Il Sapore in SUSY: Scenario Gerarchico

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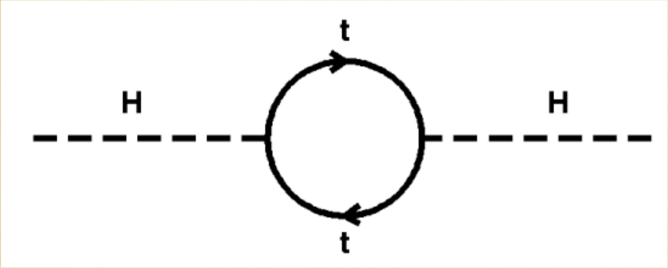
Il Problema del Sapore oltre il MS

- Assumendo che il MS sia valido fino ad una scala di energia Λ :

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM}^{\rm gauge} + \mathcal{L}_{\rm SM}^{\rm Yukawa} + \mathcal{L}_{\rm SM}^{\rm Higgs} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \ldots$$

- Limite superiore:



$$\begin{aligned} (m_h^2)_{\rm phys} &= (m_h^2)_{\rm tree} + \frac{2}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 \\ &= (m_h^2)_{\rm tree} + (0.3\Lambda)^2 \\ \Rightarrow & \quad \Lambda \lesssim 1\,\text{TeV} \end{aligned}$$

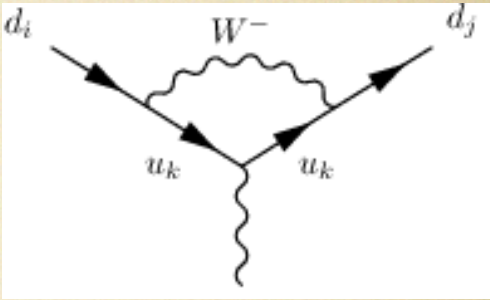
- Limiti inferiori:

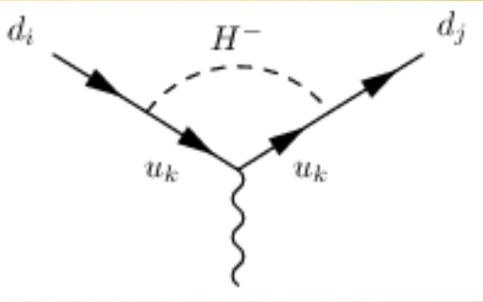
broken symmetry	operators	scale Λ
B, L	$(QQQL)/\Lambda^2$	10^{13} TeV
flavor (1,2$^{\rm nd}$ family), CP	$(\bar{d}s\bar{d}s)/\Lambda^2$	1000 TeV
flavor (2,3$^{\rm rd}$ family)	$m_b(\bar{s}\sigma_{\mu\nu}F^{\mu\nu}b)/\Lambda^2$	50 TeV

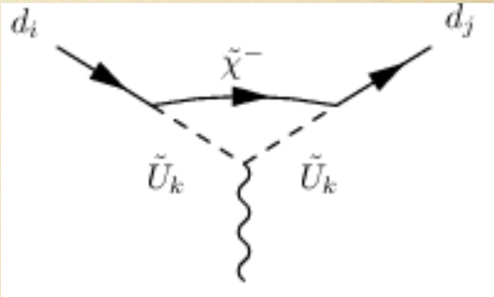
- Se $\Lambda \lesssim 1\,\text{TeV}$, la nuova fisica ha un struttura di sapore non generica

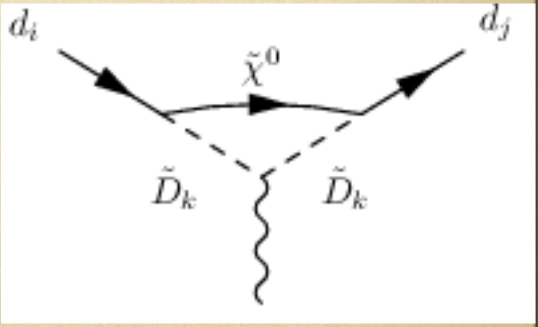
Violazione del Sapore nel MSSM

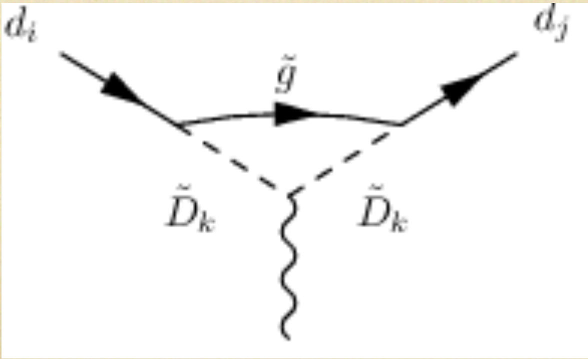
- W e quarks up
- H e quarks up
- Chargino e squarks up
- Neutralino e squarks down
- Gluino e squarks down









