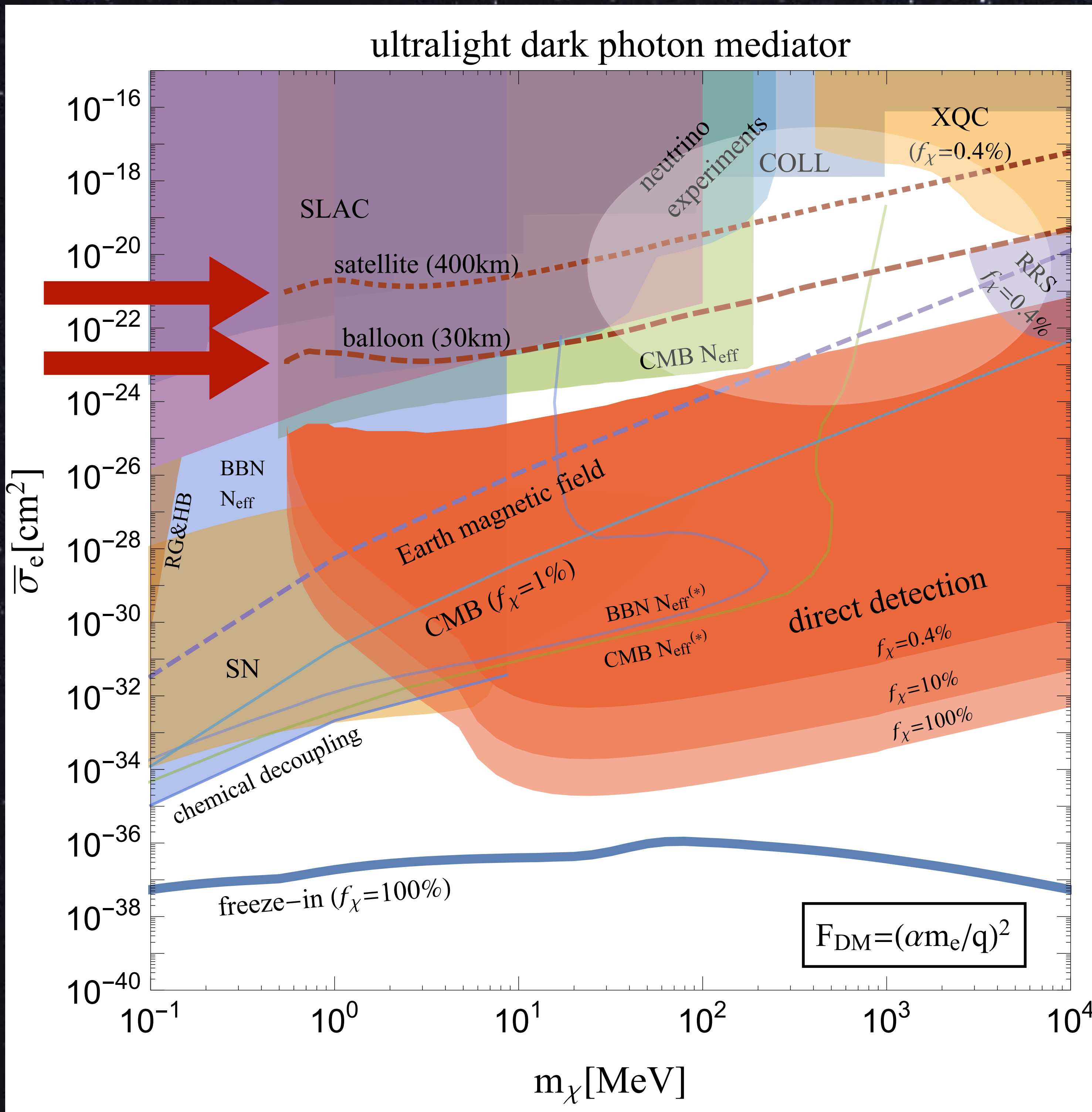
Constraint scaling with depth



Constraint scaling with exposure



Constraints on a sub-dominant component of strongly interacting DM



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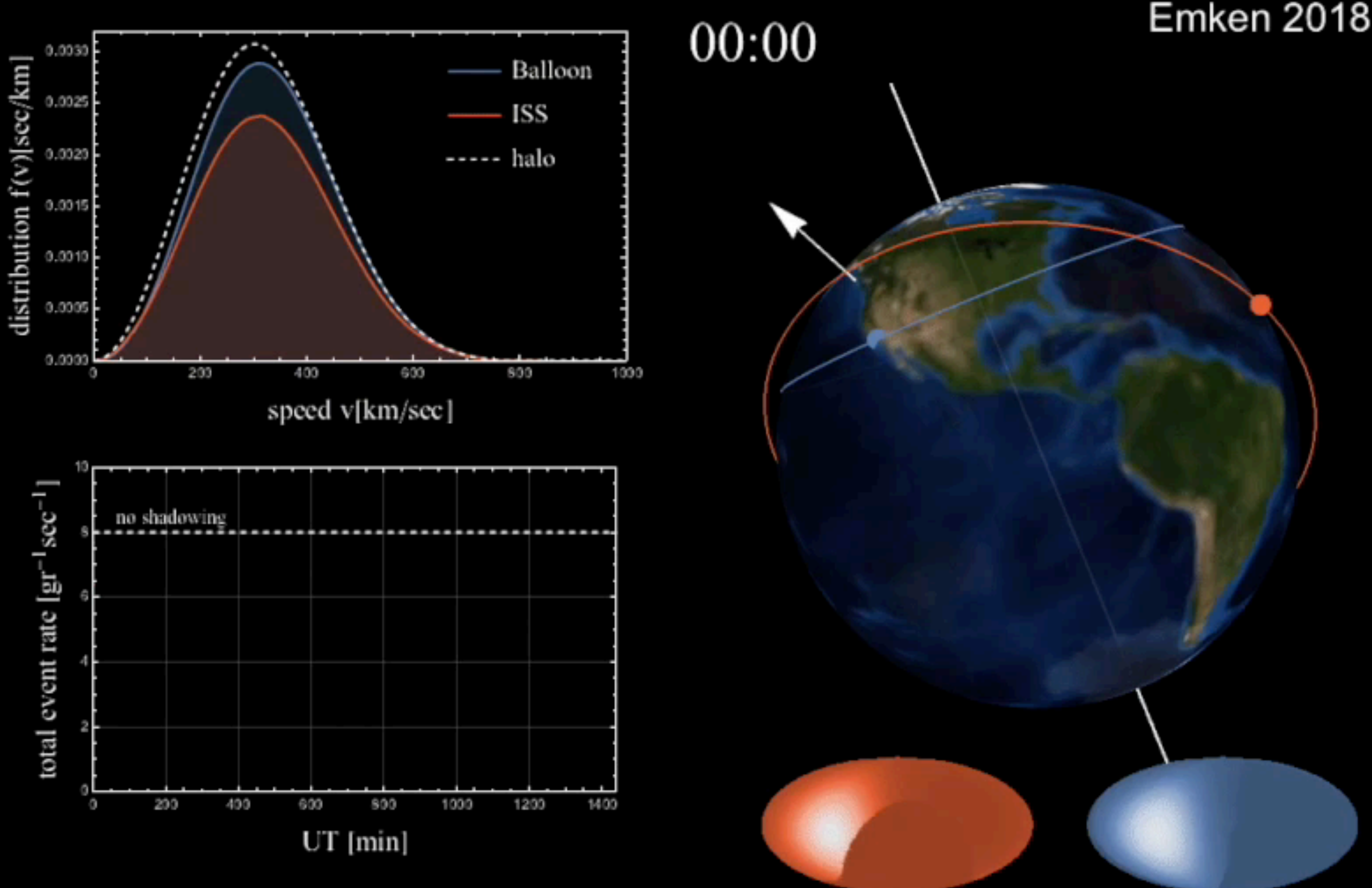
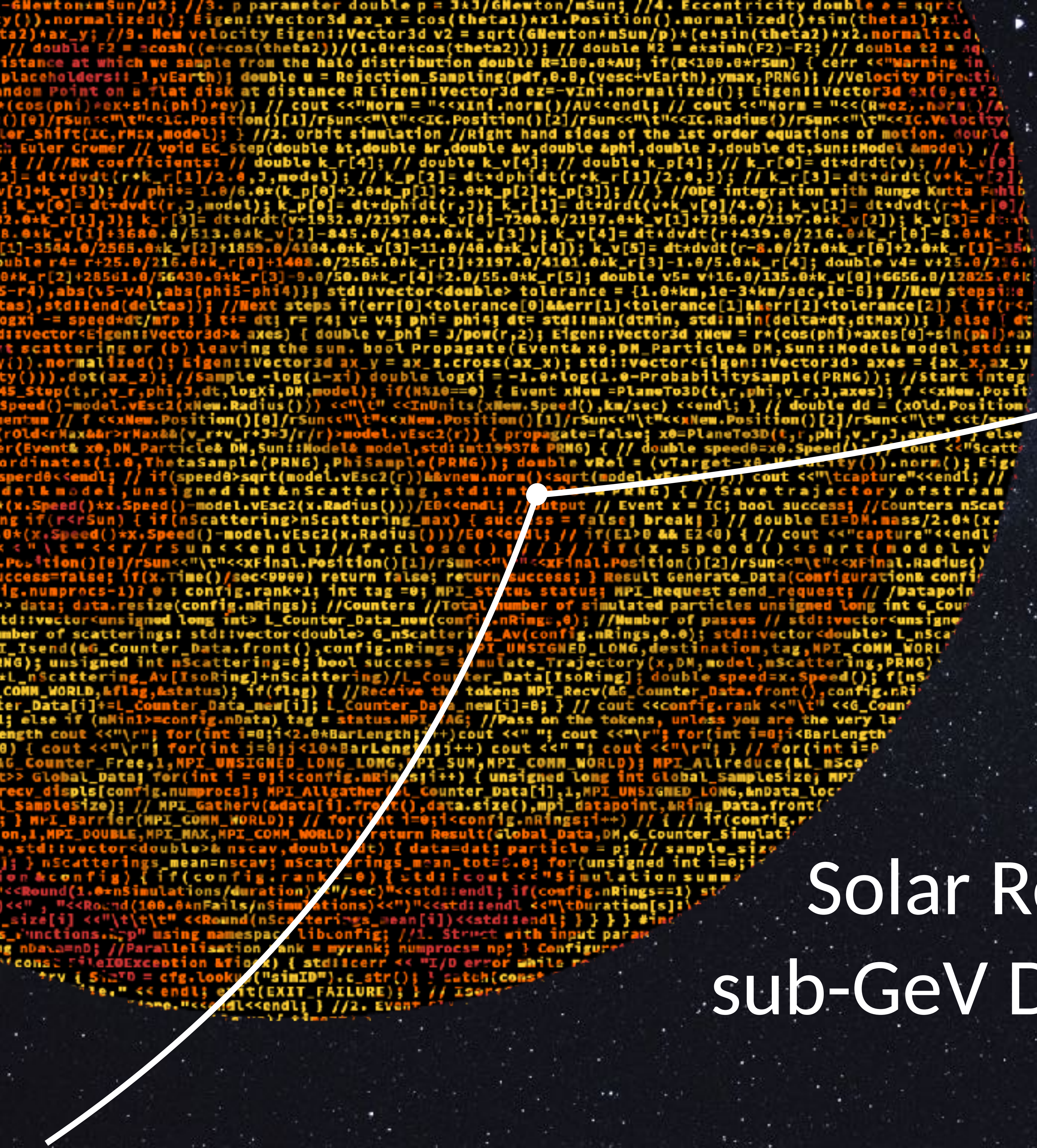

Emken 2018
00:00
distribution f(v)[sec/km]
speed v[km/sec]
Balloon
ISS
halo
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
0
200
400
600
800
1000
total event rate [gr^{-1}sec^{-1}]
UT [min]
no shadowing
0
2
4
6
8
10
0
200
400
600
800
1000
1200
1400



III.

