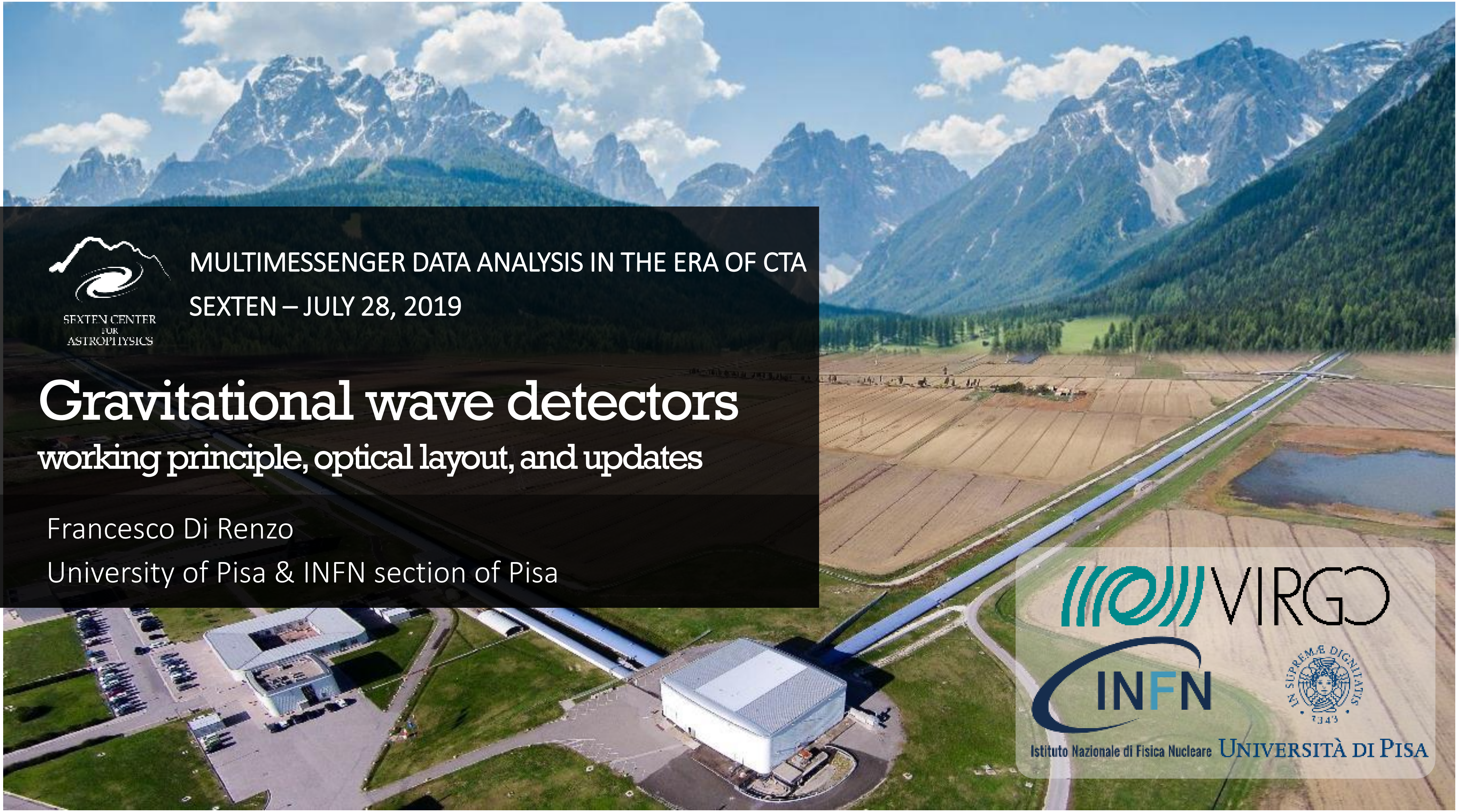


MULTIMESSENGER DATA ANALYSIS IN THE ERA OF CTA

SEXTEN – JULY 28, 2019

Gravitational wave detectors

working principle, optical layout, and updates

Francesco Di Renzo

University of Pisa & INFN section of Pisa

Plan of the presentation

1. Working principle of interferometric gravitational wave detectors:
   - “Spacetime tells matter how to move, matter tells spacetime how to curve”;
   - The “rubber ruler” puzzle;
   - Cosmological redshift of light.

2. From a simple Michelson interferometer to modern GW detectors:
   - (Simplified) optical layout and readout;
   - (Main) noise sources;
   - Sensitivity: design vs. actual.

3. Updates from the Advanced Virgo and Advanced LIGO O3 science run.

The Einstein Equations

or "spacetime tells matter how to move, matter tells spacetime how to curve". John A. Wheeler.

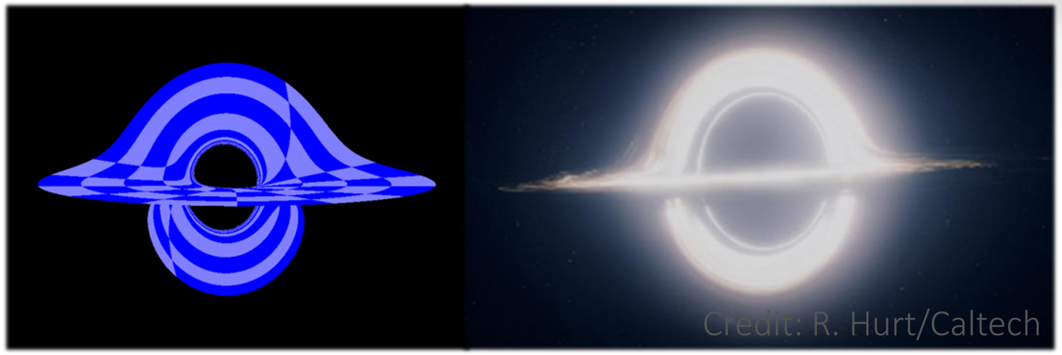
In general relativity gravity is described by the curvature of spacetime [Einstein 1915]:

Curvature of spacetime

Distribution of mass/energy

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Some constants



Linearized theory: $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$. GR admits propagating, wave-like solutions [Einstein 1916]. In *Lorentz gauge*:

D'Alambertian (wave operator)

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} + \mathcal{O}(h^2)$$

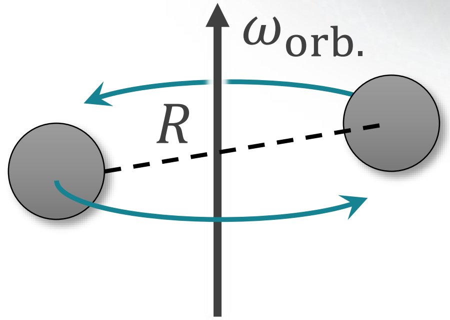
Gravitational Waves (GW): *ripples* in spacetime [Kip Thorne], caused by accelerating masses with non-constant quadrupole moment, which propagates in spacetime at speed c.

Production and free propagation of GWs

Production: accelerating mass densities:

$$h_{\mu\nu} \approx \frac{2}{r}\frac{G}{c^4}\ddot{I}_{\mu\nu}$$

Second derivative of quadrupole moment



E.g.: for an orbiting binary system $I \approx MR^2, \ddot{I} \approx 4MR^2\omega^2_{\mathrm{orb.}}$,

$$h \approx \frac{8G}{c^4}\frac{M}{r}R^2\omega^2_{\mathrm{orb.}} \approx \frac{8G}{c^2}\frac{M}{r}\left(\frac{v}{c}\right)^2 \sim \mathbf{10^{-21}}$$

Free propagation along "z-axis" in vacuum ($T_{\mu\nu} = 0$):

$$\Box h_{\mu\nu} = 0 \implies h_{\mu\nu}(t,z) = h_{\mu\nu}e^{i(kz-\omega t)} \quad \text{with} \quad \omega/c = k$$

Credit:Teviet Creighton

Interaction of GWs with test masses

and the “rubber ruler” puzzle. Peter R. Saulson.

Customary picture in the **detector reference frame**:

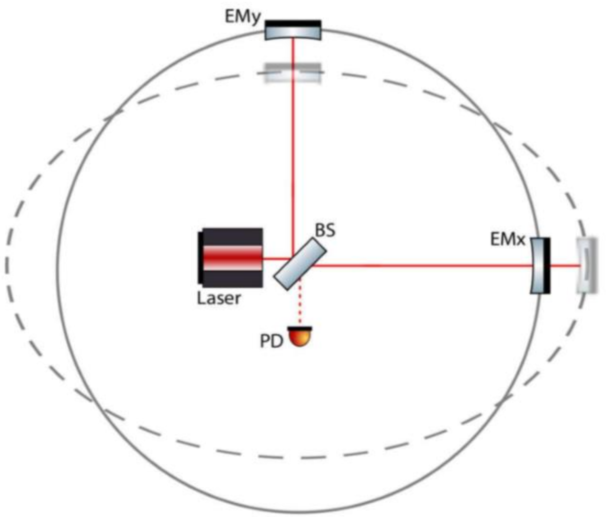
a circular array of test masses responds to an incoming GW (perpendicular to the screen):

The rubber ruler paradox: the arms of an interferometer are lengthened by a gravitational wave. The wavelength of the light in an interferometer is also lengthened by a gravitational wave, by *the same factor*.

So, how can we use light as a ruler to detect gravitational waves?

Are we using a “rubber ruler” that participates in the same distortions as the system whose distortions we are trying to measure?

Peter R. Saulson



GW interaction in the TT frame

or how to exploit gauge freedom in our favor.

Customary picture to represent GW propagating degrees of freedom:

$$h_{ij}^{\mathrm{TT}}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos[\omega(t - z/c)]$$

In this gauge, GWs are transverse and traceless (TT): h_+ and $h_\times$ d.o.f.

