Gamow-Teller excitations in the Subtracted Second RPA approach

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Outline

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- Spin-isospin nuclear excitations
- Gamow-Teller (GT) strength and the quenching problem
- Theoretical models: RPA and Second RPA
- The Subtraction method within the SRPA $\Rightarrow$ SSRPA
- Results for closed-shell nuclei
- Conclusions and Perspectives

Spin-isospin nuclear excitations

- Nuclear excitations involving the spin and isospin degree of freedom
- Tool to study the spin-isospin correlations in nuclei (spin-isospin component of the nucleon-nucleon interaction)

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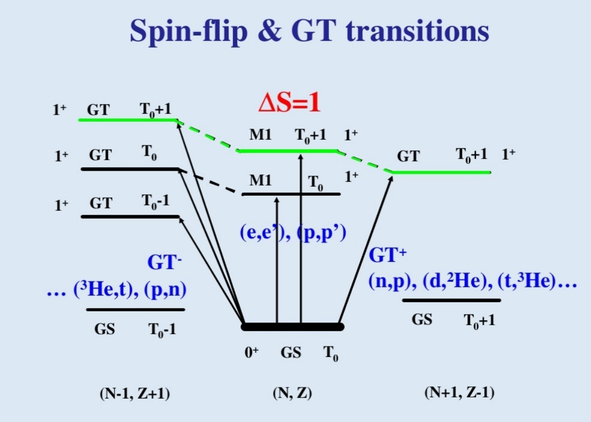
Gamow-Teller (GT) excitations

- The GT excitation is the most well-known spin-isospin mode with
  $J^{\pi} = 1^{+}$, $(\Delta L = 0, \Delta S = 1, \Delta T = 1)$
- GT closely related to the electron capture and β decay rates.
- Strong impact on the r-process nucleosynthesis
- Important also for double-β decay processes
  (especially with the two-neutrino emission)



Gamow-Teller transition

ΔS=1 ΔL=0 ΔT=1

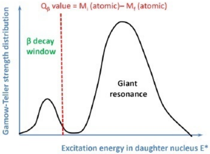
operator

$$\hat{O}_{\mathrm{GT}^-} = \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \tau_-(i)$$

Transition probability

$$B(\mathrm{GT}^-) = \sum_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2$$

$$S_{GT^-} - S_{GT^+} = 3(N - Z).$$



Courtesy of Muhsin N. Harakeh/ and of Y. Niu

The quenching problem

- Computed GT matrix elements **are larger** than the experimental ones.
- The problem is "cured" by **quenching** the strength by $q \sim 0.7$ or using effective axial constant g_A (~ 1) instead of the "bare" value ~ 1.27.

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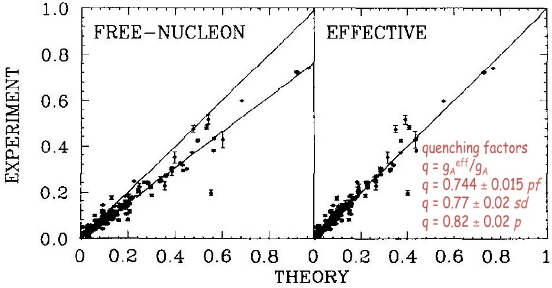
GT matrix elements deduced from the β decay



(from Brown & Wildenthal, Ann.Rev.Nucl. Part.Sci.**38**,(1988)29)

Shell Model calculations

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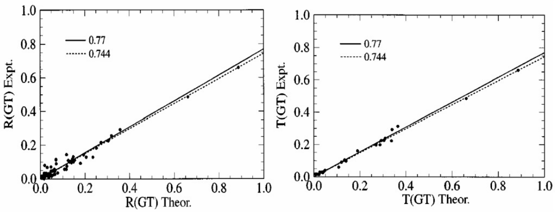
Quenching of ordinary GT β decay matrix elements in the pf shell (*A* = 41-50)
From G. Martinez-Pinedo *et al.*, Phys. Rev. C**53**, R2602 (1996).



Matrix elements scaled by $1/(N-Z)^{1/2}$

Square root of the sum of squares of m.e.