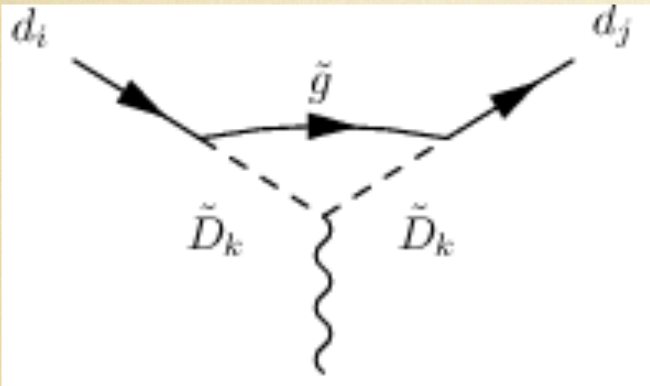


Soppressione dei Contributi SUSY

Consideriamo per esempio, il contributo del gluino ad un processo ΔF=1: $d_i^A \to d_j^B$

$A, B = \{L, R\}$
$i, j = \{1, 2, 3\}$
$i \neq j$

L'ampiezza di Feynman assume la seguente forma:



$$\mathscr{W}_{d_i^A k} f\left(x_k\right) \mathscr{W}^{\dagger}_{k d_j^B}$$

$$x_k = \frac{m^2_{\tilde{q}_k}}{m^2_{\tilde{g}}} \qquad k = \{1, ..., 6\}$$

Abbiamo soppressione se:

- Le particelle nei loops sono pesanti: $\lim_{x \to \infty} f(x) = \lim_{x \to 0} f(x) = 0$
- Gli squarks nei loops hanno massa degenere: $f\left(x\right) \mathscr{W}_{d_i^A k} \mathscr{W}^{\dagger}_{k d_j^B} = f(x) \delta_{AB} \delta_{ij} = 0$
- La matrice unitaria di mixing è circa l'indentità: $f(x_k) \delta_{A_i k} \delta_{k B_j} = f(x_k) \delta_{AB} \delta_{ij} = 0$

Un Approccio Bottom-Up

- Espansione in elementi fuori diagonale della matrice di massa:

$$\mathcal{M}^2 = \mathcal{M}_0^2 + \delta\mathcal{M}^2$$

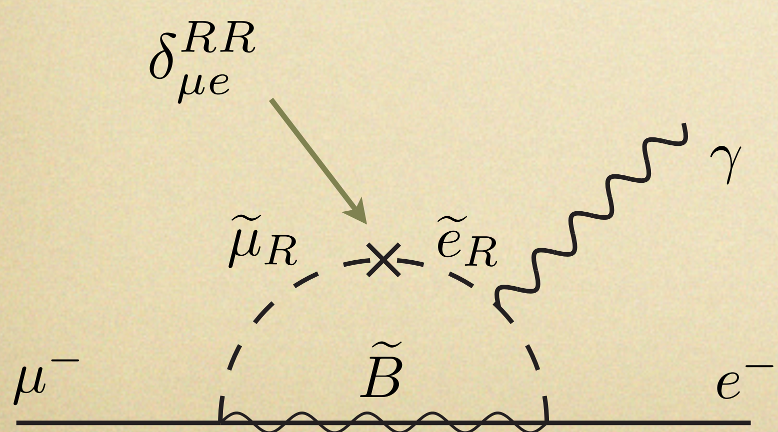
- Scenario degenere: $\mathcal{M}_0^2 = \tilde{m}^2 I_{6\times 6}$

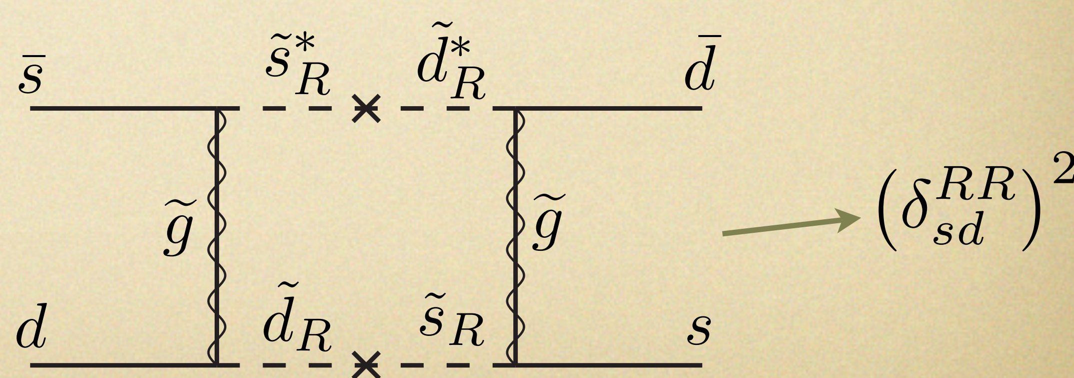
$$A(\Delta F = 1) = x f^{(1)}(x)\delta_{ij}$$

$$A(\Delta F = 2) = \frac{x^2}{3!} g^{(3)}(x)\delta_{ij}^2$$

$$\delta_{ij} \equiv \frac{(\delta\mathcal{M}^2)_{ij}}{\tilde{m}^2} \qquad x = \frac{\tilde{m}^2}{M^2}$$

