Jacobi NCSM for light (hyper)nuclei

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- Introduction
- Numerical technique
  - Jacobi no-core shell model for S=0, -1 systems
  - Similarity Renormalization Group (SRG)
- Results for $A = 3 - 6$ nuclei with chiral NN + 3N interactions
- Results for $A = 4 - 7$ Λ hypernuclei: ${}^{4}_{\Lambda}\mathrm{He}$, ${}^{5}_{\Lambda}\mathrm{He}$, ${}^{7}_{\Lambda}\mathrm{Li}$
- Summary

Ab initio (hyper)nuclear structure calculations

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Goal: solving the non-relativistic A-body Schrödinger Eq. for bound states

$$\mathrm{H}\,|\Psi_n\rangle = \mathrm{E}_n\,|\Psi_n\rangle, \qquad \mathrm{H} = \sum_{i=1}^{A} \frac{\mathbf{k}_i^2}{2m} + \sum_{i<j=1}^{A-1} \mathrm{V^{NN}}_{ij} + \sum_{i<j<k=1}^{A-1} \mathrm{V^{NNN}}_{ijk} + \sum_{i}^{A-1} \mathrm{V^{YN}}_{iY}$$

ab initio:

- nucleons and hyperons are fundamental degrees of freedom
- realistic (microscopic) nuclear interactions as in put: chiral EFT NN, 3N +YN
- controlled & improvable truncations

ab initio (few)many-body approaches:

- Faddeev-Yakubovsky integral equation, exact solutions for $A \leq 4$, arXiv:nucl-th/0004023
- Green's function Monte Carlo, require local interactions, arXiv:0804.3501
- No-core shell model (NCSM), arXiv:0904.0463
- Coupled-cluster method, arXiv:1312.7872

  } → need large model space (soft interactions)
- Lattice Monte Carlo, suitable for states with complex geometries, arXiv:0804.3501
- ...