Shell Model calculations

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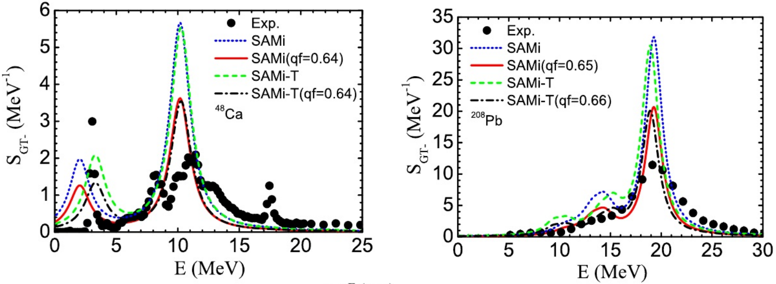

$$qf = \frac{\sum_{E_x=0}^{E_x(\max)} B(GT : E_x)_{\text{expt}}}{\sum_{E_x=0}^{E_x(\max)} B(GT)_{\text{calc}}}$$

Li-Gang Cao , Shi-Sheng Zhang, and H. Sagawa, PHYSICAL REVIEW C 100, 054324 (2019)

Skyrme-RPA calculations

The quenching problem

- Computed GT matrix elements **are larger** than the experimental ones.
- The problem is "cured" by **quenching** the strength by $q \sim 0.7$ or using effective axial constant g_A (~ 1) instead of the "bare" value ~ 1.27.

Possible causes fall in two main classes:

- **Nuclear many- body correlations that escape calculations**: (truncation of the model space, short-range correlations, multi-phonon states, multi particle-hole excitations, ...)
- **Non-nucleonic degrees of freedom**: (Many-nucleon weak currents, Δ-isobar excitations, in-medium modification of pion physics, ...)

The Random Phase Approximation (RPA)

- The RPA is a widely used approximation for the description of NEs
- Very successful especially within the Energy Density Functional framework (interactions á la Skyrme or Gogny, covariant versions)
- It provides global properties: centroid energies and total strength

However, extensions of the RPA are required for:

- Spreading Width
- Fine Structure and Strength Fragmentation
- Double exctiations and Anharmonicities, ...

The Second RPA (SRPA): more general excitation operators are introduced

Phonon Operators: RPA vs SRPA

Random Phase Approximation (RPA)

$$Q_\nu^\dagger = \underbrace{\sum_{ph} X_{ph}^{(\nu)} \underbrace{a_p^\dagger a_h}_{\text{1p-1h}} - \sum_{ph} Y_{ph}^{(\nu)} \underbrace{a_h^\dagger a_p}_{\text{1h-1p}}}_{\text{Only Landau Damping, Centroid Energy and Total Strength of GRs}}$$

Second Random Phase Approximation (SRPA)

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \underbrace{\sum_{p_1<p_2, h_1<h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2}}_{\text{2p-2h}} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2}}_{\text{2h-2p}})}_{\text{Spreading Width, Fragmentation, Double GRs and Anharmonicites, Low-Lying States}}$$

RPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^{(\nu)} a_p^\dagger a_h - \sum_{ph} Y_{ph}^{(\nu)} a_h^\dagger a_p$$

RPA Equations of Motion (1$\mapsto$ 1p1h)

$$\left(\begin{array}{cc} \mathcal{A}_{11} & \mathcal{B}_{11} \\ -\mathcal{B}_{11}^* & -\mathcal{A}_{11}^* \end{array} \right) \left(\begin{array}{c} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{array} \right) = \omega_\nu \left(\begin{array}{c} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{array} \right)$$

SRPA Excitation Operators and Equations

SRPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \sum_{p_1<p_2, h_1<h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})$$

SRPA Equations of Motion (1↦ 1p1h, 2↦ 2p2h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{21} & \mathcal{B}_{22} \\ -\mathcal{B}_{11}^* & -\mathcal{B}_{12}^* & -\mathcal{A}_{11}^* & -\mathcal{A}_{12}^* \\ -\mathcal{B}_{21}^* & -\mathcal{B}_{22}^* & -\mathcal{A}_{21}^* & -\mathcal{A}_{22}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix}$$

SRPA Excitation Operators and Equations

SRPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph}(X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \sum_{p_1<p_2, h_1<h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})$$

SRPA Equations of Motion (1↦ 1p1h, 2↦ 2p2h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{21} & \mathcal{B}_{22} \\ -\mathcal{B}_{11}^* & -\mathcal{B}_{12}^* & -\mathcal{A}_{11}^* & -\mathcal{A}_{12}^* \\ -\mathcal{B}_{21}^* & -\mathcal{B}_{22}^* & -\mathcal{A}_{21}^* & -\mathcal{A}_{22}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix}$$

Computationally very demanding

- Only recently full large scale SRPA calculations have been performed
- Several applications to Charge Conserving Excitations:
  Pygmy dipole resonance, Giant Resonances (ISGQR, IVGDR, ...)