In this frame, objects initially at rest **remain at rest**, even after the arrival of the wave. That is, they are "free falling":

Geodesic equation →

$$\left.\frac{d^2x^i}{d\tau^2}\right|_{\tau=0} = -\left.\left[\Gamma^i_{\nu\rho}(x)\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau}\right]\right|_{\tau=0} = \left.\left[\Gamma^i_{00}\left(\frac{dx^0}{d\tau}\right)^2\right]\right|_{\tau=0} = 0$$

Because, in this gauge, the Christoffel symbol $\Gamma^i_{00} = \frac{1}{2}(2\partial_0 h_{0i} - \partial_i h_{00}) = 0$. [Maggiore 2008]

Only a non-gravitational force can cause a mass to move, *i.e.*to change its coordinates.

Light travel time through the detector (TT gauge)

Suspended (test) masses can be considered free in the horizontal plane ($z = 0$).

We study in the TT gauge the propagation of light between them. Along one arm, the effect of a $+$ wave:

$$ds^2 = -c^2 dt^2 + [1 + h_+] dx^2 = 0$$

Light-like interval

$$\int dt = \frac{1}{c} \int \sqrt{1 + h_+}\, dx \approx \frac{1}{c} \int 1 + \frac{1}{2} h_+ dx$$

$$\Delta t \approx h_+ L / 2c$$

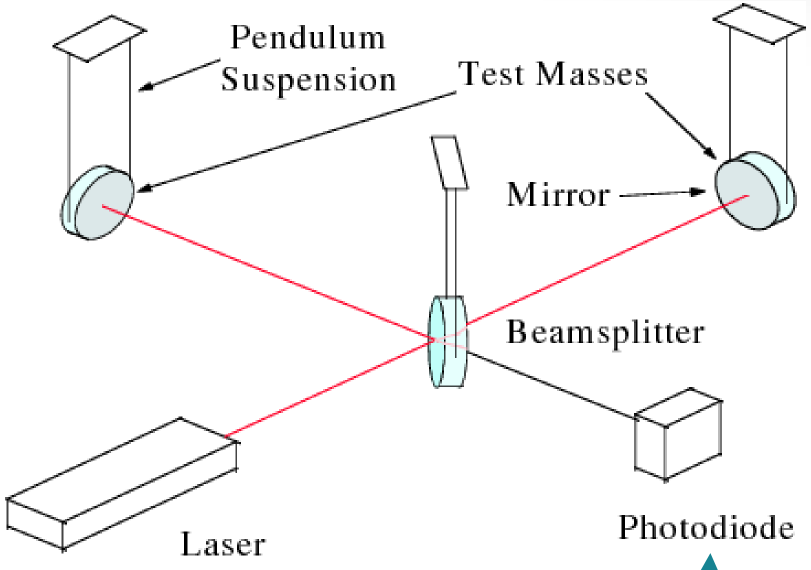
Round trip back to beam-splitter:

Opposite sign along y

$$\Delta t_x \approx h_+ L / c \qquad (\Delta t_y \approx -h_+ L / c\,)$$

Difference between x and y round-trip times:

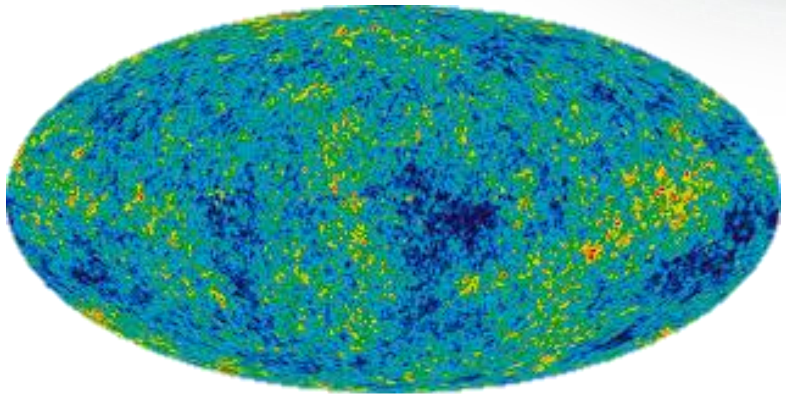
$$\boldsymbol{\Delta\tau \approx 2 h_+ L / c \qquad (\Delta\phi \approx 2 h_+ L\, 2\pi / \lambda_{\mathrm{L}})}$$



Cosmological redshift and comoving coordinates

What have in common GW detectors and the Hubble law.

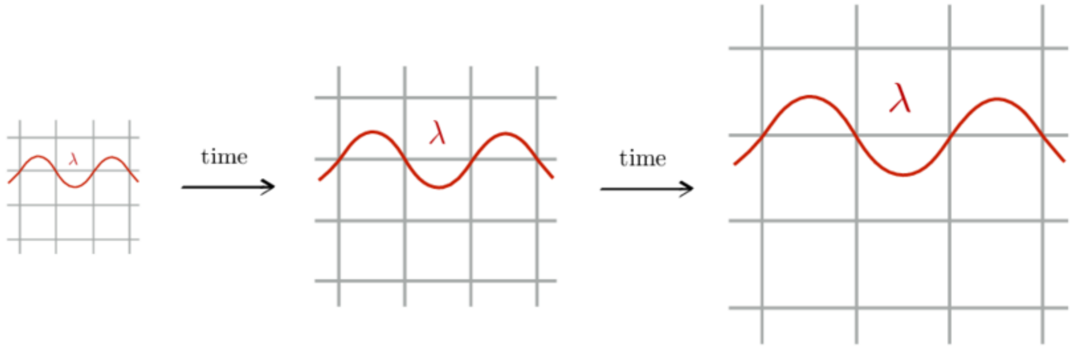
Comoving coordinates: assign constant spatial coordinate values to observers who perceive the universe as homogeneous and isotropic. Such observers "comove" with the Hubble flow.



$$ds^2 = -c^2 dt^2 + R^2(t)[dx^2 + dy^2 + dz^2]$$

where $R(t)$ is the *cosmic scale factor.*

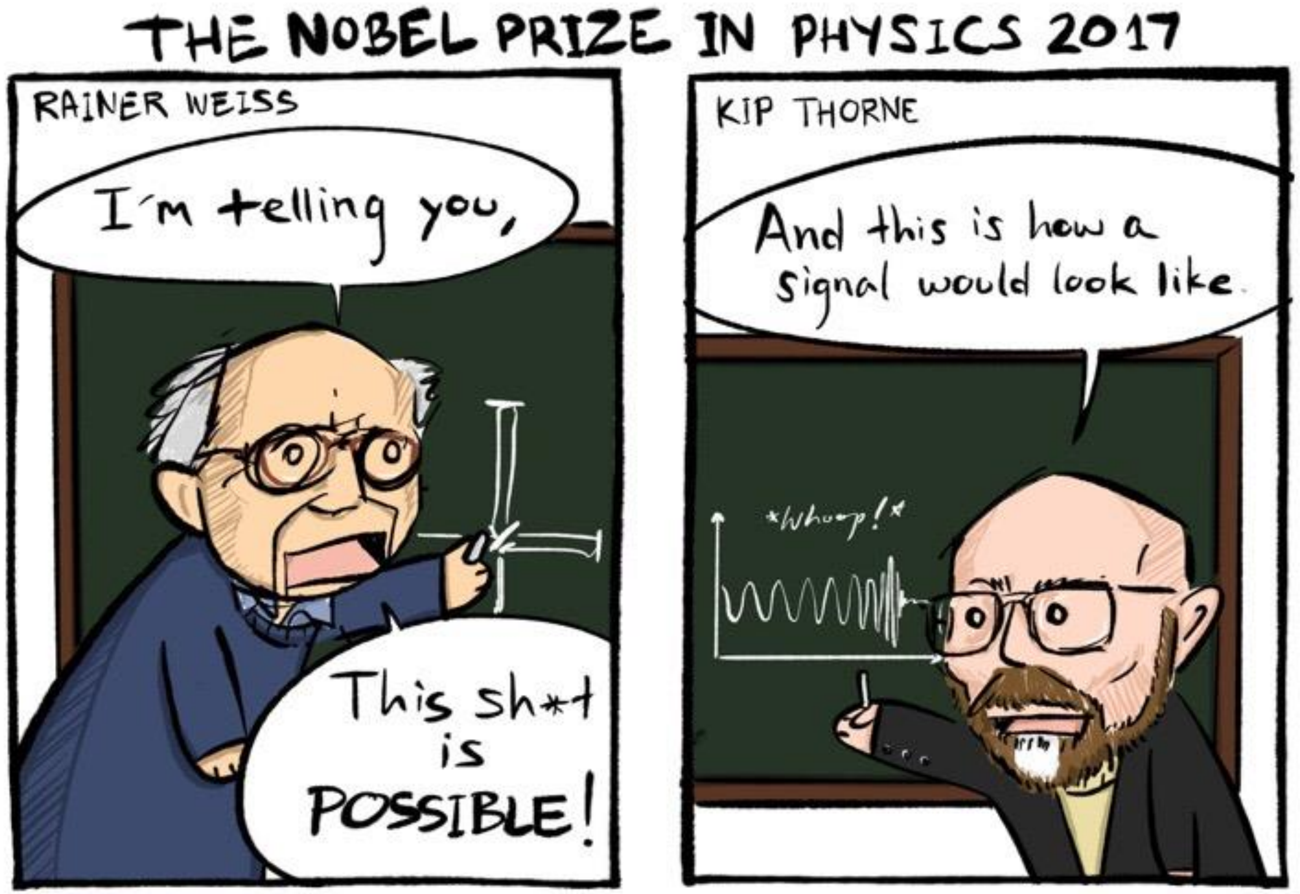
In an expanding Universe, $R(t)$ is an increasing function of time. Light emitted with wavelength λ_0 when the cosmic scale factor was $R(t_0)$ is transformed by the cosmic expansion into light we receive λ_1 when the cosmic scale factor has grown to $R(t_1)$:



