

# Recent progress in the theory of Flavour Physics

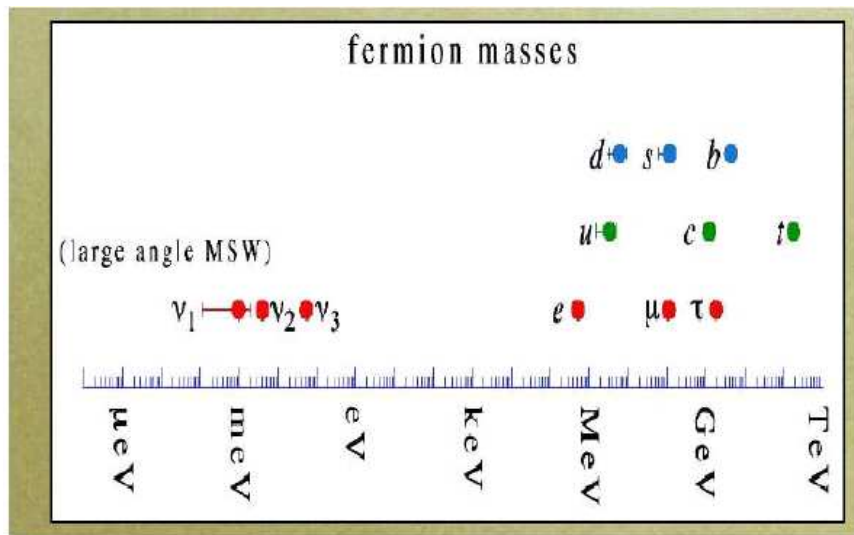
P. Paradisi



Physik Department  
Technische Universität München

IFAE 2010  
Rome, Italy  
April 08, 2010

# The fermion mass puzzle



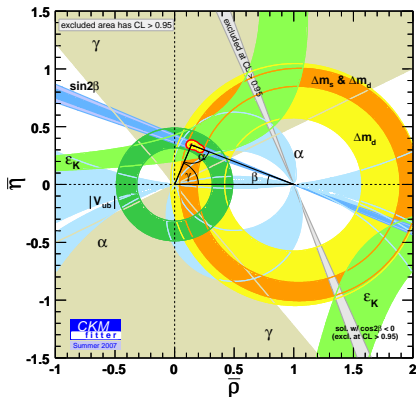
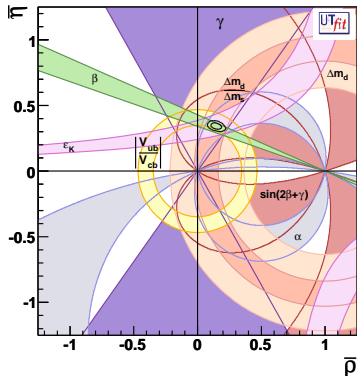
## Smallness and Hierarchy

$$\begin{aligned} Y_t &\sim 1, & Y_c &\sim 10^{-2}, & Y_u &\sim 10^{-5} \\ Y_b &\sim 10^{-2}, & Y_s &\sim 10^{-3}, & Y_d &\sim 10^{-4} \\ Y_\tau &\sim 10^{-2}, & Y_\mu &\sim 10^{-3}, & Y_e &\sim 10^{-6} \\ |V_{us}| &\sim 0.2, & |V_{cb}| &\sim 0.04, & |V_{ub}| &\sim 0.004, & \delta_{\text{KM}} &\sim 1 \end{aligned}$$

- For comparison:  $g_s \sim 1$ ,  $g \sim 0.6$ ,  $g' \sim 0.3$ ,  $\lambda \sim 1$ .
- The SM flavor parameters have structure:  
smallness and hierarchy
- Why? = The SM flavor puzzle

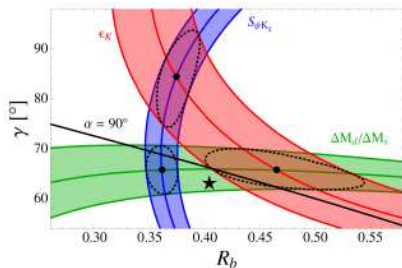
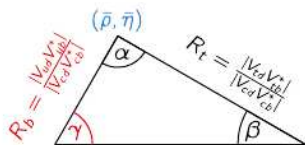
Nir

# SM success

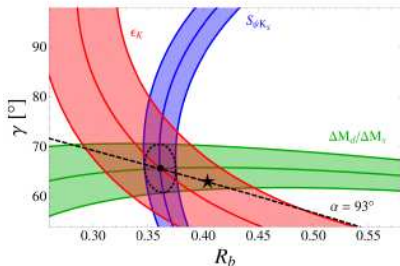
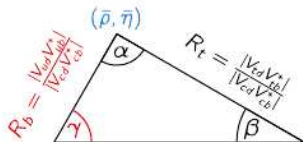


Very likely, flavour and CP violation in FC processes are dominated by the CKM mechanism (Nir)

- Recent theoretical improvements in  $\epsilon_K$  expose some tensions in the UT analysis [Lunghi & Soni, Buras & Guadagnoli]
- Look at  $\epsilon_K$ ,  $S_{\psi K_S}$  ( $\sin 2\beta$ ),  $\Delta M_d/\Delta M_s$  in the  $R_b$ - $\gamma$  plane
- $R_b$ ,  $\gamma$  can be obtained from tree-level processes



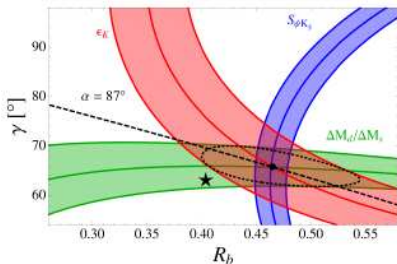
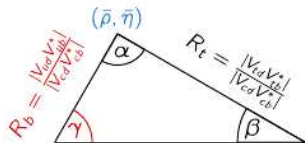
- Recent theoretical improvements in  $\epsilon_K$  expose some tensions in the UT analysis [Lunghi & Soni, Buras & Guadagnoli]
- Look at  $\epsilon_K$ ,  $S_{\psi K_S}$  ( $\sin 2\beta$ ),  $\Delta M_d/\Delta M_s$  in the  $R_b$ - $\gamma$  plane
- $R_b$ ,  $\gamma$  can be obtained from tree-level processes



Possible solutions:

- 1 +24% NP effect in  $\epsilon_K$

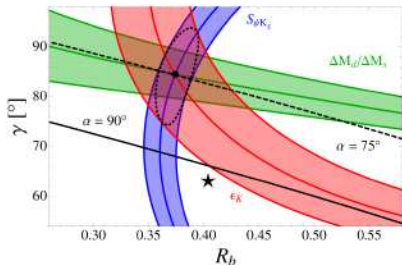
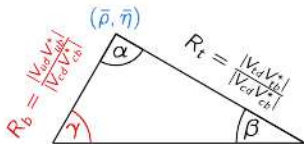
- Recent theoretical improvements in  $\epsilon_K$  expose some tensions in the UT analysis [Lunghi & Soni, Buras & Guadagnoli]
- Look at  $\epsilon_K$ ,  $S_{\psi K_S}$  ( $\sin 2\beta$ ),  $\Delta M_d/\Delta M_s$  in the  $R_b$ - $\gamma$  plane
- $R_b$ ,  $\gamma$  can be obtained from tree-level processes



Possible solutions:

- +24% NP effect in  $\epsilon_K$
- $-6.5^\circ$  NP phase in  $B_d$  mixing

- Recent theoretical improvements in  $\epsilon_K$  expose some tensions in the UT analysis [Lunghi & Soni, Buras & Guadagnoli]
- Look at  $\epsilon_K$ ,  $S_{\psi K_S}$  ( $\sin 2\beta$ ),  $\Delta M_d/\Delta M_s$  in the  $R_b$ - $\gamma$  plane
- $R_b$ ,  $\gamma$  can be obtained from tree-level processes



### Possible solutions:

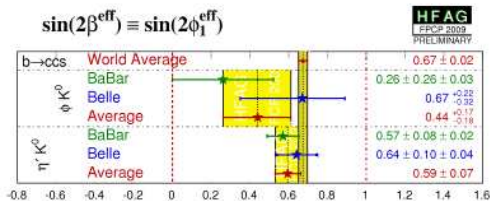
- +24% NP effect in  $\epsilon_K$
- $-6.5^\circ$  NP phase in  $B_d$  mixing
- $-22\%$  NP effect in  $\Delta M_d/\Delta M_s$  (requiring  $\alpha \sim 74^\circ$ )



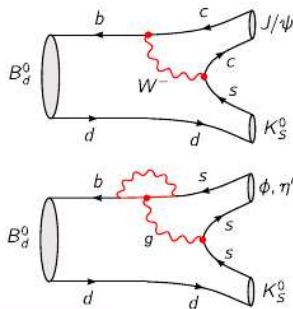
# $\sin 2\beta_{\text{eff}}$ tensions

- In the SM, mixing-induced CP asymmetries in  $B_d \rightarrow \psi K_S, \phi K_S, \eta' K_S$  all  $\approx \sin 2\beta$
- $B_d \rightarrow \psi K_S$  dominated by tree level,  $\phi K_S$  and  $\eta' K_S$  are loop-induced

Data indicate  $S_{\phi K_S} < S_{\eta' K_S} < S_{\psi K_S}$



[adapted from HFAG]



New physics in the decay amplitudes?

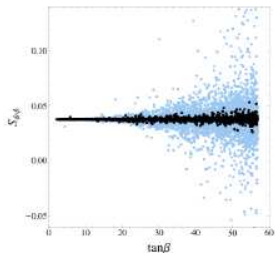
Can only be resolved at SuperB

# CPV in $B_s$ mixing

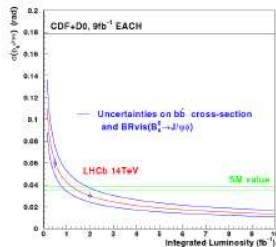
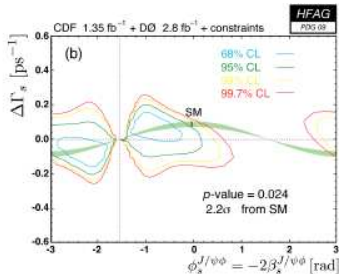
- $S_{\psi\phi}$ : mixing-induced CP asymmetry in  $B_s \rightarrow J/\psi\phi$
- $S_{\psi\phi} = \sin 2(\beta_s + \phi_{B_s}^{\text{NP}})$
- $S_{\psi\phi}^{\text{SM}} \approx 0.035$

Recent Tevatron data favour  
 $0.20 \leq S_{\psi\phi} \leq 0.98$

New physics in the  $B_s$  mixing phase?



- Sizable  $S_{\psi\phi}$  impossible in MFV MSSM
- Will be measured at LHCb



# Minimal Flavor Violation

## I. The CKM fits [constraints in the $\rho$ - $\eta$ plane]

These results are quite instructive if interpreted as bounds on the scale of new physics:

$$M(B_d - \bar{B}_d) \sim \frac{(V_{tb}^* V_{td})^2}{16 \pi^2 M_w^2} + \underbrace{c_{\text{NP}} \frac{1}{\Lambda^2}}_{\text{contribution of the new heavy degrees of freedom}}$$
  

The diagram shows the coefficient  $c_{\text{NP}}$  branching into four different physical scenarios, each leading to a constraint on the scale  $\Lambda$ :

- $\sim 1$  (tree/strong + generic flavour)  $\rightarrow \Lambda \gtrsim 2 \times 10^4 \text{ TeV [K]}$
- $\sim 1/(16 \pi^2)$  (loop + generic flavour)  $\rightarrow \Lambda \gtrsim 2 \times 10^3 \text{ TeV [K]}$
- $\sim (V_{it}^* V_{ij})^2$  (tree/strong + MFV)  $\rightarrow \Lambda \gtrsim 5 \text{ TeV [K \& B]}$
- $\sim (V_{it}^* V_{ij})^2 / (16 \pi^2)$  (loop + MFV)  $\rightarrow \Lambda \gtrsim 0.5 \text{ TeV [K \& B]}$

MFV (or something very similar at least for  $s \rightarrow d$  &  $b \rightarrow d$ ),  
is mandatory if we want to keep  $\Lambda$  in the TeV range

## The SUSY flavour problem

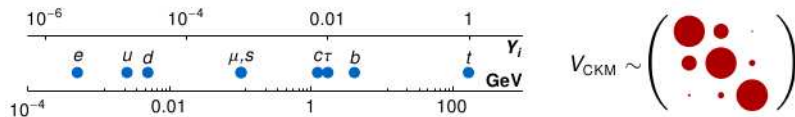
- Most of the 105 additional parameters in the MSSM violate flavour
- $O(1)$  values are strongly disfavoured by the excellent agreement of the SM with the flavour data

## Possible solutions

- 1 Decoupling
  - ▶ Sfermion mass scale very high
  - ▶ Clashes with the gauge hierarchy problem
- 2 Degeneracy
  - ▶ Sfermion masses nearly degenerate
  - ▶ Arises in models with low-scale SUSY breaking
  - ▶ Partly spoiled by RG evolution
- 3 Alignment
  - ▶ Quark and squark mass matrices aligned

# SM vs. SUSY flavour problems

Flavour violation is highly non-generic already in the SM!



The two problems should be related!

## Minimal Flavour Violation (MFV)

- Yukawa couplings are the only sources of flavour violation
- Effective theory
- Pragmatic approach
- Pessimistic phenomenology

## Flavour Models

- Flavour structure of Yukawa couplings and soft terms generated by spontaneous breaking of a flavour symmetry
- Ambitious approach
- Diverse phenomenology

# Minimal Flavour Violation

- SM without Yukawa interactions:  $SU(3)^5$  global **flavour symmetry**

$$SU(3)_u \otimes SU(3)_d \otimes SU(3)_Q \otimes SU(3)_e \otimes SU(3)_L$$

- Yukawa interactions break this symmetry
- Proposal for any New Physics model:

**Yukawa structures as the **only** sources of flavour violation**



**Minimal Flavour Violation**

**Notice that MFV allows for new CPV phases!**

## Where to look for **New Physics**?

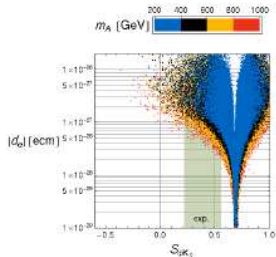
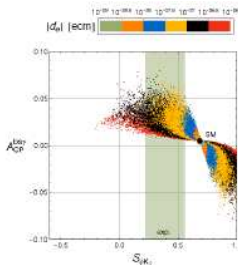
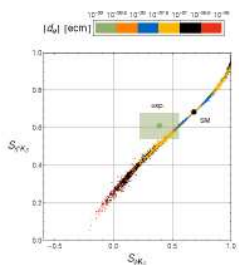
- Processes very **suppressed** or even **forbidden** in the SM
  - **FCNC** processes ( $\mu \rightarrow e\gamma$ ,  $\tau \rightarrow \mu\gamma$ ,  $B_{s,d}^0 \rightarrow \mu^+\mu^-$ ,  $K \rightarrow \pi\nu\bar{\nu}$ )
  - **CPV** effects in the electron/neutron EDMs,  $d_{e,n}...$
  - **CPV** in  $B_{s,d}$  decay/mixing amplitudes
- Processes predicted with **high precision** in the SM
  - **EWPO** as  $\Delta\rho$ ,  $(g-2)_\mu...$
  - **LU** in  $R_M^{e/\mu} = \Gamma(K(\pi) \rightarrow e\nu)/\Gamma(K(\pi) \rightarrow \mu\nu)$

# Flavour Matrix

		FLAVOUR COUPLING		
		$b \rightarrow s$ [ $-\lambda^2$ in SM]	$b \rightarrow d$ [ $-\lambda^2$ in SM]	$s \rightarrow d$ [ $-\lambda^3$ in SM]
ELECTROWEAK STRUCTURE	$\Delta F=2$ box	$\Delta M_{B_s}$ $A_{CP}(B_s \rightarrow \psi\phi), \epsilon_{B_s}$	$\Delta M_{B_d}$ $A_{CP}(B_d \rightarrow \psi K), \epsilon_{B_d}$	$\epsilon_K$
	$\Delta F=1$ 4-quark ops.	$A_{CP}(B_d \rightarrow \phi K)$	$A_{CP}(B_s \rightarrow \phi K)$	
	gluon penguin	$A_{CP}(B_d \rightarrow \phi K)$ $[\Gamma, \Delta\Gamma_{CP}](B \rightarrow X_s \gamma)$	$[\Gamma, \Delta\Gamma_{CP}](B \rightarrow \rho/\pi \gamma)$	$\Gamma(K_L \rightarrow \pi^0 l l)$
	$\gamma$ penguin	$[\Gamma, \Delta\Gamma_{CP}](B \rightarrow X_s \gamma)$ $[\Gamma, \Delta\Gamma_{CP}](B \rightarrow X_s l l)$ $A_{FB}(B \rightarrow X_s l l)$	$[\Gamma, \Delta\Gamma_{CP}](B \rightarrow \rho/\pi \gamma)$ $[\Gamma, \Delta\Gamma_{CP}](B \rightarrow \rho/\pi l l)$ $A_{FB}(B \rightarrow \rho/\pi l l)$	$\Gamma(K_L \rightarrow \pi^0 l l)$
	$Z^0$ penguin	$[\Gamma, \Delta\Gamma_{CP}](B \rightarrow X_s l l)$ $A_{FB}(B \rightarrow X_s l l)$ $\Gamma(B_s \rightarrow \mu\mu)$	$[\Gamma, \Delta\Gamma_{CP}](B \rightarrow \rho/\pi l l)$ $A_{FB}(B \rightarrow \rho/\pi l l)$ $\Gamma(B_d \rightarrow \mu\mu)$	$\Gamma(K^* \rightarrow \pi^+ \nu\nu)$ $\Gamma(K_L \rightarrow \pi^0 \nu\nu)$ $\Gamma(K_L \rightarrow \pi^0 l l)$
	$H^0$ penguin	$\Gamma(B_s \rightarrow \mu\mu)$	$\Gamma(B_d \rightarrow \mu\mu)$	



# Flavor blind MSSM $\approx$ MFV + CPV



- ▶ CP violating  $\Delta F = 0$  and  $\Delta F = 1$  dipole amplitudes can be strongly modified
- ▶  $S_{\phi K_S}$  and  $S_{\eta' K_S}$  can simultaneously be brought in **agreement with the data**
- ▶ sizeable and correlated effects in  $A_{CP}^{B \to \gamma} \simeq 1\% - 6\%$
- ▶ **lower bounds** on the electron and neutron EDMs at the level of  $d_{e,n} \gtrsim 10^{-28}$  ecm
- ▶ large and correlated effects in the CP asymmetries in  $B \rightarrow K^* \mu^+ \mu^-$  (WA, Ball, Bharucha, Buras, Straub, Wick)

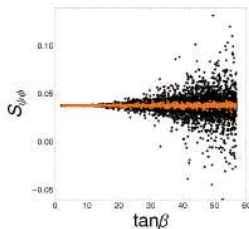
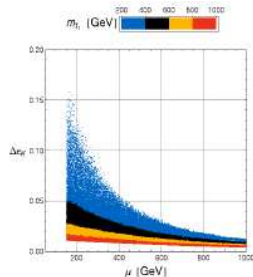
- ▶ the leading NP contributions to  $\Delta F = 2$  amplitudes are **not sensitive** to the new phases of the FBMSSM
- ▶ CP violation in meson mixing is **SM like**
- ▶ i.e. small effects in  $S_{\psi \phi}$ ,  $S_{\psi K_S}$  and  $\epsilon_K$
- ▶ in particular:  $0.03 < S_{\psi \phi} < 0.05$

A combined study of all these observables and their correlations constitutes a **very powerful test** of the FBMSSM

# Phenomenology of the flavor blind MSSM

## 1 Kaon mixing

- ▶ The mixing amplitude  $M_{12}^K$  has no sensitivity to the new flavor blind phases
- ▶ Still,  $\epsilon_K \propto \text{Im}(M_{12}^K)$  can get a **positive NP contribution** up to 15%
- ▶ But only for a **very light SUSY spectrum**:  
 $\mu, m_{\tilde{t}_1} \simeq 200\text{GeV}$



## 2 $B_d$ and $B_s$ mixing

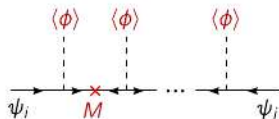
- ▶ Leading NP contributions to  $M_{12}^{d,s}$  are **insensitive to the new phases** of a FBMSSM. (at least for moderate  $\tan\beta$  ...)
- ▶ For large  $\tan\beta$ , the constraint from  $b \rightarrow s\gamma$  does not allow for sizeable effects
- ▶  $S_{\psi K_S}$  and  $S_{\psi\phi}$  are **SM like** ( $S_{\psi\phi} \simeq 0.03 - 0.05$ )

# SUSY flavour models

Main idea: hierarchies in Yukawa couplings generated by spontaneous breakdown of flavour symmetry (horizontal symmetry, family symmetry)

- Generalization of the Froggatt-Nielsen mechanism
- Yukawa hierarchies explained by different powers of small  $\epsilon$ :

$$\Rightarrow Y_{ij} \propto \left( \frac{\langle \phi \rangle}{M} \right)^{(a_i+b_j)} = \epsilon^{(a_i+b_j)}$$



- Possible to relate Yukawa matrices and sfermion mass matrices/trilinear couplings

SUSY flavour models can explain the origin of the hierarchies in the Yukawa couplings *and* solve the SUSY flavour problem

- Many different viable models exist, with abelian or non-abelian flavour symmetries

## Abelian vs. Non-abelian

- In most non-abelian models, 1st & 2nd generatio sfermions are **approximately degenerate**
  - Suppressed contributions to  $1 \leftrightarrow 2$  transitions, in particular  $D^0$ - $\bar{D}^0$  mixing
- In abelian models, sfermions of different generations need **not be degenerate**
  - $O(1)$  1-2 mass splitting leads to  $O(\lambda) (\delta_{ij}^{LL})_{12}$  in the SCKM basis
  - Large effects in  $D^0$ - $\bar{D}^0$  mixing