Large scale SRPA calculations have shown that:

- The SRPA strength distribution is sistematically shifted towards lower energies compared to the RPA one
- This shift is very strong ($\simeq$ 5 MeV), RPA description often spoiled

Origins and Causes:

1. Quasi Boson Approximation and stability problems in SRPA
2. Use of effective interactions in beyond-mean field methods

The Subtraction procedure (I. Tselyaev Phys. Rev. C 75, 024306 (2007))

- Designed for beyond RPA approaches
- It restores the Thouless theorem, e.g. instabilities are removed
- Static ($\omega = 0$) limit of the SRPA imposed to be equal to the RPA one

The Subtraction procedure

From SRPA to an Energy dependent RPA-like problem

- The SRPA problem as an energy-dependent RPA problem

$$A_{1,1'} \mapsto \tilde{A}_{1,1'}(\omega) = A^{RPA}_{1,1'} + \sum_{2,2'} A_{1,2}(\omega + i\eta - A_{2,2'})^{-1} A_{2',1'} = A^{RPA}_{1,1'} + A^{Cor}_{1,1'}(\omega)$$

The Subtraction procedure

From SRPA to an Energy dependent RPA-like problem

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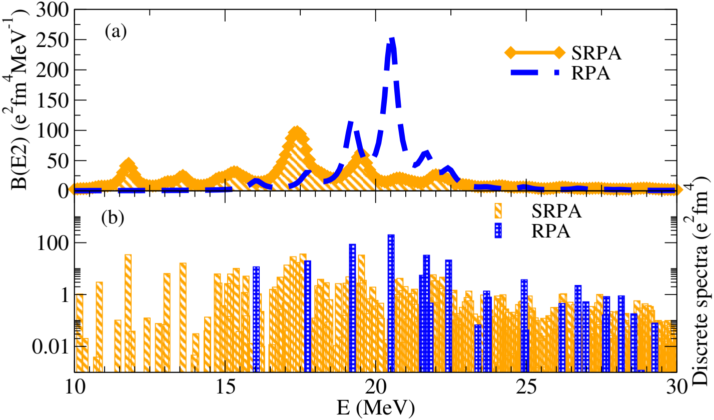
The Subtraction procedure is SRPA (SSRPA)

- Subtraction of the zero–frequency limit of the SRPA correction

$$A^{Cor}_{1,1'} \mapsto \tilde{A}^{Cor}_{1,1'}(\omega) = A_{1,1'}(\omega) - A_{1,1'}(\omega = 0) \Rightarrow$$

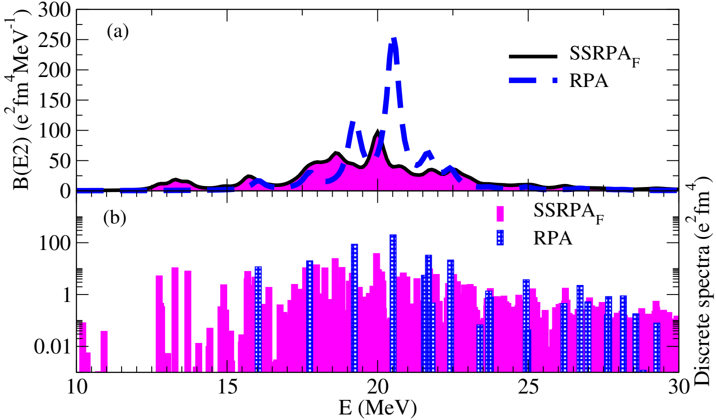
$$\tilde{A}_{1,1'}(\omega = 0) = A^{RPA}_{1,1'}$$