Solar Reflection of sub-GeV Dark Matter

The Idea of Solar Reflection of DM

- direct detection experiments can only probe DM masses above

$$m_\chi^{\min} = \frac{m_N}{\sqrt{\frac{2m_N}{E_R^{\text{thr}}}}v_{\max} - 1}$$

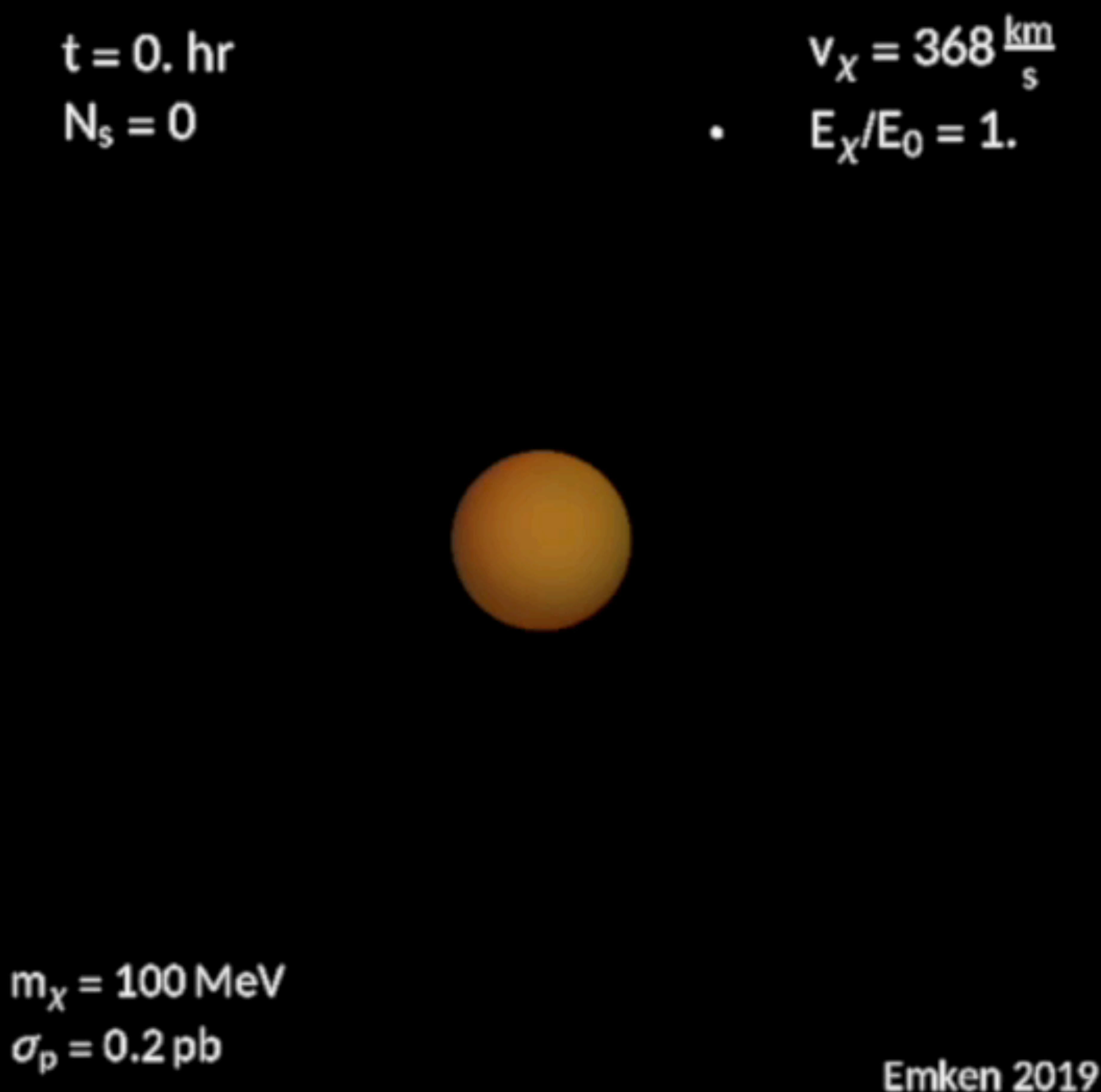
- Experimentalists have pushed this mass down by decreasing the target mass and the recoil threshold, e.g. the CRESST collaboration.

- What if some process could increase the maximum DM speed somehow?

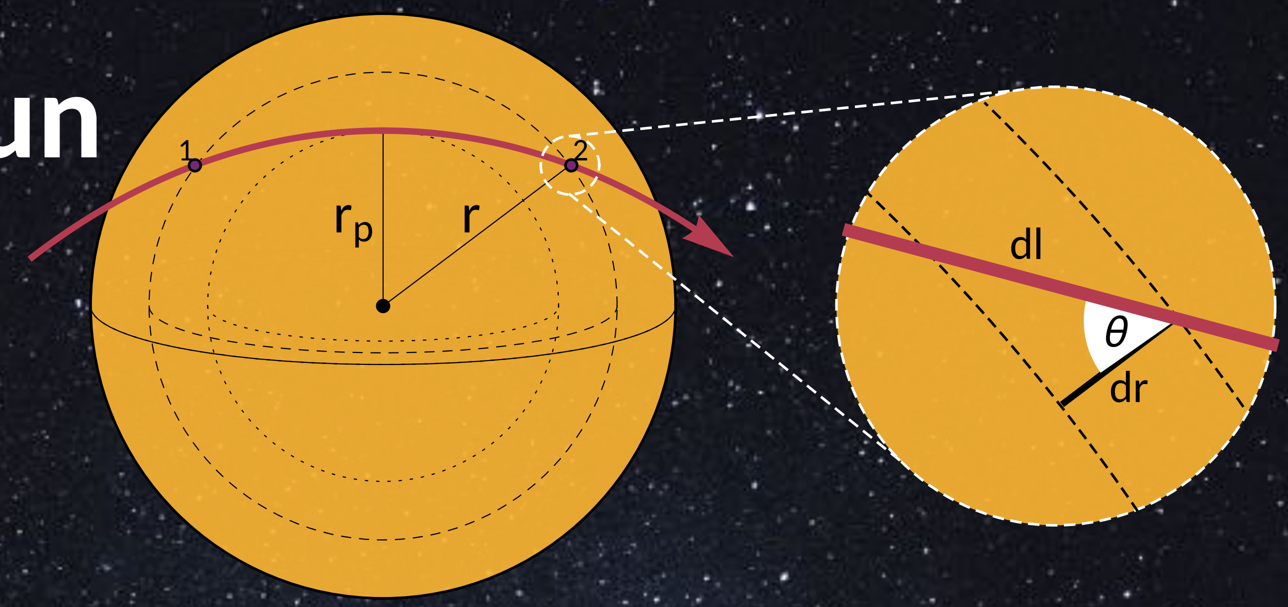
$$v_{\max} > v_{\text{esc}}^{(\text{gal})} + v_\oplus$$

- Elastic scatterings on hot solar targets inside the Sun could do just that.

H. An, M. Pospelov, J. Pradler, A. Ritz, Phys. Rev. Lett. 120 (2018), 141801


t = 0. hr
$N_s = 0$
$v_\chi = 368 \frac{km}{s}$
$E_\chi/E_0 = 1.$
$m_\chi = 100$ MeV
$\sigma_p = 0.2$ pb
Emken 2019

DM Scatterings in the Sun



- Scattering rate in a spherical shell of the Sun:

$$\mathrm{d}S = \underbrace{\mathrm{d}\Gamma}_{\text{passing rate}} \times \underbrace{\mathrm{d}P_{\text{scat}}}_{\text{scattering probability}} \times \underbrace{P_{\text{shell}}}_{\text{prob. to reach shell}}$$

- The three factors are given by

$$\mathrm{d}\Gamma = 4\pi R^2 \mathrm{d}\Phi_\chi = \pi n_\chi f_\chi(u) \frac{\mathrm{d}u\mathrm{d}J^2}{u}\,, \qquad \mathrm{d}P_{\text{scat}} = \frac{\mathrm{d}l}{w} \times \Omega(r,w)\,,$$

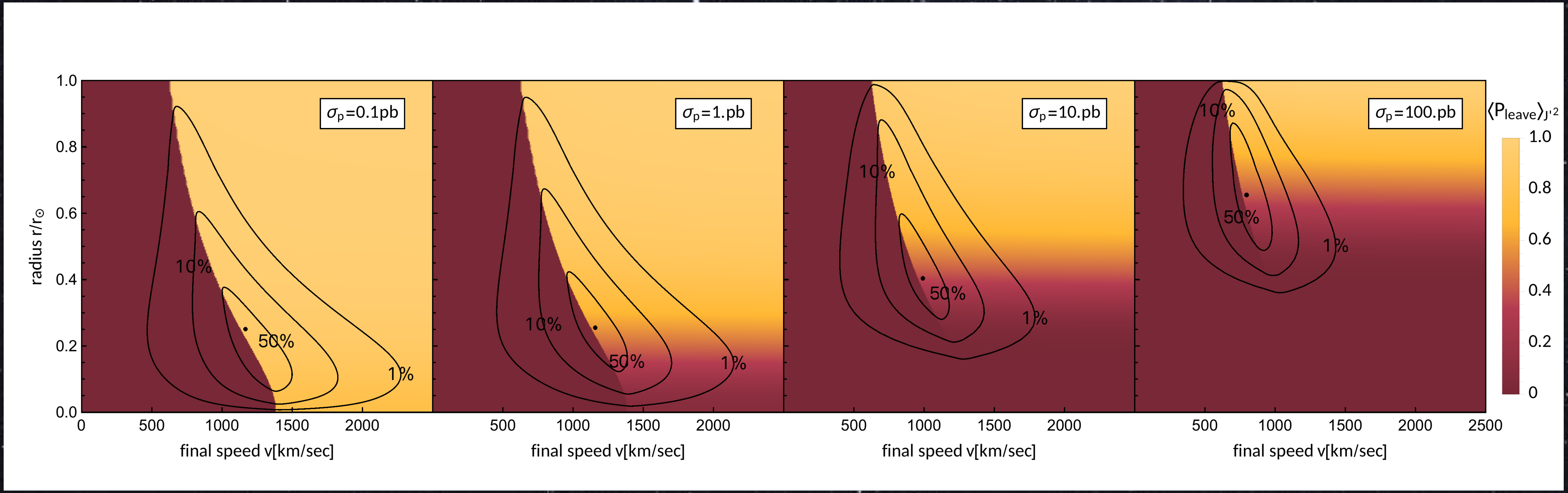
$$P_{\text{shell}}(r) = P_{\text{surv}}(r,R_\odot)\left[1 + P_{\text{surv}}(r_p,r)^2\right]\Theta\left(w(u,r_p)r_p - J\right)\,.$$

As opposed to Gould's formalism, this scattering rate applies not just to the single scattering regime.