$$\frac{\lambda_1}{\lambda_0} = \frac{R(t_1)}{R(t_0)} = 1 + z$$

Cosmological redshift of light

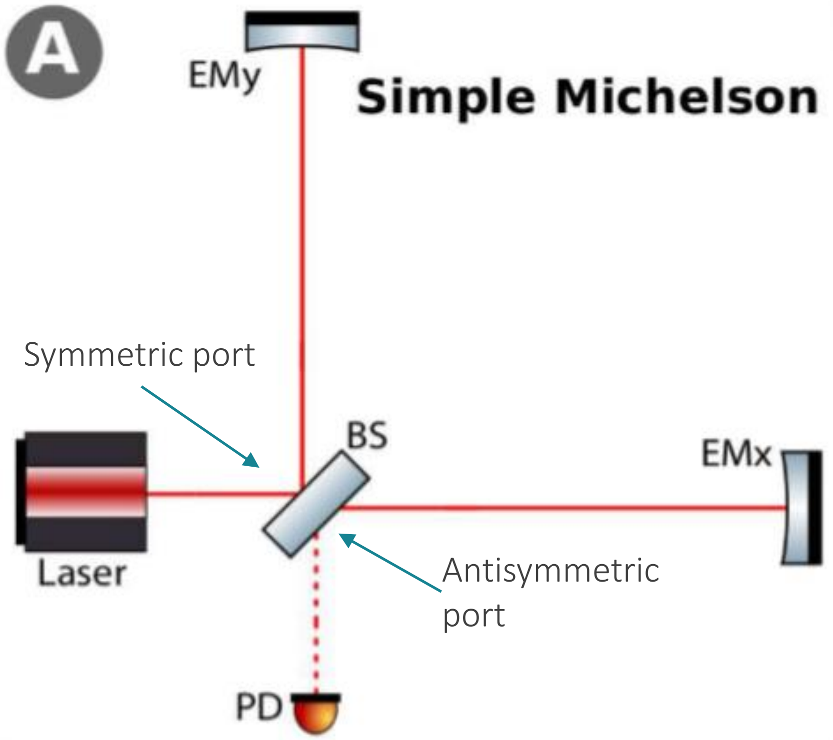
Antimatter web comics break



antimatterwebcomics.com

GW detectors: a simple Michelson interferometer

1. Laser light is split in two orthogonal paths by a beam-splitter (BS). These are reflected back toward the BS by mirrors at the ends of the arms;
2. When recombined at the BS, **they interfere**. If the arms are exactly the same length (up to $n\lambda_L$), the light returns entirely toward the laser: destructive interference;
3. If the arms differ by an amount that is not an integer number of wavelengths, then the destructive interference will be incomplete: light passes to the photodiode (PD)



Estimated sensitivity (I):

Nd:YAG laser: $\lambda_L = 1064$ nm

$$h \approx \frac{\Delta L}{L} \sim \frac{\lambda_L}{L} \sim 10^{-9}$$

$L \sim 1$ km

GW detectors: a simple Michelson interferometer

Estimated sensitivity (II):

Use photons instead of lengths. The rate at which photons arrive to the PD follows a Poisson statistic:

$$\Delta L \approx \frac{N_{\mathrm{ph.}}^{1/2}}{N_{\mathrm{ph.}}} \lambda_L$$

Number fluctuation

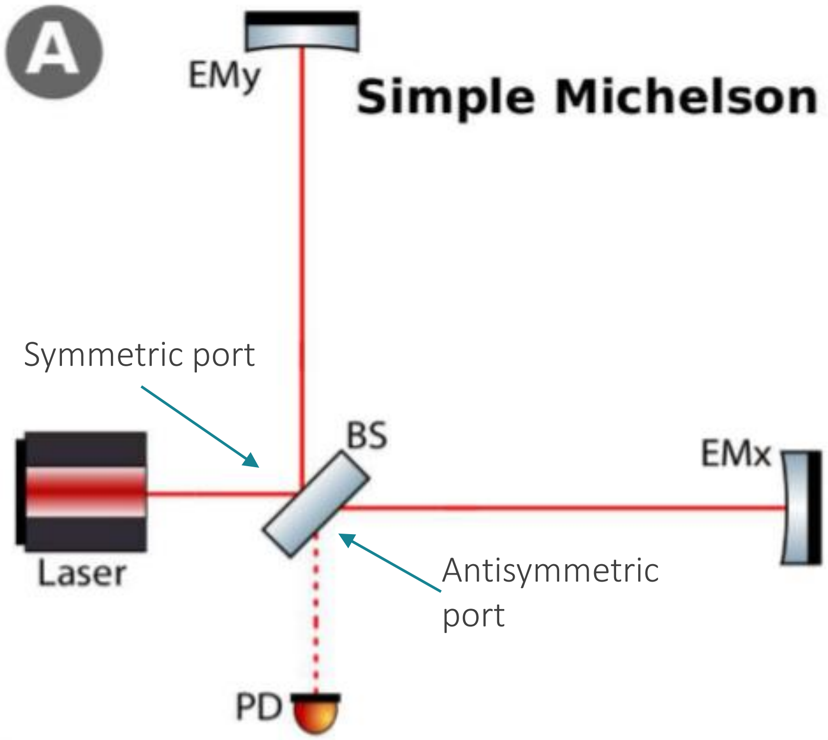
Readout time

where:

$$N_{\mathrm{ph.}} = \frac{P_L}{hc\lambda_L} \tau \sim \frac{P_L}{hc\lambda_L} \frac{1}{f_{\mathrm{gw}}}$$

For $P_L = 1$ W, $f_{\mathrm{gw}} = 300$ Hz:

$$h \approx \frac{\Delta L}{L} \sim \frac{N_{\mathrm{ph.}}^{-1/2} \lambda_L}{L} \sim 10^{-17}$$



GW detectors: a Fabry-Pérot Mich. interferometer

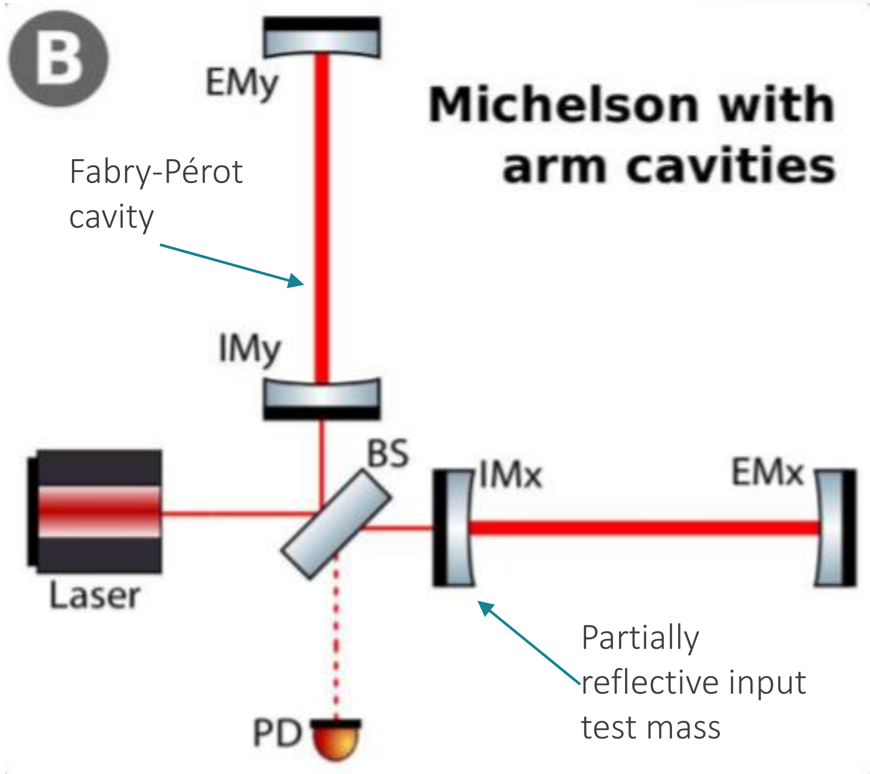
A **Fabry-Pérot interferometer** is comprised of two mirrors along an optical axis which form a cavity;

1. If the separation between the two mirrors is correctly tuned, there is a build-up of light power;
2. **In TT frame:** if the light is "detuned" by a GW, a huge amount of power leaks out of the cavity;
3. **In detector frame:** light bounces back and forth a huge number of times, increasing the "lengthening" effect of the GW on detector arms.

Estimated sensitivity:

$$h \approx \frac{\Delta L}{\lambda_{\mathrm{gw}}} \sim \frac{N_{\mathrm{ph.}}^{-1/2}\lambda_L}{\lambda_{\mathrm{gw}}} \sim 10^{-20}$$

Max effective length

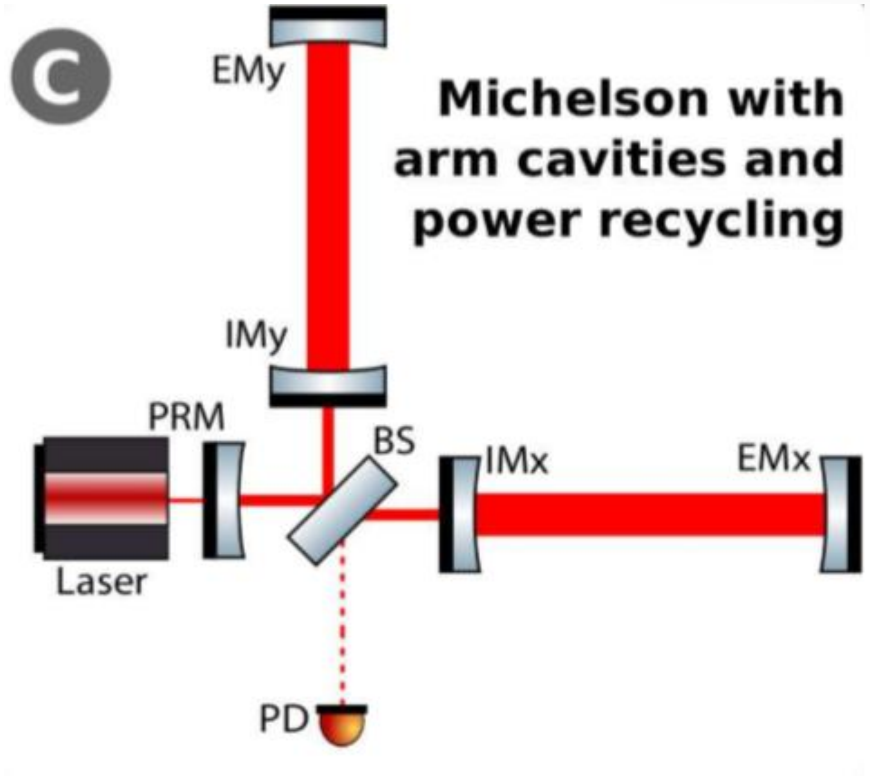


GW detectors: power recycled F.-P. Mich. interf.