$$\Rightarrow \Pi^{SSRPA}(\omega = 0) = \Pi^{RPA}$$



D. G., M. Grasso and J.Engel, Phys. Rev. C 92 , 034303 (2015)

Quadrupole Strength Distribution in ^{16}O: RPA, SRPA and SSRPA



D. G., M. Grasso and J.Engel, Phys. Rev. C 92 , 034303 (2015)

Recent extension to the charge exchange (CE) case

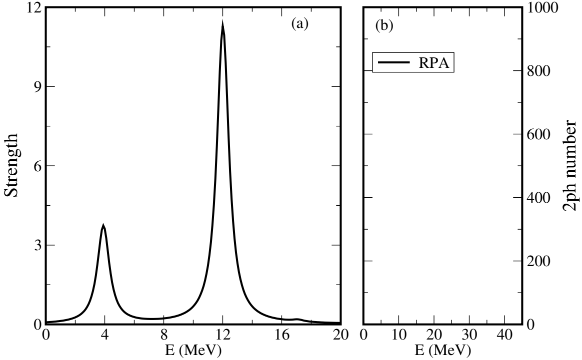
Second RPA for CE excitations

- Extension to the treatment of **CE excitations**
- **Skyrme** interaction
- **Fully Self-Consistent** scheme, e.g. same interaction employed in ground state and excited states description
- **Subtracted SRPA (SSRPA)**
  (to eliminate double counting issues and instabilities)
- First applications to ^{48}Ca (lightest double-β emitter) and ^{78}Ni in Ref [1]
- More applications (^{14}C, ^{22}O, ^{90}Zr and ^{132}Sn) in Ref [2]

More details in:

[1] D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)
[2] D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

GT$^-$ Strength Distribution ^{48}Ca, 2p2h cutoff EC (MeV) dependence



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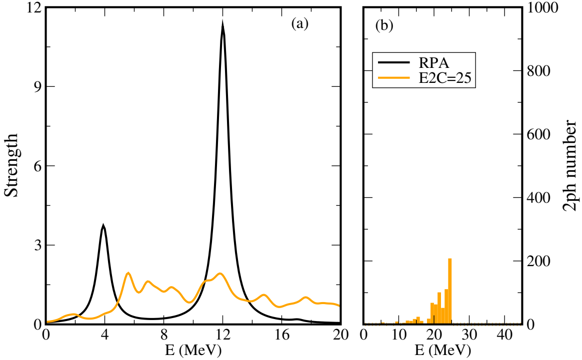

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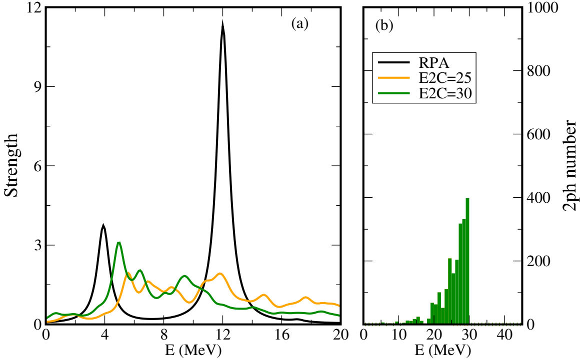

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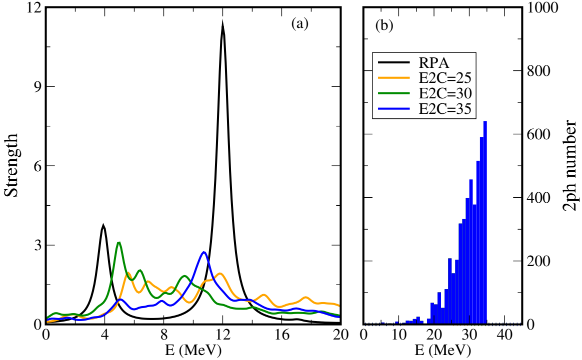