A. Gould, Astrophys. J. 321, (1987), 571
Astrophys. J. 321, (1987), 560
Astrophys. J. 388, (1992), 338

Scattering rate and leaving probability in the Sun

$$\frac{\mathrm{d}S}{\mathrm{d}r\mathrm{d}v} = \int \mathrm{d}\Gamma \frac{\mathrm{d}P_{\text{scat}}}{\mathrm{d}r\mathrm{d}v} P_{\text{shell}}(r) = \pi n_\chi \int_0^\infty \mathrm{d}u \int_0^{w(u,r)^2 r^2} \mathrm{d}J^2 \frac{f_\chi(u)}{u} \frac{\mathrm{d}\Omega}{\mathrm{d}v}(w(u,r) \to v) \left[w(u,r)^2 - \frac{J^2}{r^2} \right]^{-1/2} \times P_{\text{surv}}(r, R_\odot) \left[1 + P_{\text{surv}}(r_p, r)^2 \right]$$



Detection of Reflected Dark Matter

- DM flux of solar reflection

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}v\mathrm{d}r} = \frac{\mathrm{d}S}{\mathrm{d}v\mathrm{d}r}\langle P_{\text{leave}}(v,r)\rangle_{J^2}$$

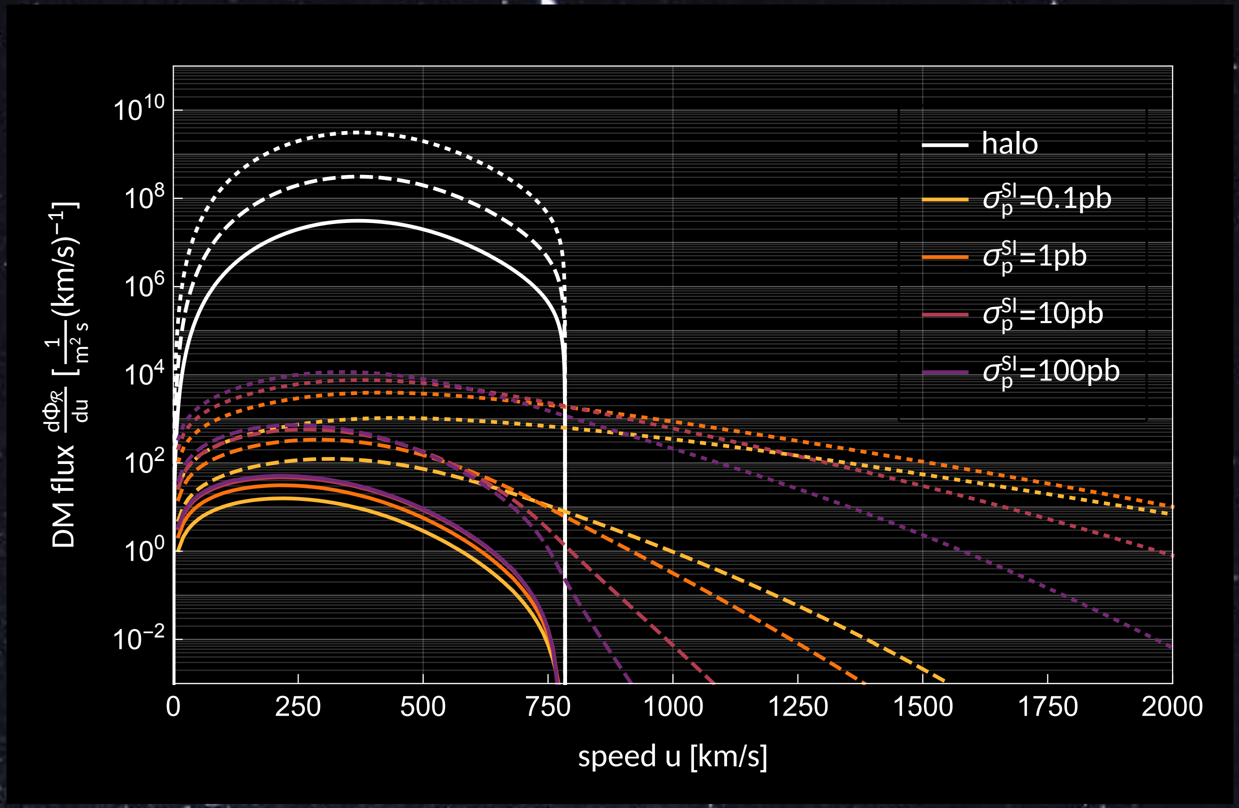
$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}u} = \int_0^{R_\odot} \mathrm{d}r\, \frac{\mathrm{d}\mathcal{R}}{\mathrm{d}v\mathrm{d}r}\frac{\mathrm{d}v}{\mathrm{d}u}\Bigg|_{v=\sqrt{u^2+v_{\text{esc}}(r)^2}}$$

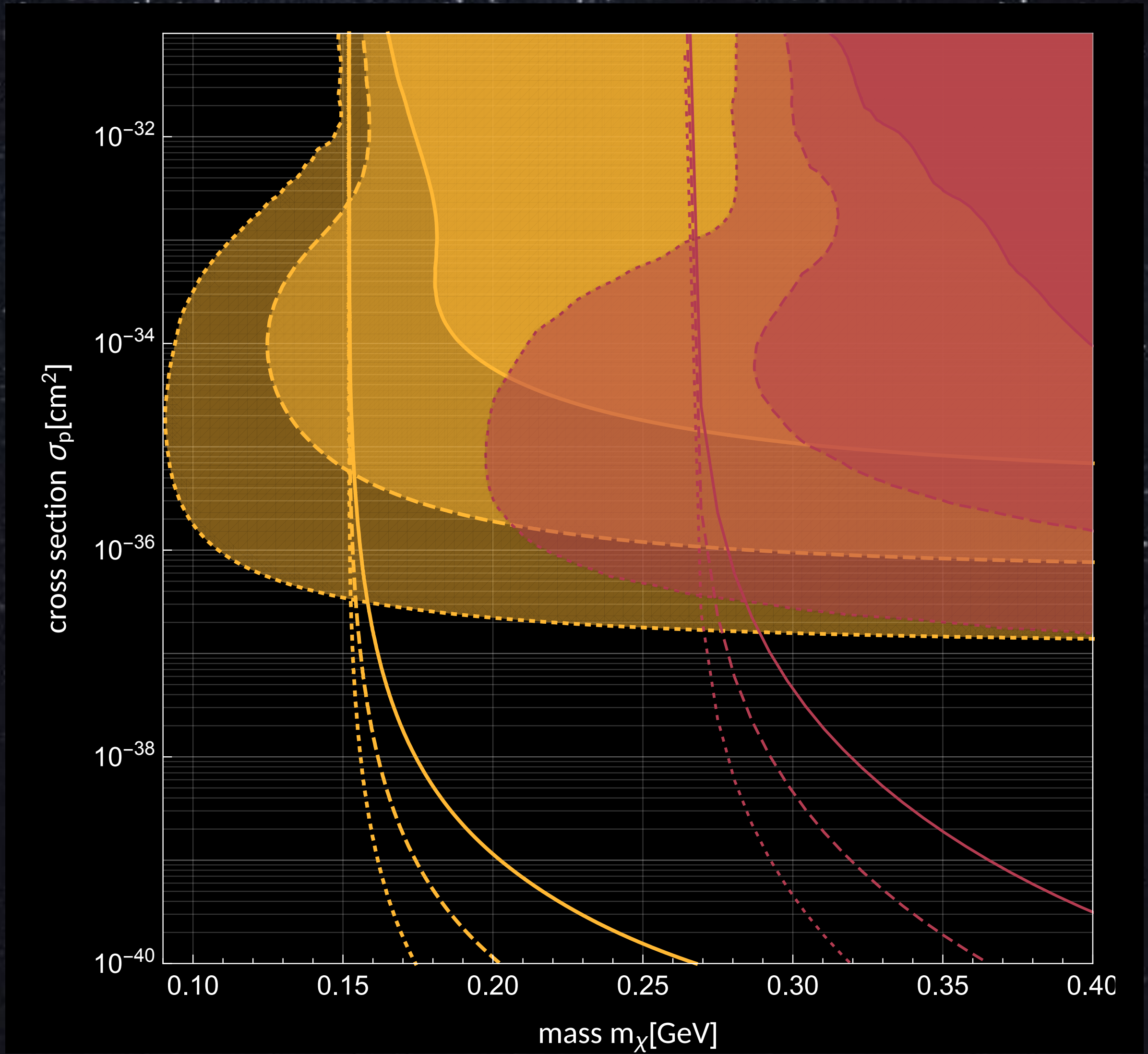
'red-shift' to the Earth's location

- Include into recoil spectrum

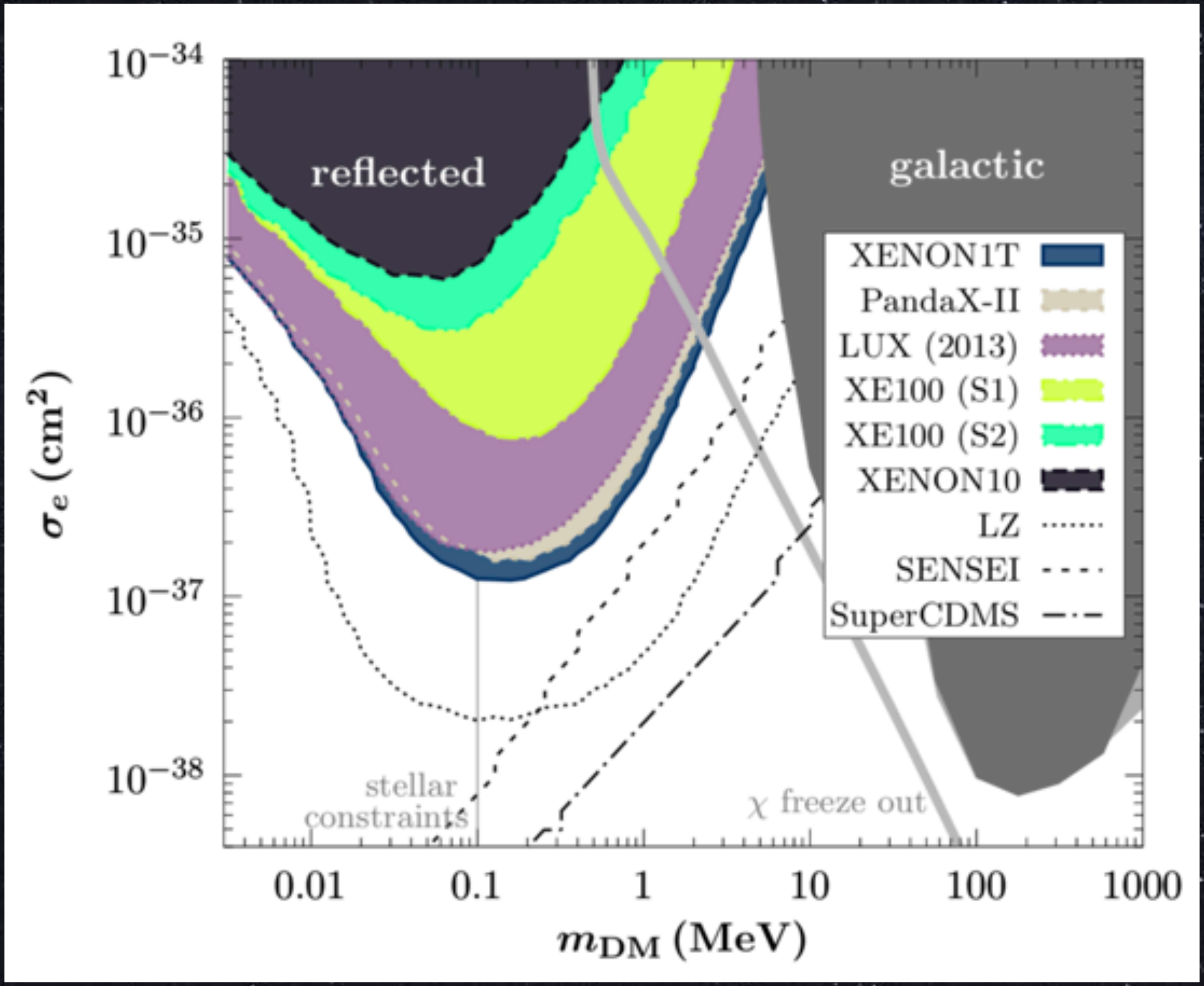
$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = \frac{1}{m_N}\int\limits_{u_{\min}(E_R)}^{\infty} \mathrm{d}u \left[\frac{\rho_\chi}{m_\chi}uf_\oplus(u) + \frac{1}{4\pi\ell^2}\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}u}\right]\frac{\mathrm{d}\sigma_N}{\mathrm{d}E_R}$$

- Compute likelihoods and constraints as usual.





