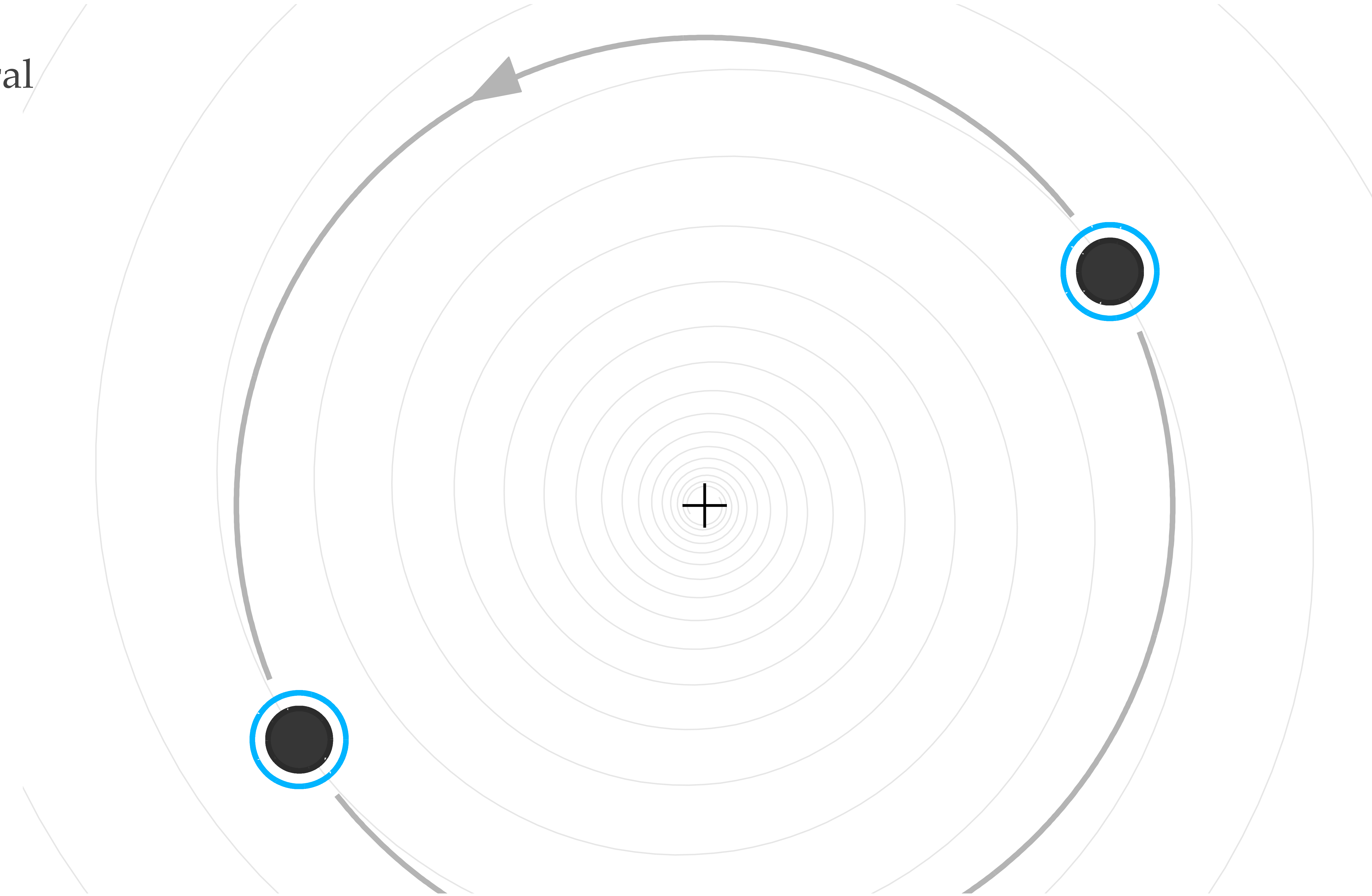
Inspiral



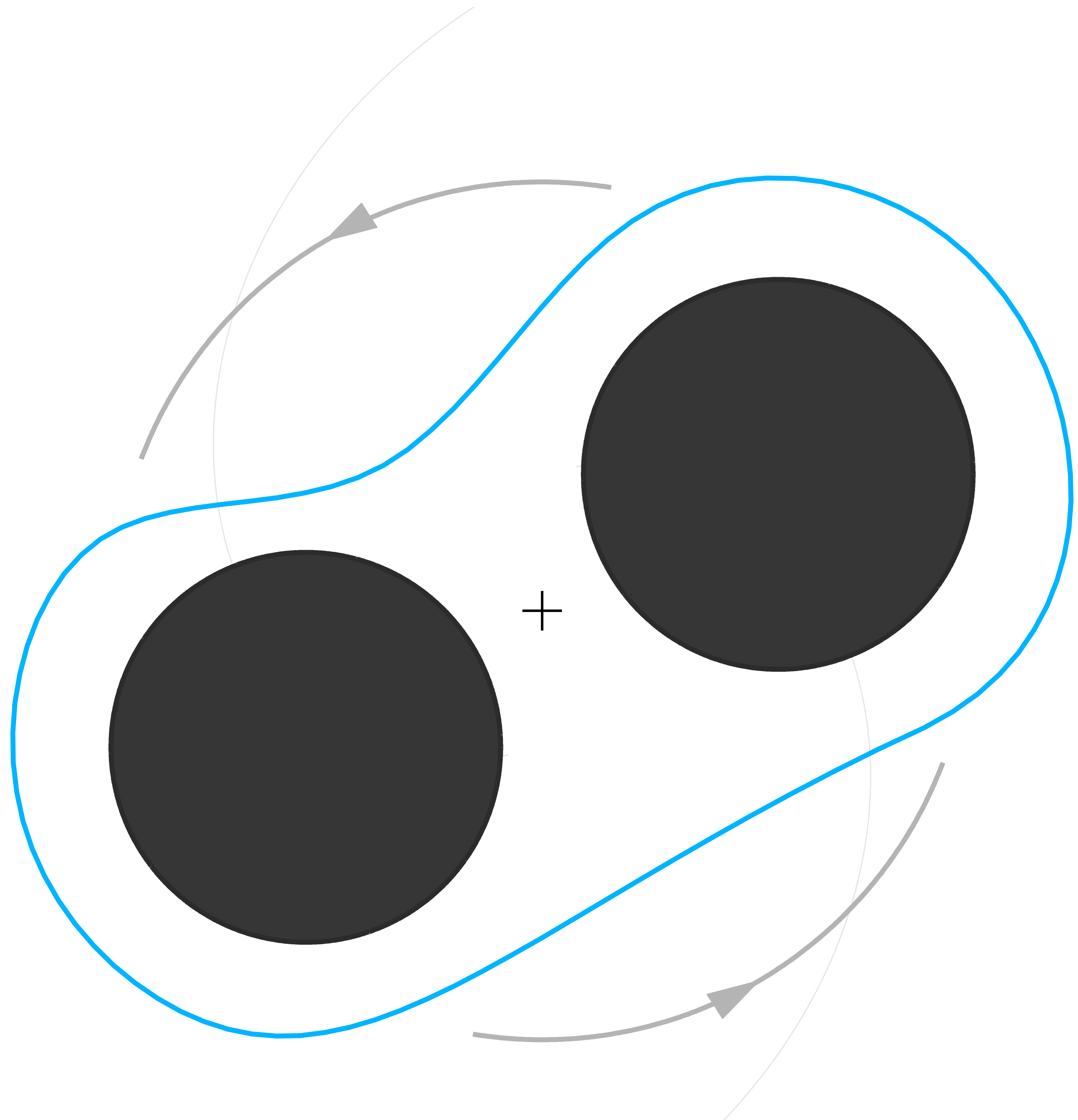
Towards a new multipolar description of gravitational waves from binary black holes

MIT KAVLI INSTITUTE

CARDIFF UNIVERSITY PRIFYSGOL CAERDYDD

Lionel London Pisa July 17th, 2019

Plunge-Merger



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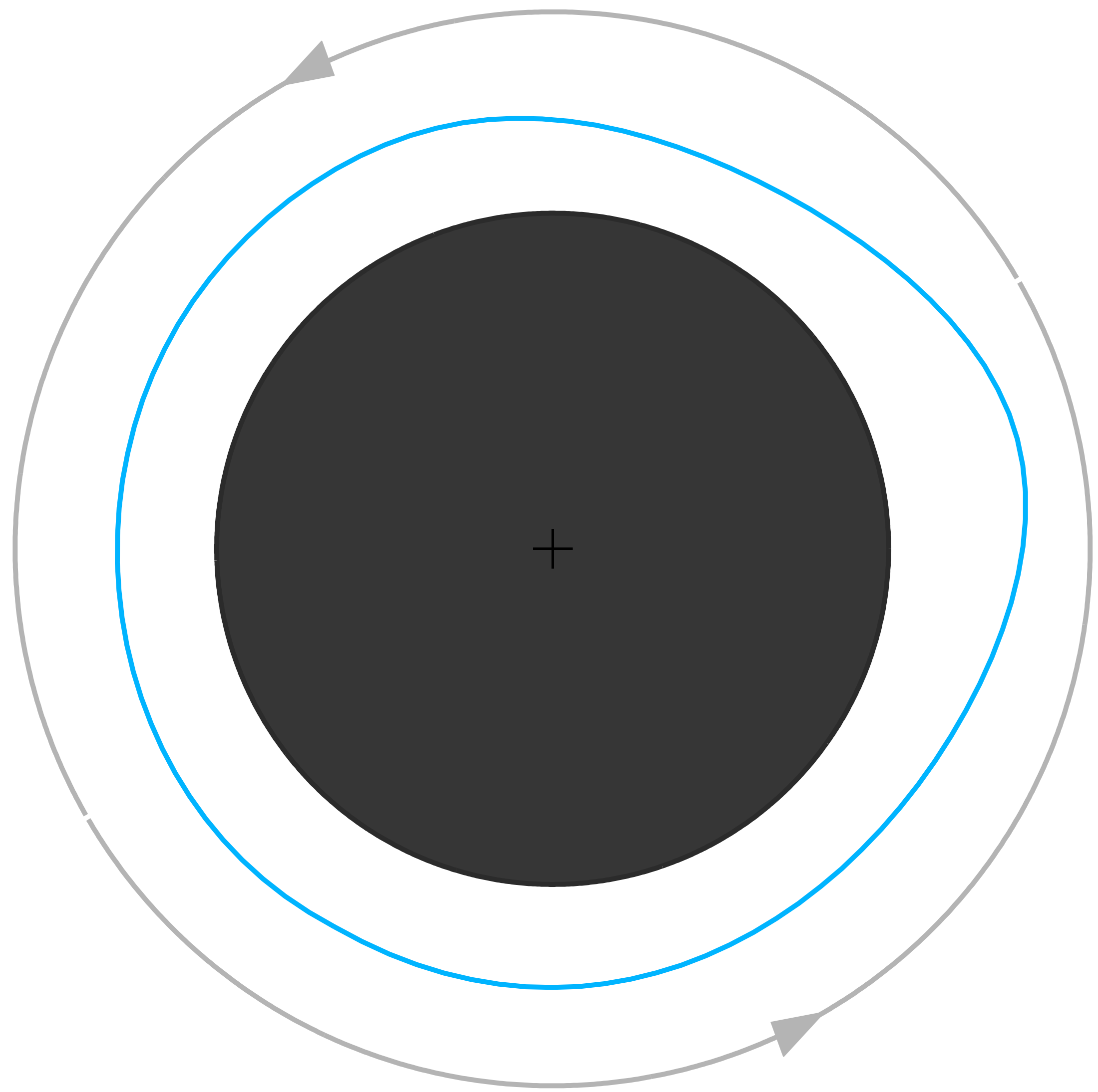
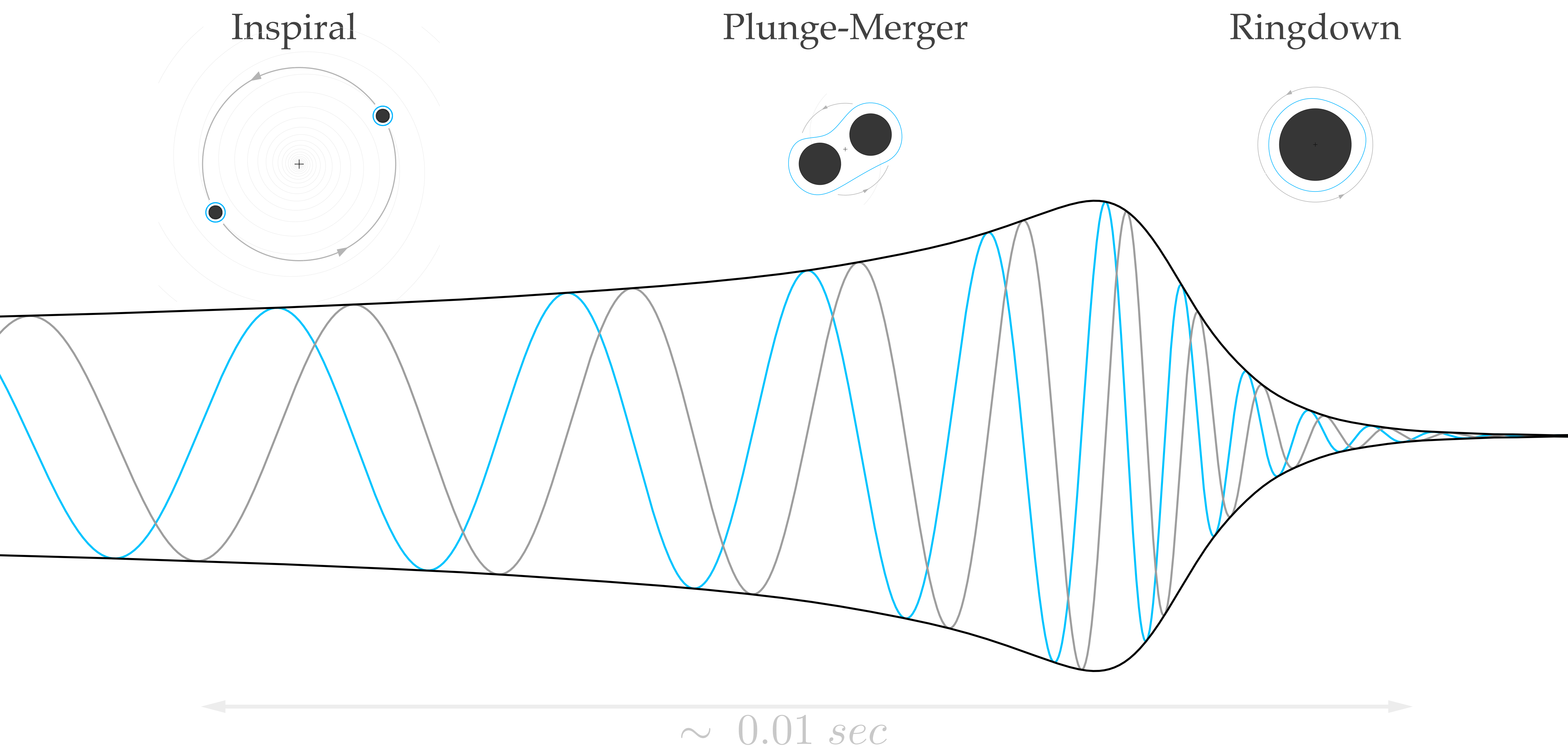
Ringdown