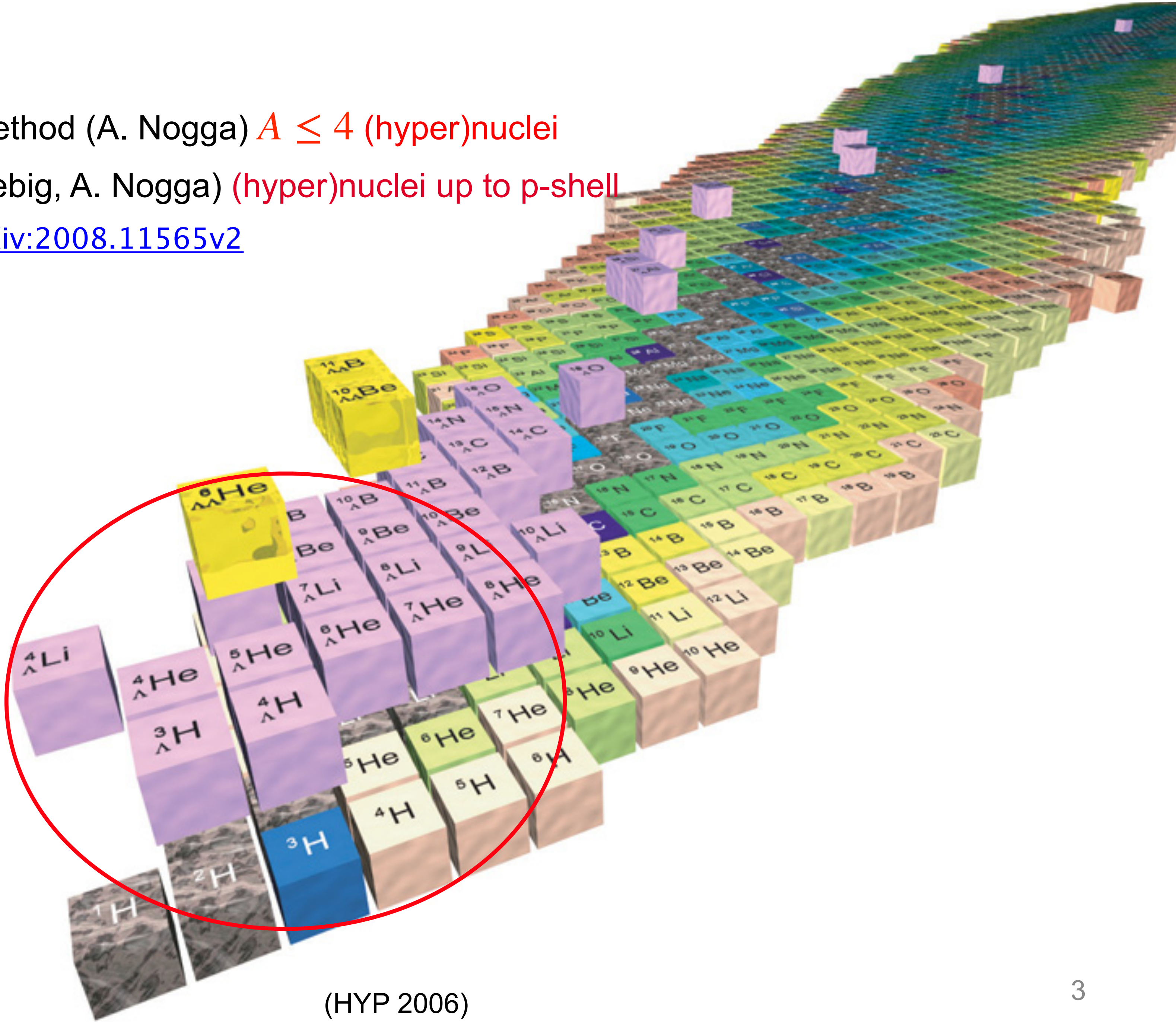
Mitglied der Helmholtz-Gemeinschaft

Ab initio (hyper)nuclear structure calculations

At our disposal:

- Faddeev-Yakubovsky method (A. Nogga) $A \leq 4$ (hyper)nuclei
- Jacobi NCSM (HL, S. Liebig, A. Nogga) (hyper)nuclei up to p-shell

  arXiv:1510.06070v1 , arXiv:2008.11565v2

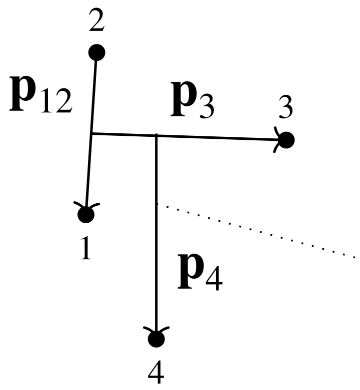


(HYP 2006)

No-core shell model (NCSM)

- Idea: represent the A-body translationally invariant (hyper)nuclear Hamiltonian

$$
\begin{aligned}
\mathrm{H}_{\mathrm{int}} &= \sum_{i=1}^{A} \frac{\mathbf{k}_i^2}{2m} + \sum_{i<j=1}^{A} \mathrm{V}_{\mathrm{ij}}^{\mathrm{NN}} + \sum_{i<j<k=1}^{A-1} \mathrm{V}_{\mathrm{ijk}}^{\mathrm{NNN]}} + \sum_{i=1}^{A-1} \mathrm{V}_{\mathrm{iY}}^{\mathrm{YN}} - \frac{\mathbf{P}^2}{2M} \\
&= \sum_{i<j=1}^{A} \frac{m_i + m_j}{M} \frac{\mathbf{p}_{ij}^2}{\mu} + \sum_{i<j=1}^{A-1} \mathrm{V}_{\mathrm{ij}}^{\mathrm{NN}} + \sum_{i<j<k=1}^{A-1} \mathrm{V}_{\mathrm{ijk}}^{\mathrm{NNN}} + \sum_{i=1}^{A-1} \mathrm{V}_{\mathrm{iY}}^{\mathrm{YN}}, \qquad \mathbf{p}_{ij} = \frac{1}{2}(\mathbf{k}_i - \mathbf{k}_j)
\end{aligned}
$$



in a basis constructed from HO functions: $\phi_{nlm}(\mathbf{p}) = \langle \mathbf{p} \,|\, nlm \rangle = R_{nl}(p) Y_{lm}(\hat{p})$

- Two approaches to construct basis states:
  - Slater determinant basis depending on single-particle coordinates (m-scheme NCSM)
    - → antisymmetric, but contain CM motion ⇒ large dimension
    - → importance truncated basis (IT-NCSM) for p-shell