Detection of Reflected Dark Matter with electrons



H. An, M. Pospelov, J. Pradler, A. Ritz, Phys. Rev. Lett. 120 (2018), 141801

Features of a solar reflection signal

- DM flux from the Sun ➡ directional detection
- **Higher exposures probe lower masses.**
- (Semi-) halo-model independent.
- A new type of annual modulation (~7% with peak in January).
- Our results so far are conservative (single scattering, only nuclear targets).

  ➡MC simulations

MC Simulations of DM Particles in the Sun

Extend the Earth simulations in **five** ways:

1. **Solar model**: density, composition, temperature.
2. **Gravity**: Solve the equations of motion numerically.
3. **Temperature**: thermal motion of the solar targets.
4. **Targets**: Include electrons.
5. **Initial conditions**: Generalize generation of ICs to account for gravitational focussing.

The mean free path inside the Sun

- When simulating DM passing through the Earth we computed the mean free path via

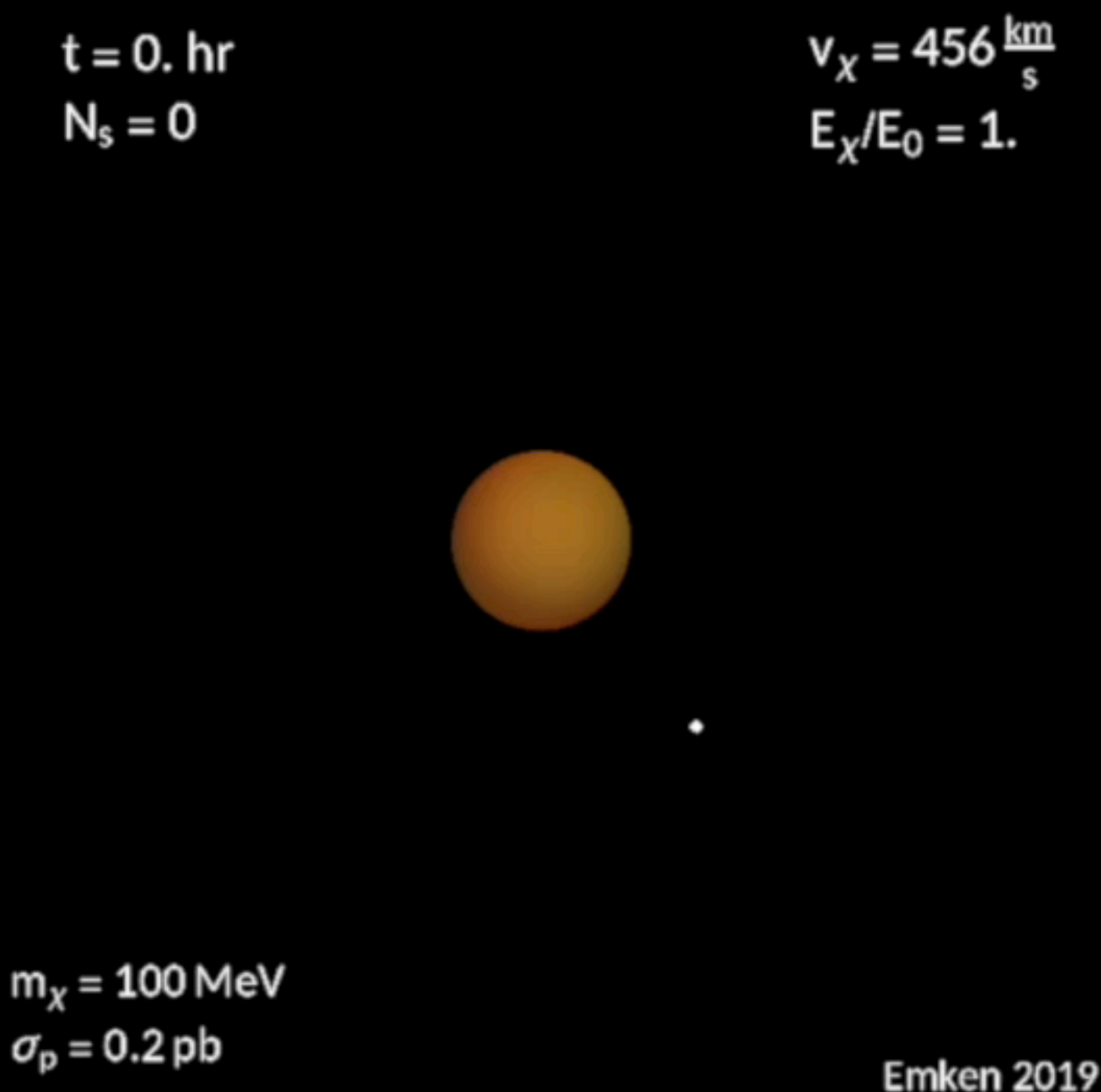
$$\lambda^{-1} = \sum_i n_i \times \sigma_i \,.$$

- However, this is only valid for the resting-targets assumption.
- Inside the Sun, the important quantity is the **scattering rate.**

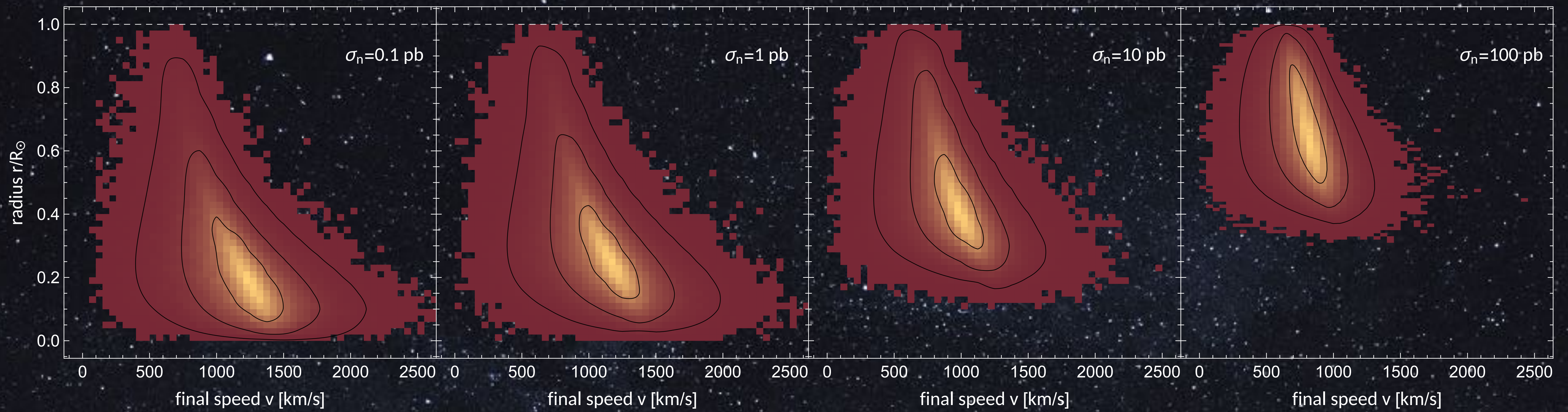
$$\Omega = \sum_i n_i \times \sigma_i \times \langle |\mathbf{w} - \mathbf{v}_i| \rangle \,.$$

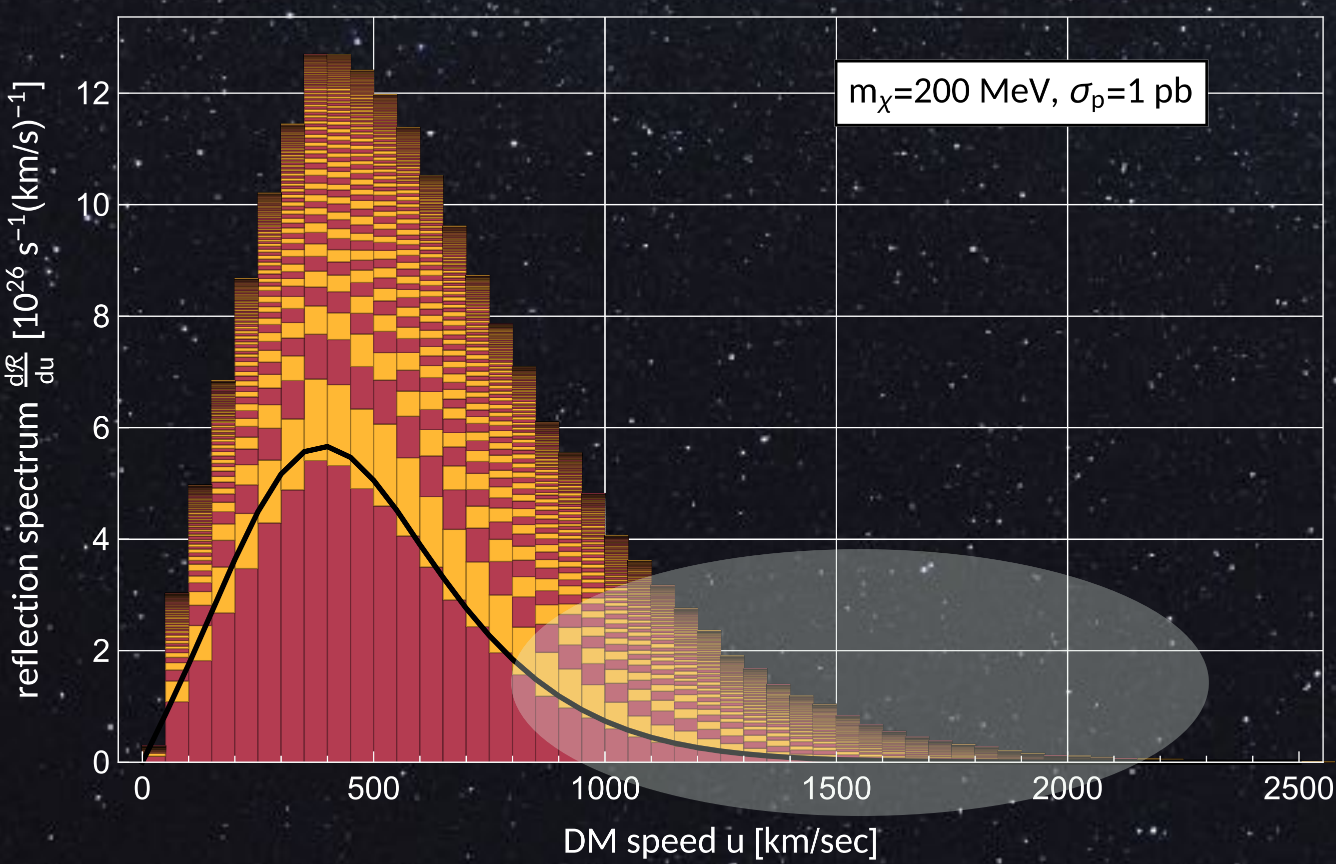
- If we still want to define a mean free path, we can do so.

$$\lambda^{-1} = \frac{\Omega}{w} = \sum_i n_i \times \sigma_i \times \frac{\langle |\mathbf{w} - \mathbf{v}_i| \rangle}{w}$$


t = 0. hr
N_s = 0
v_χ = 456 km/s
E_χ/E_0 = 1.
m_χ = 100 MeV
σ_p = 0.2 pb
Emken 2019