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Preface

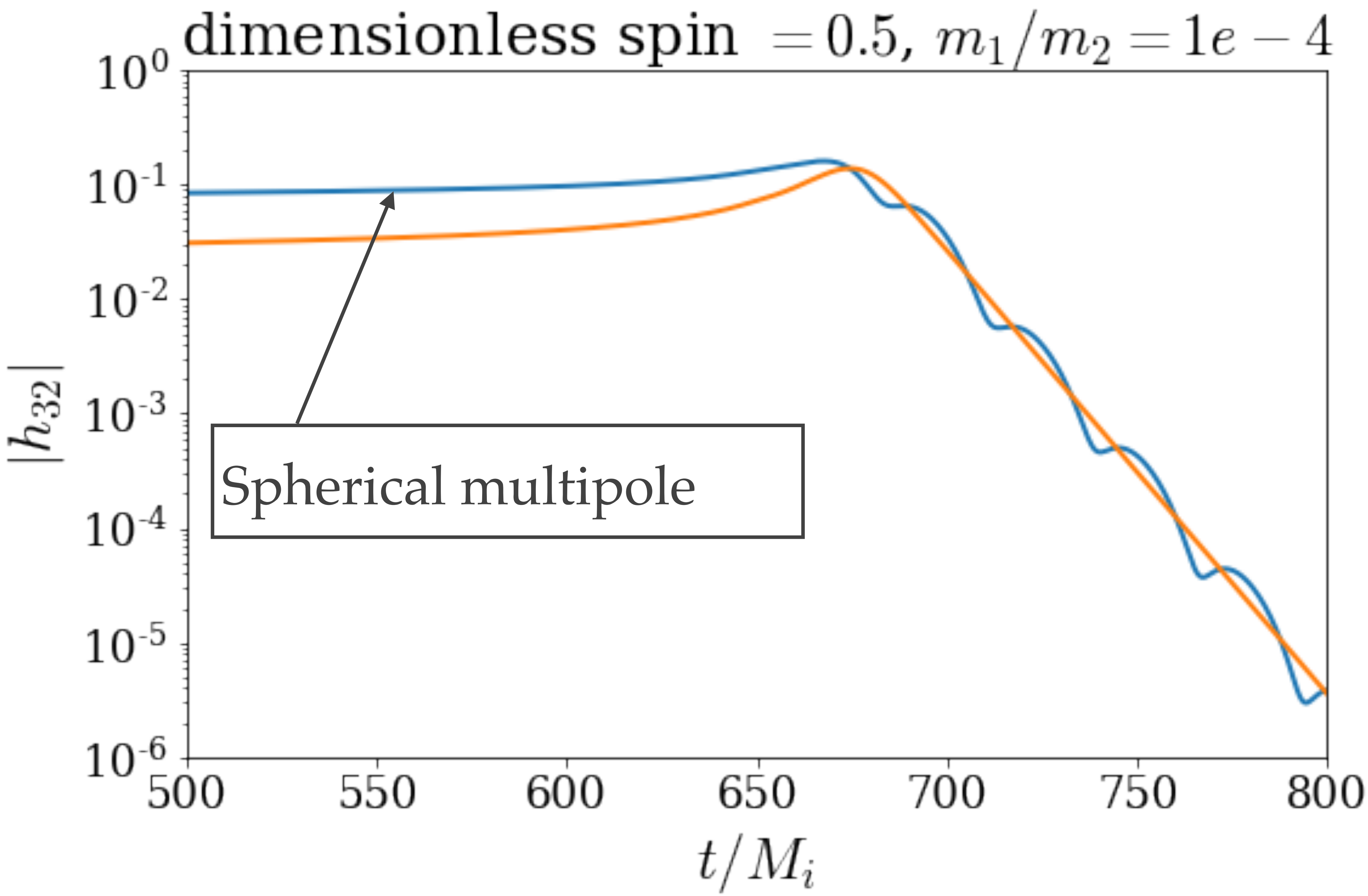
- Spherical Picture (Einstein, Penrose, Thorn +)

$$h(t) = h_+(t) - ih_\times(t) = \frac{1}{r}\sum_{l,m} {}_{-2}Y_{lm}(\theta,\phi)\, h_{lm}(t)$$

- Spheroidal Picture (Teukolsky, Press +)

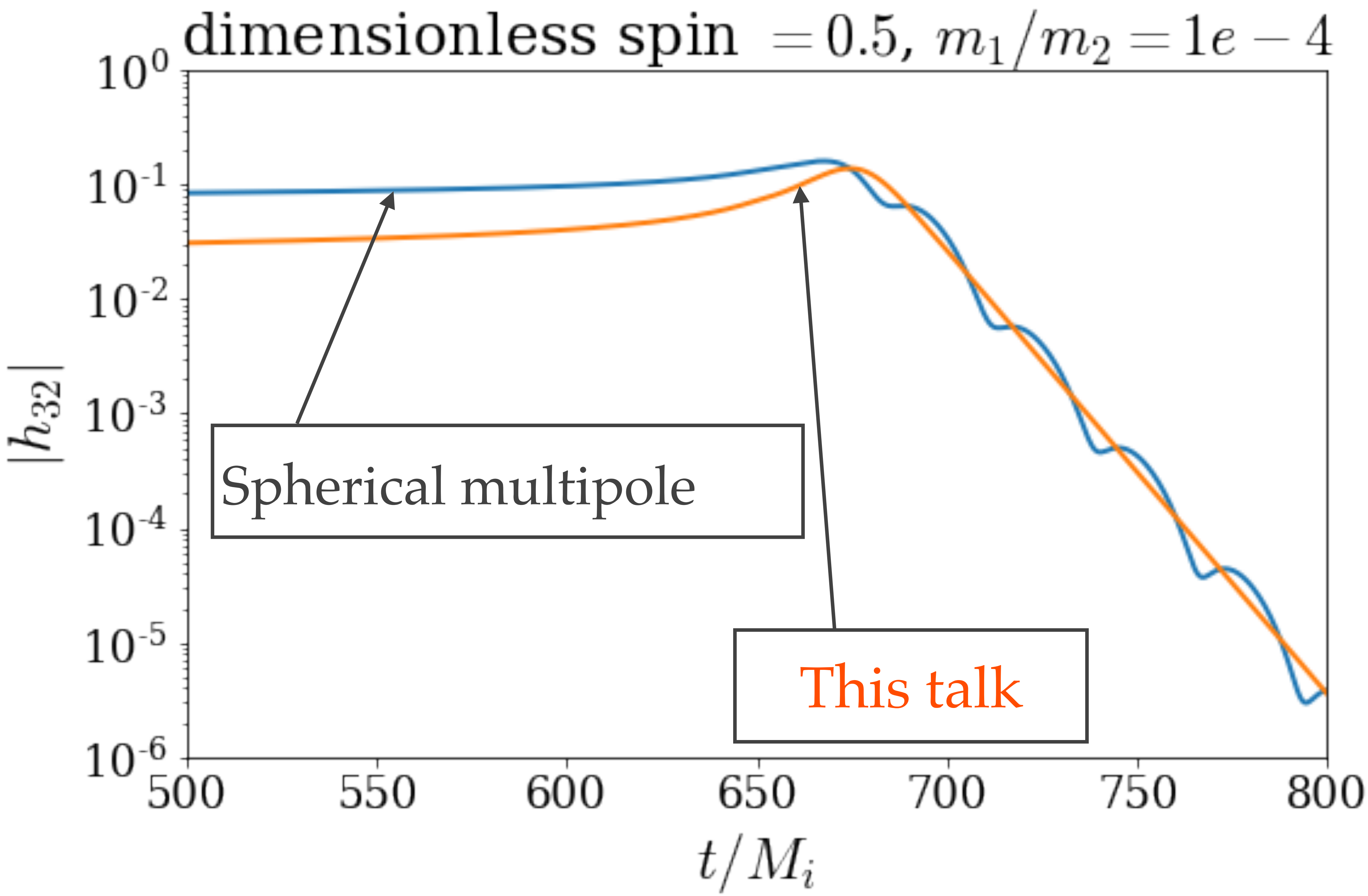
$$\boxed{h(t) = h_+ - ih_\times = \frac{1}{r}\sum_{l,m,n} h_{lmn\,-2}S_{lmn}(a\tilde{\omega}_{lmn},\theta,\phi)}$$

Preface



dimensionless spin $=0.5$, $m_1/m_2=1e-4$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$|h_{32}|$
500
550
600
650
700
750
800
t/M_i
Spherical multipole

Preface



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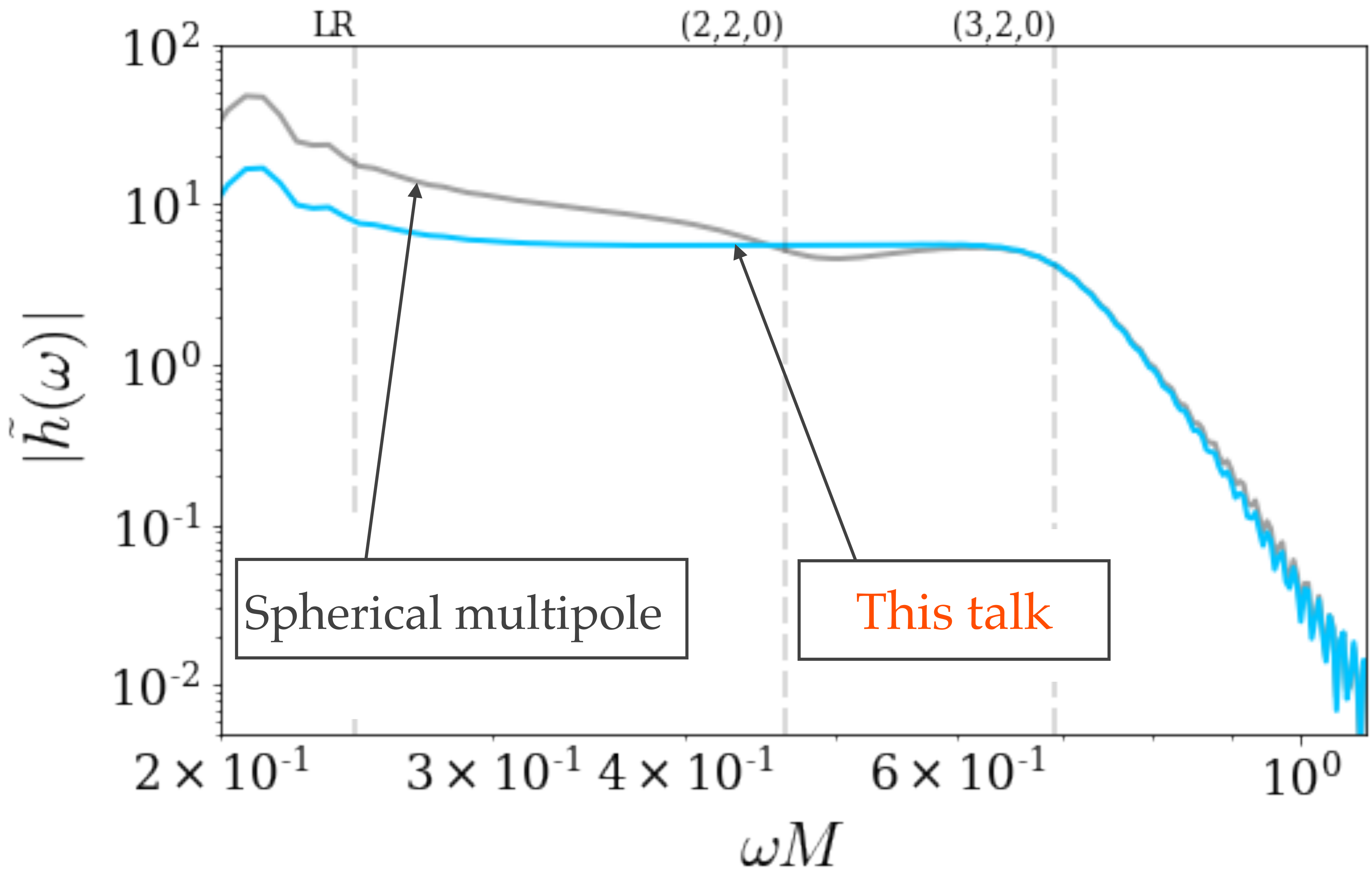
This talk

Preface



LR
(2,2,0)
(3,2,0)
10^2
10^1
10^0
10^{-1}
10^{-2}
$|\tilde{h}(\omega)|$
2×10^{-1}
3×10^{-1}
4×10^{-1}
6×10^{-1}
10^0
ωM
Spherical multipole
This talk

Overview Towards a new multipolar description

A. Foundational concepts

- General Relativity and Gravitational waves
- Black Hole Binaries, Observation & Signal Modeling
- **Core Questions**

B. Gravitational Wave Signal Models

- Review of the first higher multipole model, PhenomHM
- Missing physics ("Mode Mixing")
- Ringdown-only model, RDNP
- **Review & Questions**

C. Towards a New Multipolar Description

- Review of motivation
- Basic methodology and sample results
- **Pandora's box & final comments**