- f,g: funzioni di loop
- δ_{ij} : inserzioni di massa che violano il sapore





Lo Scenario Gerarchico

Nello scenario gerarchico la matrice di massa degli sfermioni (blocco LL o RR), ha la seguente forma :

$$\tilde{m}^2 = \begin{pmatrix} h_{11} & h_{12} & a_1 \\ h_{21} & h_{22} & a_2 \\ \bar{a}_1 & \bar{a}_2 & l_3 \end{pmatrix}$$

dove “h” è il blocco pesante, i rimanenti elementi sono leggeri

In particolare abbiamo che: $l \lesssim \mathrm{TeV}^2,\ h_1, h_2 \gg l$

La diagonalizzazione di massa avviene mediante una matrice unitaria:

$$\mathcal{M}^2 = \mathcal{W}\,\mathrm{Diag}(\tilde{m}^2_{L1}, \tilde{m}^2_{L2}, \tilde{m}^2_{R1}, \tilde{m}^2_{R2}, \tilde{m}^2_1, \tilde{m}^2_2)\mathcal{W}^\dagger$$

Decomponendo ora l’ampiezza, possiamo eliminare il contributo delle particelle pesanti:

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ai,Bj} = \mathcal{W}_{Ai,Lk}\, f\left(\frac{\tilde{m}^2_{Lk}}{m^2_{\tilde{g}}}\right)\mathcal{W}^\dagger_{Lk,Bj} + \mathcal{W}_{Ai,Rk}\, f\left(\frac{\tilde{m}^2_{Rk}}{m^2_{\tilde{g}}}\right)\mathcal{W}^\dagger_{Rk,Bj} + \mathcal{W}_{Ai,p}\, f\left(\frac{\tilde{m}^2_{p}}{m^2_{\tilde{g}}}\right)\mathcal{W}^\dagger_{p,Bj}$$

Blocco pesante LL

Blocco pesante RR

Contributo dominante della terza famiglia leggera

Lo Scenario Gerarchico

Motivazioni:

[Effective SUSY, Choen Kaplan Lepeintre Nelson '97]

- Complementare all'assunzione con spettro degenere
- Allevia il problema del sapore, senza compromettere la naturalezza [Dimopoulos-Giudice '95]
- Anche se assumiamo uno spettro degenere ad una data scala di energia, gli effetti delle correzioni radiative (Yukawa) tendono a rendere la terza famiglia leggera

$$A(\Delta F = 1) = f(x)\hat{\delta}_{ij}$$
$$A(\Delta F = 2) = g^{(1)}(x)\hat{\delta}_{ij}^2$$

$$x = \frac{\tilde{m}_3^2}{M^2} \qquad \hat{\delta}_{a3} = \frac{\mathcal{M}_{a3}^2}{\tilde{m}_h^2}$$

Ci sono solo 4 inserzioni che violano il sapore:

$$\hat{\delta}_{bd}^{LL}, \hat{\delta}_{bd}^{LL}, \hat{\delta}_{bd}^{RR}, \hat{\delta}_{bd}^{RR}$$

$$\hat{\delta}_{ij}^{LL} \equiv \hat{\delta}_{i3}^{LL}\hat{\delta}_{j3}^{LL*}$$
$$\hat{\delta}_{ij}^{LR} \equiv \frac{\mathcal{M}_{L3,R3}^2}{\tilde{m}^2}\hat{\delta}_{i3}^{LL}\hat{\delta}_{j3}^{RR*} \qquad i,j = 1,2$$
$$\hat{\delta}_{i3}^{LR} \equiv \frac{\mathcal{M}_{L3,R3}^2}{\tilde{m}^2}\hat{\delta}_{i3}^{LL}.$$

Correlazioni tra le osservabili nello scenario degenere e gerarchico possono essere molto diverse:

$$\left.\frac{A(\Delta F=2)}{[A(\Delta F=1)]^2}\right|_{\text{degenerate}} = \frac{g^{(3)}}{6g^{(1)}}\left(\frac{f}{f^{(1)}}\right)^2 \left.\frac{A(\Delta F=2)}{[A(\Delta F=1)]^2}\right|_{\text{hierarchical}}.$$

Limiti sulle Inserzioni

x=1

Hierarchy | Degeneracy

$D_0 - \bar{D}_0$ mixing

Hierarchy	Degeneracy
$\lvert\hat{\delta}^{LL}_{ut}\hat{\delta}^{LL*}_{ct}\rvert < 8.0\times 10^{-3}\left(\frac{m_{\tilde{t}}}{350\ \mathrm{GeV}}\right)$	$\lvert\delta^{LL}_{uc}\rvert < 3.4\times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)$

$B \to X_s\gamma$

Hierarchy	Degeneracy
$\lvert\mathrm{Re}\,(\hat{\delta}^{LL}_{sb})\rvert < 2.2\times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$	$\lvert\mathrm{Re}\,(\delta^{LL}_{sb})\rvert < 3.8\times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$
$\lvert\mathrm{Im}\,(\hat{\delta}^{LL}_{sb})\rvert < 6.7\times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$	$\lvert\mathrm{Im}\,(\delta^{LL}_{sb})\rvert < 1.1\times 10^{-1}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$

Δm_{B_s}

$$\left|\hat{\delta}_{sb}\right| \approx |V^*_{ts}| \approx 4\times 10^{-2}$$

Hierarchy	Degeneracy
$\lvert\mathrm{Re}\,(\hat{\delta}^{LL}_{sb})\rvert < 9.4\times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)$	$\lvert\mathrm{Re}\,(\delta^{LL}_{sb})\rvert < 4.0\times 10^{-1}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)$
$\lvert\mathrm{Im}\,(\hat{\delta}^{LL}_{sb})\rvert < 7.2\times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)$	$\lvert\mathrm{Im}\,(\delta^{LL}_{sb})\rvert < 3.1\times 10^{-1}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)$