Power recycling: a mirror (PRM) is added between the laser and the beam splitter, which creates an effective cavity between the recycling mirror and the compound mirror representing the Fabry-Pérot Michelson interferometer.

By tuning this cavity for a power build-up, we can increase the light power stored into the compound interferometer interferometer (up to ~MW in the arms).

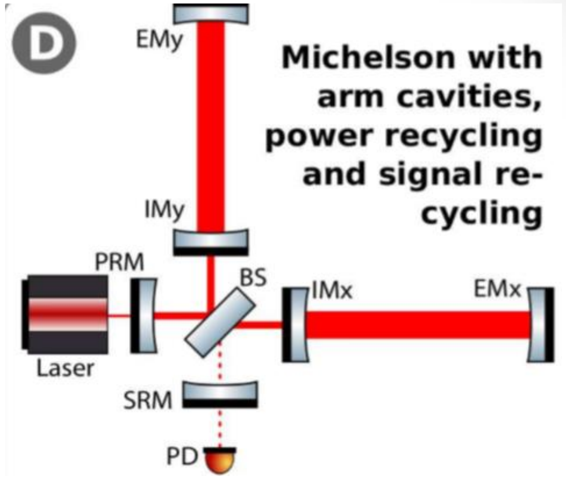
This reduces the shot noise but increases the radiation pressure noise…

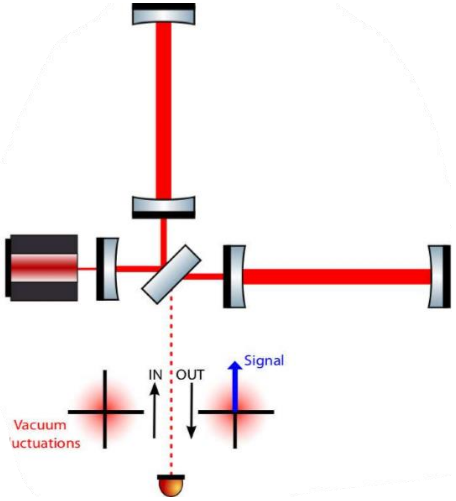


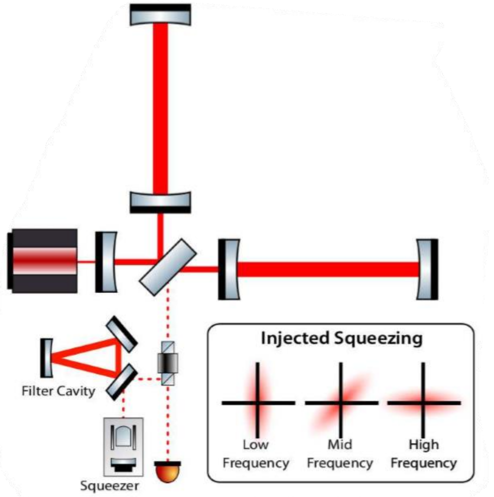
GW detectors: dual recycled F.-P. Mich. interf. w. SQZ

Signal recycling: inserting a signal recycling mirror (SRM) between the beam splitter and the output port, we can recycle the *signal sidebands* leaving the interferometer towards the output port. This will increase the sensitivity at particular frequencies.

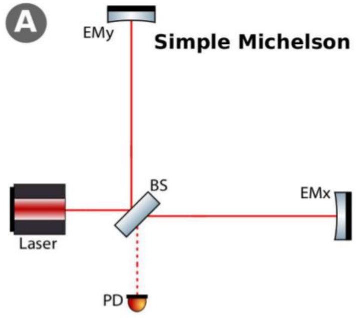
Squeezed light: quantum squeezing of the light can be used to reduce the effect of the shot noise and improve sensitivity.