Foundations Briefly on General Relativity and Gravitational Waves

On the Implications of General Relativity

Relevant Foundations

- General Relativity is a mathematical tool for describing space and time in a way that allows different observers to agree on measurable quantities in space and/or time.
- The **"mixing" of space and time** spurs the notion of "**space-time**".
- Example: The "proper", frame-invariant, path of a object moving between to locations has a length $L = \int_{\tau_a}^{\tau_b} |ds^2|^{\frac{1}{2}}$, where $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- The **metric**, $g_{\mu\nu}$, is a tool for describing the structure of space-time.
- Special case: flat-space $ds^2 = -cdt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu} x^\mu x^\nu$

Practical Implications

- Newtonian equations involving only spatial derivatives now involve ``space'' and ``time'' derivatives. For example, $\nabla^2 \phi = 4\pi G \rho$ becomes …
- … **Einstein's Equations**: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
- Einstein's Equations support "**wave equations**" for $g_{\mu\nu}$ — **Gravitational Waves**

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Gravitational Wave Radiation

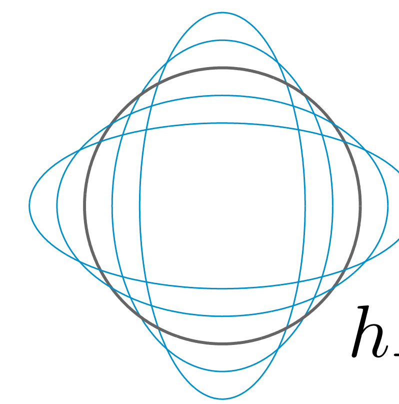
Gravitational Waves (GWs)

- In general, metric solutions to Einstein's Equations may be sought in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

background metric

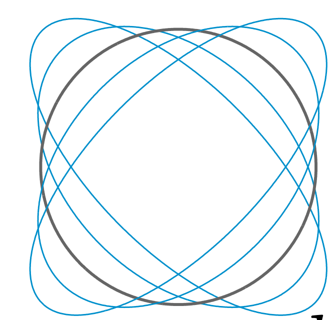
"strain" due to e.g. a black hole binary (BBH)



h_+

- There are two polarizations: h_+ and $h_\times$
- At a linear approximation, the strain satisfies a standard wave equation

$$\Box h_{\mu\nu} = \frac{8\pi}{c^4} G T_{\mu\nu}$$



$h_\times$

Gravitational Wave Radiation

- Far from an astrophysical source, we are concerned with the radiation solution

$$h(t) = h_+(t) - i h_\times(t) = \frac{1}{r} \sum_{l,m} {}_{-2}Y_{lm}(\theta, \phi)\, h_{lm}(t)$$

Gravitational Wave Radiation

Gravitational Wave Radiation

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$$h(t) = h_+(t) - i h_\times(t) = \frac{1}{r} \sum_{l,m} {}_{-2}Y_{lm}(\theta, \phi)\, h_{lm}(t)$$

Spin-weighted Spherical Harmonics

Multipole Moment

Gravitational Wave Radiation

Gravitational Wave Radiation

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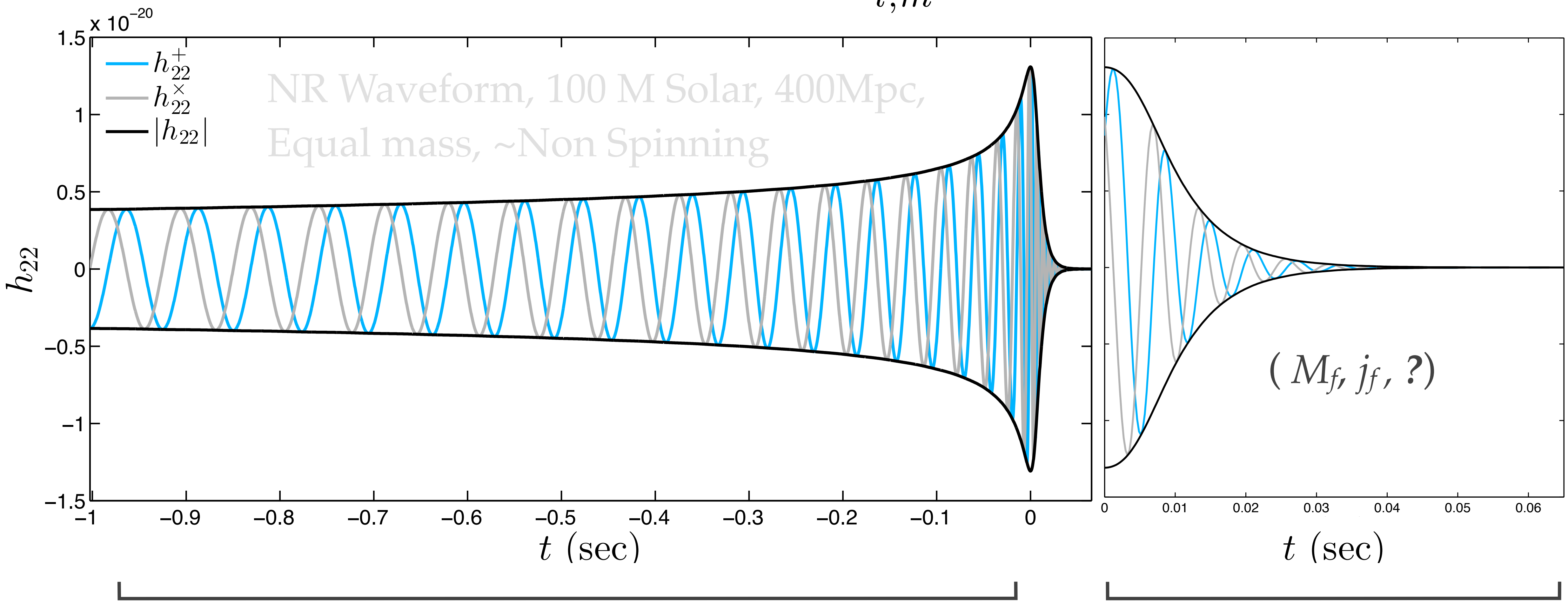
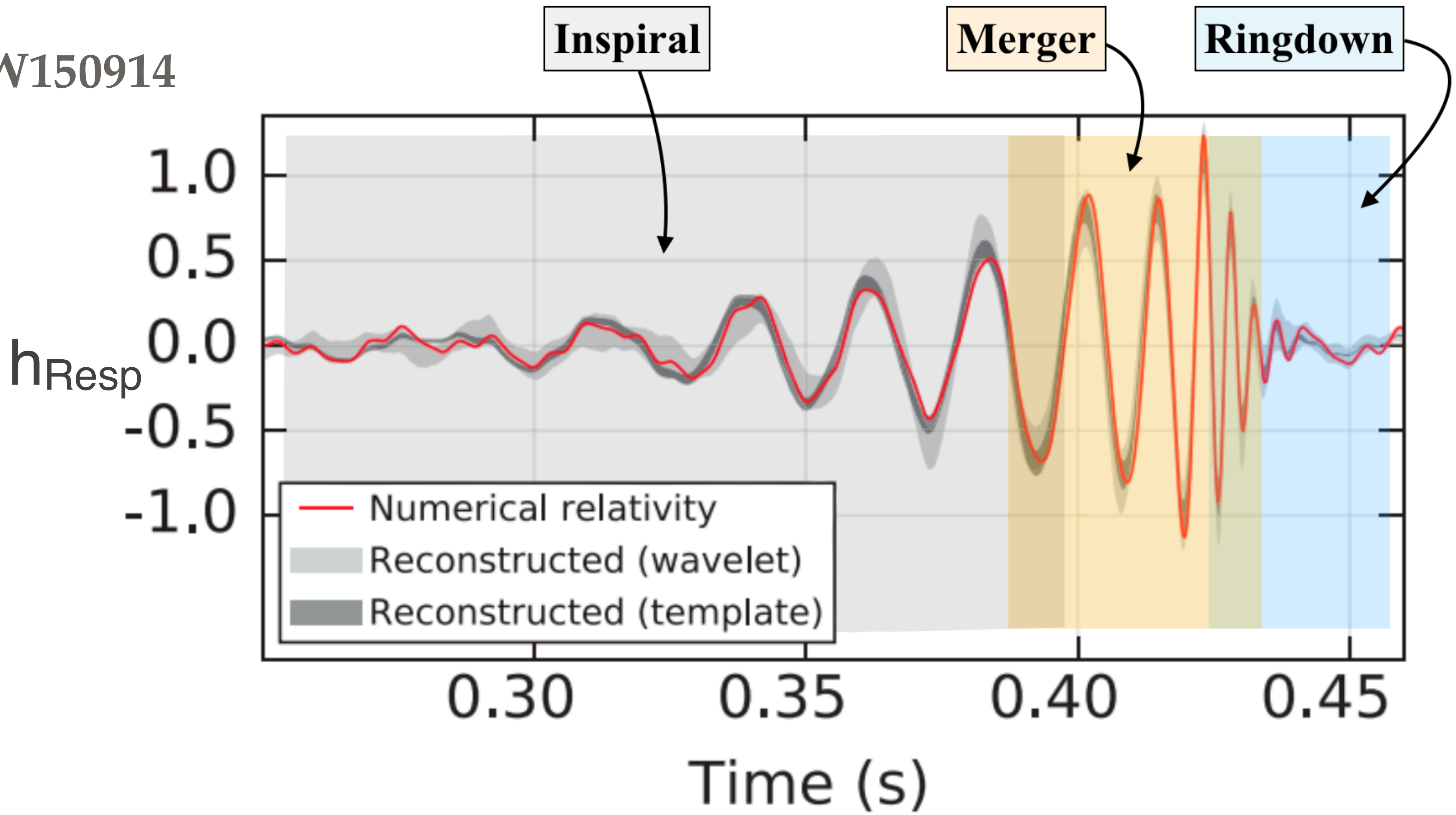


Inspiral, Merger

Ringdown (Quasinormal Modes)

Gravitational Wave Radiation

GW150914



Gravitational wave detectors such as Adv. LIGO observe a linear combination of the two GW polarizations. While the formal detection problem is highly non-trivial, it is vastly aided by prior knowledge of signal morphology. Thus GW signal models are a central effort in GW Astronomy. They enable the probing of many questions …

Gravitational Wave Signal Models

Core Questions & GW Signal Models

Signal models are our best interface between experimental data fundamental GW theory. They enable us to probe the key questions about BBH signal observations:

- **What where the physical parameters of the source? It's masses, spins?**
- **Is the signal consistent with the predictions of General Relativity?**

In tandem, the process of developing GW signal models (e.g. from costly Numerical Relativity simulations) forces us to ask deeper theoretical questions:

- **How well can we understand the parts of GW signals where analytic theory provides solutions?** (i.e. early inspiral, and late post-merger)
- Can Numerical Relativity simulations guide the way to new physical insight? **Can we analytically understand the non-linear merger?**

The recent **PhenomHM** model is a prime example of how deeper theory questions may result in improved answers to observational ones ***(London, Khan et. al. 2018)***. But this is not without limitations. The Ringdown-only model, **RDNP**, addresses mode-mixing ***(London 2018)***.

PhenomHM Motivations

PhenomHM is the first, and currently only GW signal model that:

- Applies to non-precessing Binary Black Hole (BBH) systems, and has higher multipoles
- And, is suitable for large regions of the BBH parameter space

- **Signal models with higher multipoles (modes) are needed to confront:**
  - Biases in parameter estimation (Varma, others)
  - Limited detectable volume for mass ratios greater than ~4 (Varma, Bustillo,Pekowsky, others)
- **Higher multipoles are more important for:**
  - Unequal component masses (Capano, Healy,Varma, others)
  - Moderate component spins (Varma, others)

PhenomHM Structure

- **Inspiral** (where Post-Newtonian is valid)
  - Multipole phases are related by a simple scaling

$$h_{lm}(t) = |h_{lm}| e^{i\phi_{lm}(t)}, \; \phi_{lm} \underset{\text{PN}}{\approx} m\psi_{\text{Orb}}$$

  - The Stationary-Phase-Approximation (SPA) allows this scaling to be applied to the frequency domain

$$f_{lm} \underset{\text{SPA}}{\approx} \frac{m}{2} d\phi_{22}/dt = \frac{m}{2} f_{22}$$

- **Merger-Ringdown** (Perturbed Kerr)
  - The Quasi-Normal Mode (QNM) spectrum provides relationships between the frequencies of different gravitational wave multipoles.

$$\text{(Inspiral)} \qquad \frac{m}{2} \rightarrow \frac{\omega_{lm}^{\text{QNM}}}{\omega_{22}^{\text{QNM}}} \qquad \text{(Merger-Ringdown)}$$

PhenomHM Structure

- **Bridging the gap** (inspiral—>merger—>ringdown)