First MC Results for Solar Reflection of Dark Matter



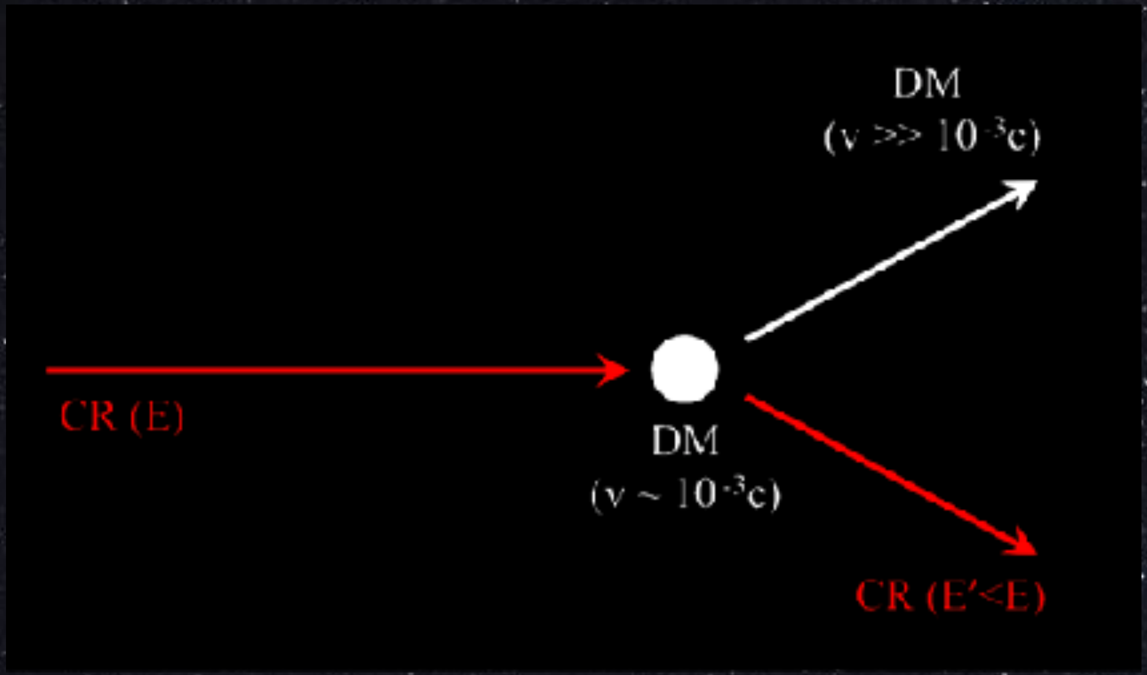


- Comparing the first scattering to our analytic results shows great agreements.
- The inclusion of multiple scatterings greatly enhances the high energy tail of the reflection spectrum.

IV.

Cosmic ray up-scattered DM

A relativistic DM flux from Cosmic Ray Scatterings

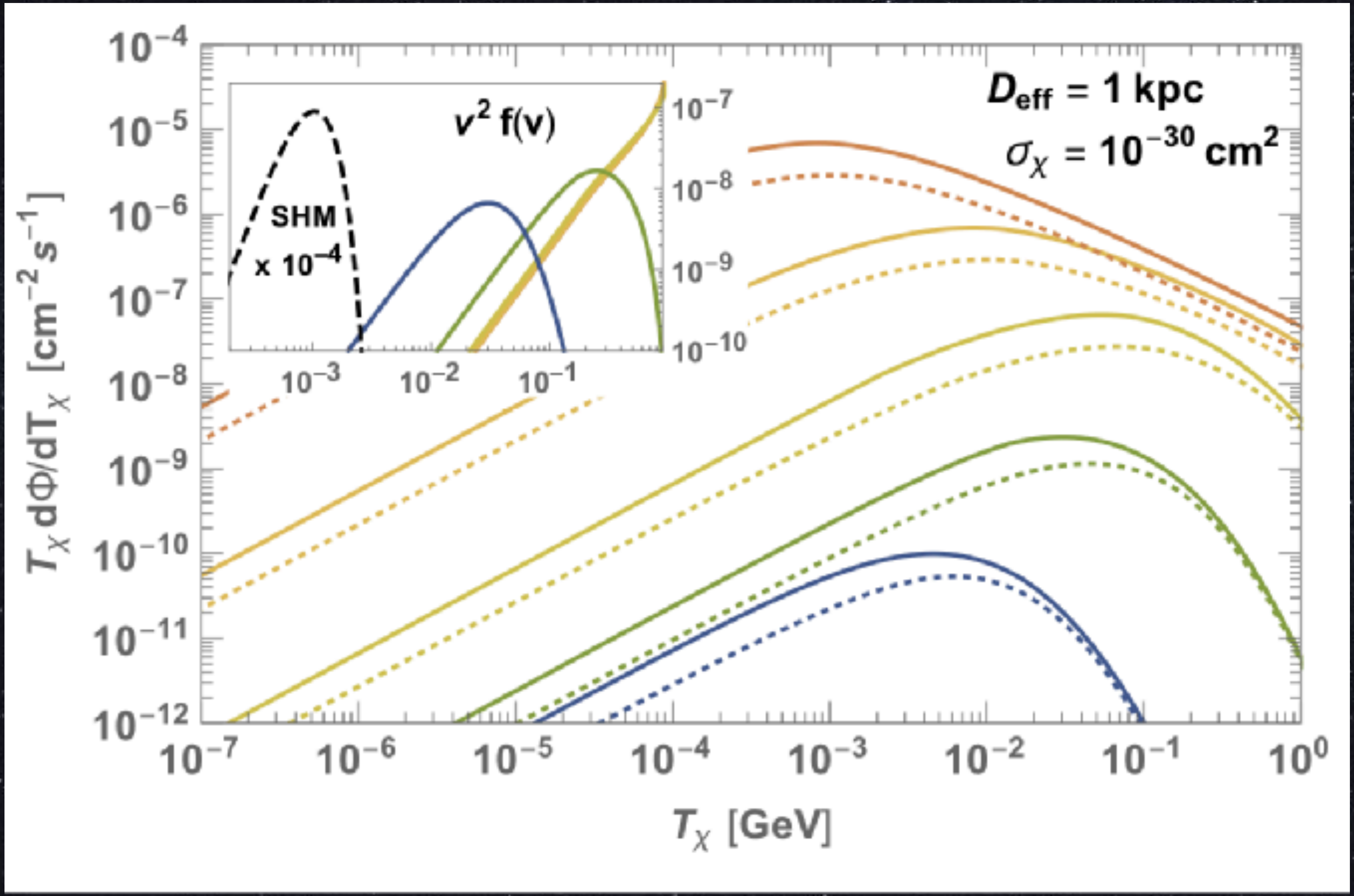


Cappiello, Beacom (2019)

- Similar effect to solar reflection.
- DM particles get accelerated to **relativistic** speeds.

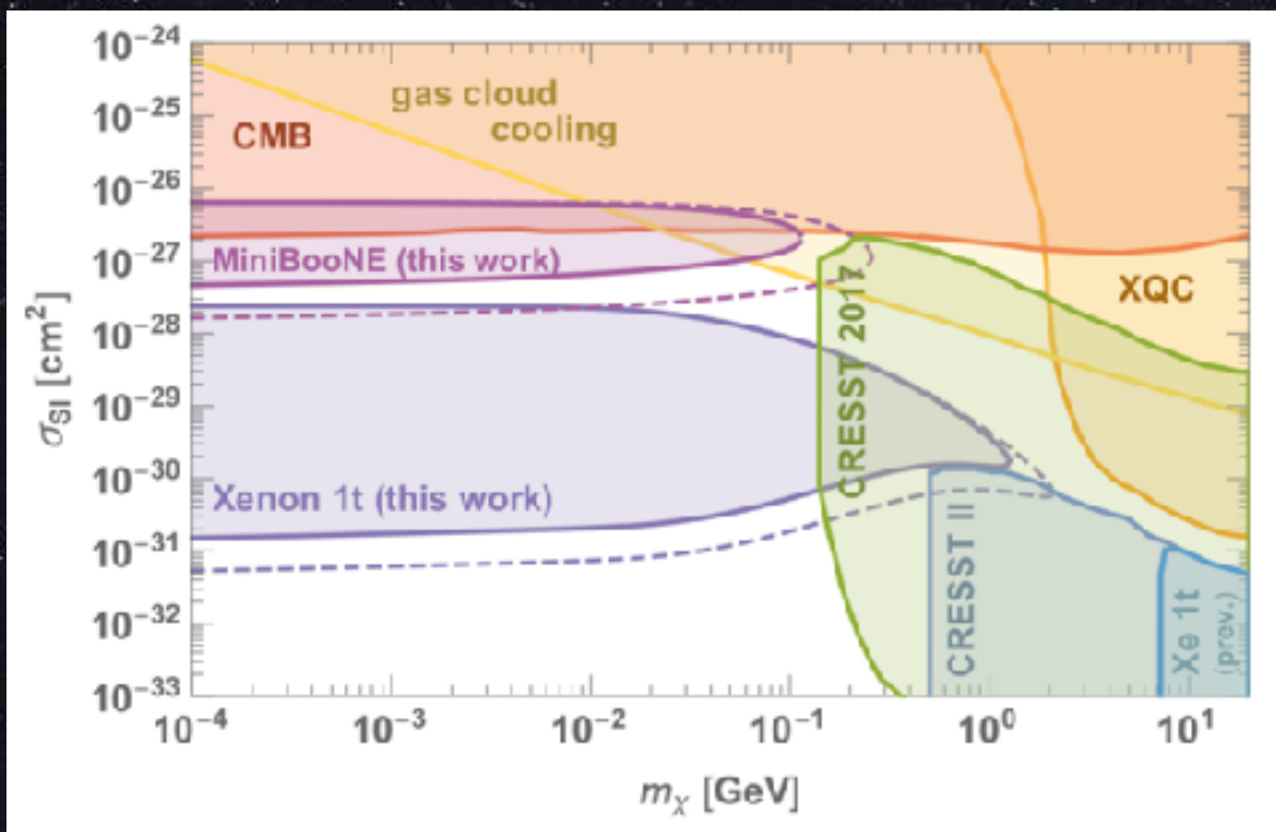
T. Bringmann, M. Pospelov, Phys.Rev.Lett. 122 (2019) no.17, 171801, [arXiv:1810.10543]
Y. Ema, F. Sala, R. Sato, Phys.Rev.Lett. 122 (2019) no.18, 181802, [arXiv:1811.00520]
C. Cappiello, J.F. Beacom, [arXiv:1906.11283]
J.B. Dent, B. Dutta, J.L. Newstead, I.M. Shoemaker, [arXiv:1907.03782]
G. Krnjaic, S.D. McDermott, [arXiv:1908.00007]

Spectrum of CR up-scattered DM

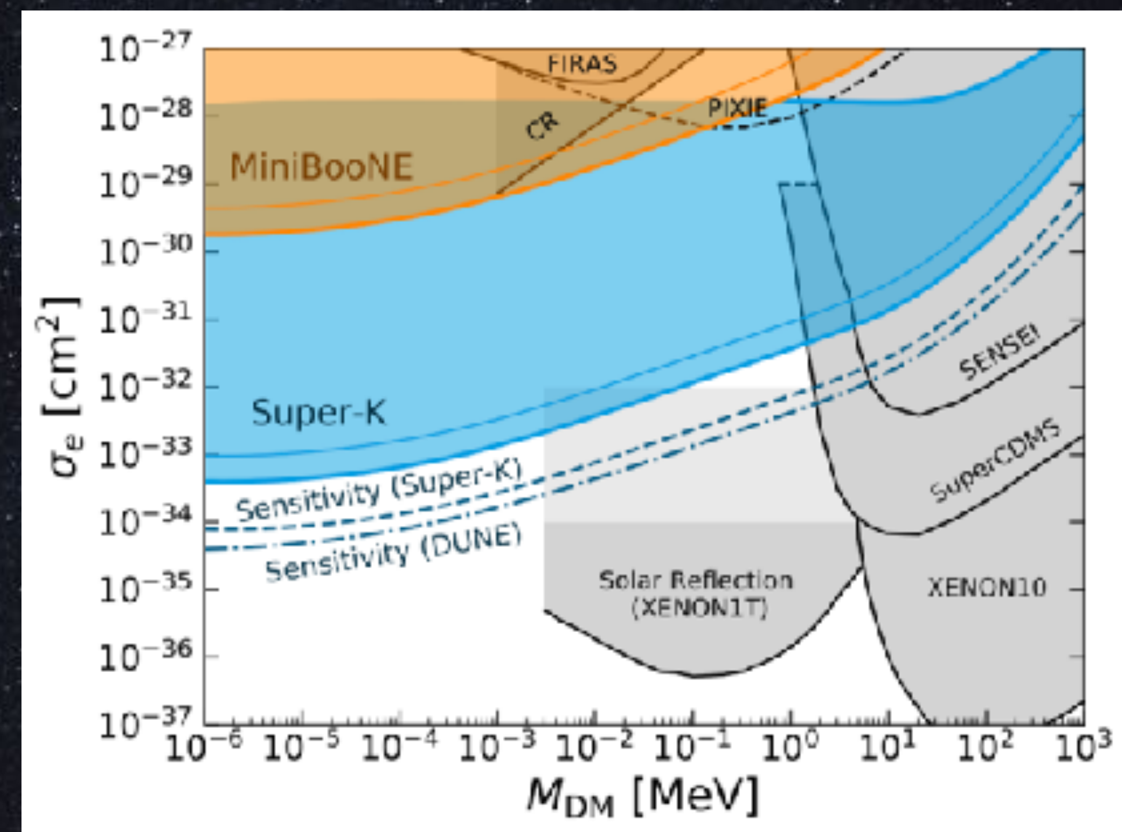


Bringmann, Pospelov (2018)