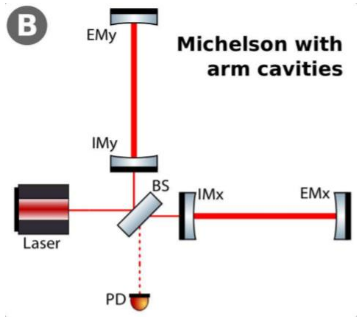


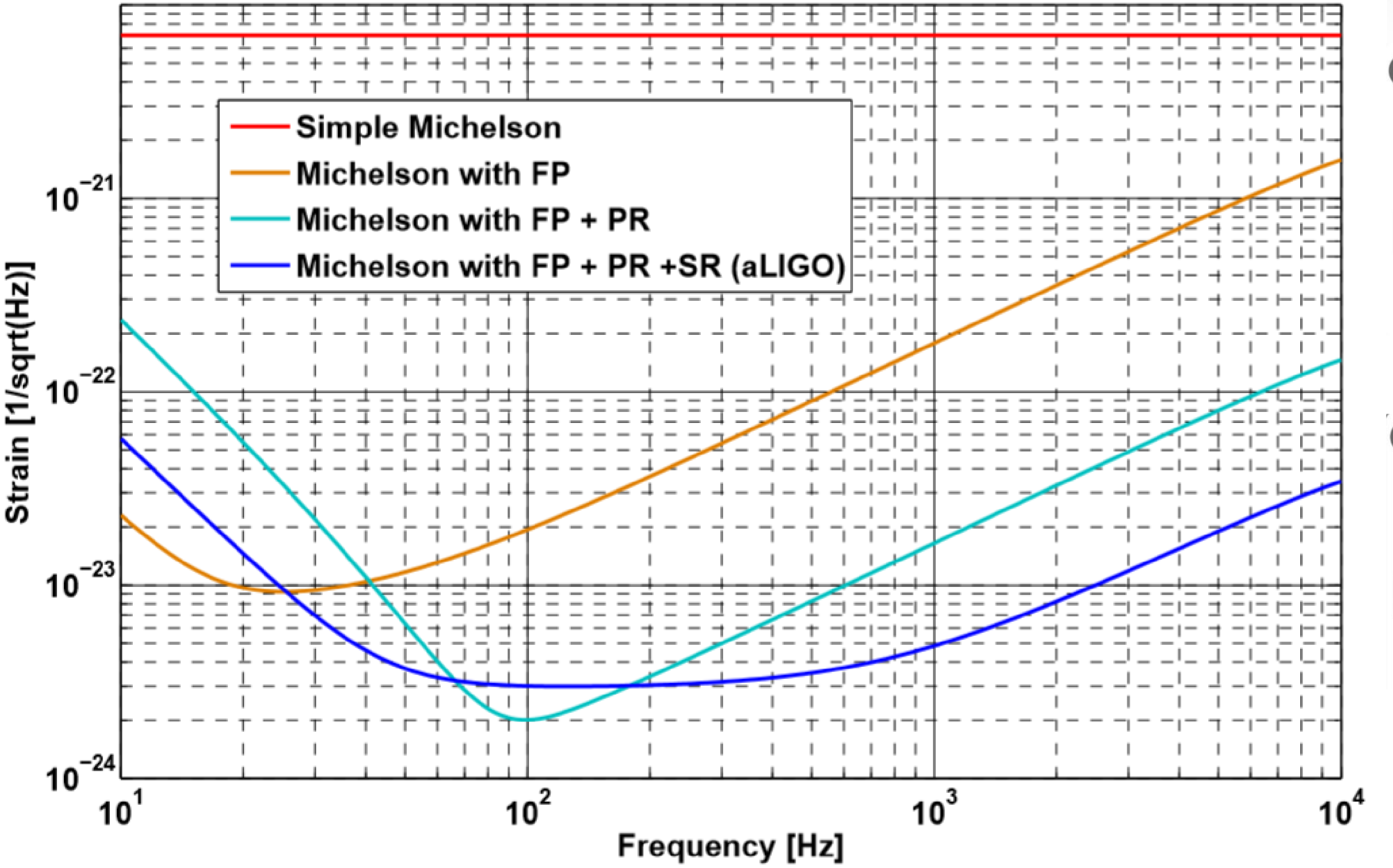


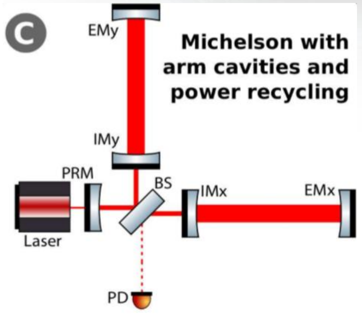


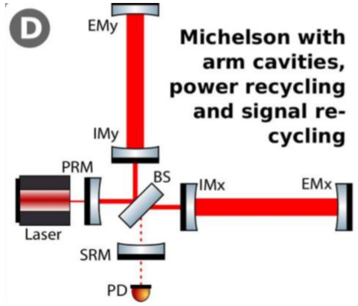
GW detectors: optical layout recap



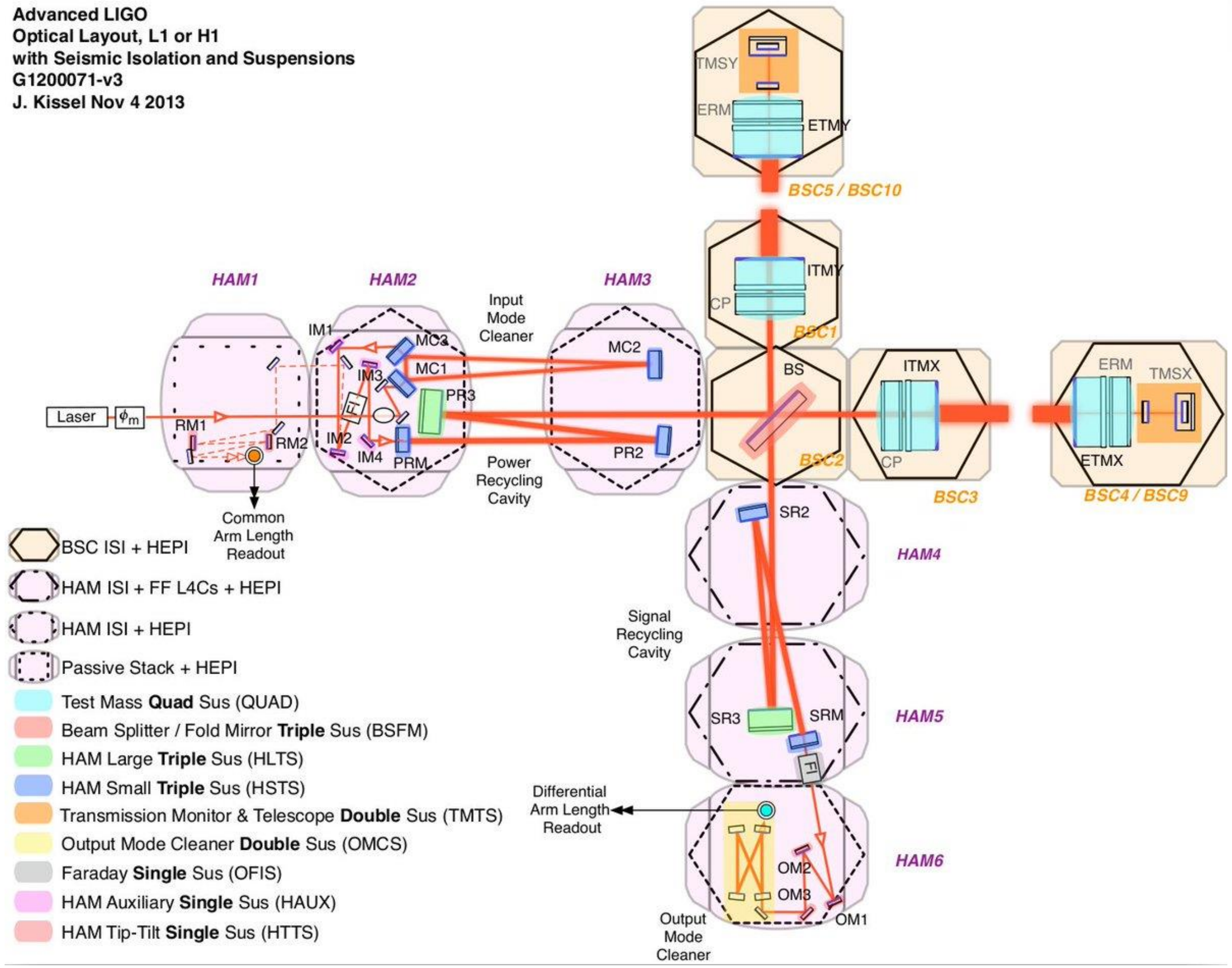