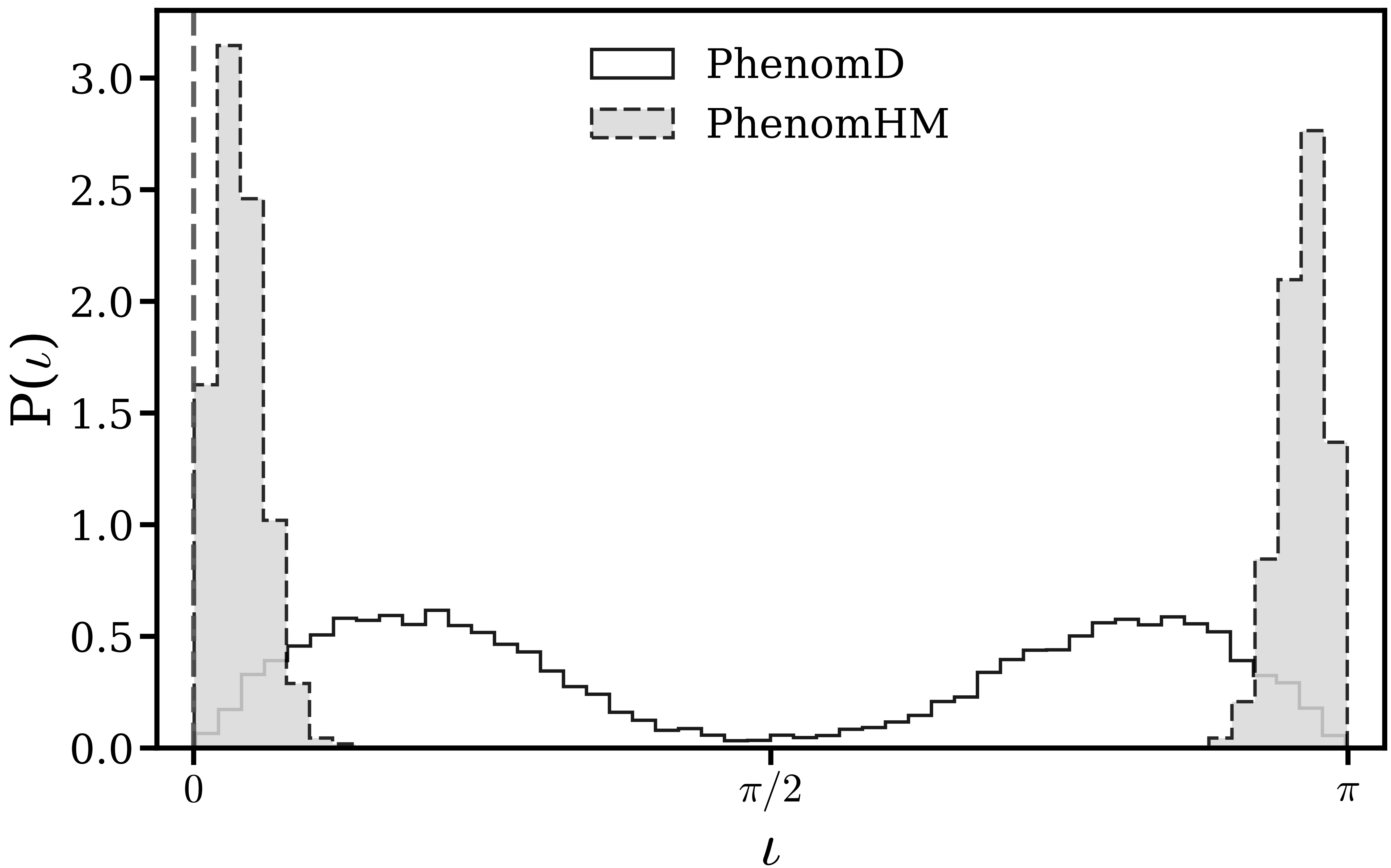
$$f_{22}(f) = \begin{cases} \frac{2}{m} f, & f \leq f_0 \\ \frac{f_{22}^{\mathrm{RD}} - 2f_0/m}{f_{\ell m}^{\mathrm{RD}} - f_0} (f - f_0) + \frac{2f_0}{m}, & f_0 < f \leq f_{\ell m}^{\mathrm{RD}} \\ f - (f_{\ell m}^{\mathrm{RD}} - f_{22}^{\mathrm{RD}}), & f > f_{\ell m}^{\mathrm{RD}}. \end{cases}$$

- **Quadrupole Mapping** (h22 —> hlm)

$$\begin{aligned} \tilde{h}_{\ell m}(f) &= A_{\ell m}(f) \times \exp\{i\,\varphi_{\ell m}(f)\} \\ &\approx |\beta_{lm}(f)|\, A_{22}(f_{22}^{A}) \times \exp\left\{i\left[\kappa\,\varphi_{22}(f_{22}^{\varphi}) + \Delta_{\ell m}\right]\right\} \end{aligned}$$

PhenomHM Results

m1/m2=4, dimensionless spin of +0.5 on each BH



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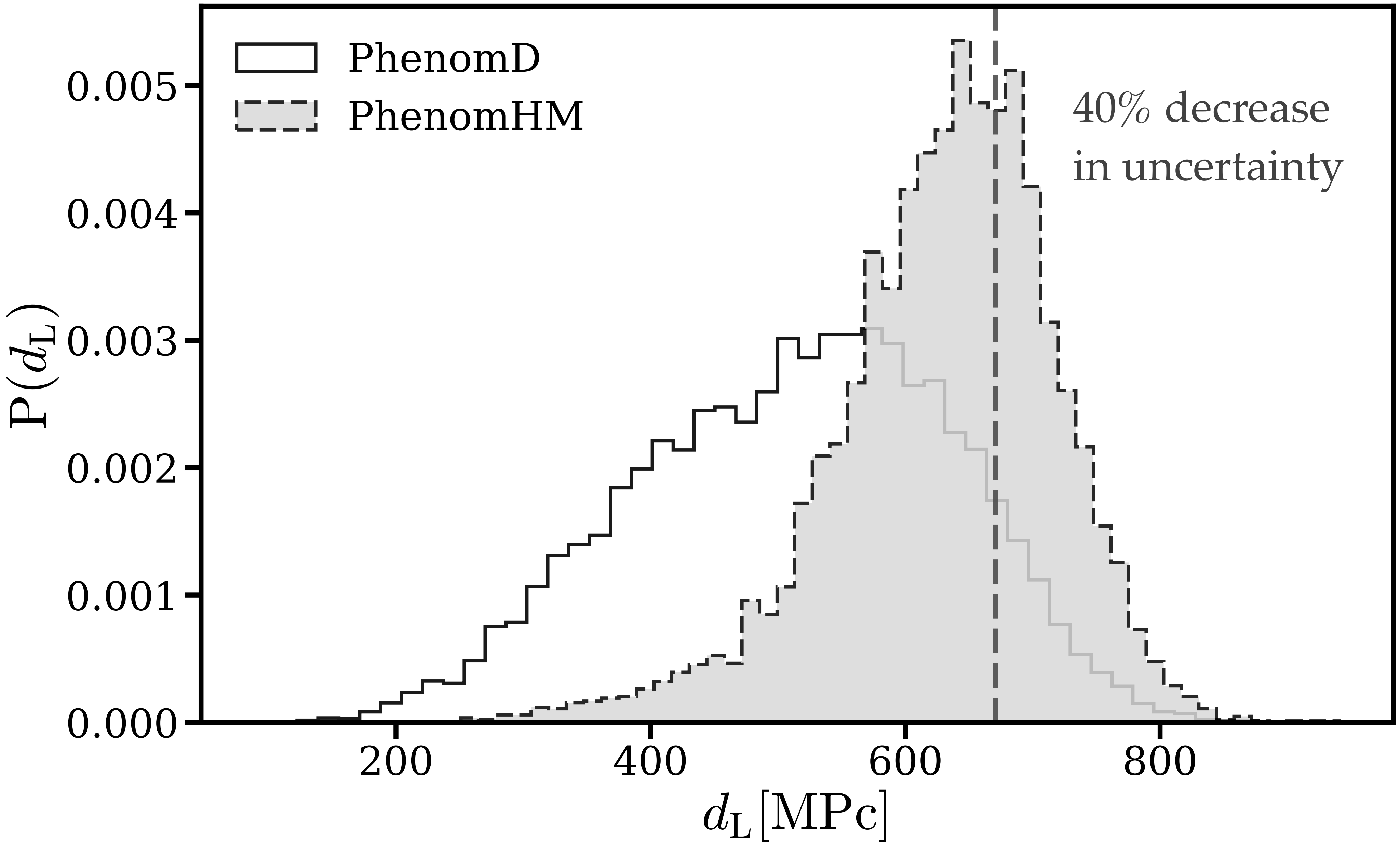
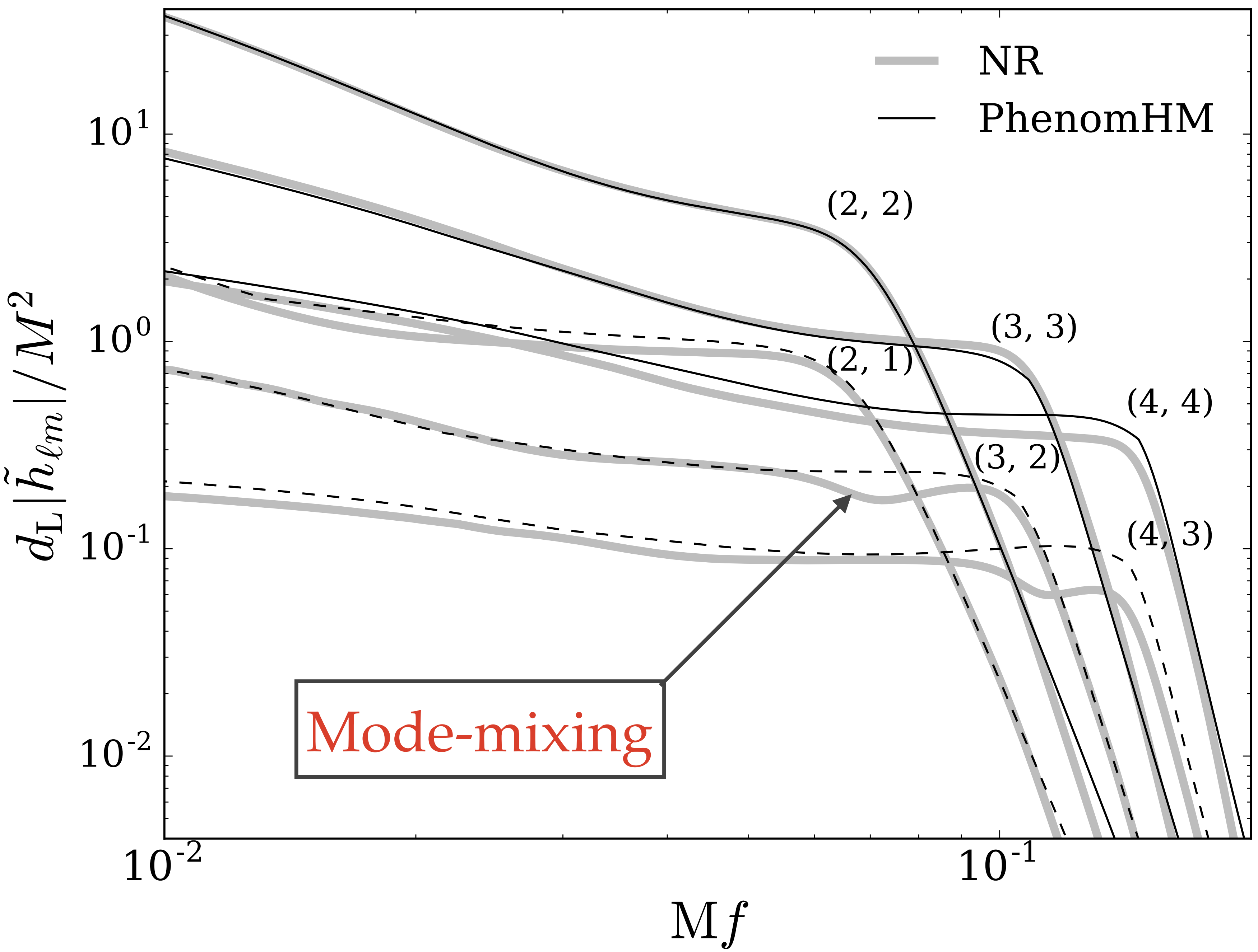


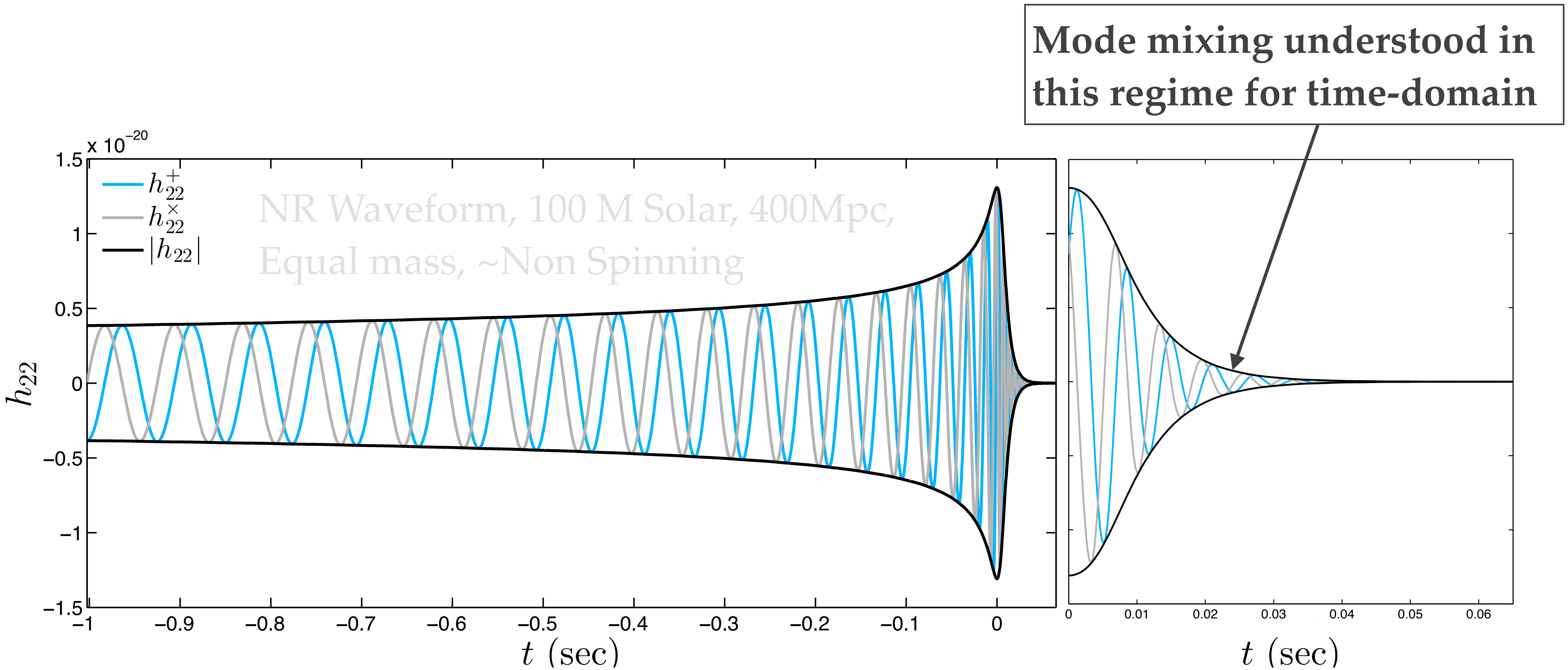
PhenomHM Limitations

- PhenomHM does not model precession. (ongoing work)
- Improvement in agreement with numerical relativity, but no robust calibration. (ongoing work)
- PhenomHM does not account for known multipolar mixing or "mode-mixing" during ringdown.
    - Mode-mixing: In the time domain, mode-mixing is well understood in perturbation theory to result from differences between **spherical** and ***spheroidal*** harmonics
- Few GW signal models account for mode mixing, most of these are ringdown-only models