## Chirality structure of flavour violating terms

- Different flavour symmetries lead to different patterns of flavour violation
- Mass insertions:  $M_d^2 = \text{diag}(\tilde{m}^2) + \tilde{m}^2 \begin{pmatrix} \delta_d^{LL} & \delta_d^{LR} \\ \delta_d^{RL} & \delta_d^{RR} \end{pmatrix}$
- $\delta^{LL}, \delta^{RR}, \delta^{LR}$  fixed by the flavour symmetry (up to  $O(1)$  factors)

# Examples of flavour models

4 representative flavour models with different chirality structures in the  $\bar{d}$  sector:

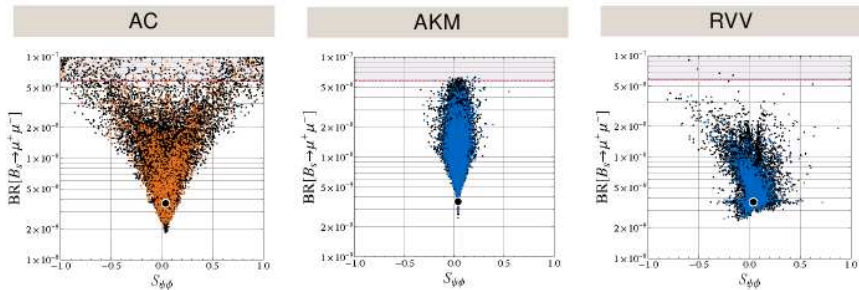
AC model [Agashe, Carone]	AKM model [Antusch, King, Malinsky]	RVV model [Ross, Velasco-Sevilla, Vives]	$\delta$ LL model [e.g. Hall, Murayama]
$U(1)$	$SU(3)$	$SU(3)$	$(S_3)^3$
Large, $O(1)$ RR mass insertions	Only CKM-like RR mass insertions	CKM-like LL & RR mass insertions	Only CKM-like LL mass insertions

$$\begin{array}{cccc}
 \delta_d^{LL} \sim \begin{pmatrix} \cdot & 0 & 0 \\ 0 & \cdot & \lambda^2 \\ 0 & \lambda^2 & \cdot \end{pmatrix} &
 \delta_d^{LL} \sim \begin{pmatrix} \cdot & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & \cdot \end{pmatrix} &
 \delta_d^{LL} \sim \begin{pmatrix} \cdot & \lambda^3 & \lambda^2 \\ \lambda^3 & \cdot & \lambda \\ \lambda^2 & \lambda & \cdot \end{pmatrix} &
 \delta_d^{LL} \sim \begin{pmatrix} \cdot & \lambda^5 & \lambda^3 \\ \lambda^5 & \cdot & \lambda^2 \\ \lambda^3 & \lambda^2 & \cdot \end{pmatrix} \\
 \\
 \delta_d^{RR} \sim \begin{pmatrix} \cdot & 0 & 0 \\ 0 & \cdot & 1 \\ 0 & 1 & \cdot \end{pmatrix} &
 \delta_d^{RR} \sim \begin{pmatrix} \cdot & \lambda^3 & \lambda^3 \\ \lambda^3 & \cdot & \lambda^2 \\ \lambda^3 & \lambda^2 & \cdot \end{pmatrix} &
 \delta_d^{RR} \sim \begin{pmatrix} \cdot & \lambda^3 & \lambda^2 \\ \lambda^3 & \cdot & \lambda \\ \lambda^2 & \lambda & \cdot \end{pmatrix} &
 \delta_d^{RR} \sim \begin{pmatrix} \cdot & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & \cdot \end{pmatrix}
 \end{array}$$

Altmannshofer et al. '09

# $Br(B_s \rightarrow \mu^+ \mu^-)$ vs. $S_{\psi\phi}$

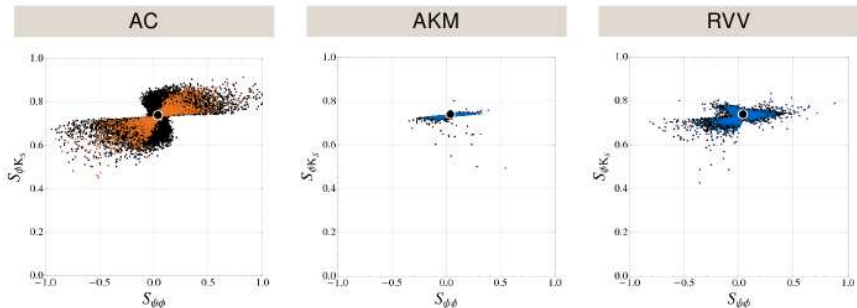
- Both observables can deviate significantly from the SM in all 3 models
- large  $S_{\psi\phi} \Rightarrow$  large  $Br(B_s \rightarrow \mu^+ \mu^-)$  in the AC and AKM models
- Correlation arises from dominance of Higgs penguin contributions



- **Orange points:** UT tension solved through contribution to  $\Delta M_d / \Delta M_s$
- **Blue points:** UT tension solved through contribution to  $\epsilon_K$
- Scan ranges:  $m_0 < 2$  TeV,  $M_{1/2} < 1$  TeV,  $|A_0| < 3m_0$ ,  $5 < \tan \beta < 55$ ,  $O(1)$  parameters varied within  $[\frac{1}{2}, 2]$

# $S_{\phi K_S}$ vs. $S_{\psi\phi}$

- In the AC model, both  $S_{\phi K_S}$  and  $S_{\psi\phi}$  can have large effects, but a simultaneous *enhancement* of  $S_{\psi\phi}$  and *suppression* of  $S_{\phi K_S}$  (as indicated by the data) is impossible
- $S_{\phi K_S}$  nearly SM-like in AKM and RVV models

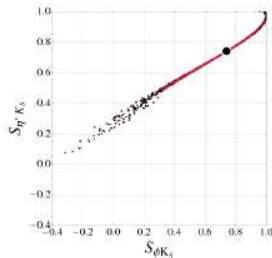
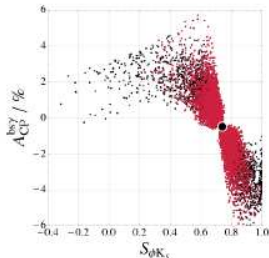
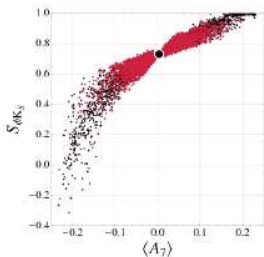


- **Orange points:** UT tension solved through contribution to  $\Delta M_d / \Delta M_s$
- **Blue points:** UT tension solved through contribution to  $\epsilon_K$
- Scan ranges:  $m_0 < 2$  TeV,  $M_{1/2} < 1$  TeV,  $|A_0| < 3m_0$ ,  $5 < \tan \beta < 55$ ,  $O(1)$  parameters varied within  $[\frac{1}{2}, 2]$

# Model with purely left-handed currents

Pattern of NP effects in the  $\delta LL$  model:

- No large effects in  $S_{\psi\phi}$
- Large, correlated effects in  $S_{\phi K_S}$ ,  $S_{\eta' K_S}$ ,  $A_{CP}(b \rightarrow s\gamma)$ ,  $\langle A_{7,8} \rangle$
- $\langle A_{7,8} \rangle$ : T-odd CP asymmetries in  $B \rightarrow K^* \ell^+ \ell^-$

