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The Null Energy Condition in Cosmology

Energy conditions in GR

Covariant ways to say: " E > 0 "

Strong Energy Condition: $(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T)\xi^{\mu}\xi^{\nu} \geq 0$

Null Energy Condition: $T_{\mu\nu} k^{\mu} k^{\nu} \geq 0$

In cosmology:
$$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

SEC is violated!

SEC: $\rho + 3p \geq 0$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad \Rightarrow \quad \ddot{a} \leq 0$$

SEC forbids acceleration

The two revolutions in cosmology in the last 25 years,
inflation + present acceleration,
are based on the violation of the SEC

What about NEC?

$T_{\mu\nu}k^\mu k^\nu \geq 0$ is insensitive to c.c. $T_{\mu\nu} = -\Lambda g_{\mu\nu}$

NEC is the only sensible energy condition,
the others can be fixed by a suitable c.c.

$$T_{\mu\nu}k^\mu k^\nu \geq 0 \quad \Rightarrow \quad \rho + p \geq 0 \qquad\qquad w \equiv \frac{p}{\rho} \qquad w \geq -1$$

$$\nabla_\mu T^{\mu 0} = 0 \quad \Rightarrow \quad \dot{\rho} = -3H(\rho + p)$$

In a spatially flat Universe: $\boxed{\text{NEC} \quad \Rightarrow \quad \dot{H} \leq 0}$

Cosmological consequences

NEC says energy density (and thus H) decreases as Universe expands

If ~~NEC~~ :

- No need for a Big Bang
- One can even have $H \to 0$ in the far past: start the Universe !
- Bouncing cosmologies. H must flip from negative to positive: $\dot{H} > 0$
- Observation of w < -1 in the present acceleration


NEC is the lid of Pandora's box

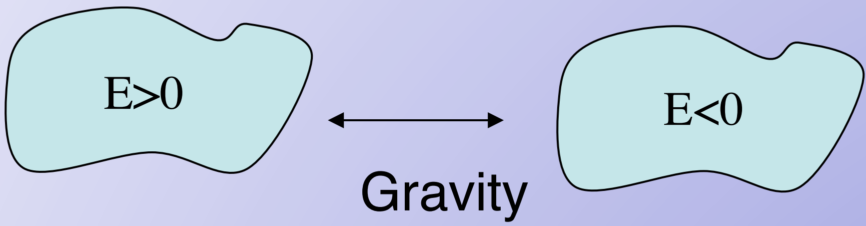
What is wrong with NEC?

Typically a ~~NEC~~ theory suffers from instabilities

E.g. States with negative energy (ghosts) will violate it

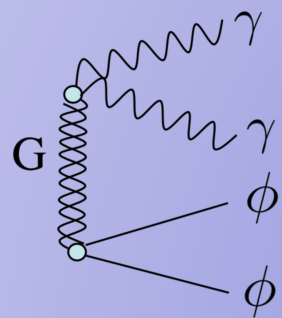
$$\mathcal{L} = -\left[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\right]$$

Classical instability:



Quantum mechanical disaster:

Vacuum decay



A no-go theorem

Dubovsky, Gregoire, Nicolis, Rattazzi, 05

Forget about gravity

Can we construct a sensible QFT with ~~NEC~~?

$$\mathcal{L} = \Lambda^4 F\left(\epsilon\frac{\phi_I}{\Lambda}, \frac{\partial^\mu\phi_I\partial_\mu\phi_J}{\Lambda^4}, \frac{\partial^2\phi_I\partial^2\phi_J}{\Lambda^6}\right)$$

$$\equiv B_{IJ}$$

Around a background: $\phi_I^B \ , \ g_{\mu\nu}^B$ $\quad |\partial_\mu| \ll \Lambda$

In the regime: $\frac{1}{\Lambda} \ll r \ll r_G = \frac{M_P}{\Lambda^2} \ ; \ \frac{1}{\epsilon\Lambda}$