PhenomHM Limitations



Aside: Mixing of Multipoles in NR Waveforms



Inspiral, Merger

Ringdown (Quasinormal Modes)

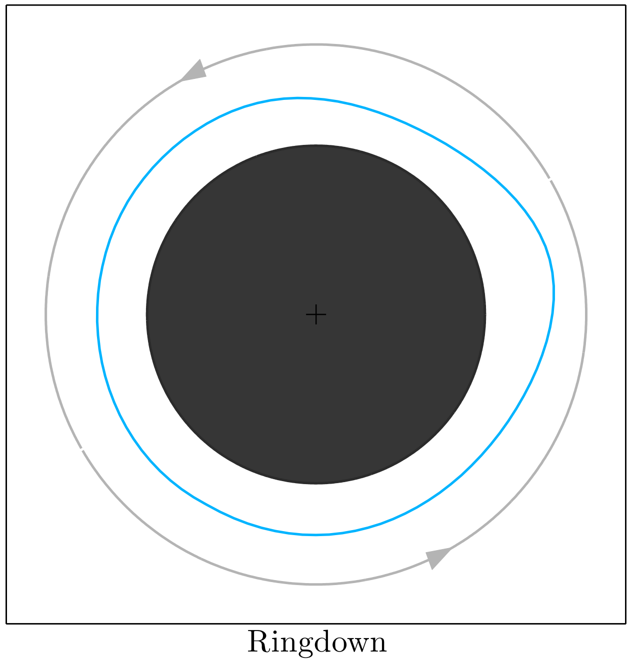
BBH Ringdown Mode-Mixing

The Working Perspective

- Numerical Relativity, Post-Newtonian Theory, LIGO signal models … (Pistorius, Baker, Damour, many others)
- Spherical Harmonic multipole moments of spin weight -2

$$rh = r(h_+ - ih_\times) = \sum_{l,m} h^{NR}_{lm} \; {}_{-2}Y_{lm}(\theta, \phi)$$

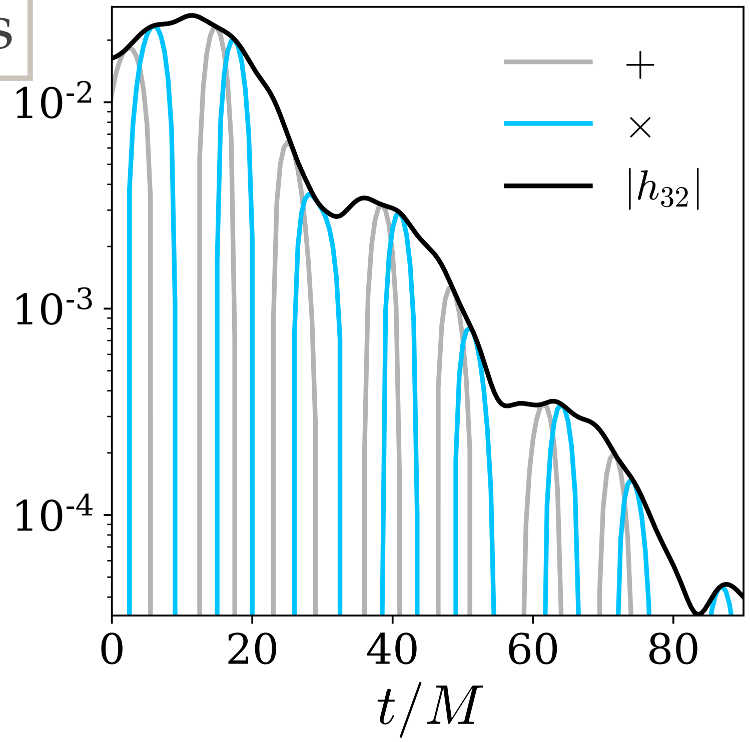
the multipole moments



Black Hole Perturbation Theory

- Perturbations of Kerr metric (spinning BHs)
- Linearized gravity = Teukolsky's Equations
- Analytically well understood:

$$h = \frac{1}{r} \sum_{\ell mn} A_{\ell mn} e^{i\tilde{\omega}_{\ell mn}(t - t_{\rm ref})} S_{\ell mn}(j\tilde{\omega}_{\ell mn}; \theta, \phi)$$



BBH Ringdown Mode-Mixing

Combining the two perspectives to understand "mode-mixing"

- **Numerical Relativity**: Spherical Harmonic Multipoles, ${}_{-2}Y_{lm}$ **(Orthogonal in *l*)**
  - $rh = r(h_+ - ih_\times) = \sum_{l,m} h^{NR}_{lm}\ {}_{-2}Y_{lm}(\theta, \phi)$
  - $h^{NR}_{lm} = \int_\Omega rh\ {}_{-2}\bar{Y}_{lm} d\Omega$
- **Perturbation Theory**: Spheroidal Harmonic Multipoles, ${}_{-2}S_{lm}$ **(Not orthogonal in *l*)**
  - $rh = \sum_{lmn} h^{PT}_{lmn}\ {}_{-2}S_{lm}(\theta, \phi; \tilde{\omega}_{lmn} j_f)$
  - $h^{PT}_{lmn} = A_{lmn}\left[e^{i\tilde{\omega}_{lmn} t}\right]$

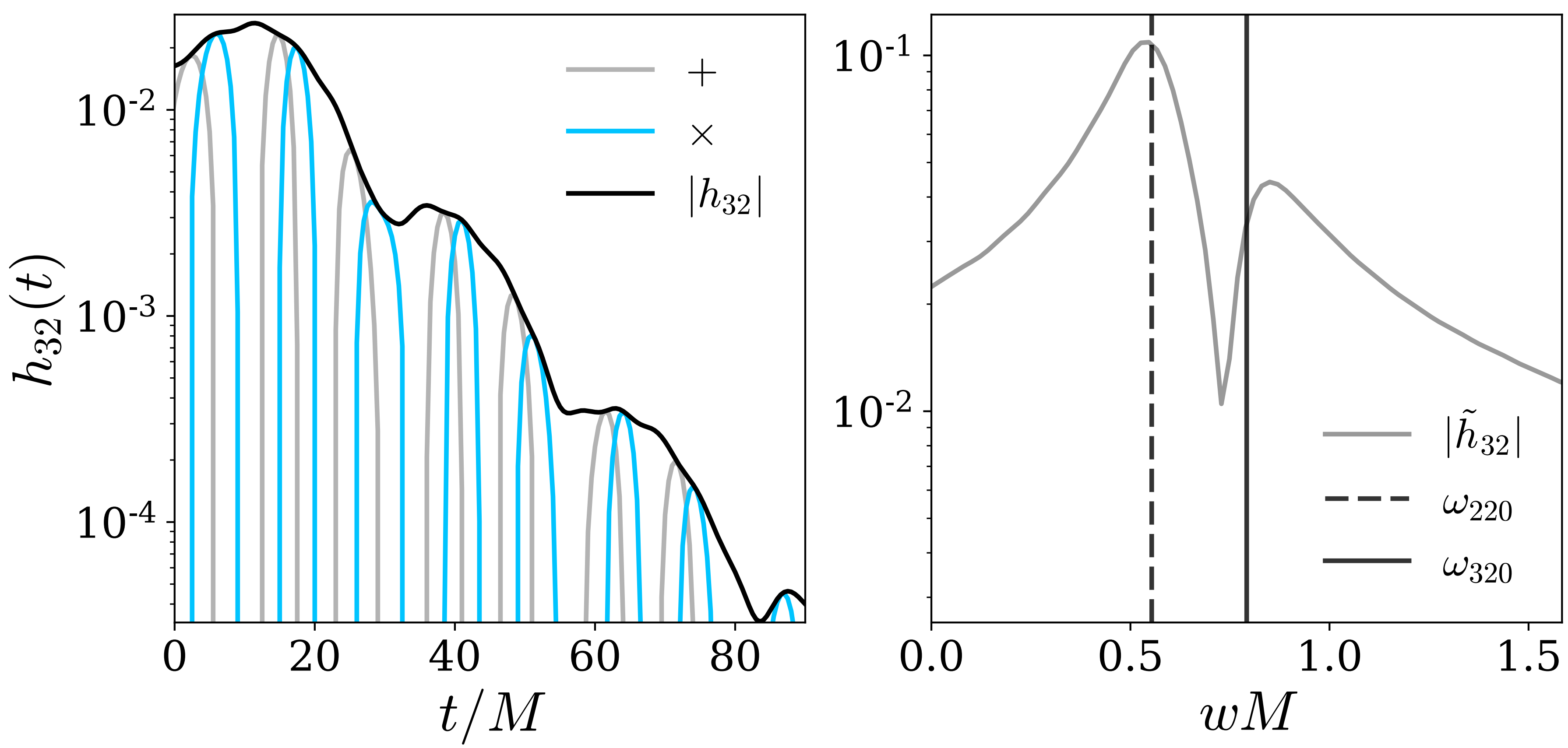
$$\rightarrow h^{NR}_{lm} = \sum_{l'n} A_{l'mn} e^{i\tilde{\omega}_{l'mn} t} \int_\Omega {}_{-2}\bar{Y}_{lm}\, {}_{-2}S_{l'mn} \mathrm{d}\Omega$$

The Spherical multipoles of NR are sums of QNMs

— i.e. "mode mixing"

BBH Ringdown Mode-Mixing

The result of these two different perspectives is **obscured** physics. Figure: NR ringdown waveform for **(*l*,*m*)=(3,2)**, equal mass, initially non-spinning.



The **mixing of QNMs due to use of a spherical basis causes "beating"**, while a simple decaying sinusoid might naively be expected. **This effect complicates the time domain modeling of post-merger higher multipoles**.

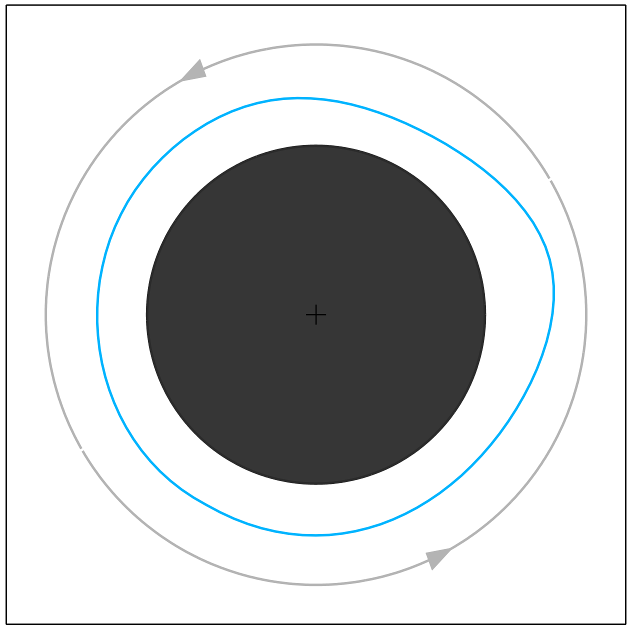
BBH Ringdown Mode-Mixing

Clashing Perspectives

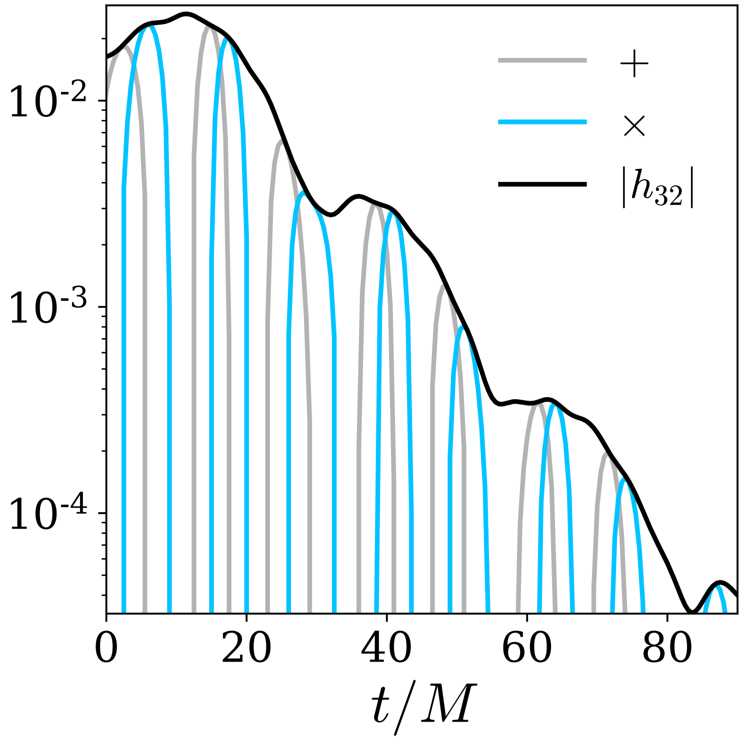
- Post-Newtonian theory and other modeling paradigms primarily use spherical harmonics, while the natural perspective of perturbation theory is to use spheroidal harmonics
- **Most GW signal models do not incorporate spheroidal harmonic information**

Learning from the Spheroidal Perspective

- Ringdown-only signal models are useful for testing GR (Carullo, Kelly, Kamaretsos, London+)
- **Example model: RDNP** (London+ 2014/2018)



Ringdown



RDNP: A QNM Signal Model for NonPrecessing Systems

- Model Overview

$$rh = r(h_+ - ih_\times) = \sum_{l,m} h^{NR}_{lm} \, {}_{-2}Y_{lm}(\theta,\phi)$$