    (**R. Wirth et al PRL (2014,2016), PRC(2018)**)

  - Jacobi basis expressed in relative Jacobi coordinate (Jacobi NCSM): $|\bigcirc\rangle \equiv |\mathcal{N}\, \mathrm{J}\, \mathrm{T}, i\rangle$ ✓
    - → preserve translational symmetry of H, no CM motion ⇒ small dimension
    - → antisymmetrization of basis states is demanding $A \leq 9$

    (**H. Le at al EPJA 8 (2020), PLB (2020)**)

Hamiltonian matrix elements in Jacobi basis

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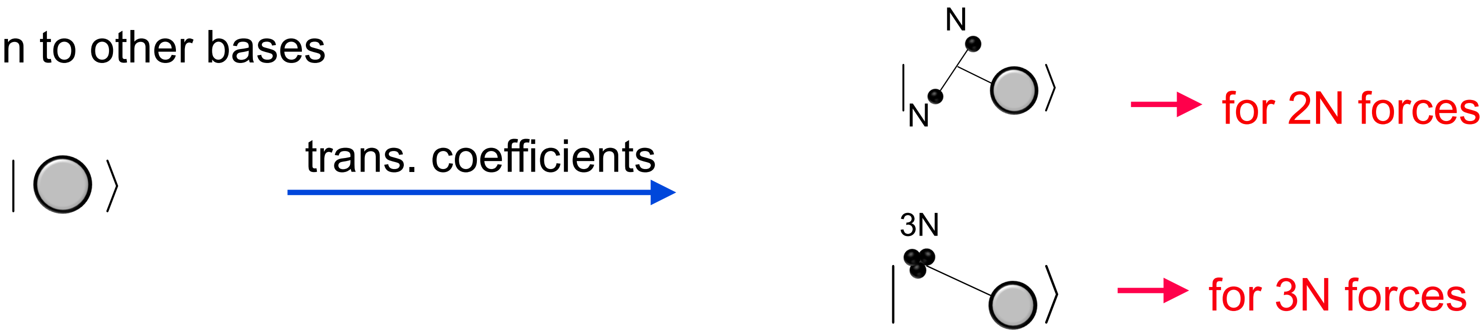
$$\mathrm{H}^{\mathrm{Nucl}} = \sum_{i<j=1}^{A} \left(\frac{2}{A}\frac{\mathbf{p}_{ij}^2}{m} + \mathrm{V}_{\mathrm{ij}}^{\mathrm{NN}}\right) + \sum_{i<j<k=1}^{A} \mathrm{V}_{\mathrm{ijk}}^{\mathrm{NNN}} = \sum_{i<j=1}^{A} h_{ij}^{NN} + \sum_{i<j<k=1}^{A} \mathrm{V}_{\mathrm{ijk}}^{\mathrm{NNN}},$$

- evaluating $\langle i\mathcal{N}JT\,|\,\mathrm{H}^{\mathrm{Nucl}}\,|\,i\mathcal{N}JT\rangle \equiv \langle\bigcirc|\ \mathrm{H}^{\mathrm{Nucl}}\ |\bigcirc\rangle$:
  - transformation to other bases



$$\Rightarrow \langle\bigcirc|\sum_{i<j=1}^{A} h_{ij}^{NN}|\bigcirc\rangle = \langle\bigcirc|\bullet\!-\!\bigcirc\rangle\langle\bullet\!-\!\bigcirc|\sum_{i<j=1}^{A} h_{ij}^{NN}|\bullet\!-\!\bigcirc\rangle\langle\bullet\!-\!\bigcirc|\bigcirc\rangle$$

$$= \langle\bigcirc|\bullet\!-\!\bigcirc\rangle\ \delta_{core(A-2)}\binom{A}{2}\underbrace{\langle\alpha_{12}|h_{ij}^{NN}|\alpha'_{12}\rangle}_{\text{2-body matrix element}}\ \langle\bullet\!-\!\bigcirc|\bigcirc\rangle$$

arXiv:1510.06070v1

- basis truncation:

$$\mathcal{N} = \mathcal{N}_{A-2} + N_{2N} + 2n_{\lambda_{2N}} + \lambda_{2N} = \mathcal{N}_{A-3} + N_{3N} + \lambda_{3N} + 2n_{\lambda_{3N}} \le \mathcal{N}_{max} \Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$$