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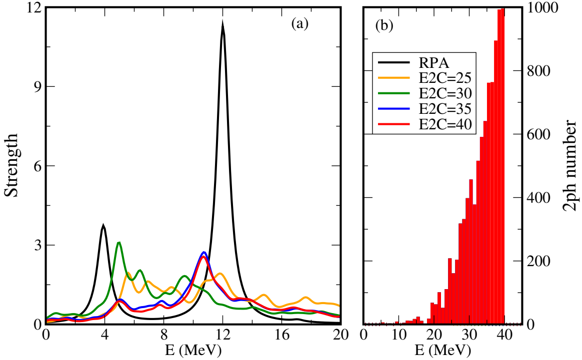
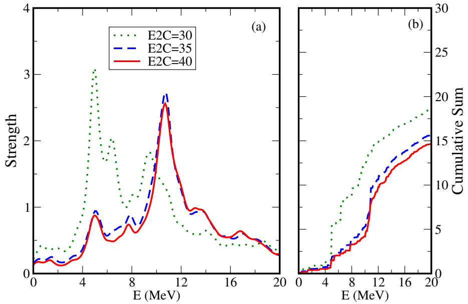
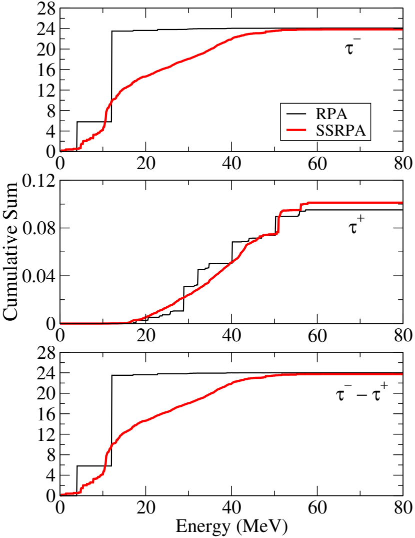


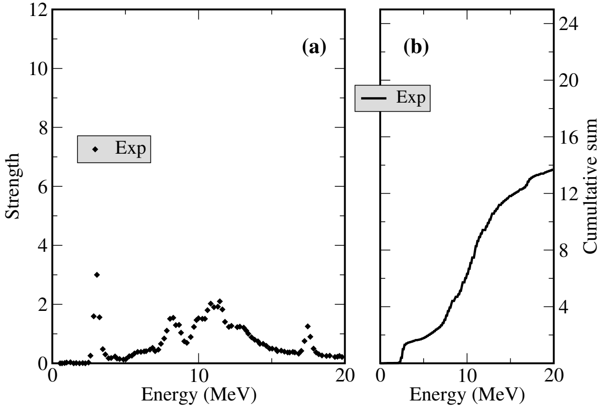


SSRPA results, 2p-2h cutoff E2C value in MeV units.

GT$^-$ Strength Distribution ^{48}Ca, sum rules in the two channels



GT$^-$ Strength Distribution ^{48}Ca, SGII interaction



(a) GT$^-$ strength in RPA and SSRPA compared with (GT$^-$ plus IVSM) data.
(b) Cumulative strengths up to 20 MeV.
Data from: K. Yako *et al.*, Phys. Rev. Lett. 103, 012503 (2009)
From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

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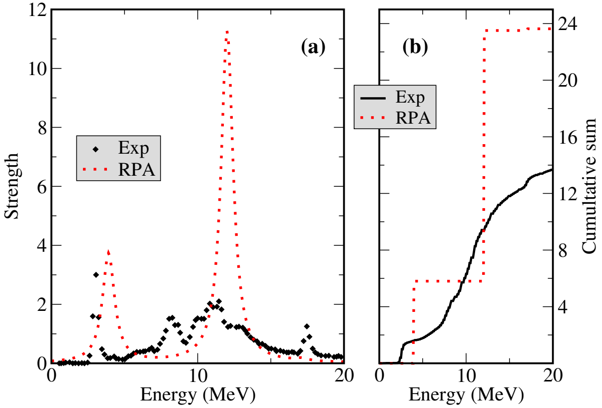

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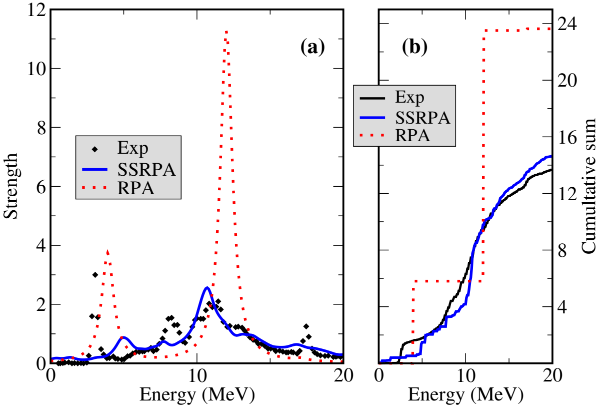