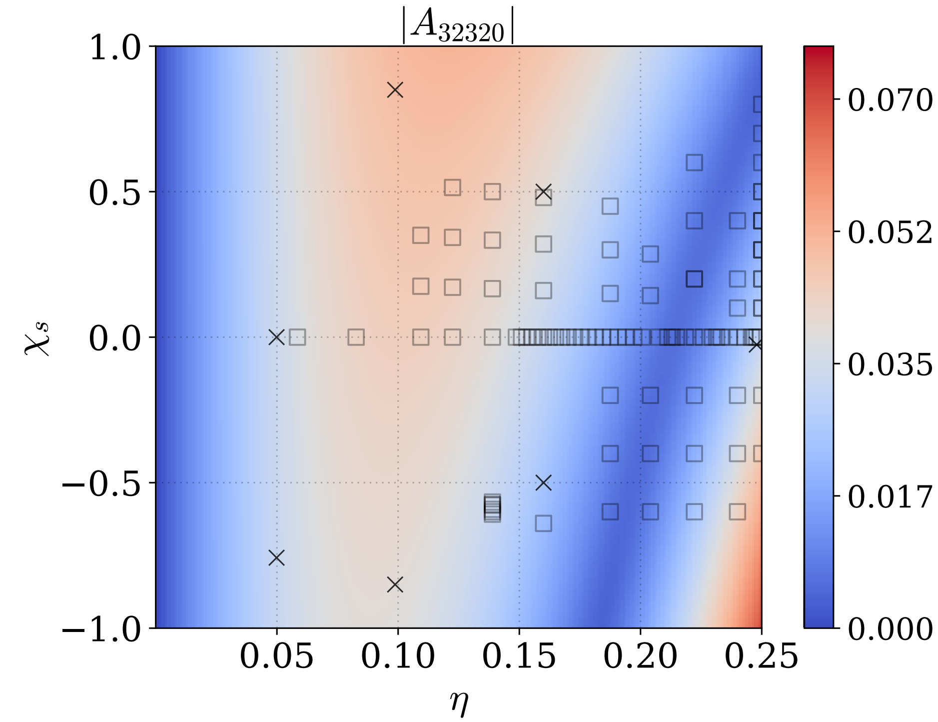
$$h^{NR}_{lm} = \sum_{l'n} A_{l'mn} e^{i\tilde{\omega}_{l'mn}t} \underbrace{\int_\Omega {}_{-2}\bar{Y}_{lm} \, {}_{-2}S_{l'mn} \mathrm{d}\Omega}_{\text{Spherical-Spheroidal Inner-product}}$$

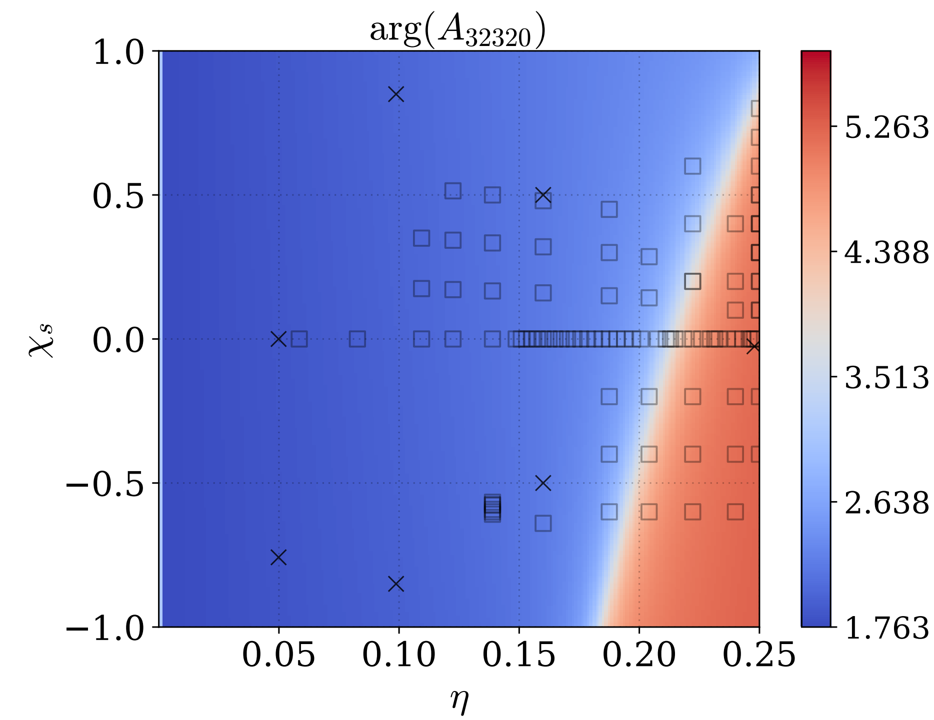
Quasinormal mode amplitudes, $A_{l'mn}$, are modeled over initial binary parameters: masses, spins

RDNP: A QNM Signal Model for NonPrecessing Systems

Sample Amplitudes

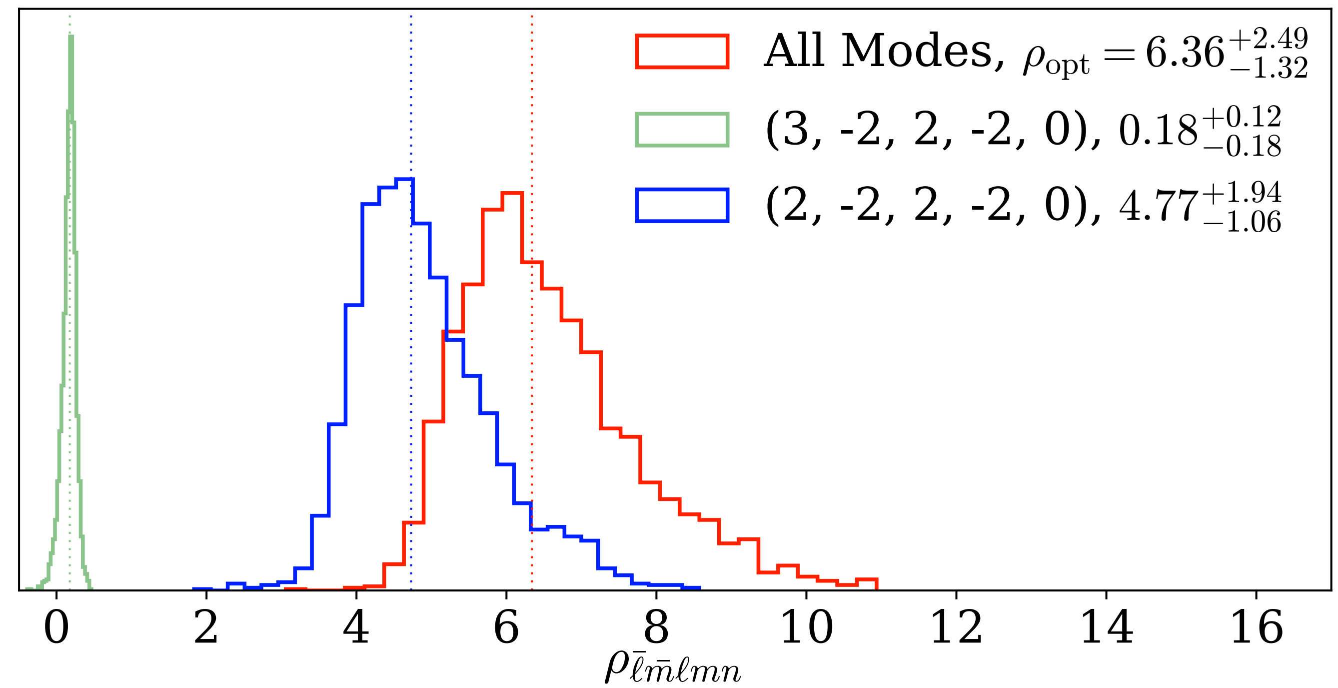
$$
\begin{aligned}
A_{220} &= \eta\,(\,-0.6537\,\chi_s \;+\; (-4.0071)\,) \\
A_{210} &= \eta\,(\,2.3488\,e^{2.6631i}\,\delta \;+\; (0.8011\,e^{5.7070i})\,\chi_a \\
&\quad +\; (3.5828\,e^{5.5223i})\,\eta\,\delta \;+\; (1.1774\,e^{0.4254i})\,\chi_s\,\delta \\
&\quad +\; (0.6260\,e^{5.3457i})\,\chi_s\chi_a\,) \\
A_{330} &= \eta\,(\,2.6412\,e^{2.9880i}\,\delta \;+\; (1.6030\,e^{0.6655i})\,\delta^2 \\
&\quad +\; (1.0354\,e^{3.6096i})\,\chi_s\,\delta \;+\; (0.4911\,e^{4.7347i})\,\chi_a{}^2\,) \\
A_{320} &= \eta\,(\,2.5707\,e^{4.1427i}\,\eta \;+\; (9.4216\,e^{0.8076i})\,\eta^2 \\
&\quad +\; (0.5973\,e^{2.1816i})\,\eta\,\chi_s \;+\; (0.2104\,e^{4.9043i})\,\chi_a{}^2 \\
&\quad +\; (0.4417\,e^{5.4544i})\,\chi_a\,\delta \;+\; (0.9439\,e^{1.7614i})\,\delta^2\,)
\end{aligned}
$$

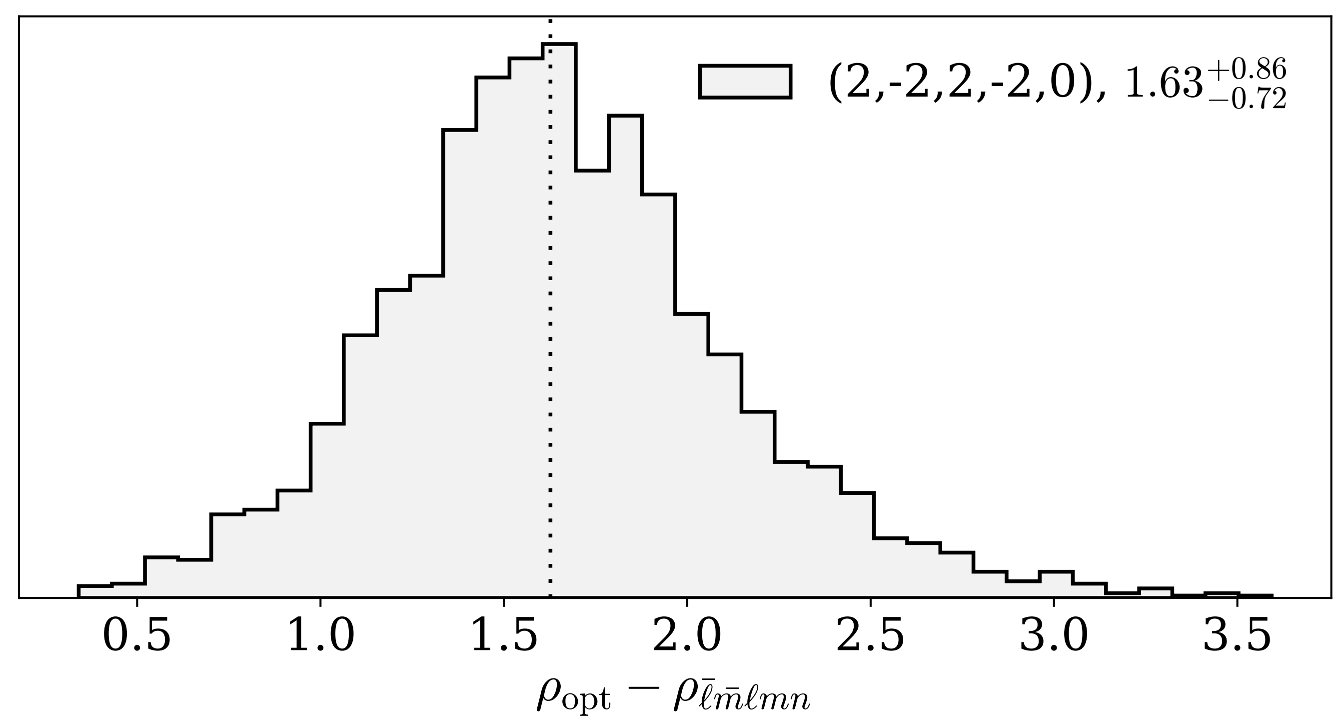




Sample Application: GR predictions for GW150914

- Enables quantification of **GR predictions for current and future detections**
- **e.g.** GW150914 likely had a ringdown SNR of about 6.4
- **About 1.6 of this SNR can be attributed to the presence of more than 1 QNM**
- This SNR ~1 effect suggests that events like GW150914 can be of cumulative use for testing GR.





Core Questions

Can a **change in perspective** enable an **advance in physical understanding**?

For gravitational waves (GWs) and black hole binaries (BBHs), can we better understand:

- The GR manifold of solutions for experimental tests?
- The imprint of source properties on GW signals?

For BBH Ringdown: Yes

Let's Review & Ask Questions

Gravitational Waves

- Typically represented as ripples atop a Minkowski ("flat") background metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- This approach leads to (at low order) a spherical harmonic representation of the radiation: $rh = r(h_+ - ih_\times) = \sum_{l,m} h_{lm}^{NR} \, {}_{-2}Y_{lm}(\theta, \phi)$

Signal Models

- **Most** GW Inspirl-Merger-Ringdown models to date use the spherical harmonic representation

Mode-Mixing in Merger-Ringdown

- BH perturbation theory describes the ***spheroidal*** harmonics as the natural basis <u>for time domain ringdown</u>

Let's Review & Ask Questions

Mode-Mixing in Merger-Ringdown

- BH perturbation theory describes the ***spheroidal*** harmonics as the natural basis for time domain ringdown.
- The spherical harmonics are only the natural basis when there is **no angular momentum** in the space-time.

Primary Question

- **Is there a natural (dynamical) multipolar basis for GWs from BBHs?**

Towards a New Multipolar Description

Towards the use of Spheroidal Harmonics to describe GWs from BBH coalescence

Spheroidal Picture Motivations

Insight from Misner-Thorne-Wheeler (MTW)

- Chapter 19 (second page) — far away from a general astrophysical source, and neglecting self-gravity, the metric is determined by the source's mass, and it's angular momentum.
- This is consistent with the Kerr metric **in the large r limit**.

This (+other) suggests that Kerr may be a more appropriate background metric far from the source

- This is definitely true for time domain ringdown, and must generalize to merger and inspiral in a continuous way.
- **But how?** MTW, as well as work in general azimuthally symmetric spacetimes suggest that **large-r-Kerr** is the appropriate metric.