B^0_d–$\bar{B}^0_d$ mixing

$$\left|\hat{\delta}_{db}\right| \approx |V^*_{td}| \approx 8\times 10^{-3}$$

Hierarchy	Degeneracy
$\lvert\mathrm{Re}\,(\hat{\delta}^{LL}_{db})\rvert < 4.3\times 10^{-3}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)$	$\lvert\mathrm{Re}\,(\delta^{LL}_{db})\rvert < 1.8\times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)$
$\lvert\mathrm{Im}\,(\hat{\delta}^{LL}_{db})\rvert < 7.3\times 10^{-3}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)$	$\lvert\mathrm{Im}\,(\delta^{LL}_{db})\rvert < 3.1\times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)$

Δm_K

$$\left|\hat{\delta}_{db}\hat{\delta}_{bs}\right| \approx 3\times 10^{-4}$$

Hierarchy	Degeneracy
$\sqrt{\lvert\mathrm{Re}\,(\hat{\delta}^{LL}_{db}\hat{\delta}^{LL*}_{sb})^2\rvert} < 1.0\times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)$	$\sqrt{\lvert\mathrm{Re}\,(\delta^{LL}_{ds})^2\rvert} < 4.2\times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)$

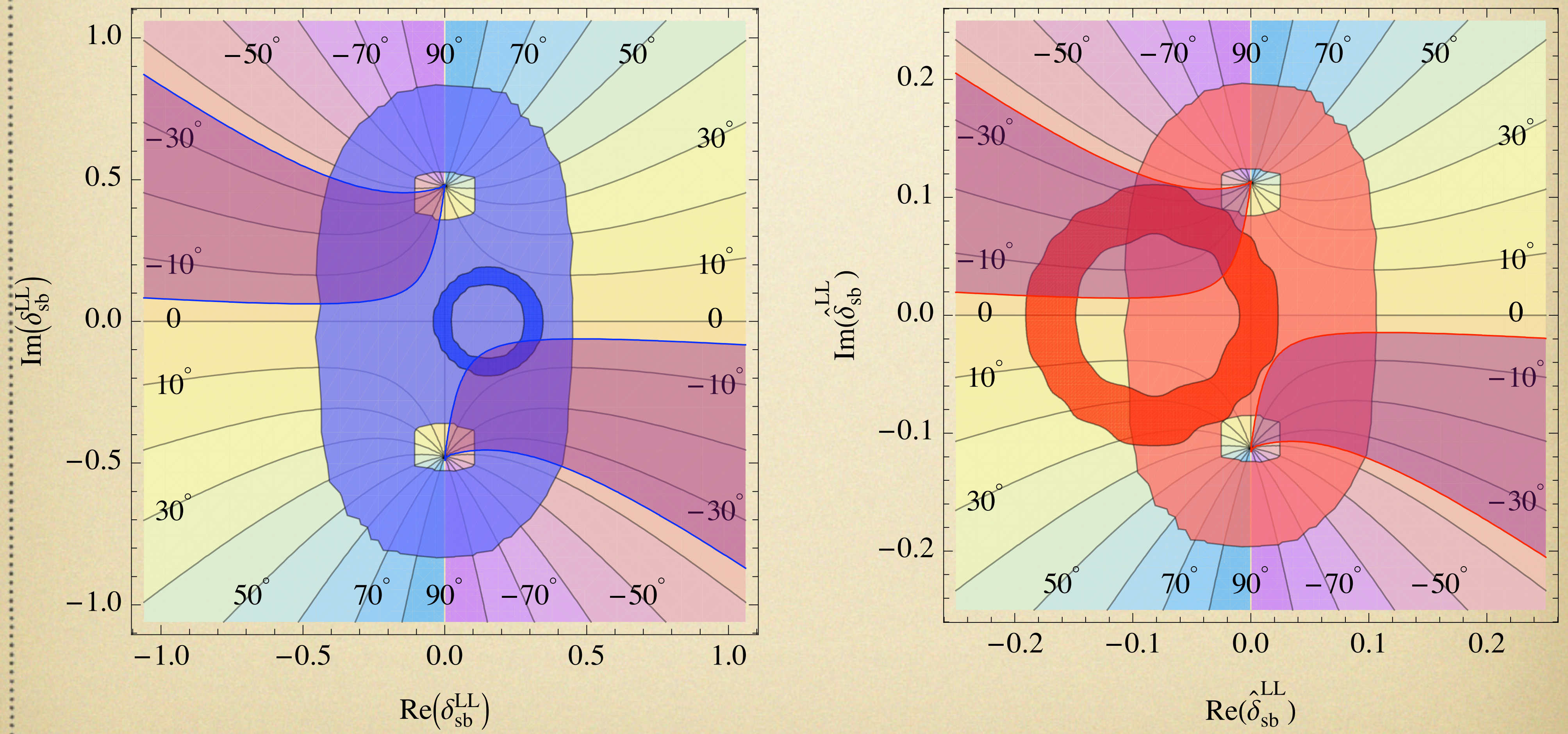
ϵ_K

Hierarchy	Degeneracy
$\sqrt{\lvert\mathrm{Im}\,(\hat{\delta}^{LL}_{db}\hat{\delta}^{LL*}_{sb})^2\rvert} < 4.4\times 10^{-4}\left(\frac{m_{\tilde{b}}}{350\ \mathrm{GeV}}\right)$	$\sqrt{\lvert\mathrm{Im}\,(\delta^{LL}_{ds})^2\rvert} < 1.8\times 10^{-3}\left(\frac{m_{\tilde{q}}}{350\ \mathrm{GeV}}\right)$