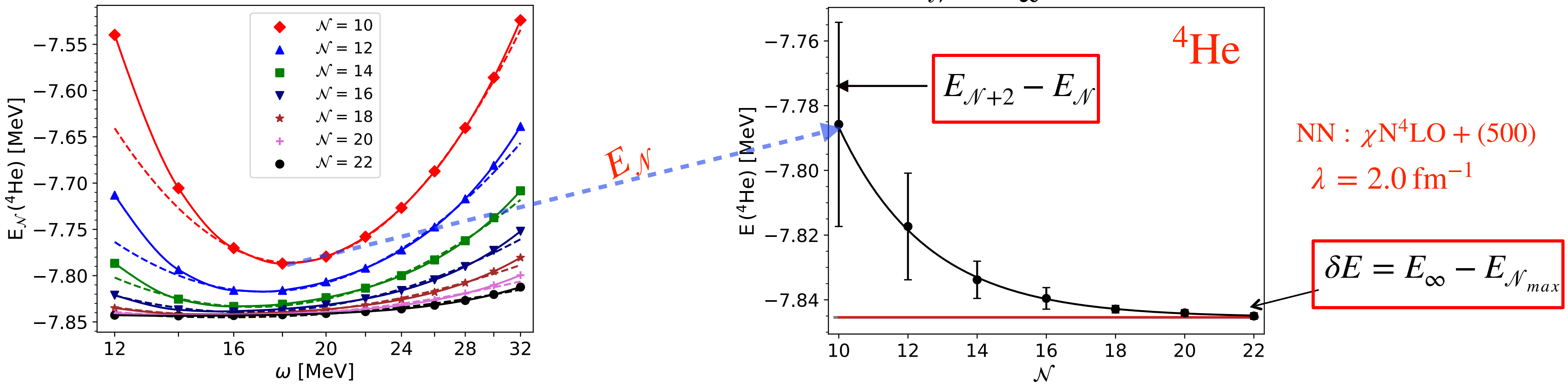
→ require $\mathcal{N}_{max} \to \infty$ extrapolation

Extrapolation of binding energies

- Two parameters in NCSM: HO-ω, and $\mathcal{N}$ $\Rightarrow$ $E_b = E_b(\omega, \mathcal{N})$

$$E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$$

$$E_{\mathcal{N}} = E_{\infty} + Ae^{-b\mathcal{N}}$$



- ω_{opt} shifts to smaller values as $\mathcal{N}$ increases
- ω-dependence energy curves flatten with increasing $\mathcal{N}$

- $E_{\mathcal{N}}$ converges to E_{∞} strictly from above

Chiral NN & YN interactions

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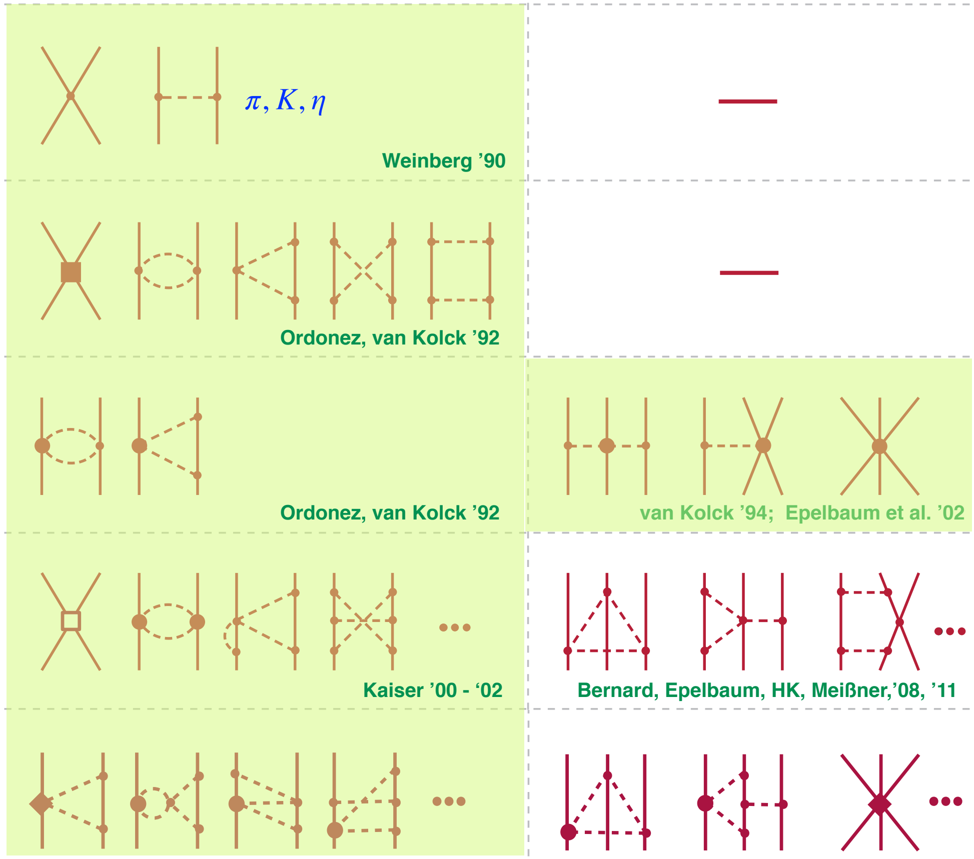