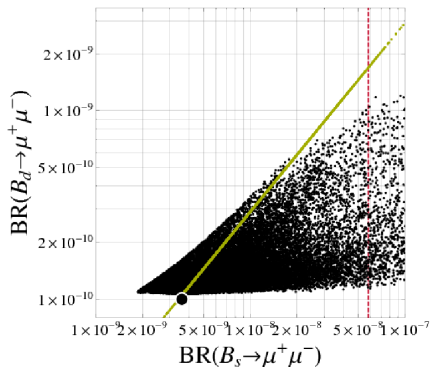


- Scan ranges:  $m_0 < 2$  TeV,  $M_{1/2} < 1$  TeV,  $|A_0| < 3m_0$ ,  $5 < \tan \beta < 55$ ,  $O(1)$  parameters varied within  $[\frac{1}{2}, 2]$

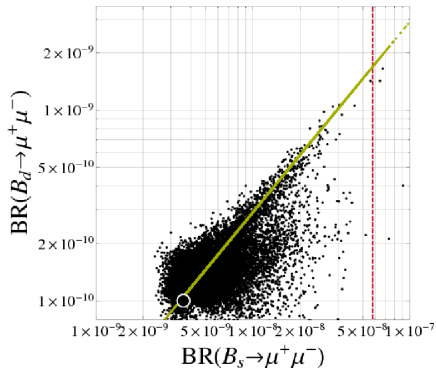


# $Br(B_s \rightarrow \mu^+ \mu^-)$ vs. $Br(B_d \rightarrow \mu^+ \mu^-)$

## Abelian (AC)



## Non abelian (RVV)



$$Br(B_s \rightarrow \mu^+ \mu^-) / Br(B_d \rightarrow \mu^+ \mu^-) = |V_{ts} / V_{td}|^2 \text{ in MFV models}$$

**CPV** in  $D^0 - \bar{D}^0 \sim ((V_{cb} V_{ub})/(V_{cs} V_{us})) \sim \mathbf{10^{-3}}$  in the **SM**

- $\langle D^0 | \mathcal{H}_{\text{eff}} | \bar{D}^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}, \quad |D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$
- $\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}}, \quad \phi = \text{Arg}(q/p)$
- $x = \frac{\Delta M_D}{\Gamma} = 2\tau \text{Re} \left[ \frac{q}{p} (M_{12} - \frac{i}{2} \Gamma_{12}) \right]$
- $y = \frac{\Delta \Gamma}{2\Gamma} = -2\tau \text{Im} \left[ \frac{q}{p} (M_{12} - \frac{i}{2} \Gamma_{12}) \right]$

$$S_f = 2\Delta Y_f = \frac{1}{\Gamma_D} \left( \hat{\Gamma}_{\bar{D}^0 \rightarrow f} - \hat{\Gamma}_{D^0 \rightarrow f} \right)$$

$$\eta_f^{\text{CP}} S_f = x \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) \sin \phi - y \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \cos \phi$$

$$a_{\text{SL}} = \frac{\Gamma(D^0 \rightarrow K^+ \ell^- \nu) - \Gamma(\bar{D}^0 \rightarrow K^- \ell^+ \nu)}{\Gamma(D^0 \rightarrow K^+ \ell^- \nu) + \Gamma(\bar{D}^0 \rightarrow K^- \ell^+ \nu)} = \frac{|q|^4 - |p|^4}{|q|^4 + |p|^4}$$

# CPV in D-physics vs. neutron EDM in SUSY

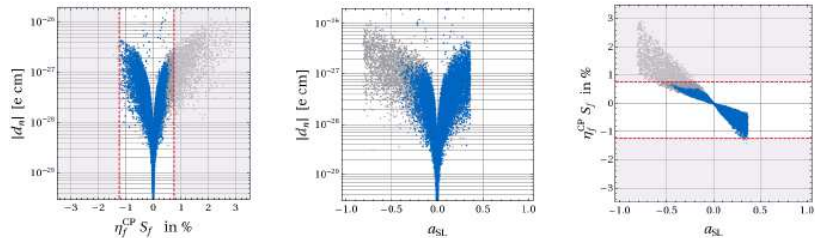


FIG. 3: Correlations between  $d_n$  and  $S_f$  (left),  $d_n$  and  $a_{SL}$  (middle) and  $a_{SL}$  and  $S_f$  (right) in SUSY alignment models. Gray points satisfy the constraints (8)-(10) while blue points further satisfy the constraint (11) from  $\phi$ . Dashed lines stand for the allowed range (18) for  $S_f$ .

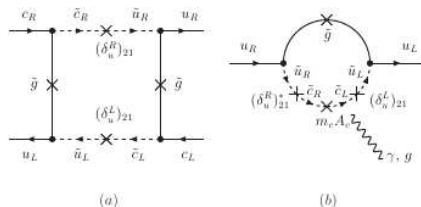


FIG. 2: Examples of relevant Feynman diagrams contributing (a) to  $D^0 - \bar{D}^0$  mixing and (b) to the up quark (C)EDM in SUSY alignment models.

# $K \rightarrow \pi \nu \bar{\nu}$ in the SM

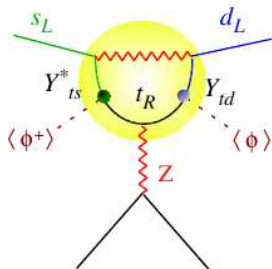
- $K \rightarrow \pi \nu \bar{\nu}$  processes offer a unique possibility in probing the underlying **flavour mixing mechanism**:
  - No SM tree-level contributions (**FCNC decays**);
  - One-loop SM contributions CKM-suppressed ( $V_{ts}^* V_{td} \sim \lambda^5$ );
  - High precision of the SM prediction thanks to short distance (e.w.) dynamics dominance:

$$\mathcal{H}_{\text{eff}}^{(s,d)} = \sum_{l=e,\mu,\tau} V_{ts}^* V_{td} [X_L(\bar{s}d)_{V-A} + X_R(\bar{s}d)_{V+A}] (\bar{\nu}_l \nu_l)_{V-A}$$

$$Br(K \rightarrow \pi \nu \bar{\nu}) \sim |X = X_L + X_R|^2$$

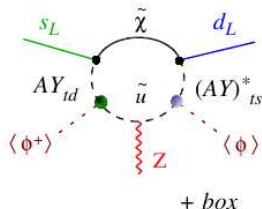
$$X \sim c_{\text{SM}} \frac{y_t^2 V_{ts}^* V_{td}}{16\pi^2 M_W^2}$$