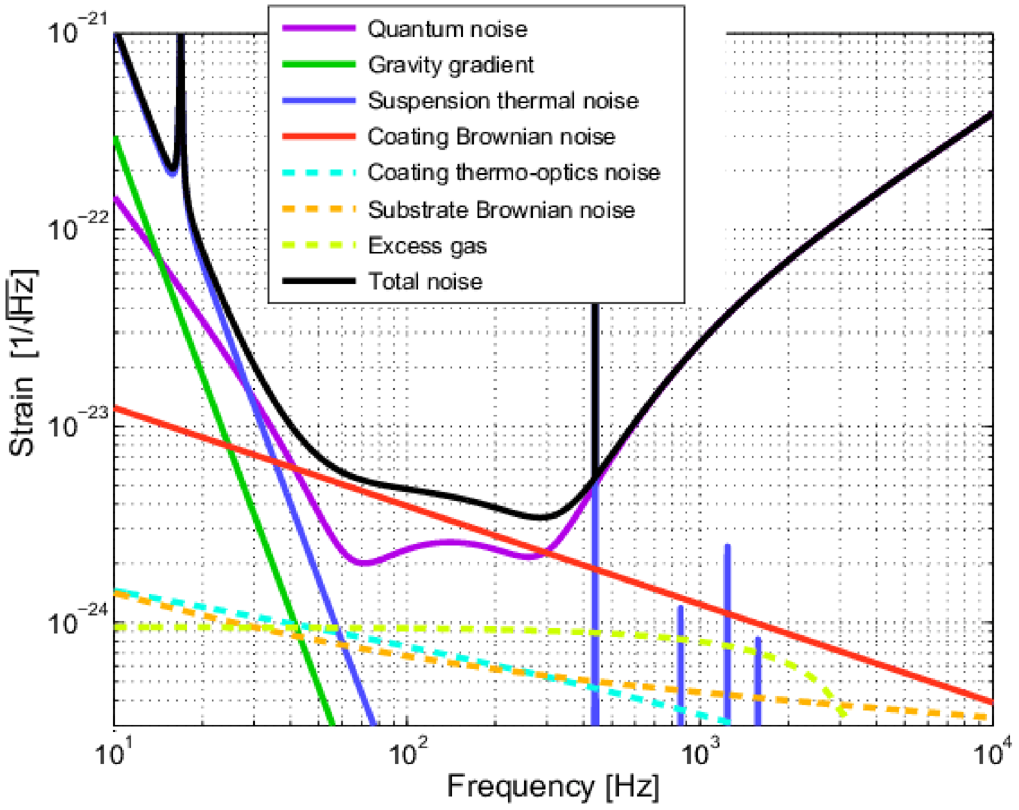


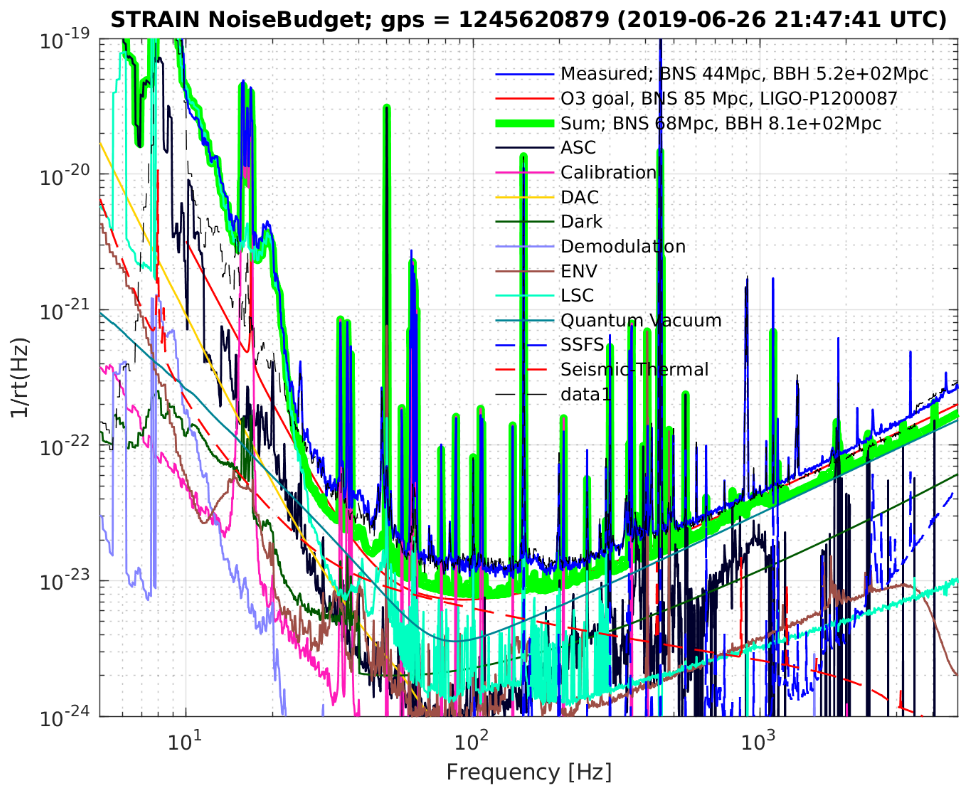


Optical layout in reality…

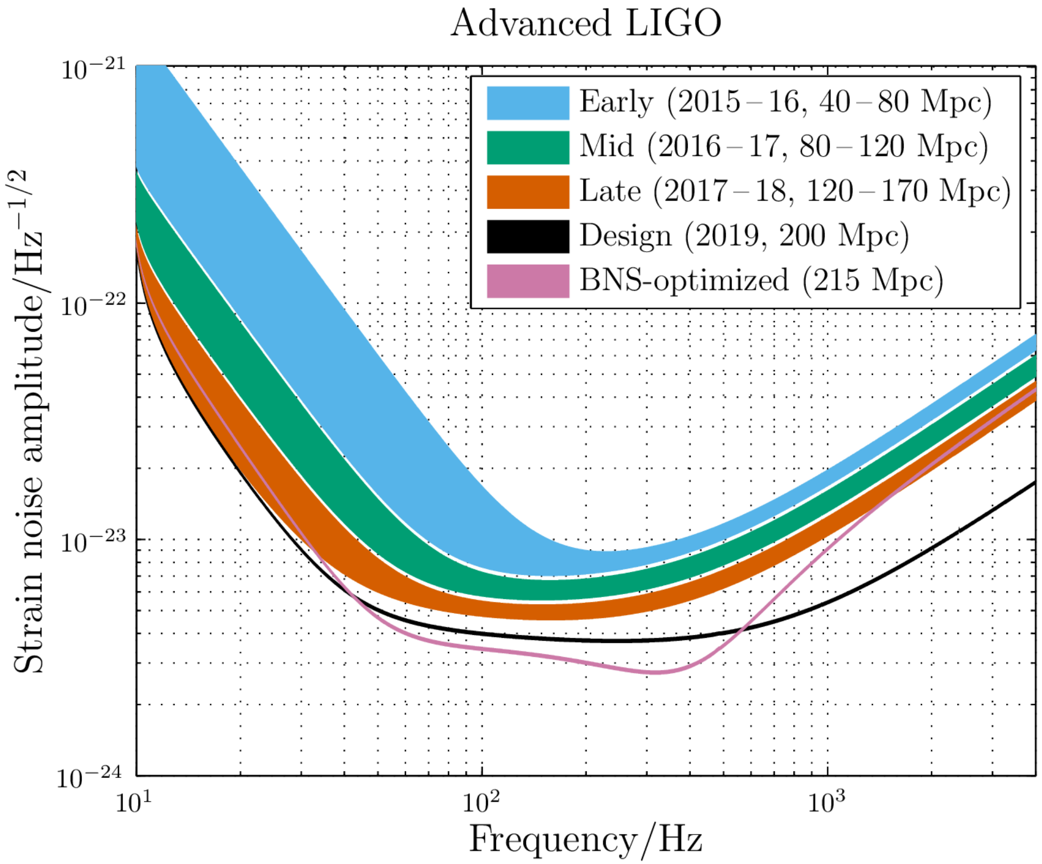


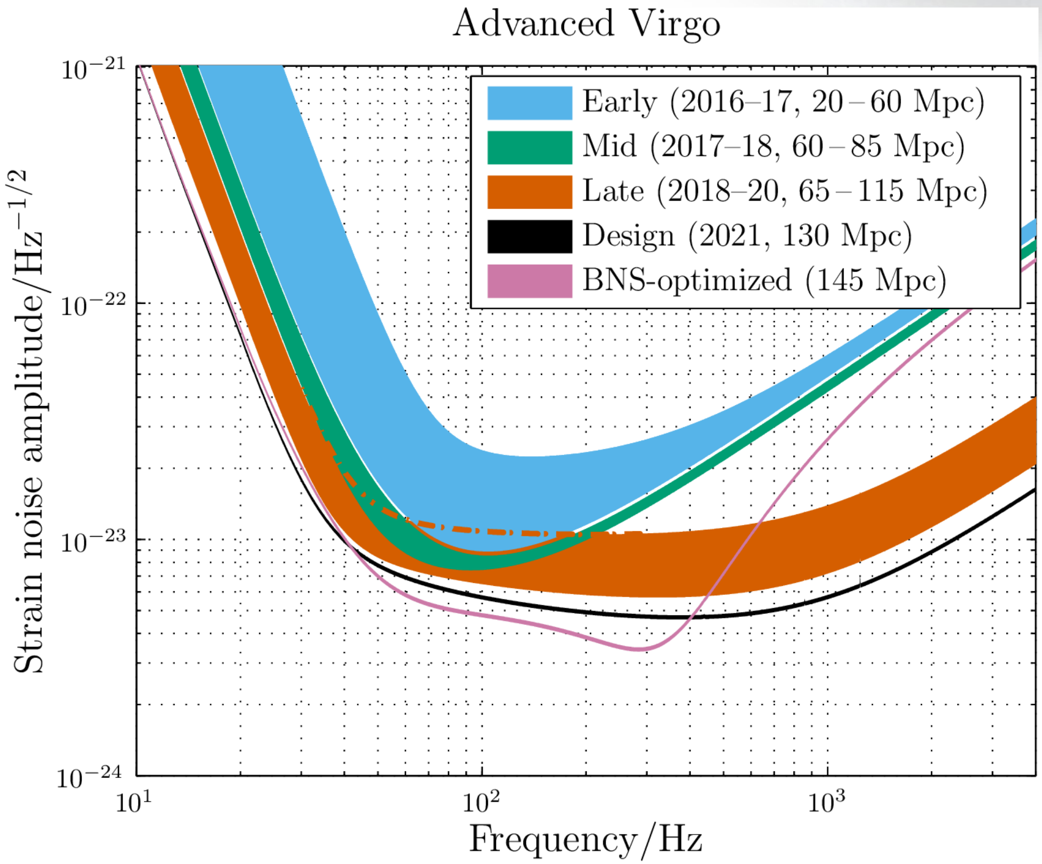
Noise budget: fundamental vs. actual





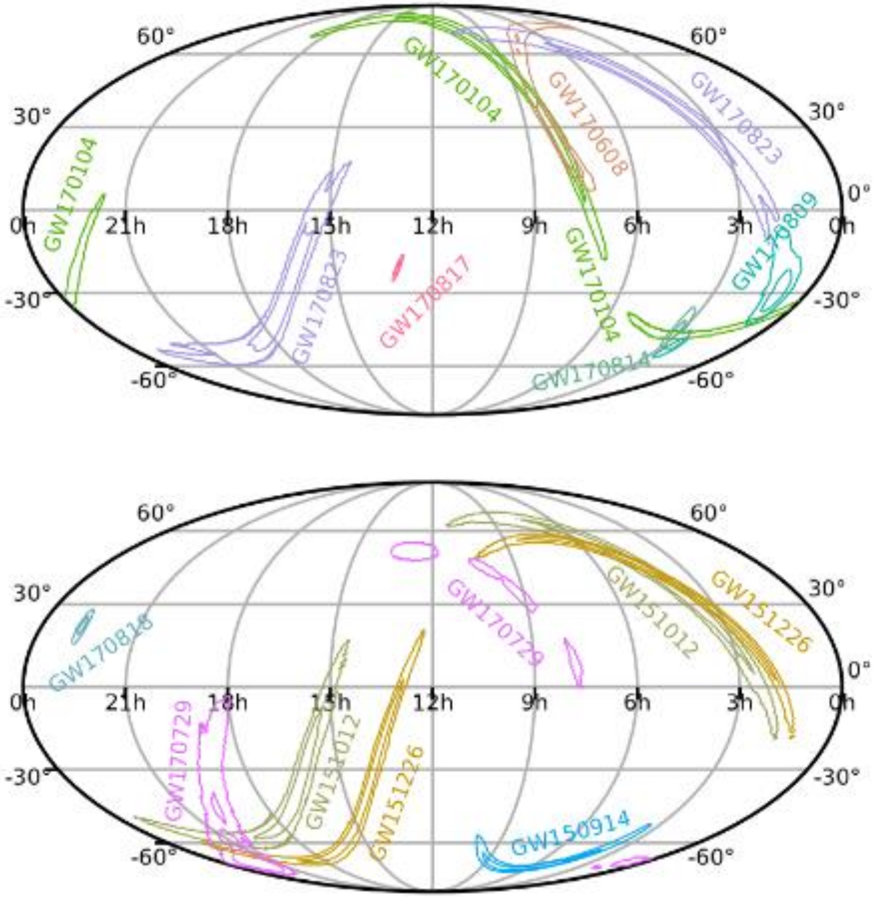