	BB force	3B force	
LO (Q^0)	π, K, η — Weinberg '90	—	2 (5) NN (YN) LECs short range parameters
NLO (Q^2)	Ordonez, van Kolck '92	—	+7 (+23) NN (YN) LECs
N^2LO (Q^3)	Ordonez, van Kolck '92	van Kolck '94; Epelbaum et al. '02	+2 YN LECs
N^3LO (Q^4)	Kaiser '00 - '02	Bernard, Epelbaum, HK, Meißner,'08, '11	+12 NN LECs
N^4LO (Q^5)	Entem, Kaiser, Machleidt, Nosyk '15 Epelbaum, HK, Meißner '15	Girlanda, Kievsky, Viviani '11 HK, Gasparyan, Epelbaum '12,'13	+5 NN LECs

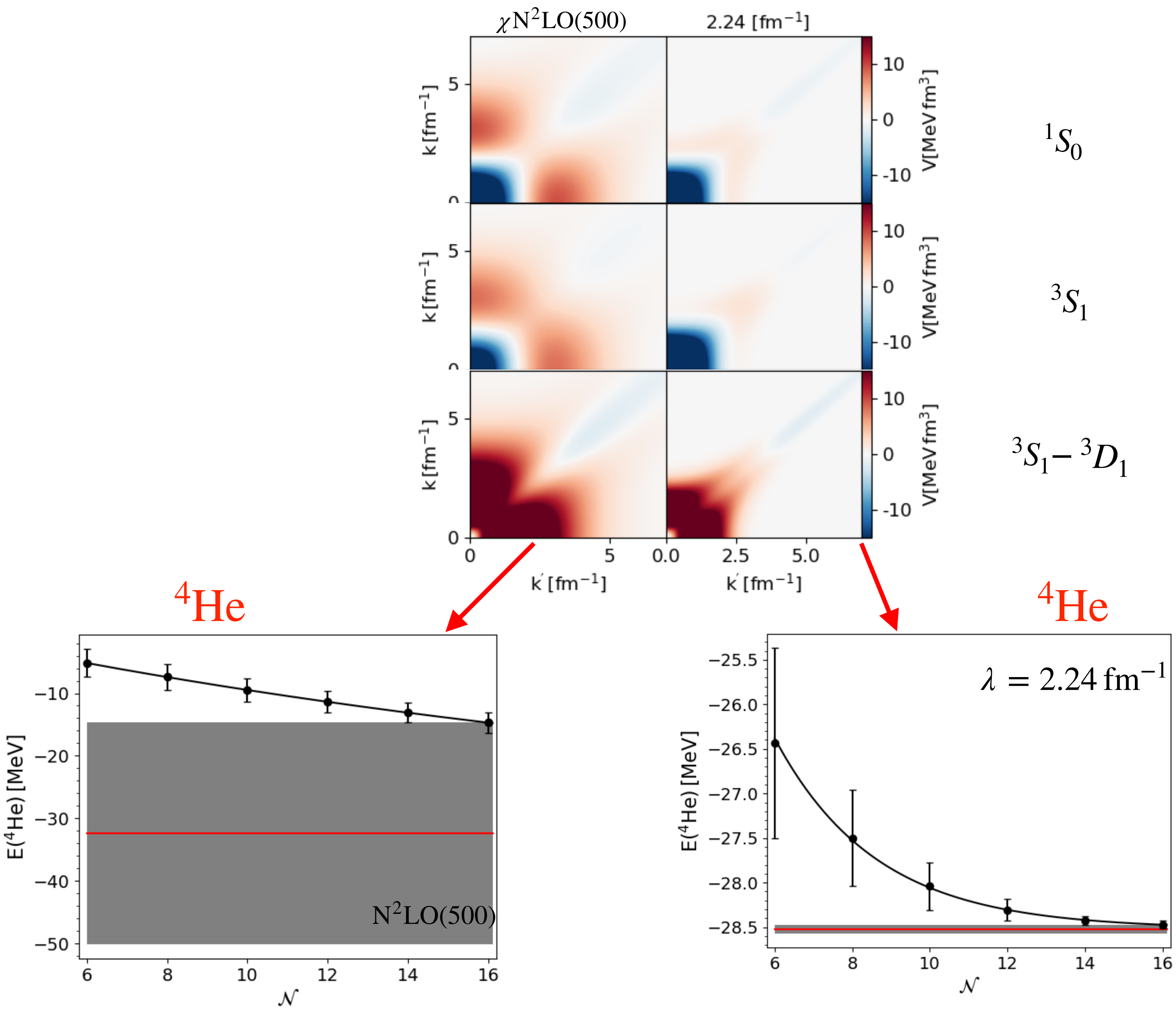
(adapted from K. Hermann CD workshop, 18th November 2021)