$\Delta M_{B_s}\,,\ B \to X_s\gamma\ ,\ \phi_{B_s}$

Scenario Degenere

Scenario Gerarchico



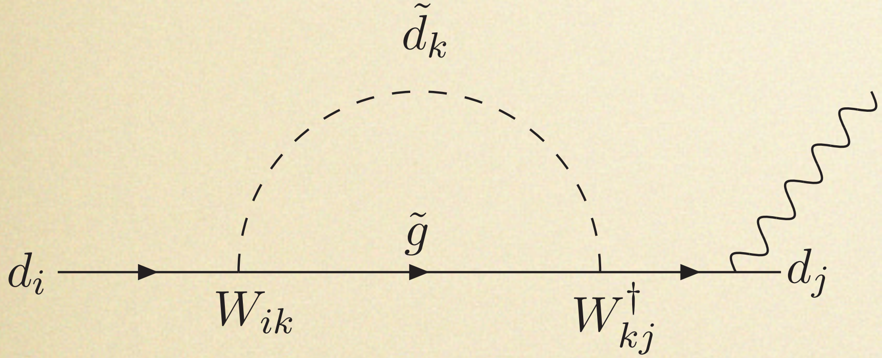
$\tilde{m} = M_{\tilde{g}} = \mu = 350\ \text{GeV}, \qquad \tan\beta = 10, \qquad A = 0$

Scenario Gerarchico: Conclusioni

- E' possibile avere le prime due famiglie di sfermioni pesanti senza compromettere la naturalezza della massa dell'Higgs
- Si allevia il problema del sapore nel MSSM
- Utile scenario di confronto per modelli in cui l'assunzione di uno spettro degenere non è valida
- Offre predizioni peculiari e correlazioni tra diverse osservabili

FCNC processes

- Consider the chirality conserving case $\left\{ \begin{array}{c} d_i \to d_j \\ i \neq j \end{array} \right.$



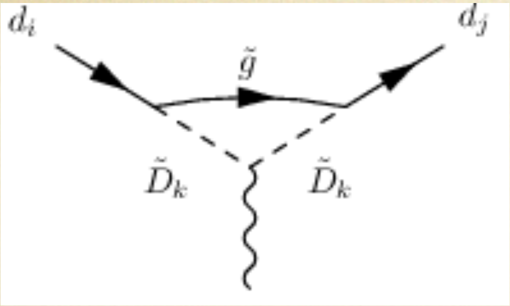
$$\sum_{k=1}^{3} W_{ik} f(x_k) W^\dagger_{kj} \qquad x_k \equiv \frac{m^2_{\tilde{d}_k}}{m^2_{\tilde{g}}}$$

- In the hierarchical case: $\sum_{k=1}^{3} W_{ik} f(x_k) W^\dagger_{kj} \approx f(x_3) W_{i3} W^\dagger_{3j} \equiv f(x_3) \hat{\delta}_{ij}$
- We can define the flavor violating parameters:

$$\left\{ \begin{array}{c} \hat{\delta}_{13} = W_{13} W^\dagger_{33} \approx W_{13} \\ \hat{\delta}_{23} = W_{23} W^\dagger_{33} \approx W_{23} \\ \hat{\delta}_{12} = W_{13} W^\dagger_{32} \approx \hat{\delta}_{13} \hat{\delta}^*_{23} \end{array} \right.$$

We expect a suppression in the contribution to Kaon Physics and a correlation between the parameters

Natural size of the insertions



- $\mathcal{W}_{A_i k}$ is the mixing between the fermion "Ai" and the k-th sfermion mass eigenstate
- In a general basis we have:

$$V = U_L^u U_L^{d\dagger} \qquad \mathcal{W} = \begin{pmatrix} U_L & 0 \\ 0 & U_R \end{pmatrix} \mathcal{W}', \quad U_R M U_L^\dagger = \text{diagonal}, \quad \mathcal{W}'^\dagger \mathcal{M}^2 \mathcal{W}' = \text{diagonal}.$$