Neglect higher derivative and potential

Stress-energy tensor and Lagrangian for perturbations

$$T_{\mu\nu} \sim F_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J$$

$$\phi^I \to \phi^I + \pi^I \qquad \mathcal{L} \sim \left[F_{IJ} \eta_{\mu\nu} + 2 F_{IK,JL} \partial_\mu \phi^K \partial_\nu \phi^L \right] \partial^\mu \pi^I \partial^\nu \pi^J$$

No gradient or ghost instabilities: $\begin{cases} \dot{\pi}^2 + (\nabla \pi)^2 \\ -\dot{\pi}^2 + (\nabla \pi)^2 \end{cases}$

Stability + isotropy $\longrightarrow$ NEC

Stability + subluminality $\longrightarrow$ NEC

Evading the assumptions

$$\mathcal{L} = \Lambda^4 F\left(\epsilon\frac{\phi_I}{\Lambda}, \frac{\partial^\mu\phi_I\partial_\mu\phi_J}{\Lambda^4}, \frac{\partial^2\phi_I\partial^2\phi_J}{\Lambda^6}\right)$$

Why did we neglect higher derivative?

- They are irrelevant at low energy
- They describe new (pathological) degrees of freedom

$$-(\partial\phi)^2 + \frac{1}{M^2}(\Box\phi)^2 \to -(\partial\phi)^2 + (\partial\chi)^2 + M^2\chi^2$$

- When they are important EFT

1^{st} case: Ghost Condensate

Arkani-Hamed, Cheng, Luty, Mukhoyama, 03

HD operators are not always irrelevant at low energy

Degenerate dispersion relation: $\dot{\pi}^2 + 0 \times (\nabla\pi)^2$

Higher dimension operators: $(\Box\phi)^2 \rightarrow \underbrace{\ddot{\pi}^2 + (\nabla\dot{\pi})^2}_{\text{Higher dim.}} + (\nabla^2\pi)^2$

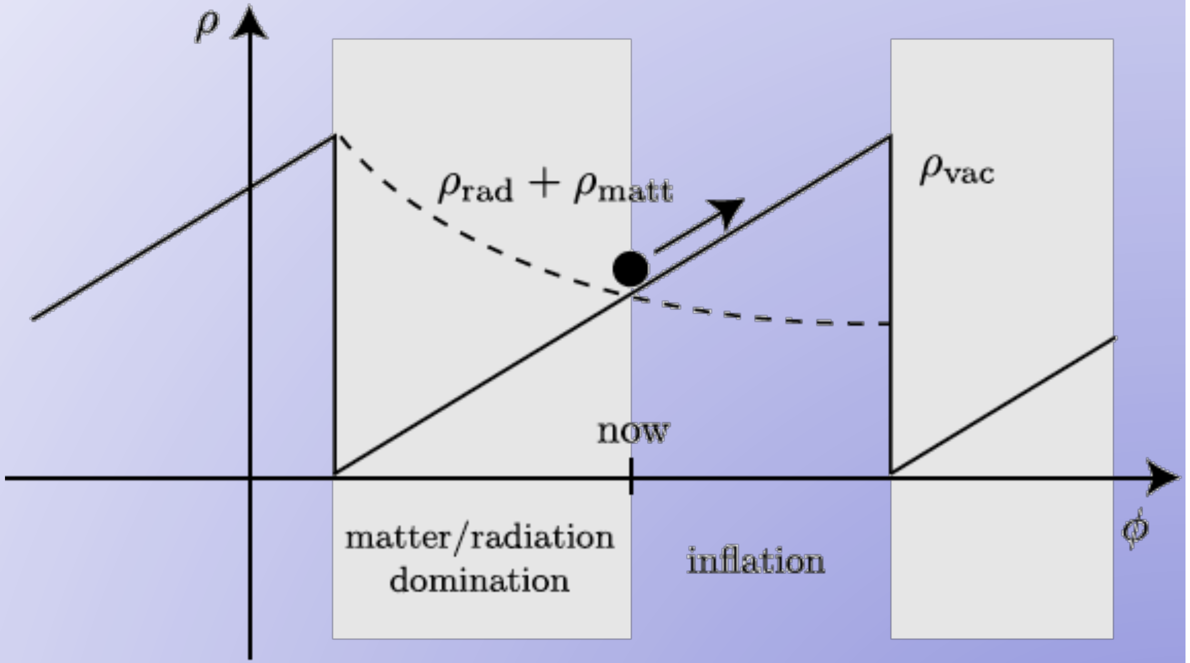
Non relativistic scaling: consistent derivative expansion