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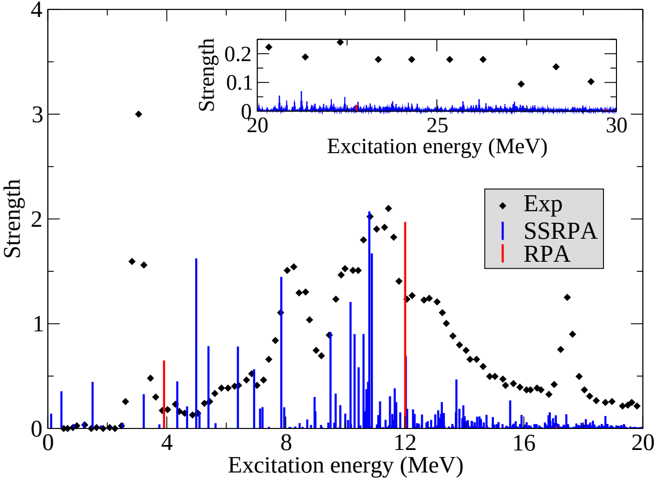
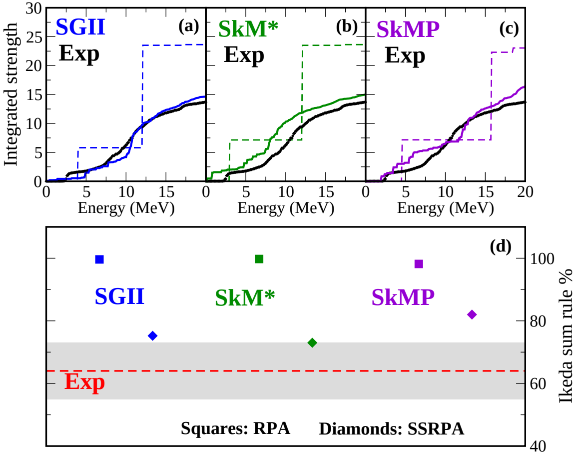


Experimental GT$^-$ in MeV^{-1} and discrete RPA and SSRPA strength distributions (no units) obtained with the Skyrme parameterization SGII, for ^{48}Ca . The RPA strength has been divided by nine and the SSRPA strength by two.

From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

GT$^-$ Strength Distribution ^{48}Ca



(a), (b), (c) Strengths integrated up to 20 MeV with different parameterizations.

(d) RPA and SSRPA percentages of the Ikeda sum rule below 30 MeV compared with the experimental one.

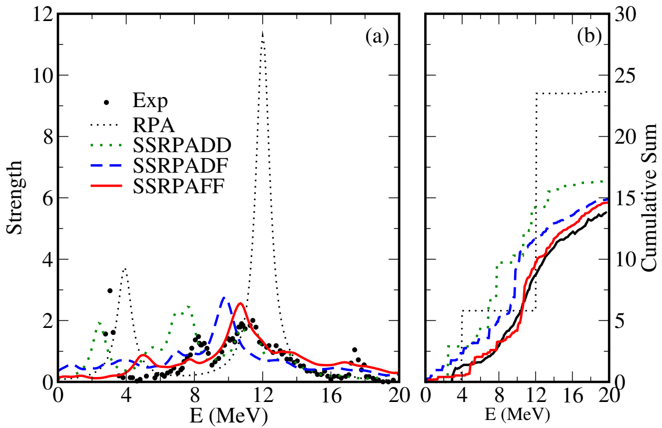
From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

Numerical complexity

- The most demanding task is related to the treatment of the $A_{22'}$ matrix
- The number of 2p-2h configurations can be very large$\simeq 10^7, 10^8$
- We need to calculate the "full" spectrum
- Most demanding tasks:
  a) subtraction procedure, $A_{22'}$ inversion
  b) diagonalization of the SSRPA eigenvalue problem
- Strong simplification if $A_{22'}$ is assumed to be diagonal

Different calculation scheme:

1. SSRPADD: $A_{22'}$ is Diagonal both in a) and b)
2. SSRPADF: $A_{22'}$ is Diagonal both in a) and Full in b)
3. SSRPAFF: $A_{22'}$ is Full both in a) and in b)



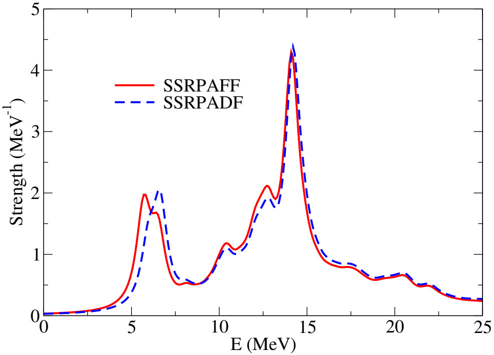
Comparison between SSRPADD, SSRPADF and SSRPAFF results .