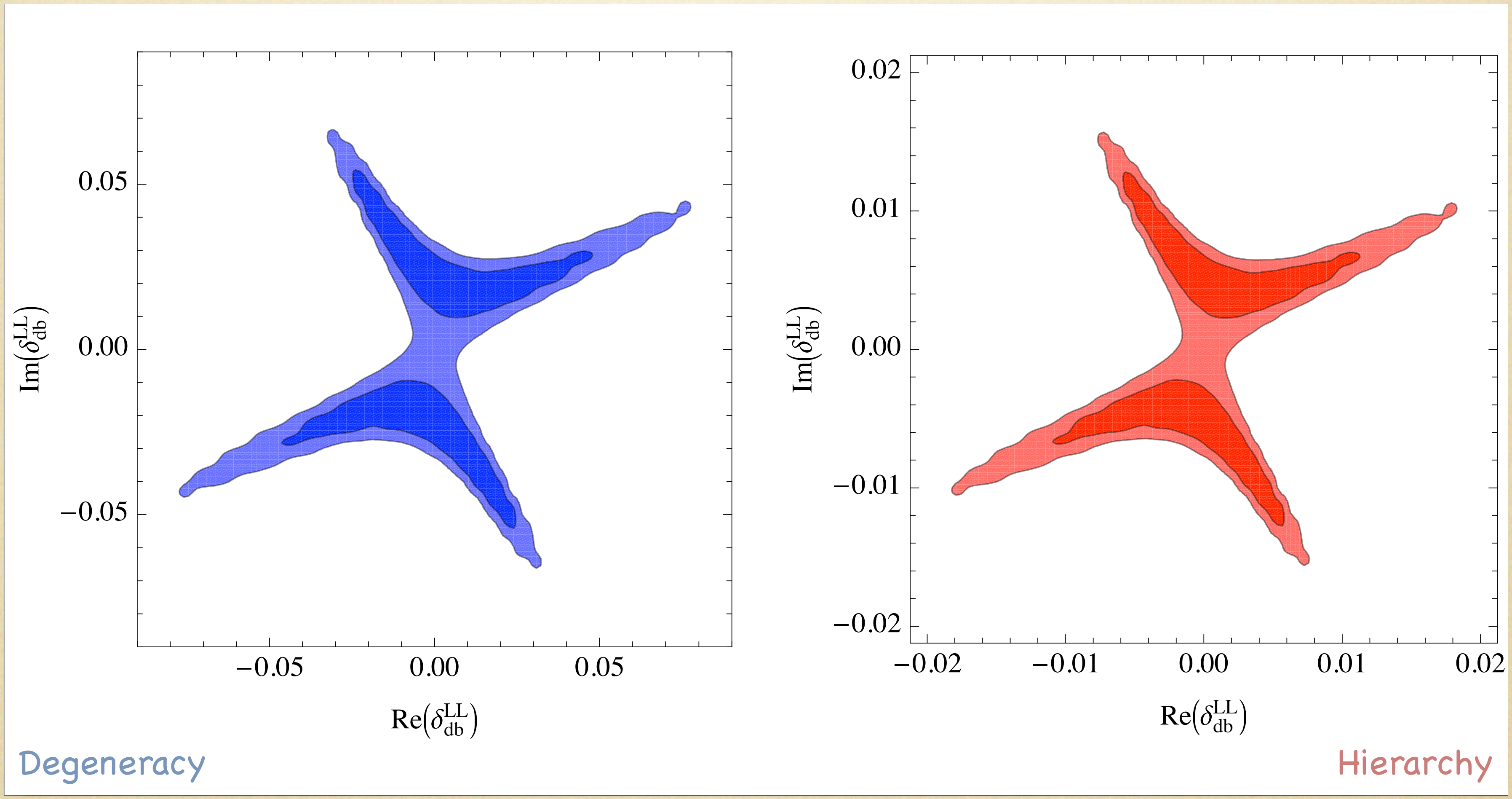
- We now focus on LL insertion in the down sector. In the hierarchical scenario we have that: $\hat{\delta}_{ij}^{LL} = \mathcal{W}_{i3} \mathcal{W}_{3j}^\dagger$
- In particular $\hat{\delta}_{i3}^{LL} = \mathcal{W}_{i3} \mathcal{W}_{33}^\dagger \approx \mathcal{W}_{i3} = \left(U_L^d \mathcal{W}_L'\right)_{i3}$
- Now we assume that the CKM matrix is dominated by the rotation in the down sector:

$$\hat{\delta}_{i3}^{LL} \approx (V^\dagger \mathcal{W}_L')_{i3} = \sum_{k=1}^{3} V_{ki}^* (\mathcal{W}_L')_{k3}$$