$$X_L^{\text{SM}} = 1.464 \pm 0.041, \quad X_R^{\text{SM}} = 0$$

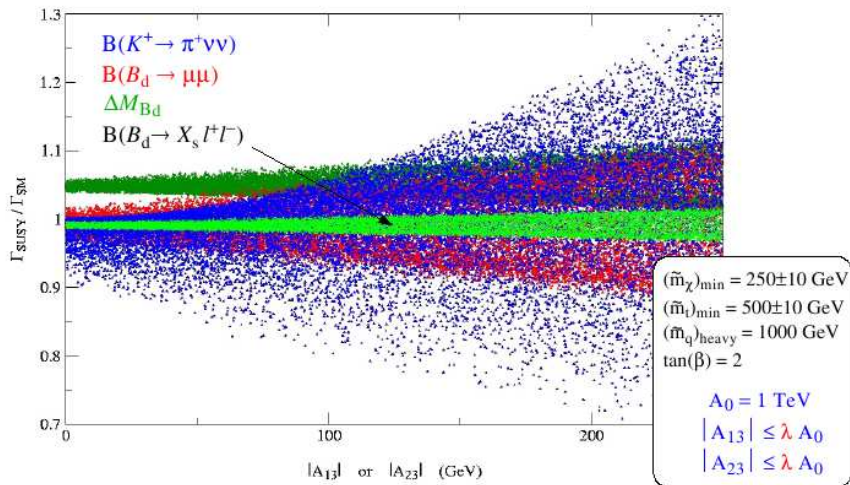


# $K \rightarrow \pi \nu \bar{\nu}$ and NP

- $K \rightarrow \pi \nu \bar{\nu}$  has a **high sensitivity to NP** effects of many theories as **SUSY, LHT, Z'** models.....
- Large **NP** effects only if  $\delta_{21} \approx V_{ts}^* V_{td}$  (**beyond MFV**)
- The dominant effects to  $K \rightarrow \pi \nu \bar{\nu}$  arise from  $\tilde{\chi}/\tilde{u}$  diagrams with double-MIA **[Colangelo, Isidori '98]**.
- Gluino-type amplitudes (LL, RR and LR-down squarks type mixings) essentially negligible contrary to  $\epsilon_K, b \rightarrow s \gamma, B^0 - \bar{B}^0$
- Minor effects within pure MFV.
- The maximal sensitivity to the up-type trilinear terms is obtained for
  - **Light stop and charginos**
  - **small  $\tan \beta$**

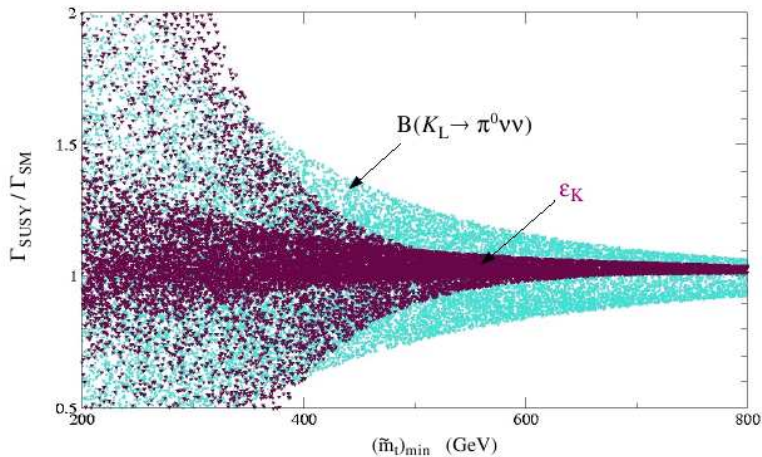


# Chargino mediated $K \rightarrow \pi \nu \bar{\nu}$



Isidori et al., '06

# Chargino mediated $K \rightarrow \pi \nu \bar{\nu}$



Isidori et al., '06

- **Neutrino Oscillation**  $\Rightarrow m_{\nu_i} \neq m_{\nu_j} \Rightarrow$  **LFV**
- **see-saw**:  $m_\nu = \frac{(m_\nu^D)^2}{M_R} \sim \text{eV}$ ,  $M_R \sim 10^{14-16} \Rightarrow m_\nu^D \sim m_{\text{top}}$
- **LFV** transitions like  $\mu \rightarrow e\gamma$  @ 1 loop with exchange of

- $W$  and  $\nu$  in the **SM** framework (**GIM**)

$$\text{Br}(\mu \rightarrow e\gamma) \sim \frac{m_\nu^4}{M_W^4} \leq 10^{-50} \quad m_\nu \sim \text{eV}$$

- $\tilde{W}$  and  $\tilde{\nu}$  in the **MSSM** framework (**SUPER-GIM**)

$$\text{Br}(\mu \rightarrow e\gamma) \sim \frac{m_\nu^D{}^4}{\tilde{m}^4} \leq 10^{-11} \quad m_\nu^D \sim m_{\text{top}}$$

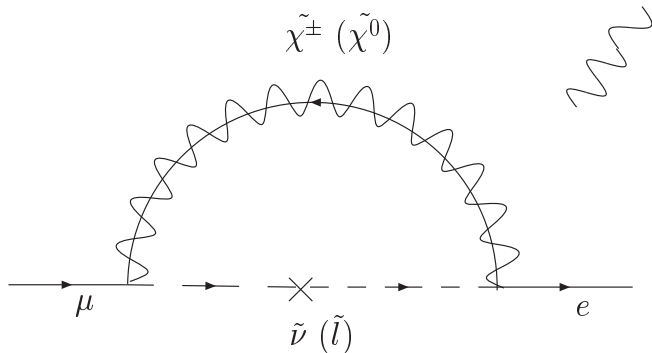
⇓

- **LFV** signals are undetectable (**detectable**) in the SM (**MSSM**)



## LFV interactions – leptons/sleptons/gauginos

$$\mathcal{L} = \bar{l}_i \left( C_{ijA}^R P_R + C_{ijA}^L P_L \right) \tilde{\chi}_A^- \tilde{\nu}_j + \bar{l}_i \left( N_{ijA}^R P_R + N_{ijA}^L P_L \right) \tilde{\chi}_A^0 \tilde{l}_j. \quad (1)$$



$$\left. \frac{BR(l_i \rightarrow l_j \gamma)}{BR(l_i \rightarrow l_j \nu_i \bar{\nu}_j)} \right|_{\text{Gauge}} \simeq \frac{\alpha_{el}}{20\pi} \left( \frac{m_W^4}{m_{SUSY}^4} \right) \left( \delta_{LL}^{21} \right)^2 t_\beta^2 \quad \delta_{LL} \sim h^\nu h^{\nu\dagger}$$

## RG induced LFV interactions in SUSY see-saw

- SUSY see-saw superpotential (MSSM + RN)

$$W = h^e L e^c H_1 + h^\nu L \nu^c H_2 + M_R \nu^c \nu^c + \mu H_1 H_2,$$

$$\mathcal{M}_\nu = -h^\nu M_R^{-1} h^{\nu T} v_2^2,$$

$$M_\ell^2 = \begin{pmatrix} m_L^2(1 + \delta_{LL}^{ij}) & (A - \mu t_\beta)m_\ell + m_L m_R \delta_{LR}^{ij} \\ (A - \mu t_\beta)m_\ell + m_L m_R \delta_{LR}^{ij \dagger} & m_R^2(1 + \delta_{RR}^{ij}) \end{pmatrix}$$

- If  $h^e = h_{ij}^e \delta_{ij}$  and  $M_R = M_{Rij} \delta_{ij} \Rightarrow h^\nu \neq h_{ij}^\nu \delta_{ij}$  in general.

$$\delta_{LL}^{ij} \approx -\frac{3}{8\pi^2} (h^\nu h^{\nu \dagger})_{ij} \ln \frac{M_X}{M_R},$$

[Borzumati & Masiero, '86]

$h^\nu$  is unknown  $\Rightarrow$  No model independent predictions for LFV

$$h^\nu = U_{\text{MNS}}^* \mathcal{D}_{\sqrt{\mathcal{M}_\nu}} R^T \mathcal{D}_{\sqrt{M_R}} \frac{1}{V_2},$$