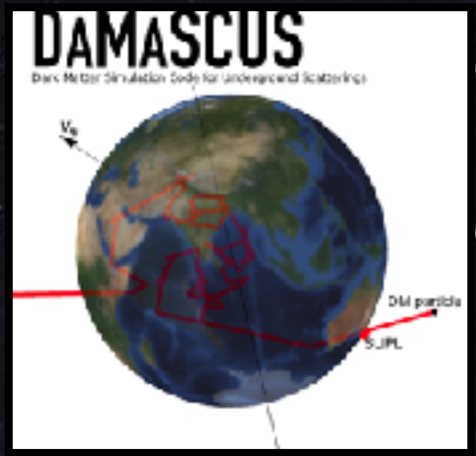
Constraints from CR up-scattered DM



Bringmann, Pospelov (2018)



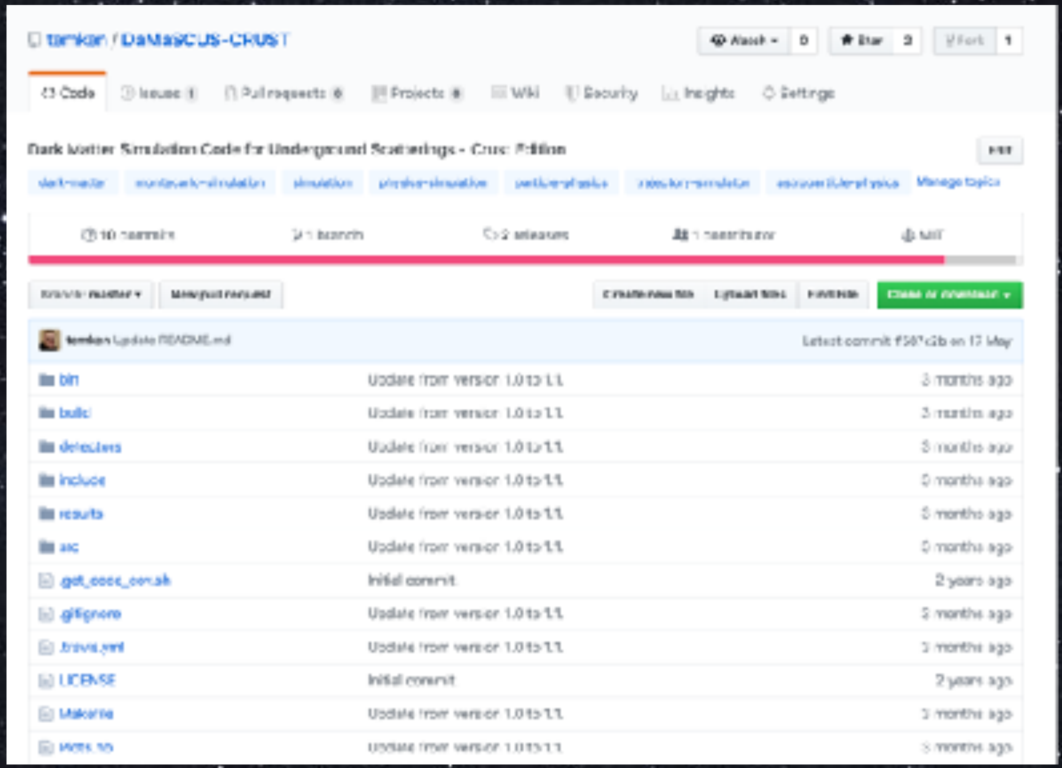
Ema, Sala, Sato, (2018)



DaMaSCUS

Dark Matter Simulation Code for Underground Scatterings

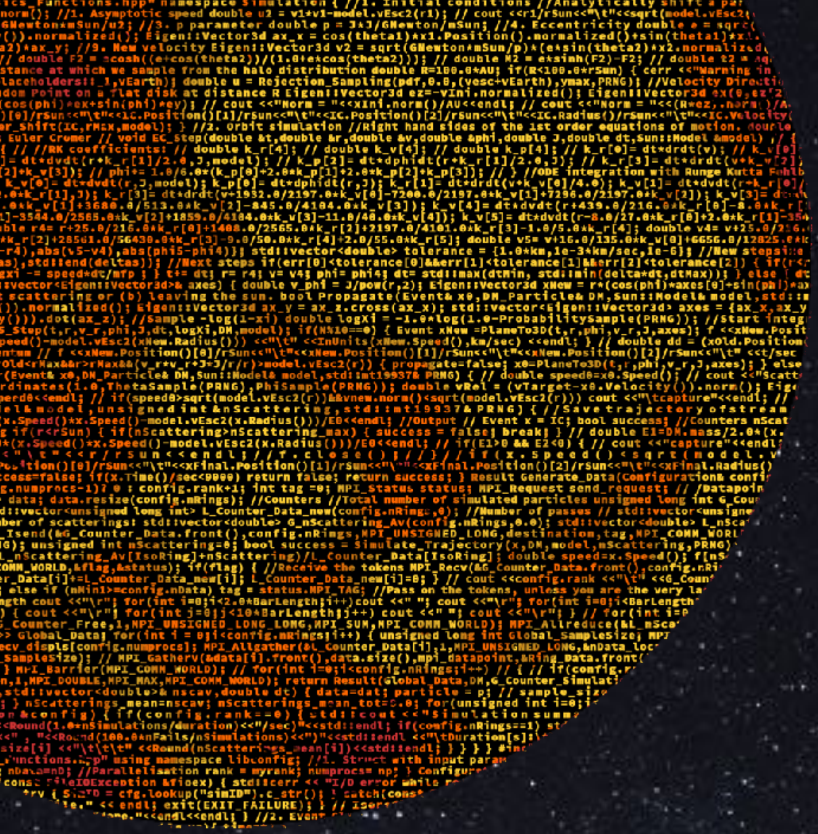
- Written in C++.
- Fully parallelized with *MPI*.
- Results were generated on the ABACUS2.0 supercomputer.
- The code is public.
- Two versions available:
    1. DaMaSCUS
    2. DaMaSCUS-CRUST



http://github.com/temken/

VI.
Summary

- Pre-detection scatterings change the DM properties inside a laboratory. This can have important implications for DM searches:
    1. Diurnal modulations.
    2. Loss of sensitivity due to overburden scatterings.
    3. Extended sensitivity to lower masses via solar reflection or cosmic ray up-scatterings.
- Multiple scatterings in a medium are best described by MC simulations and

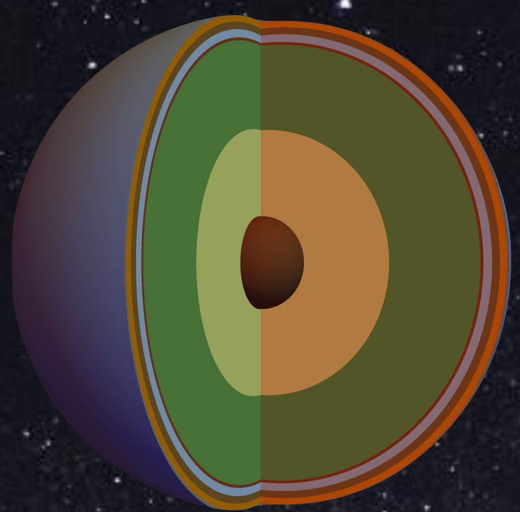


Thank you!