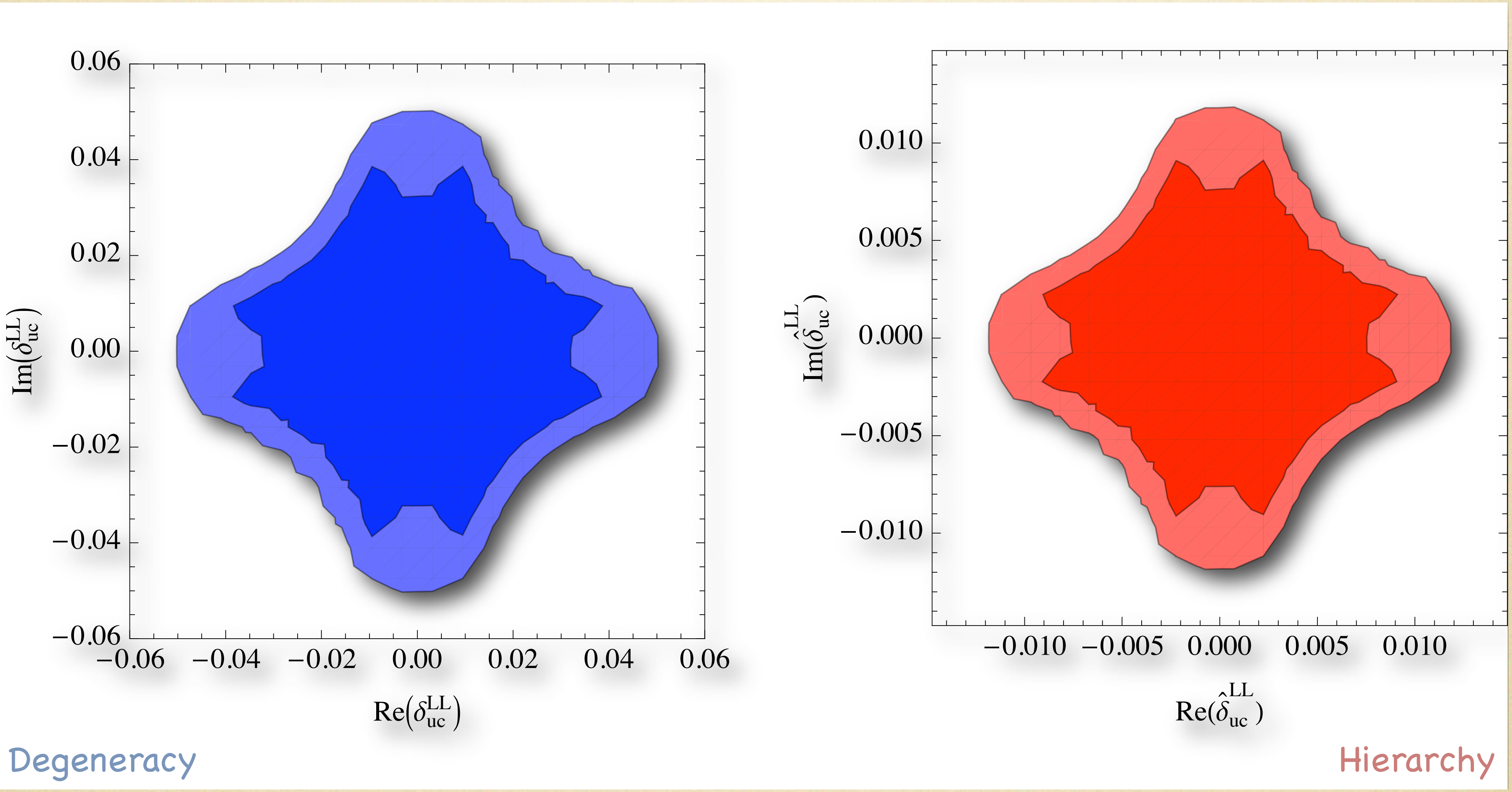
- Barring accidental cancellations:

$$\left|\hat{\delta}_{i3}\right| \geq |V_{3i}^* (\mathcal{W}_L')_{33}| \approx |V_{3i}^*|$$

B-d Transition



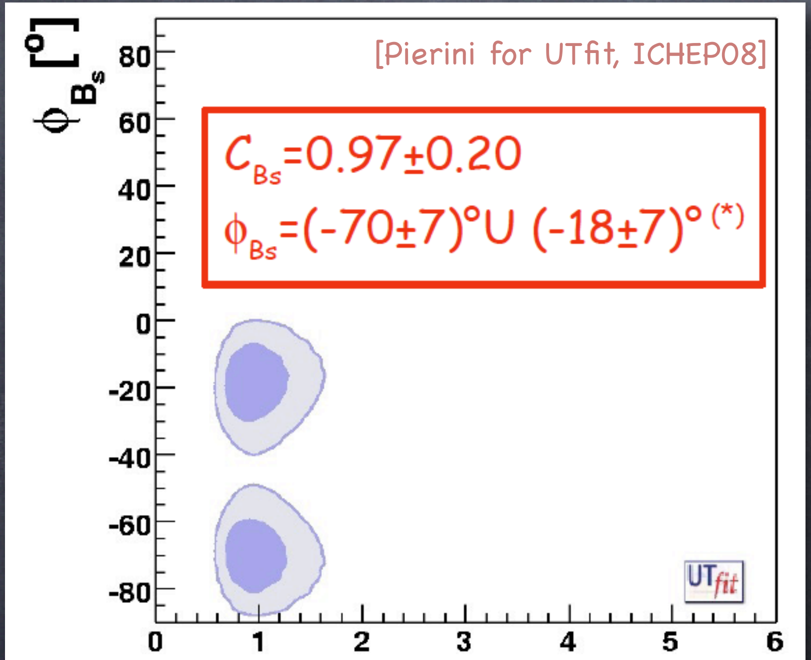
D Mixing


0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.06 −0.04 −0.02 0.00 0.02 0.04 0.06
$\mathrm{Re}(\delta_{uc}^{LL})$
$\mathrm{Im}(\delta_{uc}^{LL})$
Degeneracy
0.010
0.005
0.000
−0.005
−0.010
−0.010 −0.005 0.000 0.005 0.010
$\mathrm{Re}(\hat{\delta}_{uc}^{LL})$
$\mathrm{Im}(\hat{\delta}_{uc}^{LL})$
Hierarchy