$R^\dagger R = 1 \Rightarrow$  **three angles** and **three phases**

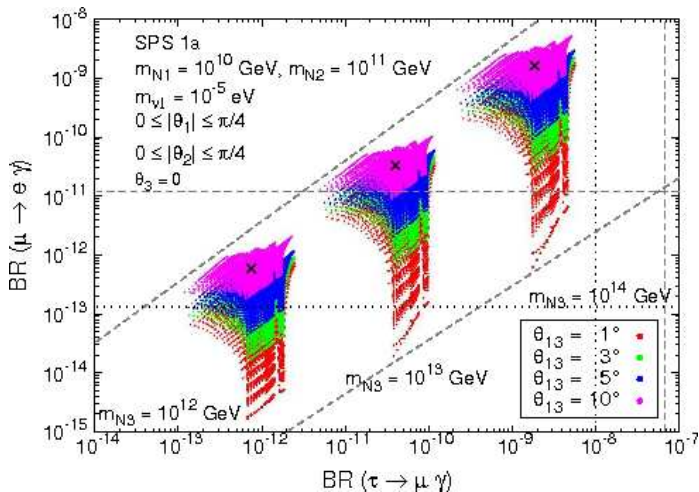
- $\nu_L$  &  $\nu_R$  hierarchical (and R real)

$$\frac{B(\mu \rightarrow e\gamma)}{B(\tau \rightarrow \mu\gamma)} \sim \frac{|U_{e3}|^2}{B(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)}$$

- $\nu_L$  hierarchical and  $\nu_R$  degenerate (and R real)

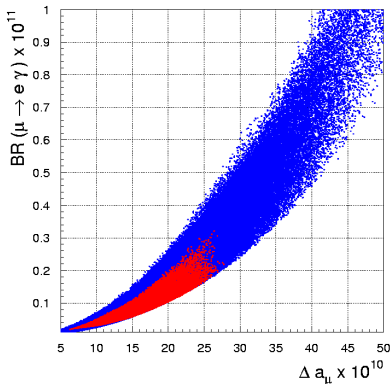
$$\frac{B(\mu \rightarrow e\gamma)}{B(\tau \rightarrow \mu\gamma)} \sim \frac{|s_{12}c_{12}(m_{\text{sol}}/m_{\text{atm}}) + U_{e3}|^2}{B(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)}$$

# $\mu \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$ in SUSY see-saw

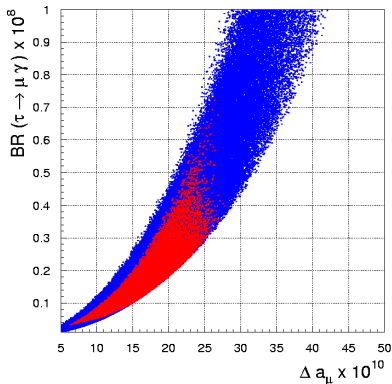


Herrero et al., '06

# $(g - 2)_\mu$ vs $l_i \rightarrow l_j \gamma$



$$|\delta_{LL}^{12}| = 10^{-4} \text{ and } |\delta_{LL}^{23}| = 10^{-2},$$



**Isidori et al., 07**

$$BR(l_i \rightarrow l_j \gamma) \approx \left[ \frac{\Delta a_\mu}{30 \times 10^{-10}} \right]^2 \times \begin{cases} 2 \times 10^{-4} |\delta_{LL}^{12}|^2 & [\mu \rightarrow e] \\ 5 \times 10^{-5} |\delta_{LL}^{23}|^2 & [\tau \rightarrow \mu] \end{cases}$$

## RG induced Flavor Violating interactions in SUSY GUTs

- **SUSY SU(5)** [Barbieri & Hall, '95]

$$(\delta_{LL}^{\tilde{q}})_{ij} \sim h^u h^{u\dagger}_{ij} \sim h_t^2 V_{CKM}^{ik} V_{CKM}^{kj*} \rightarrow (\delta_{RR}^{\tilde{\ell}})_{ij} \simeq (\delta_{LL}^{\tilde{q}})_{ij}$$

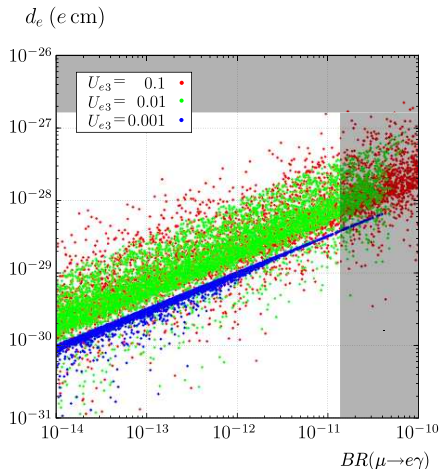
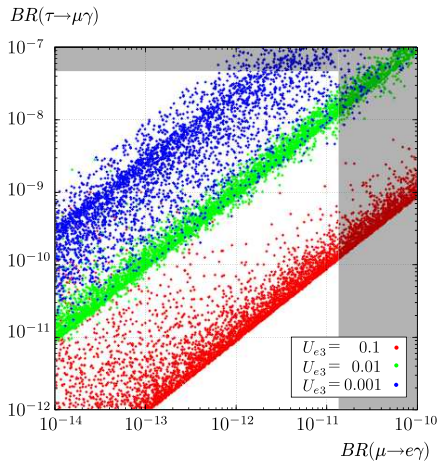
- **SUSY SU(5)+RN** [Yanagida et al., '95]

$$(\delta_{LL}^{\tilde{\ell}})_{ij} \sim (h^\nu h^{\nu\dagger})_{ij} \quad \& \quad (\delta_{RR}^{\tilde{\ell}})_{ij} \sim (h^u h^{u\dagger})_{ij}$$

- **SUSY SU(5)+RN** [Moroi, '00] & **SO(10)** [Chang et al., 02]

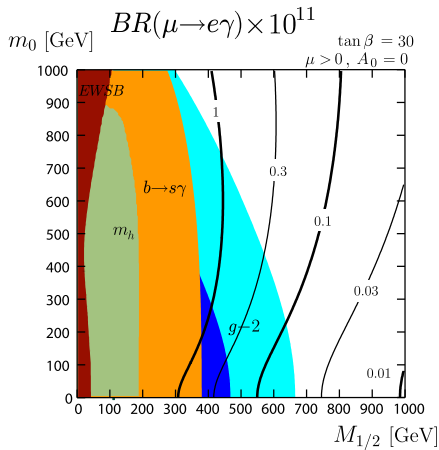
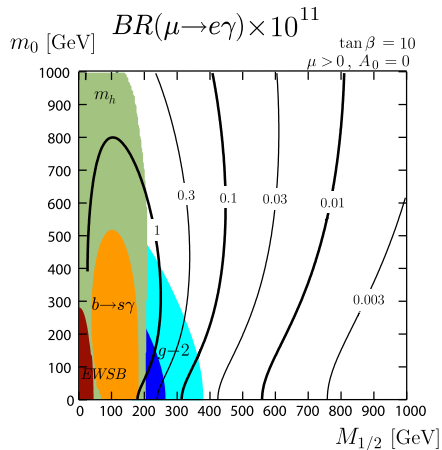
$$\sin \theta_{\mu\tau} \sim \frac{\sqrt{2}}{2} \Rightarrow (\delta_{LL}^{\tilde{\nu}})_{23} \sim 1 \Rightarrow (\delta_{RR}^{\tilde{q}})_{23} \sim 1$$