- LECs are determined via a fit to experiment:
  - 5000 NN data + deutron → NN forces up to N^4LO+, 3NF up to N^3LO **(Bochum, Bonn, Idaho,…)**
  - 37 YN data, no YN bound state → YN forces up to NLO (**NLO13, NLO19 J. Haidenbauer, U-G. Meißner,..)**

→ additional constraints are expected from studying **(light) hypernuclei or lattice simulations**

Convergence of E with respect to $\mathcal{N}$

- BB interactions contain short-range and tensor correlations that couple low- and high-momentum states → NCSM calculations converge slowly



Similarity Renormalization Group (SRG)

Idea: continuously apply unitary transformation to H to suppress off-diagonal matrix elements

⟶ observables are conserved due to unitarity of transformation

F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$\frac{dV(s)}{ds} = \left[\left[T_{rel}, V(s)\right], H(s)\right], \qquad H(s) = T_{rel} + V(s); \qquad T_{rel} = T_{12} + T_3 = T_{23} + T_1 = T_{31} + T_2$$

$$V(s) = V_{12}(s) + V_{13}(s) + V_{23}(s) + V_{123}(s) + \ldots$$

- separate SRG flow equations for NN (YN) and 3N interactions:

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= [[T_{ij}, V_{ij}], T_{ij} + V_{ij}], \\
\frac{dV_{123}}{ds} &= [[T_{12}, V_{12}], V_{31} + V_{23} + V_{123}] \\
&+ [[T_{31}, V_{31}], V_{12} + V_{23} + V_{123}] \\
&+ [[T_{23}, V_{23}], V_{12} + V_{31} + V_{123}] \\
&+ [[T_{\text{rel}}, V_{123}], H_s] \; .
\end{aligned}$$

Eqs.(1)