Briefly on Spheroidal Harmonics

Definition

- Spheroidal harmonics are the angular eigenfunctions of Einstein's Equations at linear order, when Kerr is used as the background
- They are functions of the usual spherical polar angles, as well as the source's intrinsic frequency & angular momentum ***(a=J/M^2)***

Leaver 1986 (An analytic representation for quasi-normal modes of Kerr black holes)

$$u = \cos(\theta)\ ,\ S_{lm}(u) = e^{a\omega u}(1+u)^{\frac{1}{2}|m-s|}(1-u)^{\frac{1}{2}|m+s|}\sum_{n=0}^{\infty} a_n(1+u)^n$$

A Basic Question

- Can one calculate $a\omega$ for all of inspiral-merger and ringdown?
- Note that here, ω is complex (the image part is related to damping)

Calculating system spin (a) and frequency

A Basic Question

- Can one calculate $a\omega$ for all of inspiral-merger and ringdown?
- Note that here, ω is complex! (the image part is related to damping)

Spin & <u>Mode</u> Frequency (<u>using NR simulation data</u>)

- The spin (**total angular momentum**) and mode frequency of the system can be calculated either from the waveforms, or (in inspiral) from Post-Newtonian
- During inspiral, the total angular momentum is proportional to <u>*r^(1/2)*</u>
- Towards inspiral, the real part of $a\omega$ —> Complex Infinity
- Frequencies, ω, are determined by spacetime boundary conditions

Spheroidal IMR Method Overview

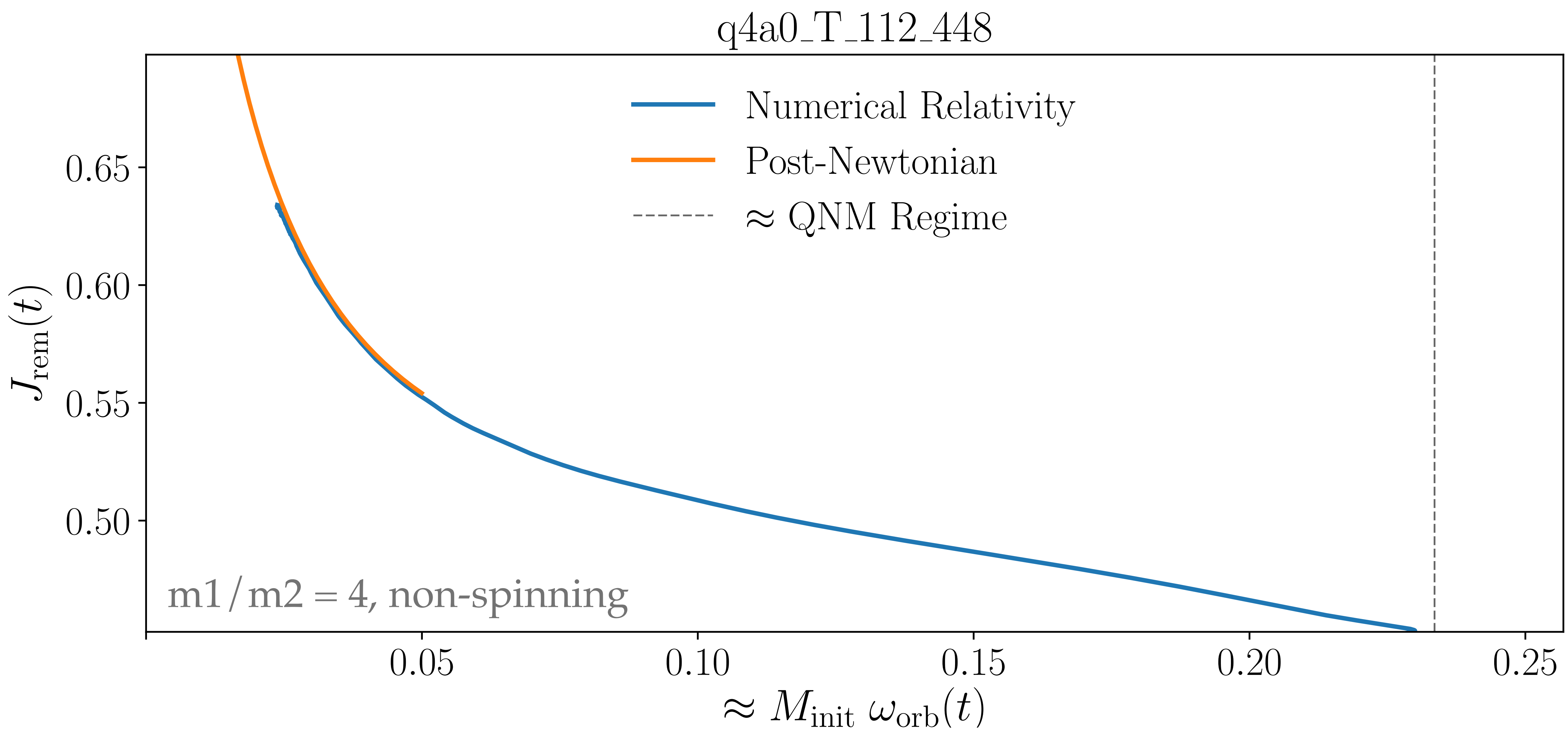
Low Level Goal

- Given an NR simulation, use the given spherical harmonic multipole moments to estimate the spheroidal ones

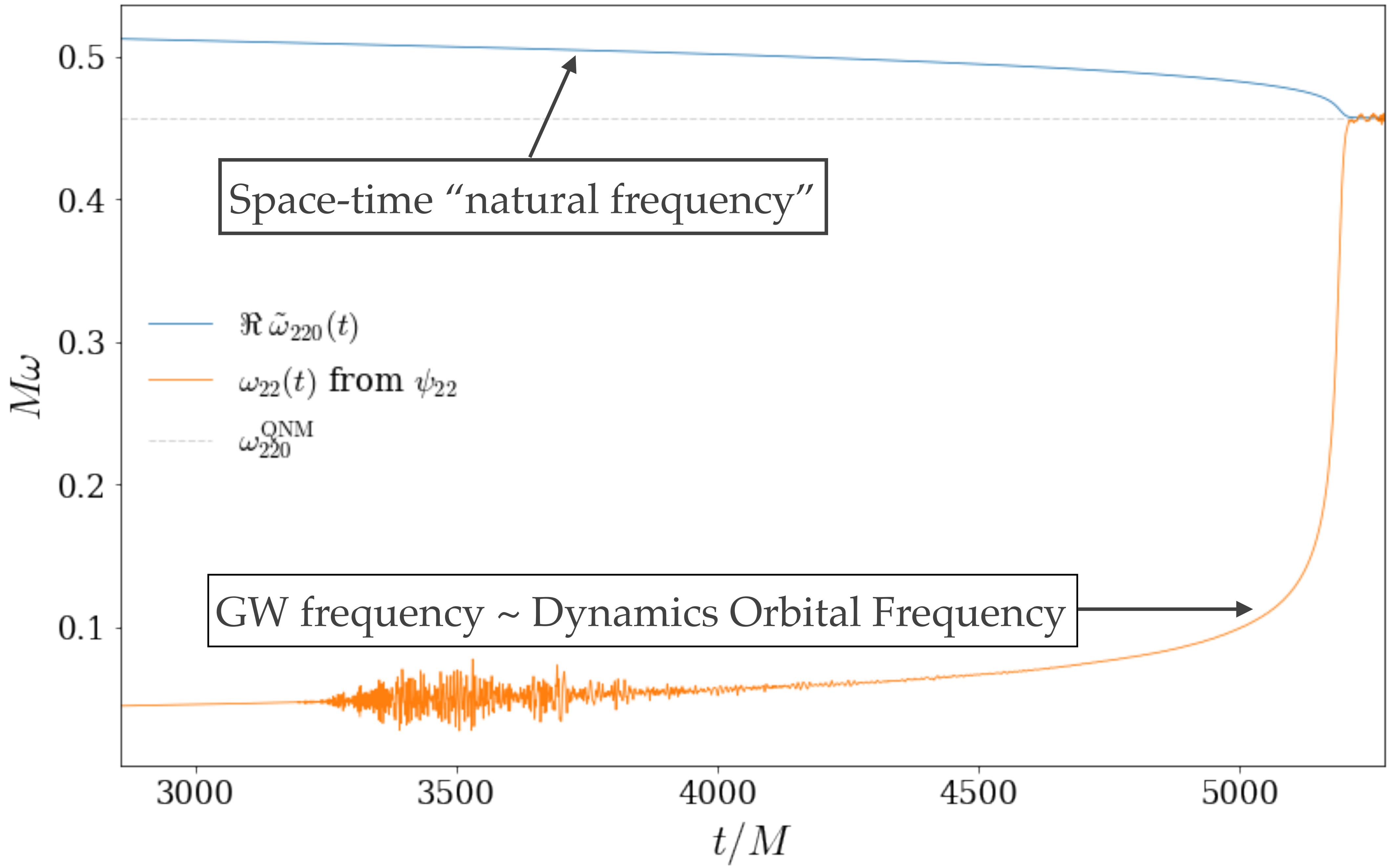
Outline of Method

- Use Numerical Relativity data to estimate the system frequency (per mode) and total angular momentum.
- Input these parameters into Leaver's analytic representation for the spheroidal harmonics — **a set of coupled transcendental equations must be solved**
- Use solutions to the aforementioned equations to calculate dynamical spheroidal functions and related moments.

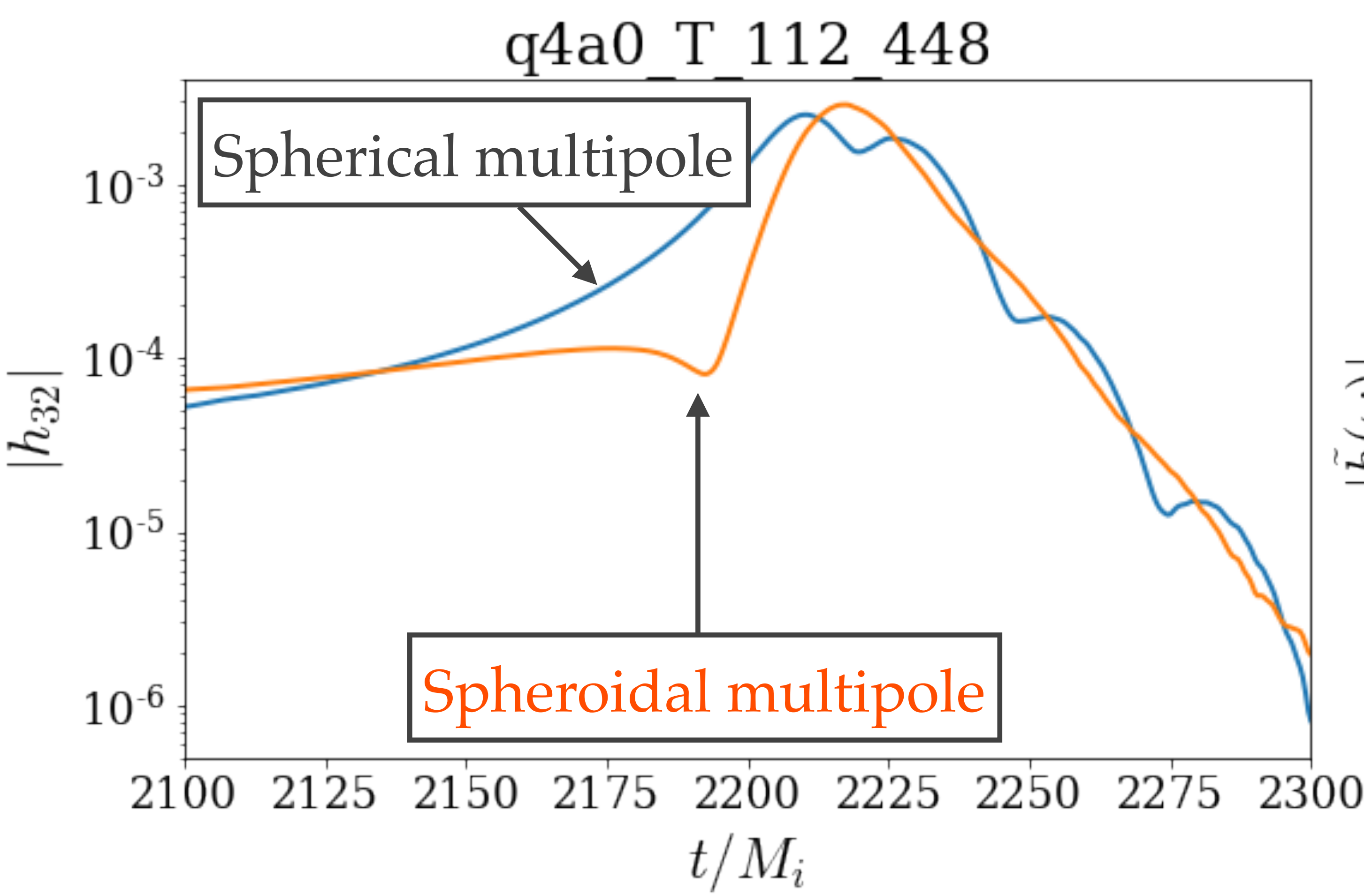
Spheroidal Picture Example Angular Momentum