# $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ in SUSY SU(5)+RN



Hisano et al. '09

# BR( $\mu \rightarrow e\gamma$ ) in $SU(5)_{RN}$ and the LHC reach



hierarchical  $\nu_L$  and  $N_R$ ,  $U_{e3} = 0.1$ ,  $M_{N_3} = 10^{-13}$  GeV

Hisano et al. '09

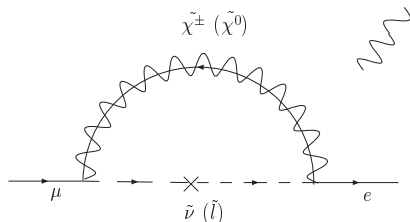
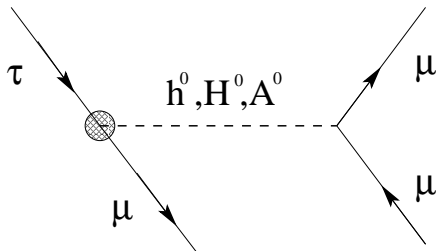


- LFV Yukawa Interactions (if  $\delta_{ij} = \tilde{m}_{ij}^2 / \tilde{m}^2 \neq 0$ ):

$$\begin{aligned}
 -\mathcal{L} &\simeq (2G_F^2)^{\frac{1}{4}} \frac{m_\tau}{c_\beta^2} \left( \Delta_L^{3j} \bar{\tau}_R l_L^j + \Delta_R^{3j} \bar{\tau}_L l_R^j \right) (c_{\beta-\alpha} h^0 - s_{\beta-\alpha} H^0 - iA^0) \\
 &+ (8G_F^2)^{\frac{1}{4}} \frac{m_\tau}{c_\beta^2} \left( \Delta_L^{3j} \bar{\tau}_R \nu_L^j + \Delta_R^{3j} \nu_L^\tau \bar{l}_R^j \right) H^\pm + h.c. \\
 \Delta_{3j} &\sim \frac{\alpha_2}{4\pi} \delta_{3j}
 \end{aligned}$$

- Higgs (gaugino)** mediated LFV effects decouple as  $m_H \rightarrow \infty$  ( $m_{SUSY} \rightarrow \infty$ ),
- Key ingredients in the Higgs mediated LFV:
  - $\tan \beta \sim 50$
  - $\delta_{3j} \sim \mathcal{O}(1)$  and  $m_{SUSY} \geq 1 \text{ TeV}$

$\tau \rightarrow l_j X$  ( $X = \gamma, \eta, f_0, l_j l_j (l_k l_k)$ )



$$\frac{BR(\tau \rightarrow 3\mu)}{BR(\tau \rightarrow \mu\nu\bar{\nu})} \simeq \left(\frac{\alpha_2}{48\pi}\right)^2 \left(\frac{m_\tau m_\mu}{M_H^2}\right)^2 \delta_{32}^2 t_\beta^6 \quad \frac{BR(\tau \rightarrow \mu\gamma)}{BR(\tau \rightarrow \mu\nu\bar{\nu})} \simeq \frac{\alpha_{el}}{20\pi} \frac{m_W^4}{\tilde{m}^4} \delta_{32}^2 t_\beta^2$$

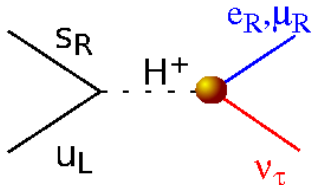
If  $t_\beta \sim 50$  and  $M_H \ll \tilde{m}$ , i.e.  $M_H \sim m_W$  and  $\tilde{m} \sim \text{TeV}$

$\Downarrow$

$$\frac{BR(\tau \rightarrow 3\mu)}{BR(\tau \rightarrow \mu\gamma)} \approx \alpha_{el}$$

# $\mu - e$ universality in $K \rightarrow l\nu$

$$R_K = (1 + \Delta r_K^{e-\mu}) = \frac{\sum_i K \rightarrow e\nu_i}{\sum_i K \rightarrow \mu\nu_i} \simeq \frac{\Gamma_{SM}(K \rightarrow e\nu_e) + \Gamma(K \rightarrow e\nu_\tau)}{\Gamma_{SM}(K \rightarrow \mu\nu_\mu)}$$



$$eH^\pm \nu_\tau \rightarrow \frac{g_2}{\sqrt{2}} \frac{m_\tau}{M_W} \Delta_R^{31} \tan^2 \beta$$

$$\Delta_R^{31} \sim \frac{\alpha_2}{4\pi} \delta_{RR}^{31}$$




$$\Delta_R^{31} \sim 5 \cdot 10^{-4} \quad t_\beta = 40 \quad M_{H^\pm} = 500 \text{ GeV}$$

$$\Delta r_{K \text{ SUSY}}^{e-\mu} \simeq \left( \frac{m_K^4}{M_{H^\pm}^4} \right) \left( \frac{m_\tau^2}{m_e^2} \right) |\Delta_R^{31}|^2 \tan^6 \beta \approx 10^{-2}$$

$$\Delta r_{K \text{ SUSY}}^{e-\mu} \approx 10^{-2} \implies Br^{th.(exp.)}(\tau \rightarrow eX) \leq 10^{-10(-7)}$$

Masiero, P.P., Petronzio, '05,'08

# “DNA-Flavour Test”

	GMSSM	AC	RVV2	AKM	$\delta$ LL	FBMSSM	
$S_{\phi K_S}$	★★★	★★★	●●	■	★★★	★★★	
$A_{CP}(B \rightarrow X_S \gamma)$	★★★	■	■	■	★★★	★★★	
$B \rightarrow K^{(*)} \nu \bar{\nu}$	●●	■	■	■	■	■	
$\tau \rightarrow \mu \gamma$	★★★	★★★	★★★	■	★★★	★★★	
$D^0 - \bar{D}^0$	★★★	★★★	■	■	■	■	
$A_{7,8}(B \rightarrow K^* \mu^+ \mu^-)$	★★★	■	■	■	★★★	★★★	
$A_9(B \rightarrow K^* \mu^+ \mu^-)$	★★★	■	■	■	■	■	
$S_{\psi \phi}$	★★★	★★★	★★★	★★★	■	■	
$B_s \rightarrow \mu^+ \mu^-$	★★★	★★★	★★★	★★★	★★★	★★★	
$\epsilon_K$	★★★	■	★★★	★★★	■	■	
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	★★★	■	■	■	■	■	
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	★★★	■	■	■	■	■	
$\mu \rightarrow e \gamma$	★★★	★★★	★★★	★★★	★★★	★★★	
$\mu + N \rightarrow e + N$	★★★	★★★	★★★	★★★	★★★	★★★	
$d_n$	★★★	★★★	★★★	★★★	●●	★★★	
$d_e$	★★★	★★★	★★★	●●	■	★★★	
$(g-2)_\mu$	★★★	★★★	★★★	●●	★★★	★★★	

Altmannshofer et al. '09

## ► Flavour physics in the LHC era

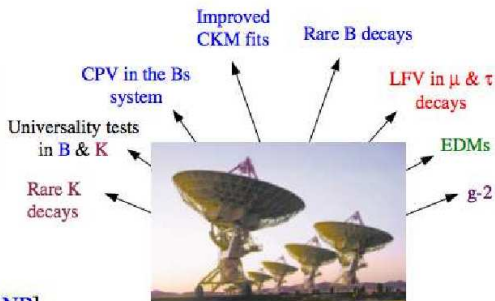
### LHC [high $p_T$ ]

A *unique* effort toward the high-energy frontier



[to determine the energy scale of NP]

### Flavour physics



A *collective* effort toward the high-intensity frontier

[to determine the flavour structure of NP]

# Murayama's view