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

GT$^-$ Strength Distribution for ^{90}Zr, SSRPADF vs SSRPAFF

Calculations fo heavier systems

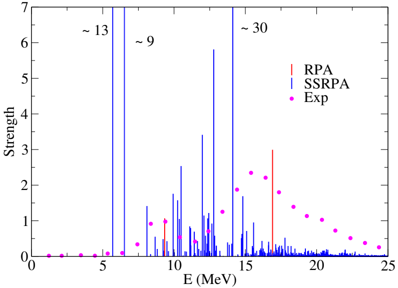
For heavier nuclei, the inversion of the A_{22} becomes prohibitive.
We checked that the diagonal approximation provides relieable results



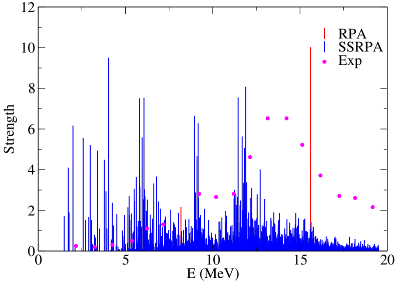
GT$^-$ strengths obtained for the nucleus ^{90}Zr in MeV^{-1}. SSRPAFF and SSRPADF spectra are shown.

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

GT^- Strength Distribution for ^{90}Zr and ^{132}Sn , discrete spectrum



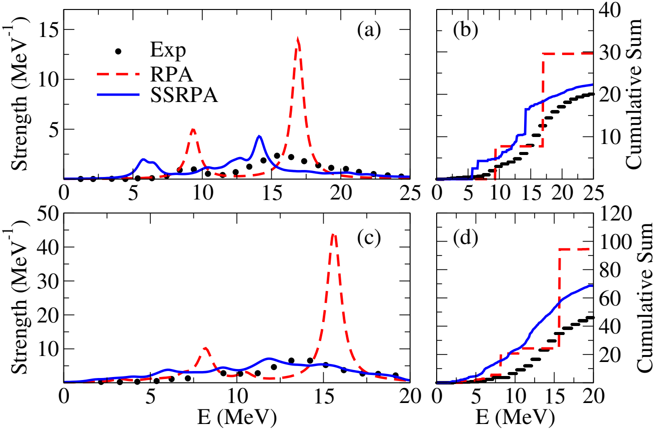
Strength Distribution for ^{90}Zr: RPA is divided by 7 and the SSRPA is multiplied by 5.



Strength Distribution for ^{132}Sn: RPA is divided by 7 and the SSRPA is multiplied by 14.

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

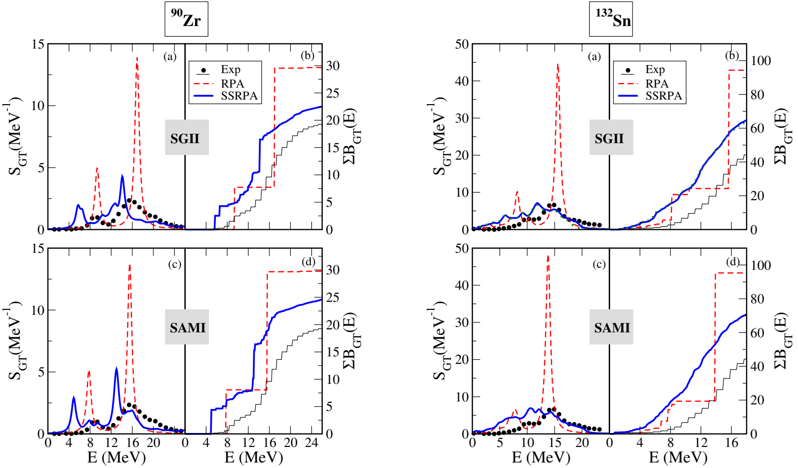
GT^{-} Strength Distribution for ^{90}Zr and ^{132}Sn



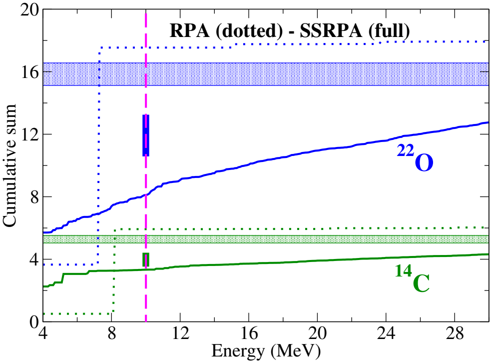
(a) GT^{-} strength and sumulative sum for ^{90}Zr (upper panels) and ^{132}Sn (lower panels).