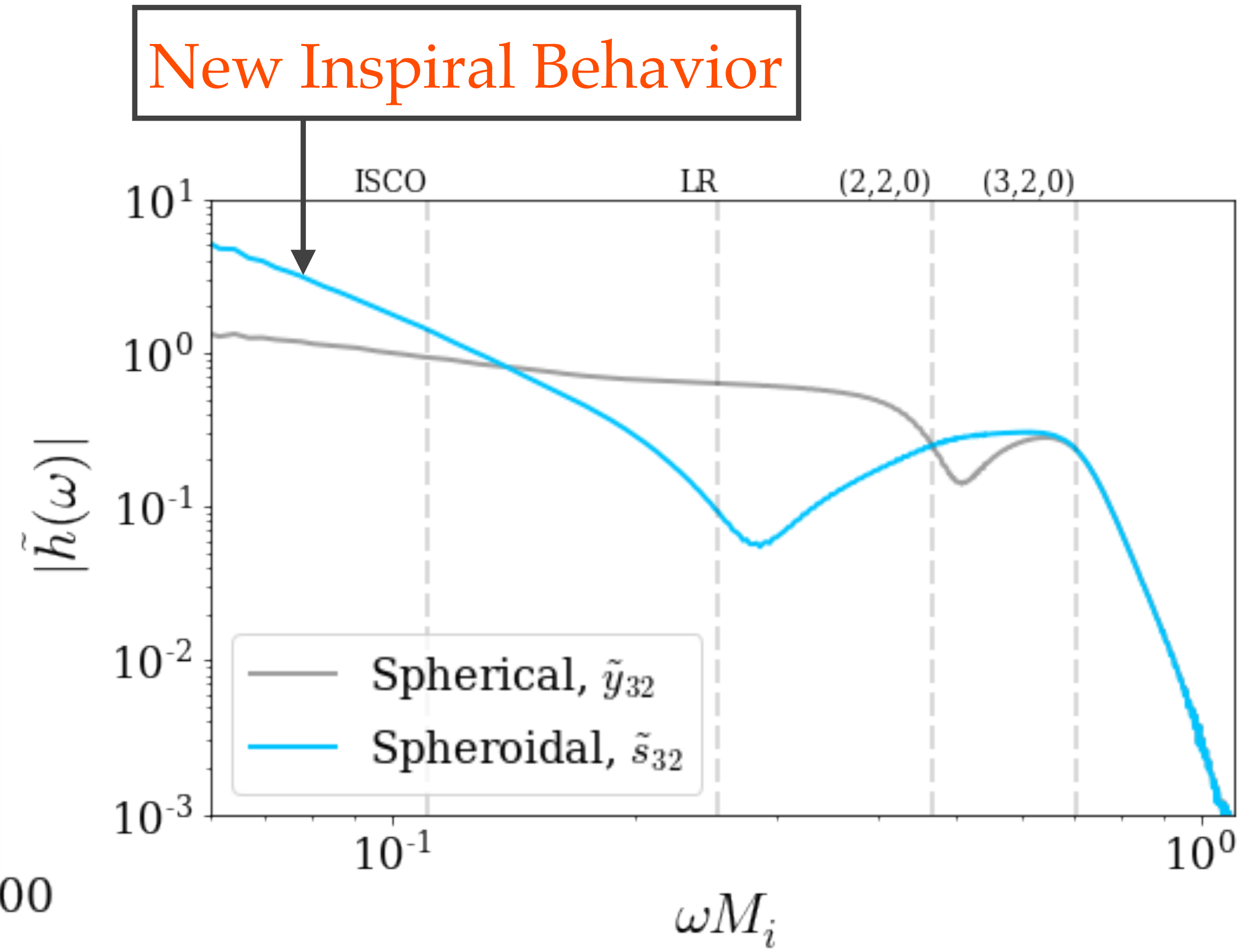


Spheroidal Picture System Frequencies

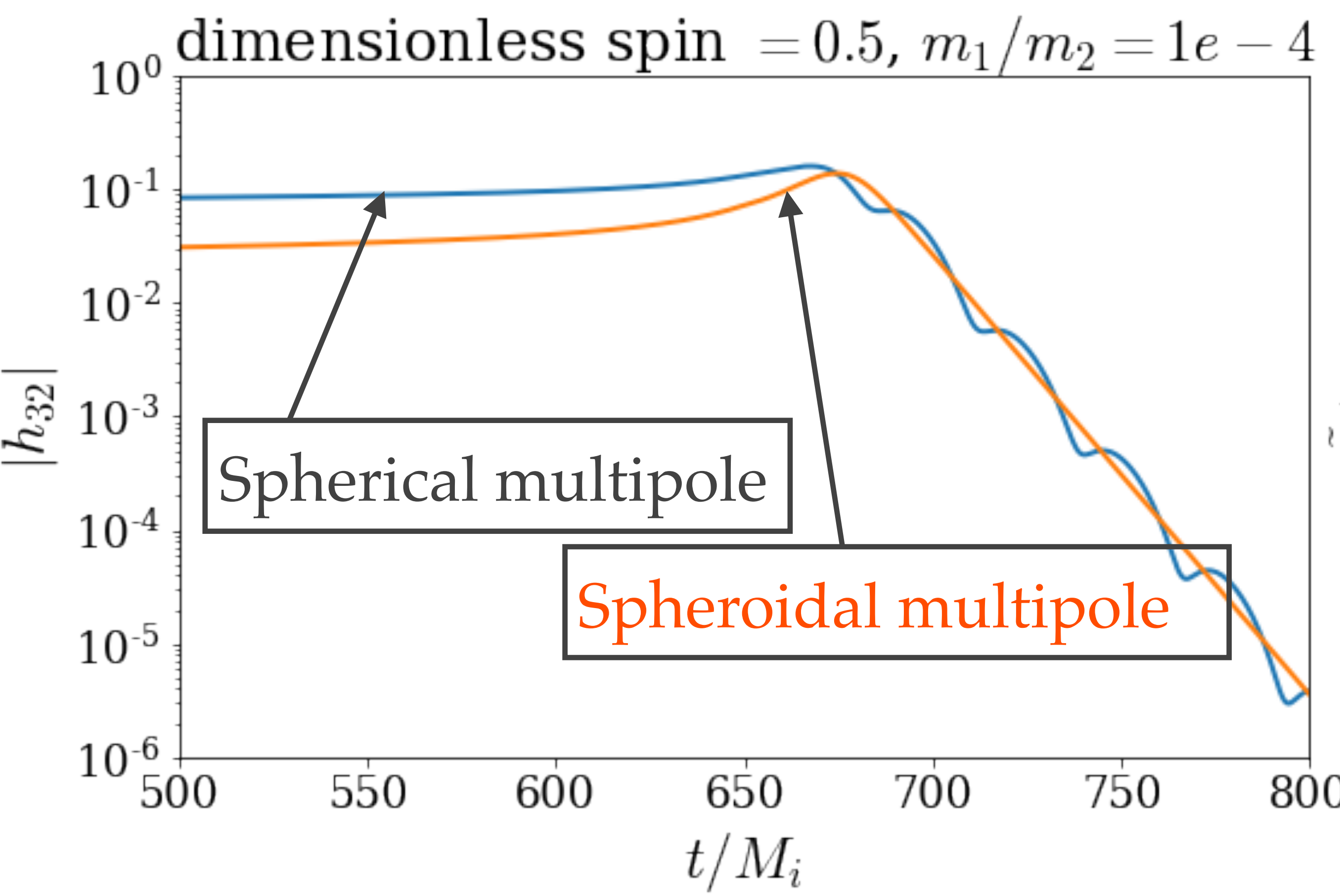


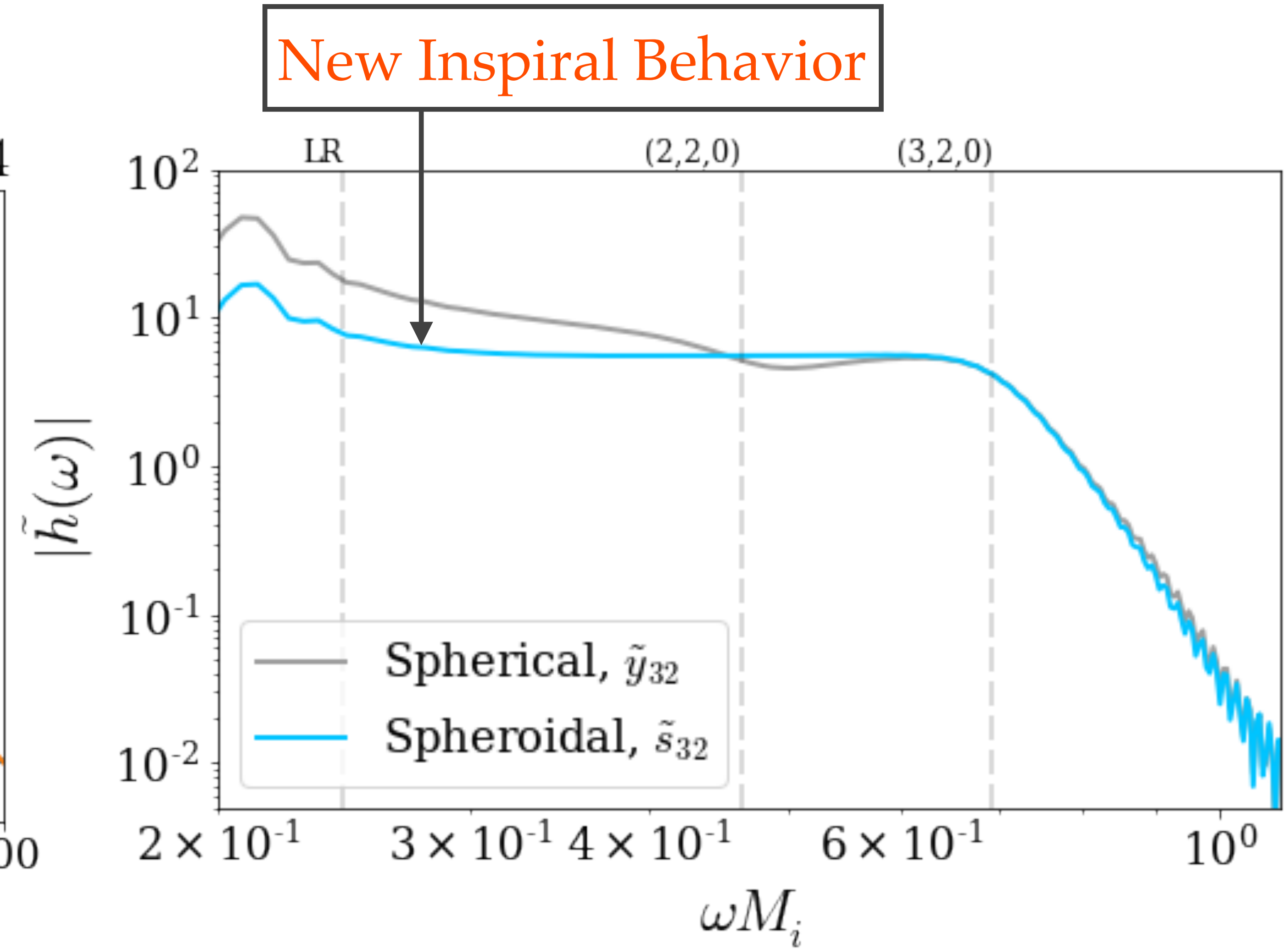
Spheroidal Picture Example End Result





Spheroidal Picture EMRI Example





Concluding remarks & Pandora's Box

Review

There is a long history of viewing GWs as propagating atop a flat background metric; however, this is not likely the most appropriate perspective for BBH systems. A high-level method for a spheroidal harmonic picture has been presented. A publication is in prep.

Unexpected Consequences (Pandora's Box)

- **Time varying "QNM" frequency**
- A generalization of Carter's constant
- **A new, low-level PN might be developed**
- Open mathematical problem related to Spheroidal Harmonics
- **Cleanest for EMRI** cases

Core Questions

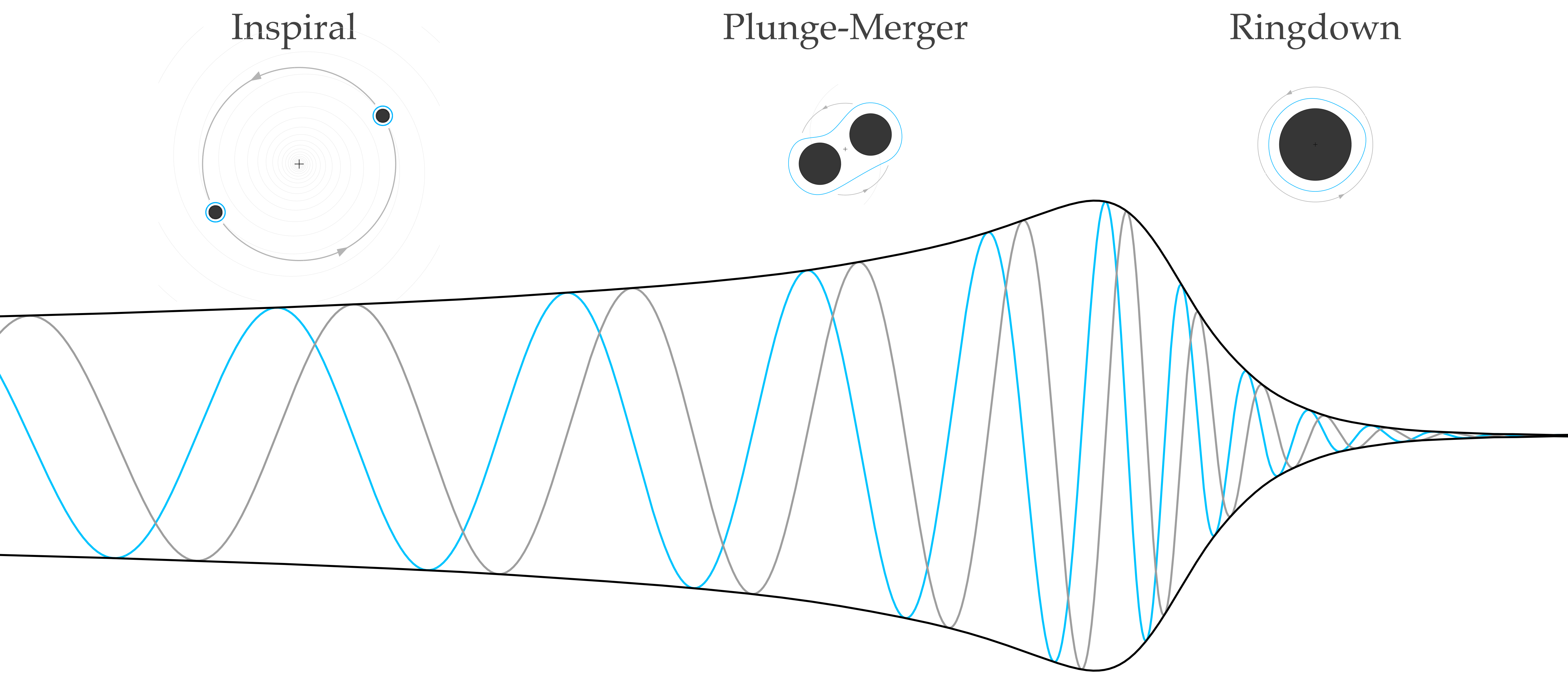
Can a **change in perspective** enable an **advance in physical understanding**?

For gravitational waves (GWs) and black hole binaries (BBHs), can we better understand:

- The GR manifold of solutions for experimental tests?
- The imprint of source properties on GW signals?

For BBH Coalescence: Perhaps Yes

(Paper in preparation)



Towards a new multipolar description of gravitational waves from binary black holes

Thanks!