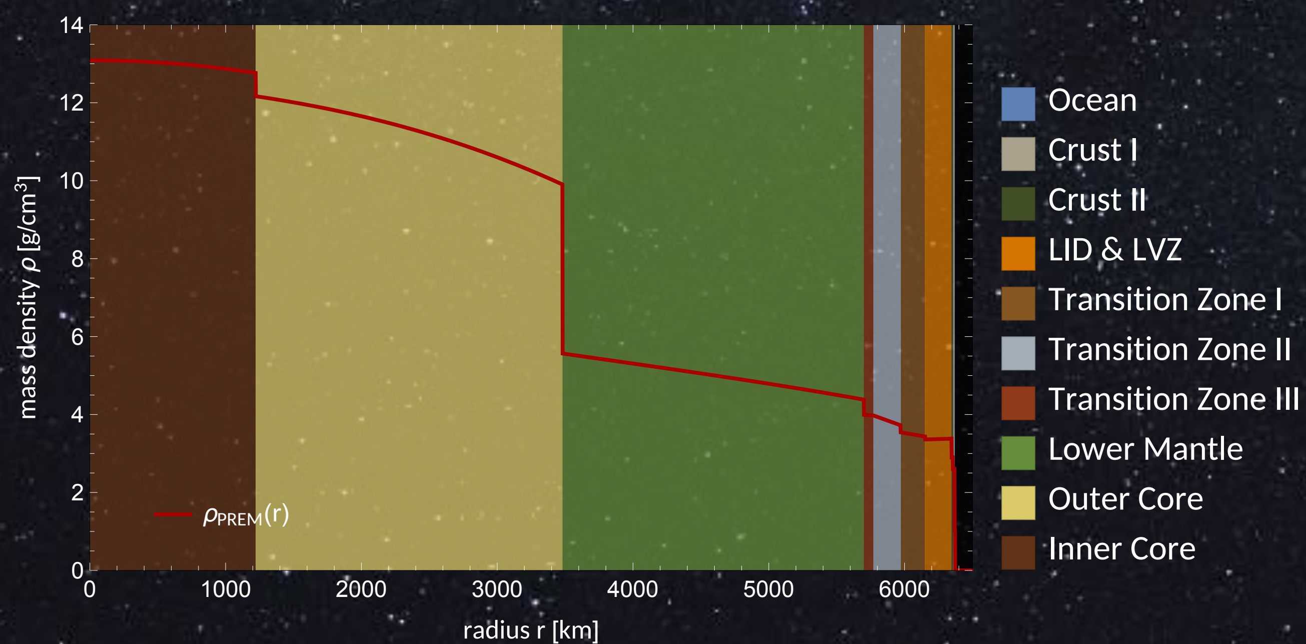
VII.
Backup Slides

Modelling the Earth

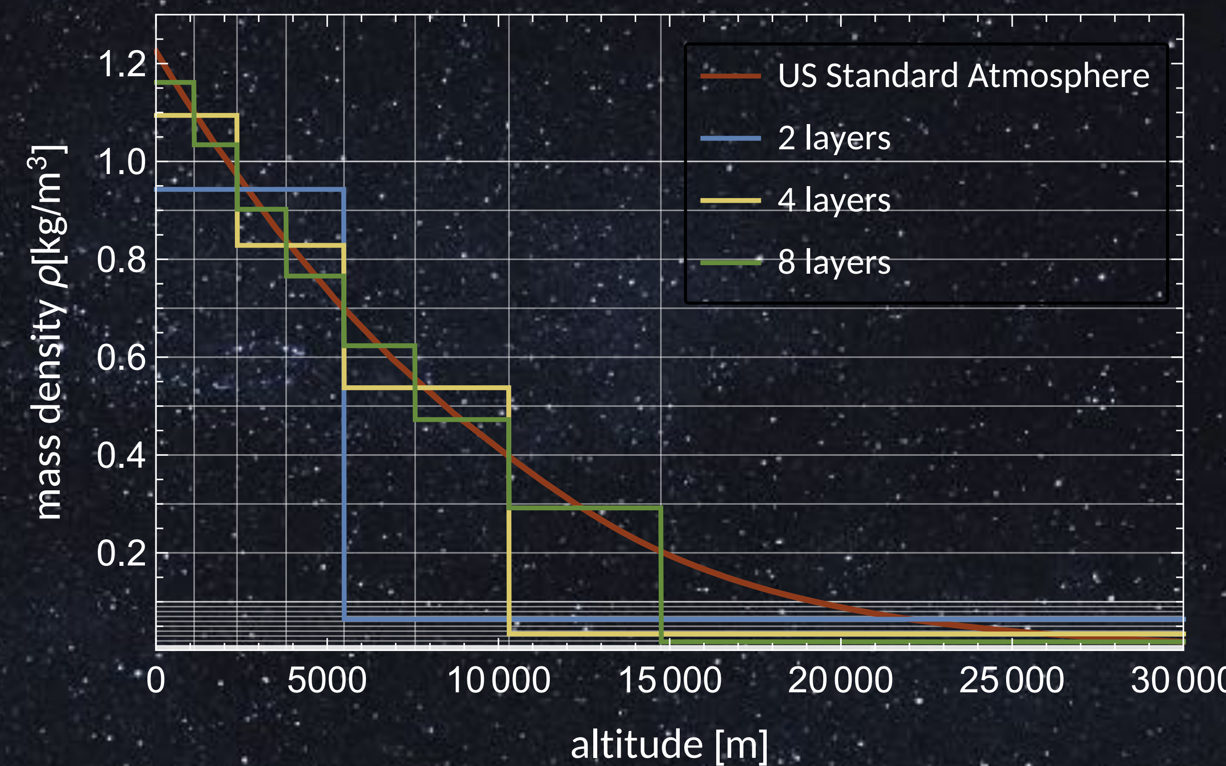
Preliminary Reference Earth Model



- 10 mechanical layers
- 2 compositional layers



US Standard Atmosphere



Earth Crust

- constant mass density

$$\rho = 2.7\,\mathrm{g\,cm}^{-3}$$

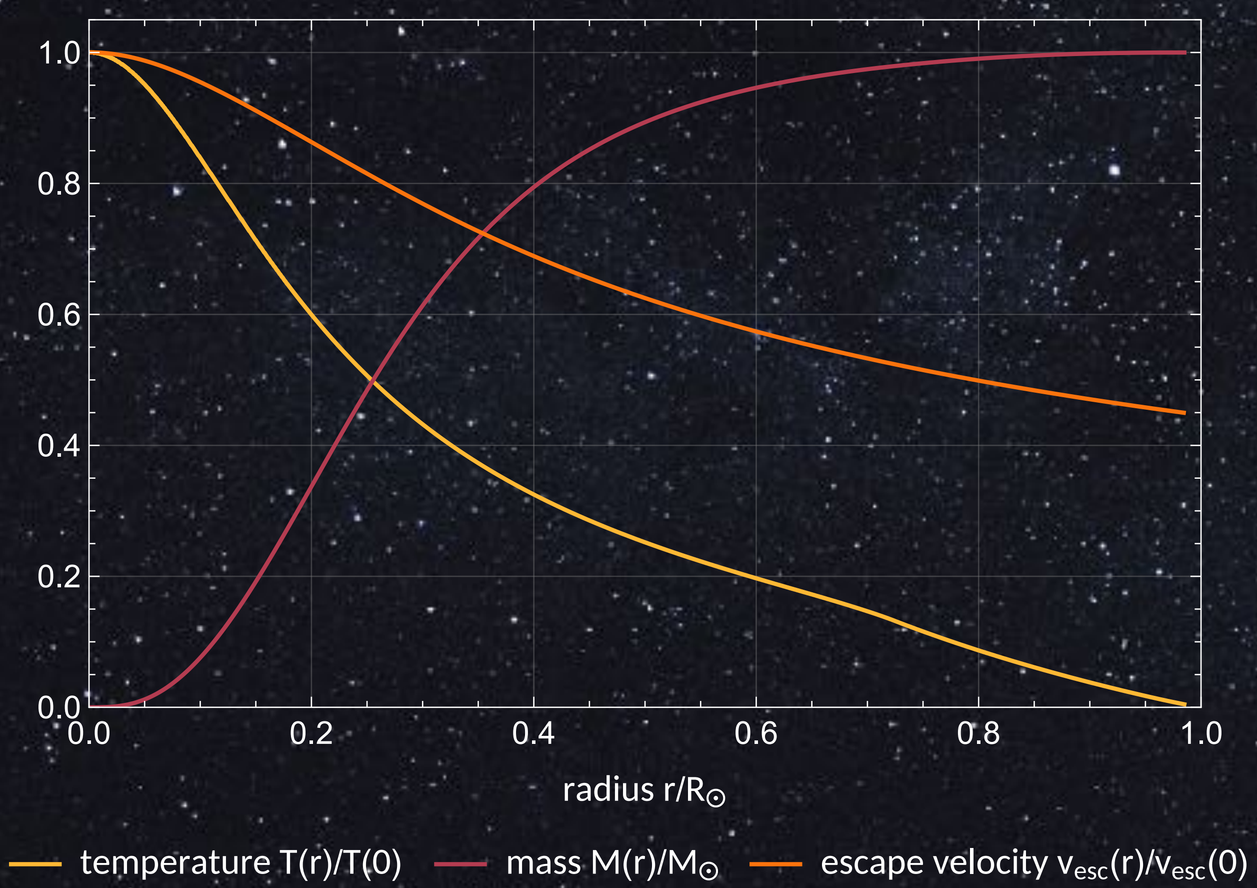
- composition

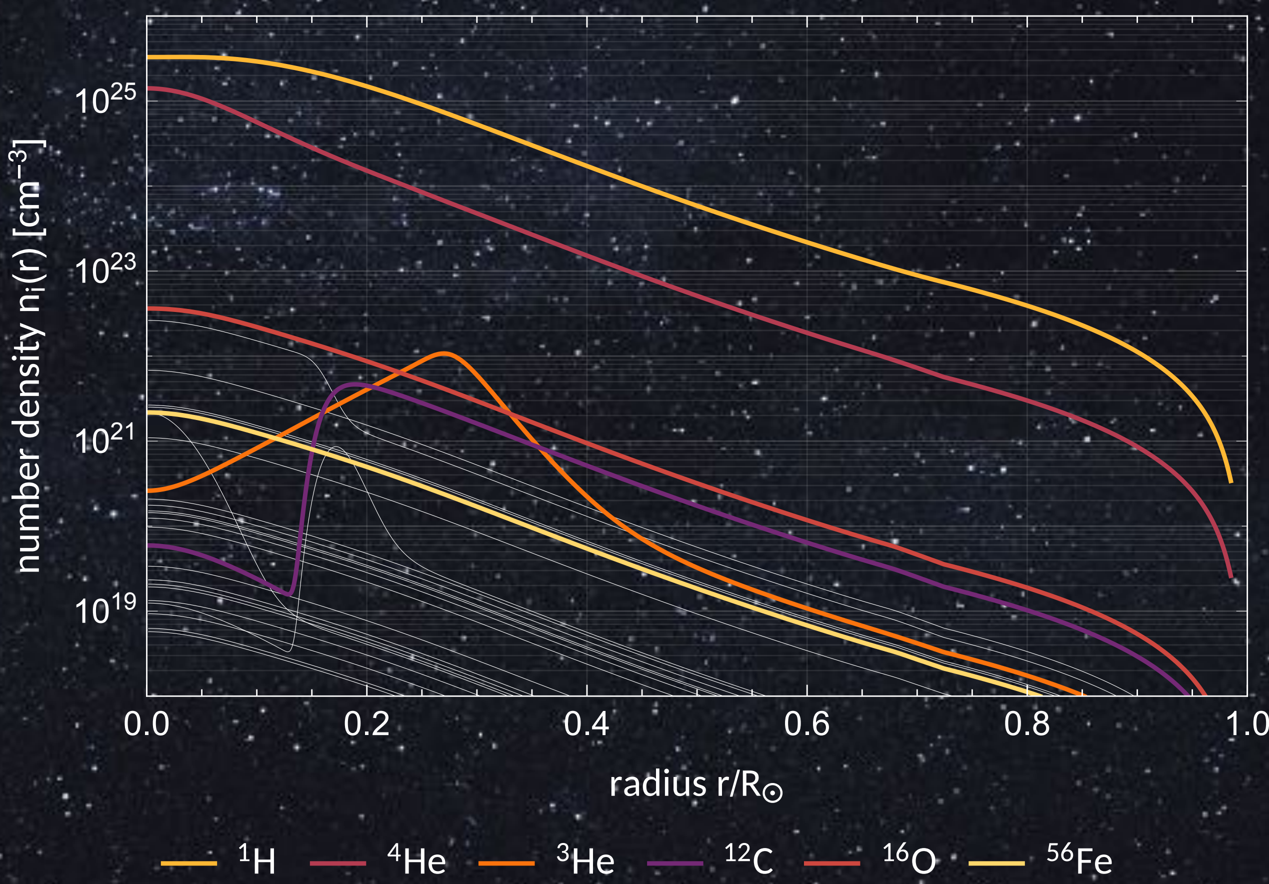
W. McDonough, Treatise on Geochemistry 2 (2003) 568
A.M. Dziewonski, D.L. Anderson, Phys. Earth Planet. Interiors 25 (1981) 297-356
R. Rudnick, S. Gao, Treatise on Geochemistry, p. 1-64, Oxford, (2003)
U.S. Government Printing Office, U.S. Standard Atmosphere, (1976)