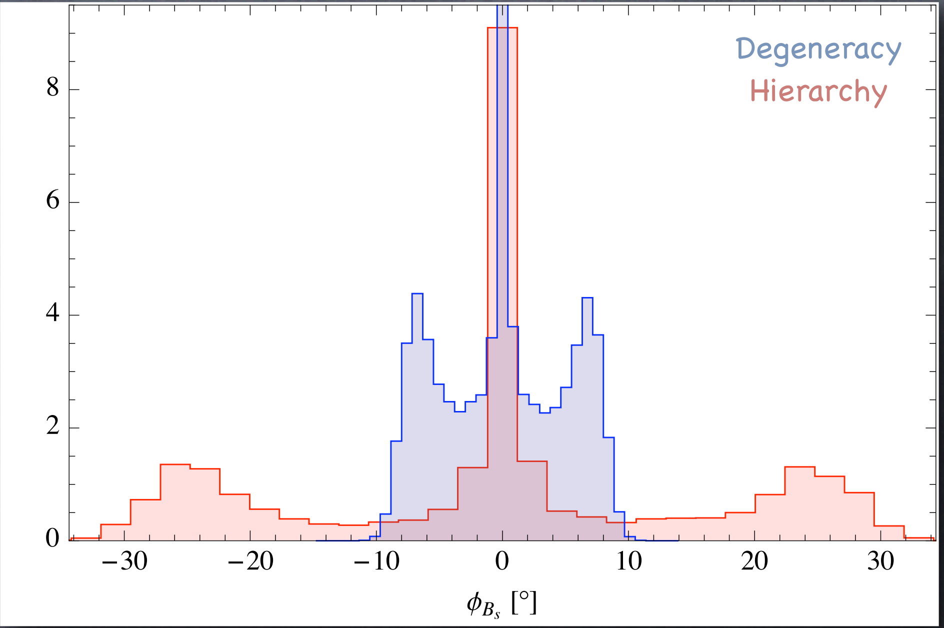
ϕ_{Bs}

$$\langle B_s|H_{\rm eff}^{\rm full}|\bar{B}_s\rangle = C_{B_s} e^{2i\phi_{B_s}} \langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle$$

$$\langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle = A_s^{\rm SM} e^{-2i\beta_s} \quad \beta_s = \arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = 0.018 \pm 0.001$$



(*) Significance reduced from σ~3.0 to σ~2.5 due to the correlation between ΔΓs and φs in D0 likelihood



Inputs

Parameter	Value	Gaussian (σ)	Uniform ($\frac{\Delta}{2}$)	Reference
$\lvert\varepsilon_K\rvert$	2.229×10^{-3}	0.012×10^{-3}	–	[19]
Δm_K (ps^{-1})	5.292×10^{-3}	0.009×10^{-3}	–	[19]
BR($B \to X_s\gamma$)	3.55×10^{-4}	0.26×10^{-4}	–	[20]
Δm_{B_s} (ps^{-1})	17.77	0.12	–	[19]
Δm_{B_d} (ps^{-1})	0.507	0.005	–	[19]
$\lvert M_{12}^D\rvert$ (ps^{-1})	7.7×10^{-3}	2.5×10^{-3}	–	[21]
$\bar{\rho}$	0.167	0.051	–	[22]
$\bar{\eta}$	0.386	0.035	–	[22]
λ	0.2255	0.010	–	[19]
$\lvert V_{cb}\rvert$	41.2×10^{-3}	1.1×10^{-3}	–	[19]
F_K (GeV)	0.160	–	–	[19]
F_{B_d} (MeV)	189	27	–	[23]
$F_{B_s}\sqrt{B_s}$ (MeV)	262	35	–	[23]
F_D (MeV)	201	3	17	[24]
$\hat{B}_K$	0.79	0.04	0.08	[24]
B_1^B	0.88	0.04	0.10	[24]
η_{cc}	0.47	0.04	–	[25]
η_{ct}	0.5765	0.0065	–	[25]
η_{tt}	1.43	0.23	–	[25]
$\overline{m}_t$ (GeV)	161.2	1.7	–	[24]
$\overline{m}_b$ (GeV)	4.21	0.08	–	[24]
$\overline{m}_c$ (GeV)	1.224	0.057	–	[26]

Soft Masses and RGE

$$\mathbf{m^2_{\tilde{d}}} = \begin{pmatrix} \overbrace{m^2_Q}^{\text{soft}} + m^2_d + \Delta_{\tilde{d}_L} & v(a^*_d \sin\beta - \mu y_d \cos\beta) \\ v(a_d \sin\beta - \mu^* y_d \cos\beta) & \underbrace{m^2_{\bar{d}}}_{\text{soft}} + m^2_d + \Delta_{\tilde{d}_R} \end{pmatrix}$$

Roughly speaking, the sfermion mass matrix is dominated by the soft terms, because all the others are suppressed by the EW scale. For the soft terms the RGE are:

First and second family: $$16\pi^2 \frac{d}{dt} m^2_{\phi_i} = -\sum_{a=1,2,3} 8 C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} Y_i g_1^2 S$$

Third family Left: $$16\pi^2 \frac{d}{dt} m^2_{Q_3} = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 S$$

Third family Right: $$16\pi^2 \frac{d}{dt} m^2_{\bar{d}_3} = 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 + \frac{2}{5} g_1^2 S$$

Also if we start with degenerate condition at very high energy, we end up with a split situation because of the Yukawa!!