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

GT$^-$ Strength Distribution ^{90}Zr and ^{132}Sn, interaction dependence



GT$^-$ Strength Distribution for the nuclei ^{22}O (blue) and ^{14}C (green): SSRPA versus *ab initio* Coupled Cluster including two-body currents [1].



The blue and green horizontal areas represent the reduction of the total Ikeda sum rule $S_{GT^-} - S_{GT^+}$ from *ab initio* results [1].
The blue and green vertical intervals correspond to a reduction of (70-80 %) of the sum rule exhausted at 10 MeV.
[1] A. Ekstrom *et al.* Phys. Rev. Lett. 113, 262504 (2014)
See also Gysbers et al. Nature Phys. 15 428 (2019)

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

Beta-decay Life time

Beta-decay Half-life

$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z,\omega) dE}.$$

$$f(Z,\omega) = \int_{m_e c^2}^{\omega} p_e E_e (\omega - E_e)^2 F_0(Z+1, E_e) dE_e,$$

Fermi function of the emitted electron

Gamow-Teller transition

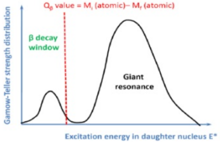
ΔS=1 ΔL=0 ΔT=1

operator

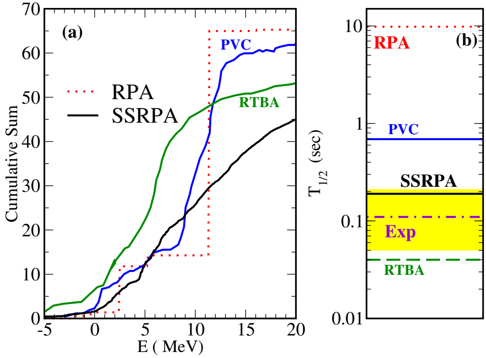
$$\hat{O}_{\mathrm{GT}^-} = \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \tau_-(i)$$

Transition probability

$$B(\mathrm{GT}^-) = \sum_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2$$



GT$^-$ Strength Distribution and β-decay half-life ^{78}Ni



(a) Cumulative sum for the nucleus ^{78}Ni within the SSRPA, PVC and RTBA models;
(b) β-decay half-life for ^{78}Ni. **No quenching, bare $g_a = 1.27$;**
Data from: P. T. Hosmer *et al.* Phys. Rev. Lett. 94, 112501 (2005)
PVC: Y. F. Niu, G. Coló and E. Vigezzi, Phys. Rev. C 90, 054328 (2014)
RTBA:C. Robin and E. Litvinova, Phys. Rev. C 98, 051301(R), 2018

From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

Conclusions

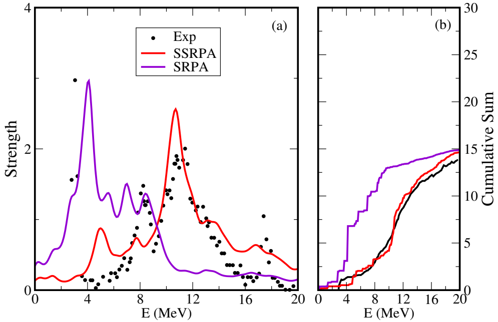
- First applications of the CE-SSRPA model to GT strength
- Inclusion of 2p2h configurations produces quenching of the strength
- Better agreement with data without *ad hoc* quenching factors
- Other effects might contribute to quenching (especially for heavy systems)
- Improved description of β-decay half-life in ^{78}Ni
  (also compared with other beyond-mean-field approaches)

Perspectives

- More systematic calculations of β-decay half-life (in progress)
- Extension to open-shell nuclei, pairing correlations with the Second QRPA
- SSRPA calculations for double-beta decays studies

Thanks For Your Attention !!!

Backup Slides



Subtracted SRPA (SSRPA) versus SRPA (no subtraction).