Modelling the Sun

Standard Solar Model AGSS09

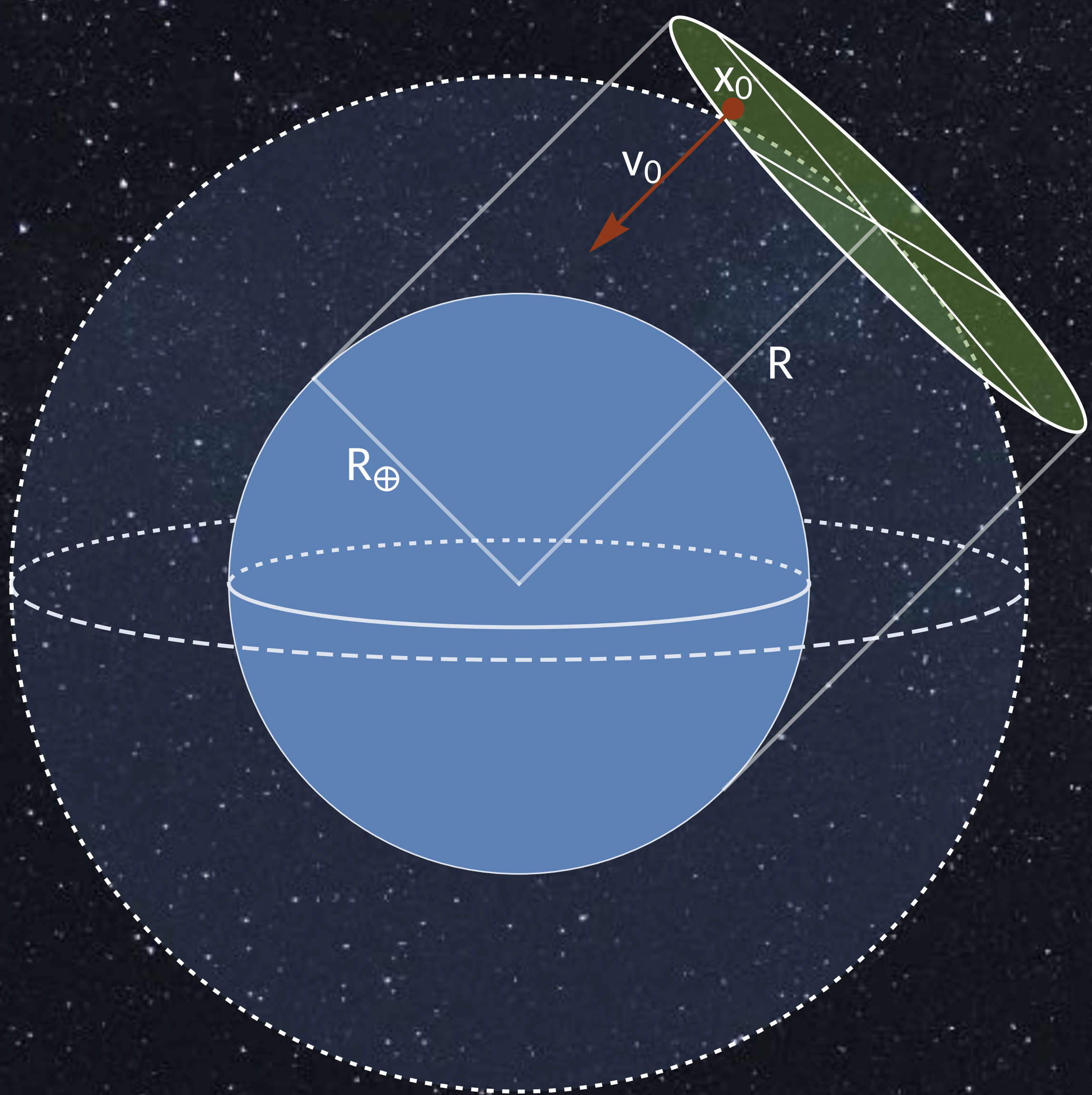
A. Serenelli et al., Astrophys. J. 705 (2009), L123-L127





- Mass Function $M(r)$
- Temperature $T(r)$
- Nuclear composition
- Number densities

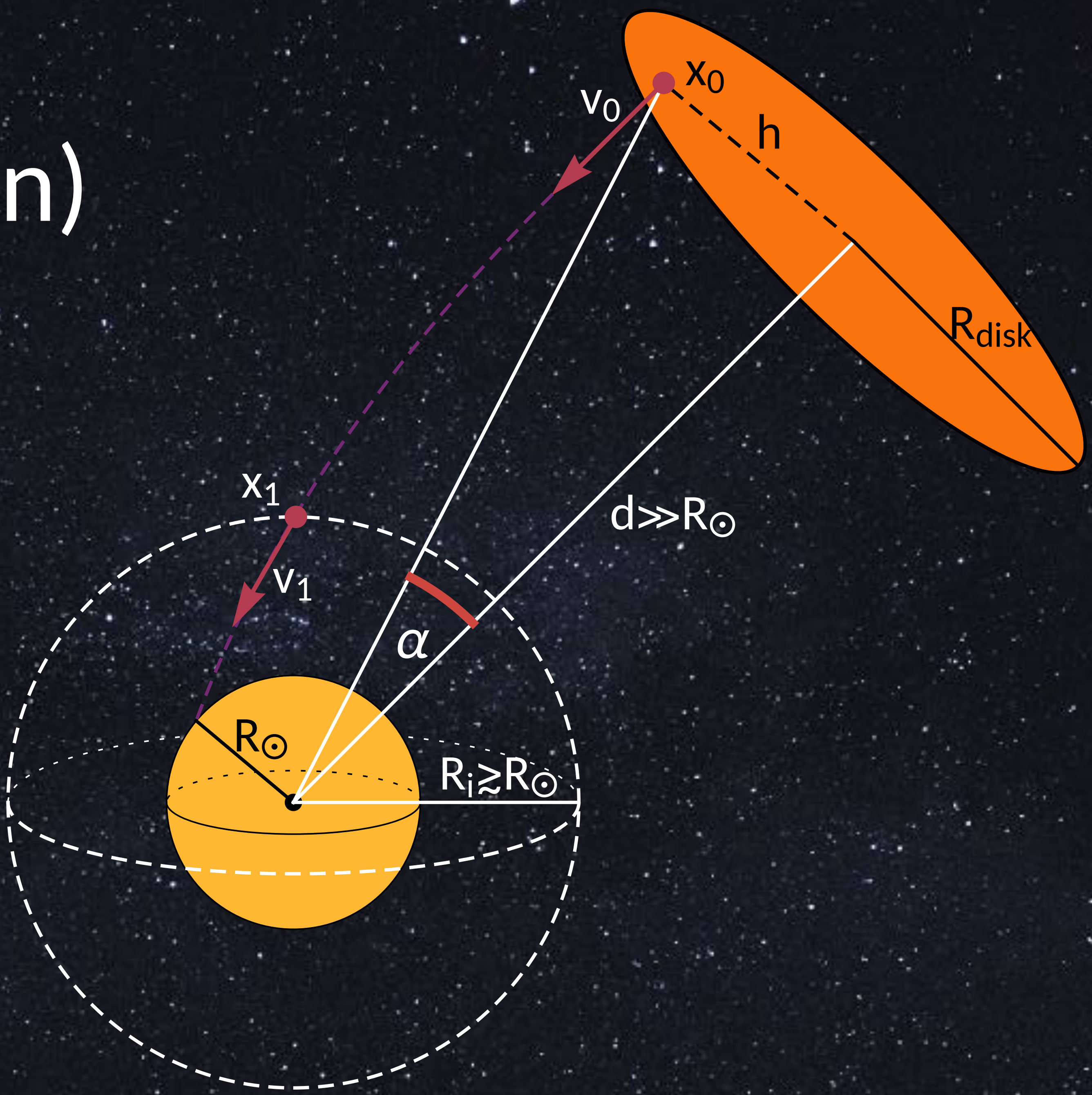
Initial Conditions (Earth)



1. **Initial velocity**: Sample from the isotropic halo distribution, and boost into the Earth's rest frame.
2. **Initial position**: To ensure a homogenous distribution of initial DM positions, and an Earth "hit", all particles start from a circular disk, or from within a cylinder parallel to $\mathbf{v}_0$.

Initial Conditions (Sun)

1. Sample a velocity u far away from the Sun from the halo.
2. Sample the initial position on a disk.
3. Propagate the particle analytically on its hyperbolic Kepler orbit to a location close to the Sun.



$$R_{\rm disk}(u) = \sqrt{1 + \frac{v_{\rm esc}^2(R_\odot)}{u^2}} R_\odot + \mathcal{O}\left(\frac{v_{\rm esc}^2(r_0)}{u^2}\right)$$

$$R_{\rm disk}(u) > R_\odot$$

gravitational focusing

Kernel Density Estimation (KDE)

A non-parametric method to estimate an unknown PDF based on data.

For a data set $\{x_1, x_2, \ldots, x_N\}$ we can estimate the PDF via

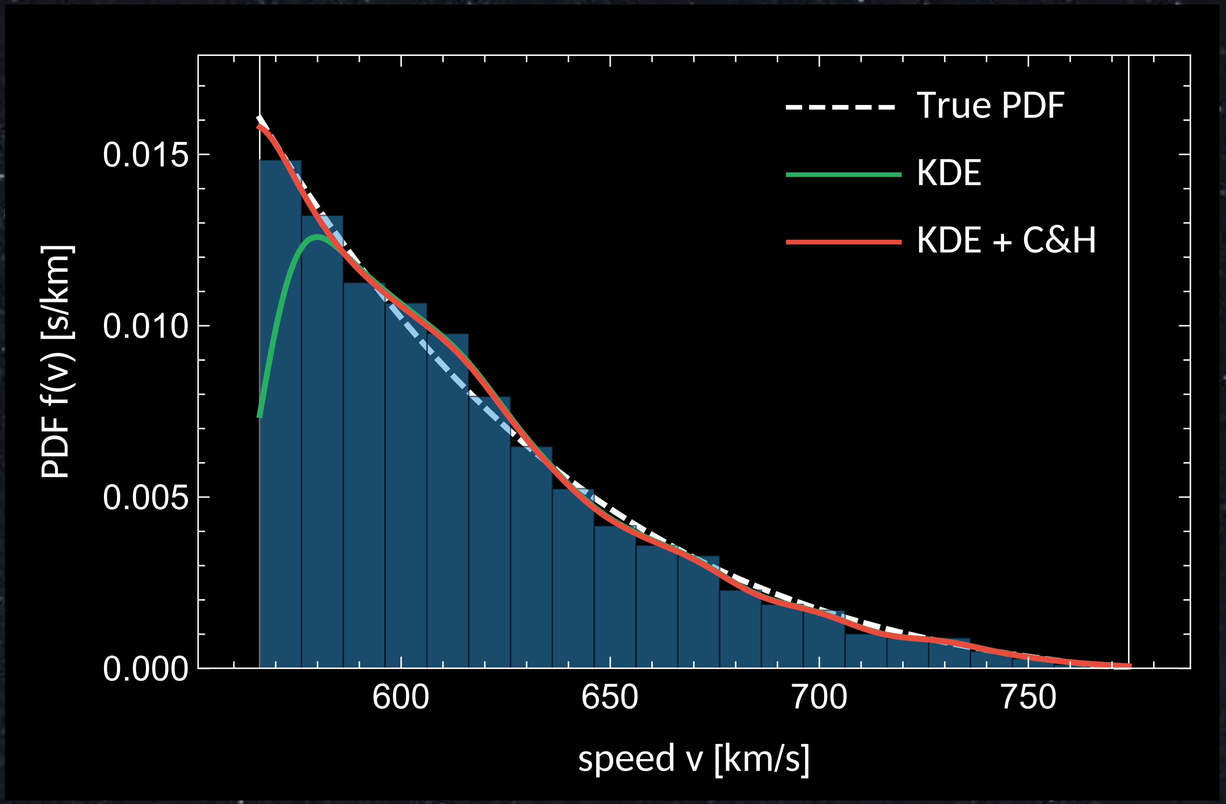
$$\hat{f}_h(x) = \frac{1}{h} \sum_{i=1}^{N} K\left(\frac{x - x_i}{h}\right) .$$

E.g. with a Gaussian Kernel,

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right) .$$

We set the bandwidth h using Silverman's rule of thumb,

$$h = \left(\frac{4}{3N}\right)^{1/5} \hat{\sigma} .$$



The bias at the domain's boundary has to be compensated, e.g. by a pseudo-data method by Cowling and Hall

R. Karunamuni, T. Alberts, Statistical Methodology 2 (2005), 191
A. Cowling, P. Hall, Journal of the Royal Statistical Society, B58 (1996), 551

Runge-Kutta-Fehlberg (RK45)

E. Fehlberg, NASA technical report, NASA, 1969

Adaptive method for the numerical solution of 1st order ODEs.

$$\dot{y} = f(t, y)\,, \quad y(t_0) = y_0\,,$$

Requires the same number of function evaluation of RK6, but is rather a combination of RK4,

$$y_{k+1} = y_k + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4101}k_4 - \frac{1}{5}k_5\,,$$

and RK5,

$$\tilde{y}_{k+1} = y_k + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6\,.$$

The comparison of both yields an estimate of the error, and allows to choose the step size adaptively.

$$\Delta t_{k+1} = 0.84 \left(\frac{\epsilon}{\left| y_{k+1} - \tilde{y}_{k+1} \right|} \right)^{1/4} \Delta t_k$$