1^{st} counterexample

Small deformation of the GC theory leads to consistent ~~NEC~~

Creminelli, Luty, Nicolis, Senatore, 06

The Pandora's box is open!



Bouncing cosmologies:

Creminelli, Senatore, 07
Buckbinder, Khoury, Ovrut, 07

2^{nd} case: Galileon

Nicolis, Rattazzi, Trincherini, 08

HD terms do not always lead to new (pathological) dof

EOM with 2 derivatives and not more: $\frac{\delta \mathcal{L}_\pi}{\delta \pi} = F(\partial_\mu \partial_\nu \pi)$

Galilean symmetry: $\pi(x) \to \pi(x) + c + b_\mu x^\mu$

The lowest dim Galilean operators are ok: $\mathcal{L}^{(n)} \sim \partial^{2n-2} \pi^n$

$$\mathcal{L}^{(2)} = -\frac{1}{2}(\partial \pi)^2 \qquad \mathcal{L}^{(3)} = -\frac{1}{2}(\partial \pi)^2 \Box \pi$$

There are only 5 in total (in 4d)!

From the Galileon to the dilaton

Technically it is easier to extend Galilean symmetry + Poincare'
to the conformal group SO(4,2)

$$D: \qquad \pi(x) \to \pi'(x) \equiv \pi(\lambda x) + \log \lambda$$

$$K_\mu: \qquad \pi(x) \to \pi'(x) \equiv \pi\big(x + (c\, x^2 - 2(c\cdot x)x)\big) - 2c_\mu x^\mu$$

$$g_{\mu\nu}^{(\pi)} \equiv e^{2\pi(x)}\eta_{\mu\nu}$$

Let us study solutions: SO(4,2) --> SO(4,1)

$$e^{\pi_{\rm dS}} = -\frac{1}{H_0 t} \qquad -\infty < t < 0$$

Galilean Genesis

Nicolis, Rattazzi, Trincherini, 09
Creminelli, Nicolis, Trincherini, in prep.

$$\mathcal{S}_\pi = \int d^4x\,\sqrt{-g}\left[f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{f^3}{2\Lambda^3}(\partial\pi)^4\right]$$

$$e^{\pi_{\rm dS}} = -\frac{1}{H_0 t} \qquad -\infty < t < 0 \qquad H_0^2 = \frac{2\Lambda^3}{3f}$$

- No instability: $\mathcal{L}_{\hat\pi} = -\frac{f^2}{H_0^2}\frac{1}{t^4}(\partial\hat\pi)^2$

- Brutal violation of NEC: $\rho = 0 \qquad p \propto -\frac{1}{t^4}$

$$H \propto \frac{1}{|t|^3} \qquad a \propto \exp\left(\frac{1}{t^2}\right)$$

The Universe starts from ~ Minkowski, genesis at t ~ 0 when we exit EFT description

Scale invariance from the fake de Sitter

In inflation, scale invariance comes from symmetries of dS

Here $g^{(\pi)}_{\mu\nu} \equiv e^{2\pi(x)}\eta_{\mu\nu}$ is dS

A spectator massless scalar will behave as in dS !

Signatures:

- Local non-Gaussianities
- Unobservable, strongly blue GWs
- Possible isocurvature

Conclusions

1. NEC is a crucial constraint on cosmology
2. Generically associated with instabilities
3. There are counterexamples: Ghost Condensate, Galileon
4. Alternative to inflation are possible
5. Swampland? Superluminality, unitarity constraints

Either way it is an important issue !!