Scheduled sensitivity upgrades

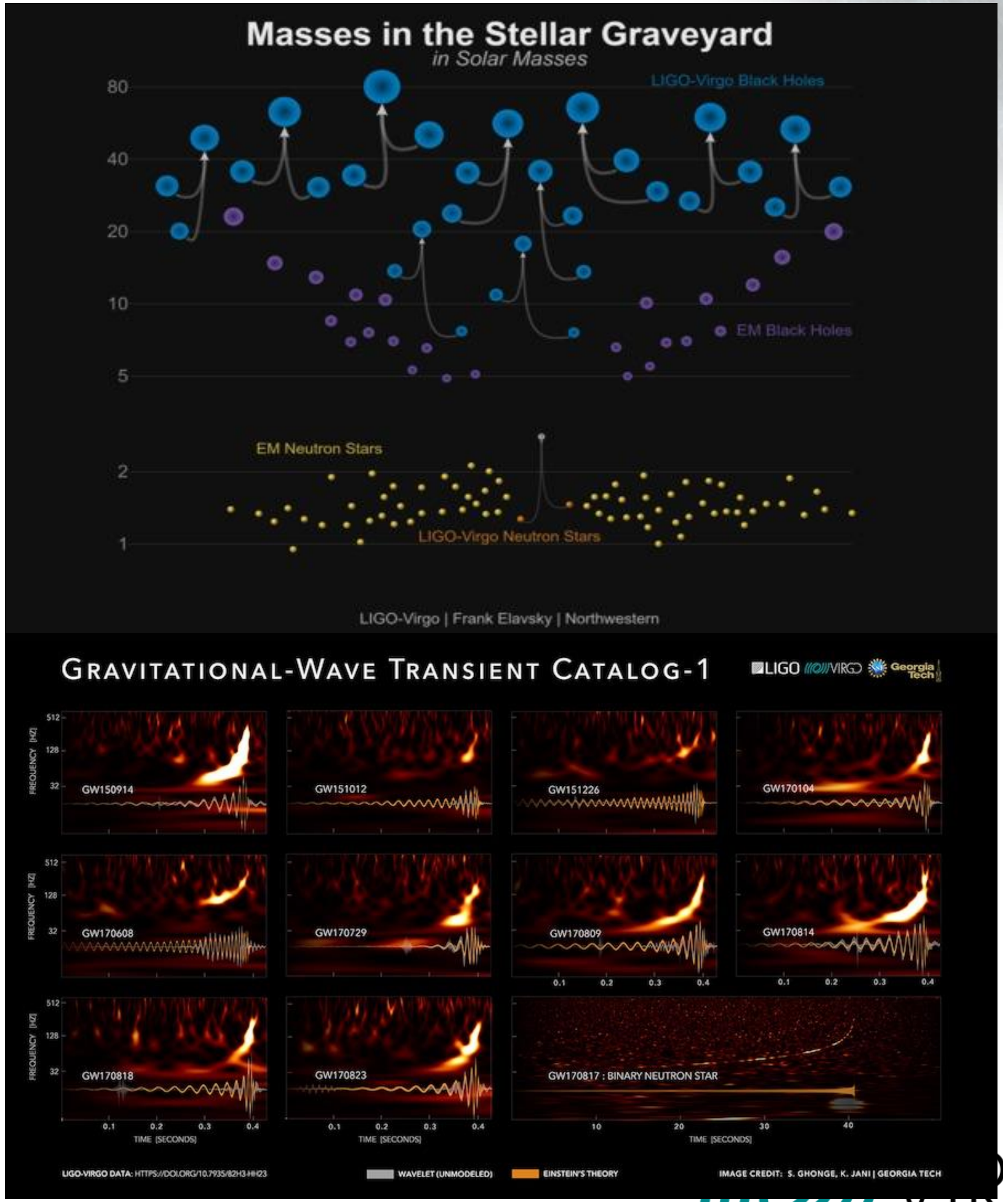




Aasi et al. 2013

Summary of GWTC-1



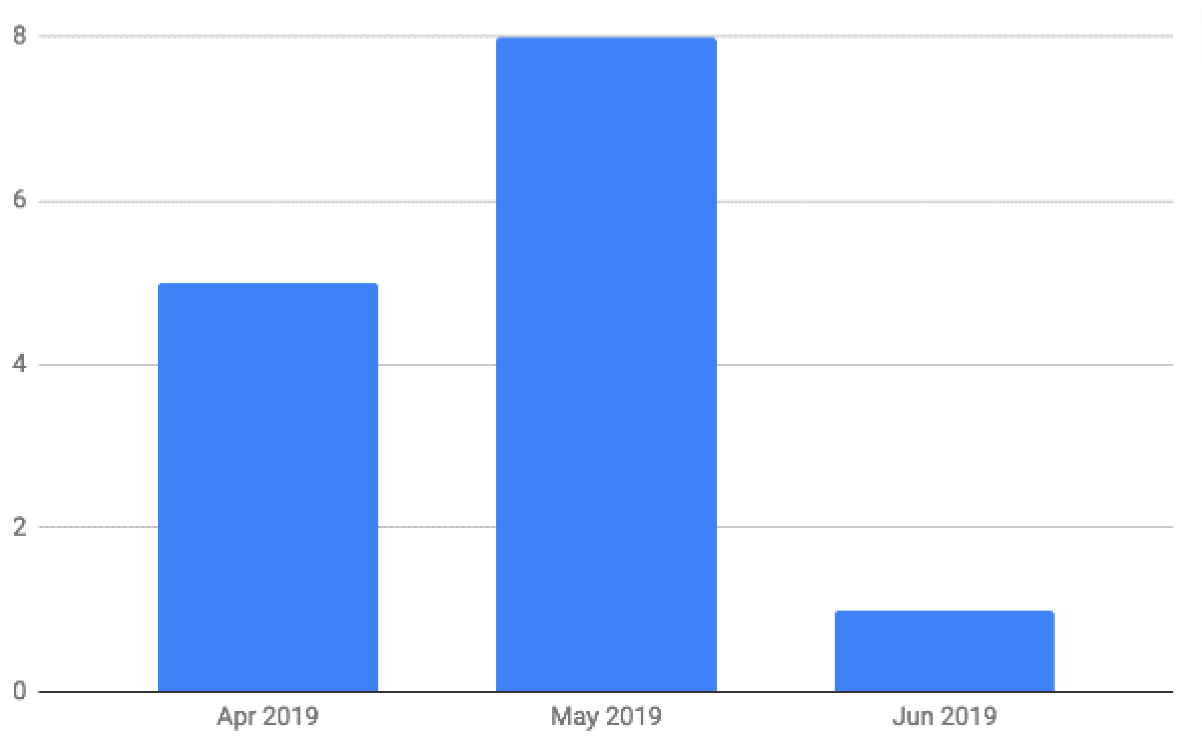


GWTC-1: https://arxiv.org/pdf/1811.12907.pdf

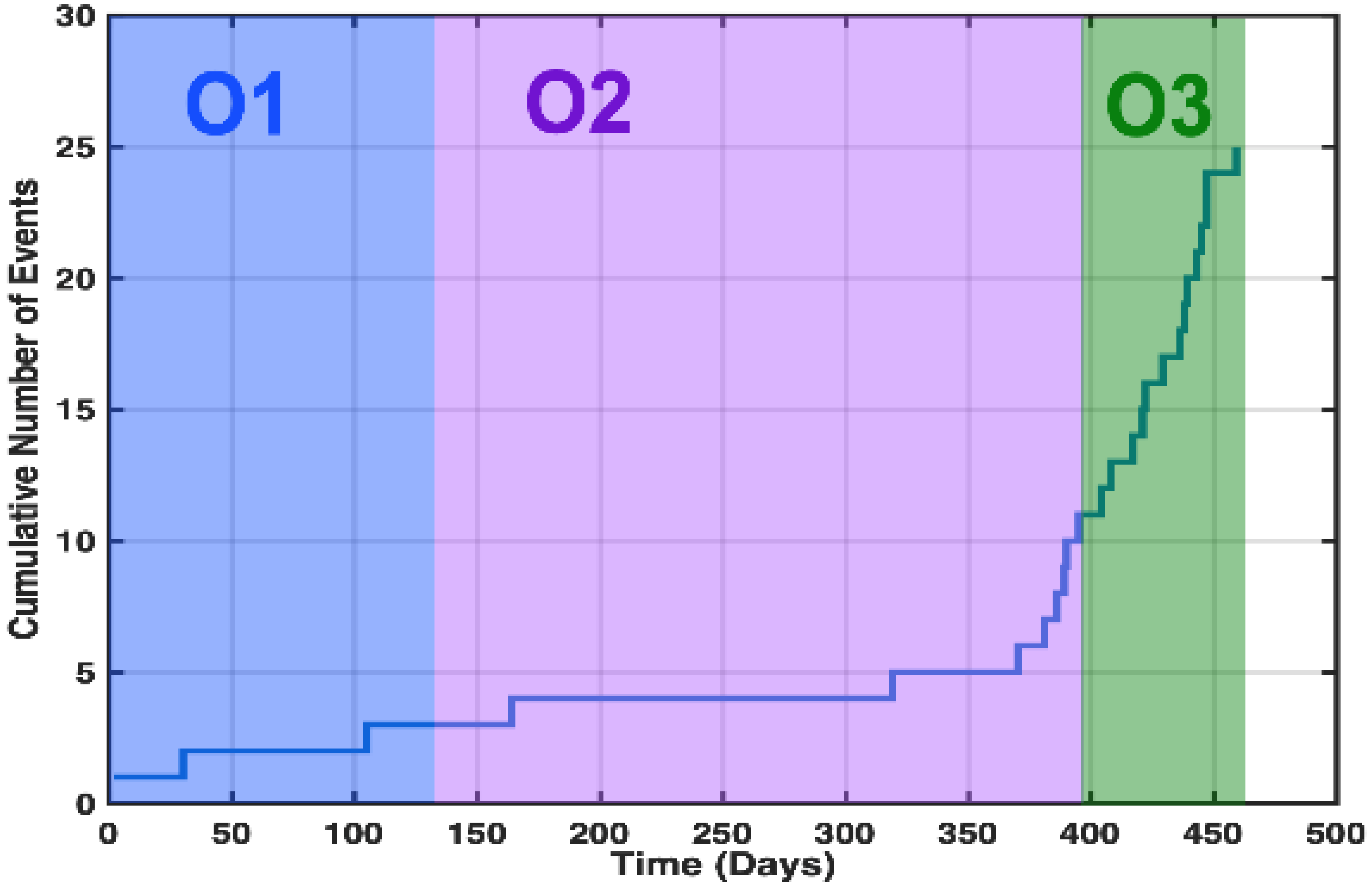
Pop: https://arxiv.org/pdf/1811.12940.pdf

Alerts sent during O3

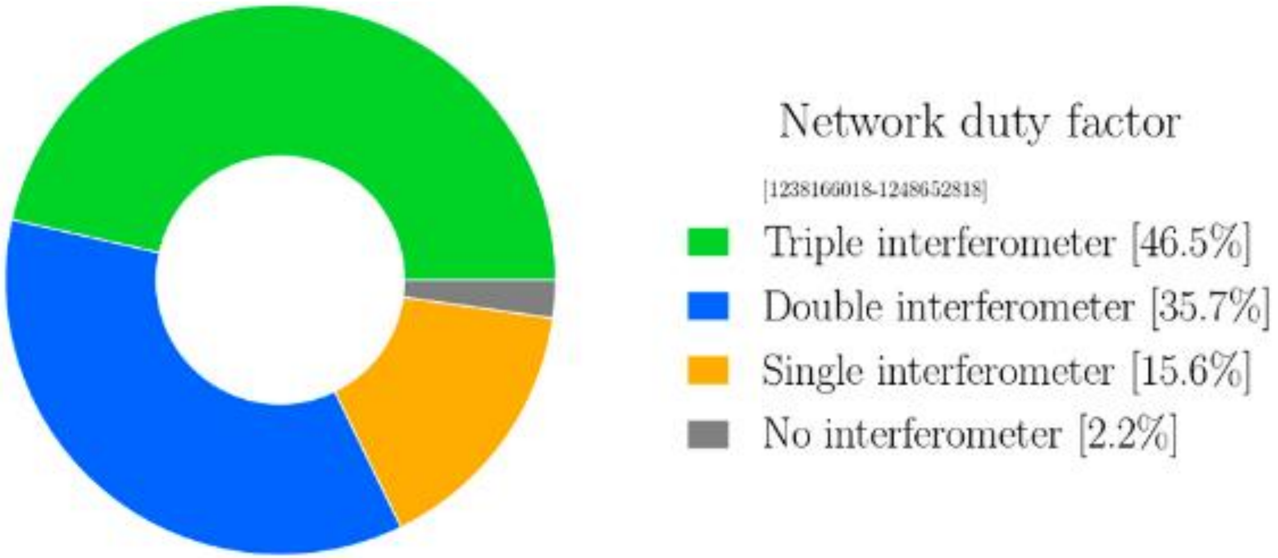
- S190524q (retracted due to non-stationary noise in the highest sensitivity detector L1)
- S190602aq, the last potential BBH

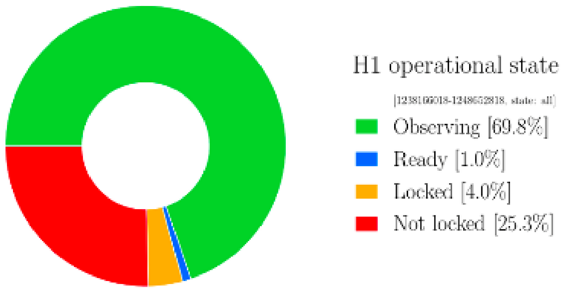


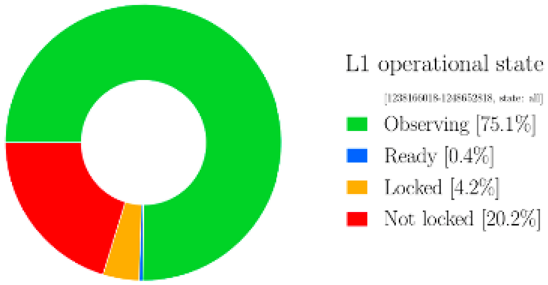
Number of O3 and O2+O1 detections vs run time

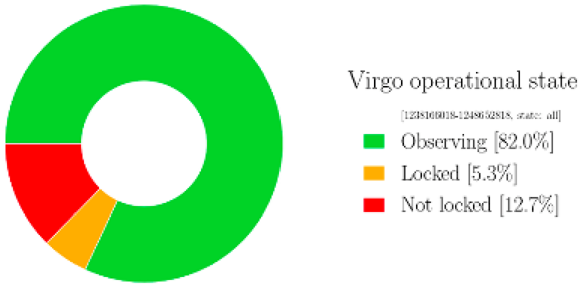


Detector Performance: O3 Cumulative Duty Factor

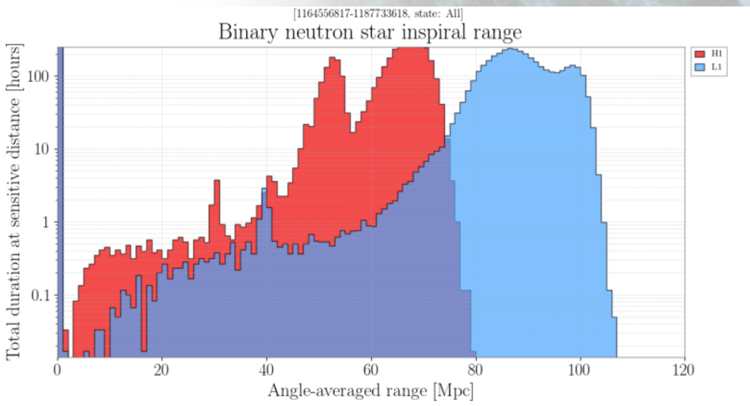






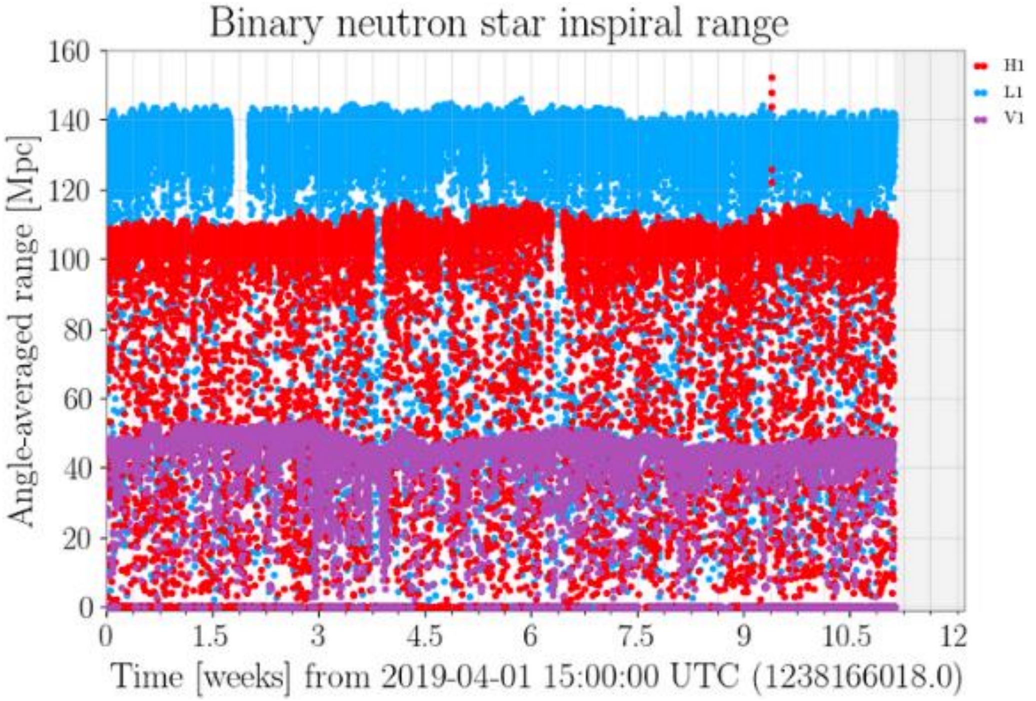


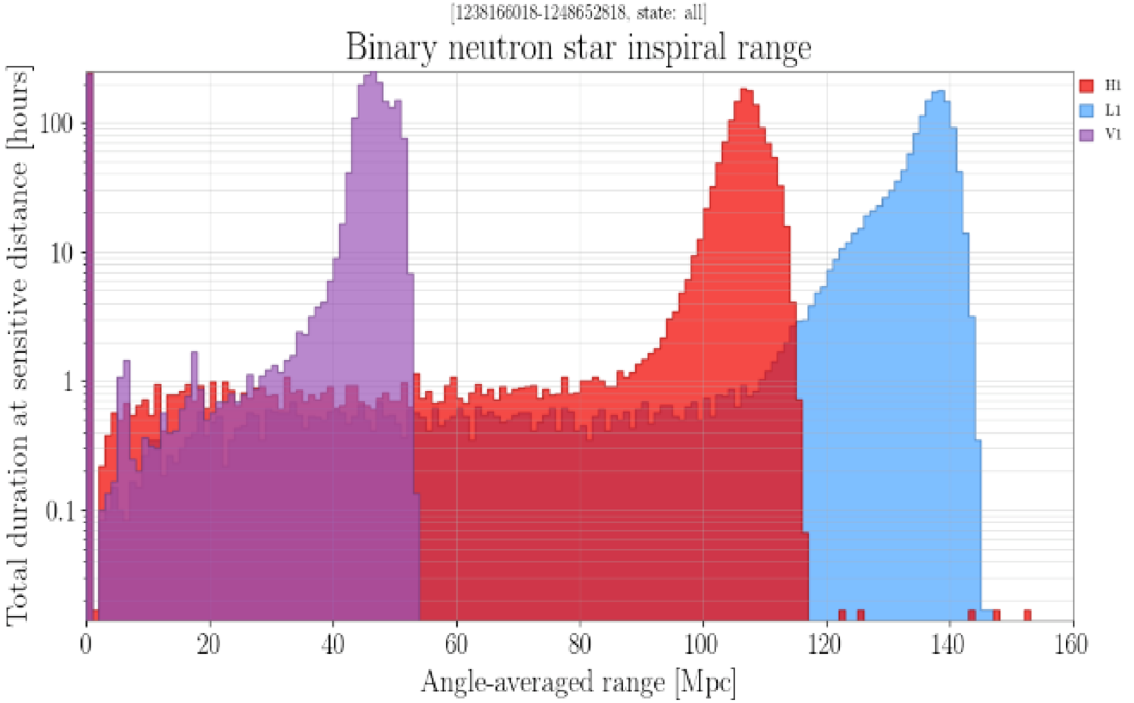
Detector Performance: BNS range



BNS range [Bassan 2014]:

$$\frac{d_{\text{range}}}{1\ \text{Mpc}} = 0.86 \times 10^{-20} \left(\frac{\mathcal{M}}{M_{\odot}}\right)^{5/6} \sqrt{\int_{f_{\min}}^{f_{\text{ISCO}}} \frac{f^{-7/3}}{|h(f)|^2}\, df}$$





Thank you for the attention

SEXTEN CENTER FOR ASTROPHYSICS

Francesco Di Renzo
University of Pisa & INFN section of Pisa
francesco.direnzo@pi.infn.it
www.pi.infn.it/~fdirenzo

Image credit: https://www.tre-cime.info/it/sesto.html

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