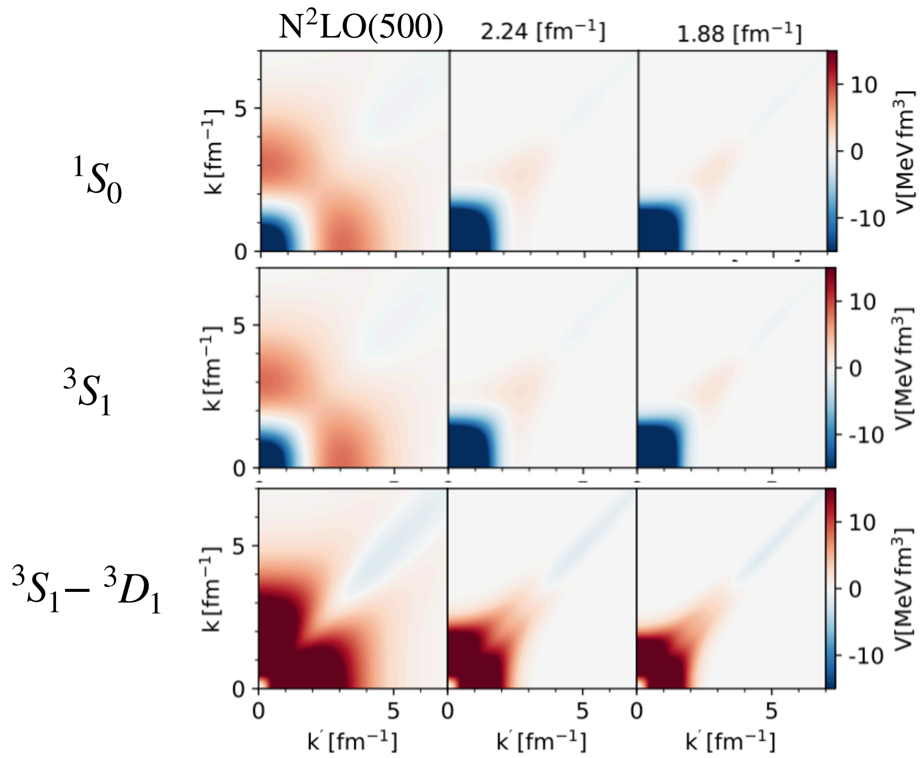
⇒ no disconnected terms in $\frac{dV_{123}}{ds}$: dangerous delta functions are cancelled

- Eqs.(1) are solved by projecting on a partial-wave decomposed Jacobi-momentum basis

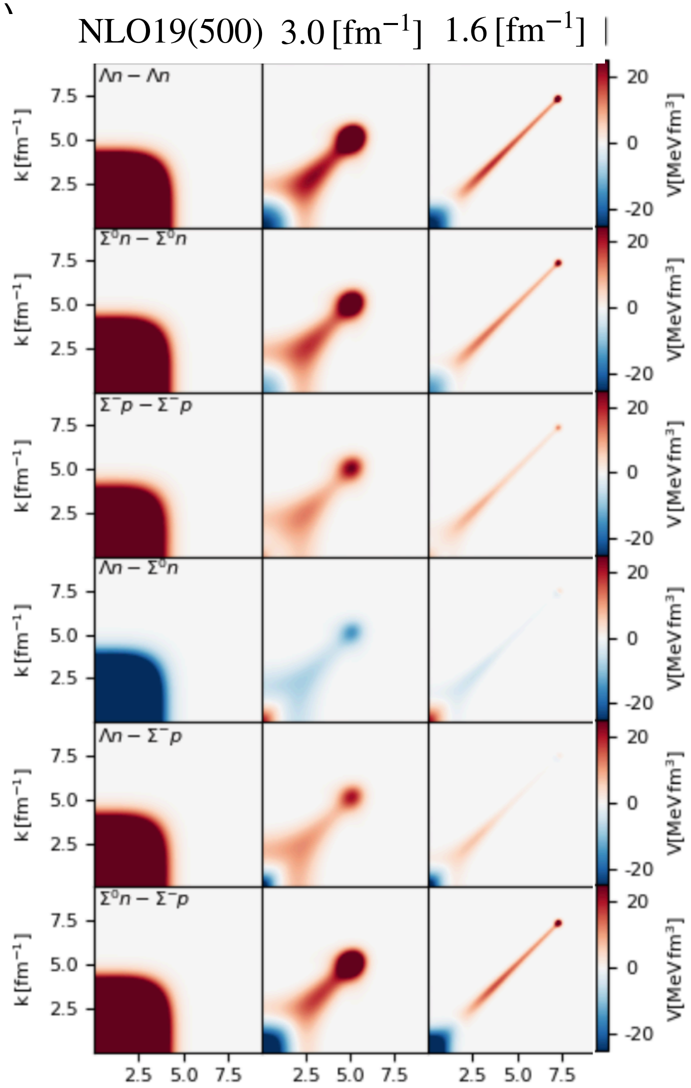
SRG evolution of NN, YN

- $\lambda = (4\mu^2/s)^{1/4}$, $[\lambda] = [p]$: $\lambda \sim$ width of the band-diagonal structure of V in p-space

(S.K. Bogner et al. PRC 75 (2007)`



NN: χN^{2}LO(500)

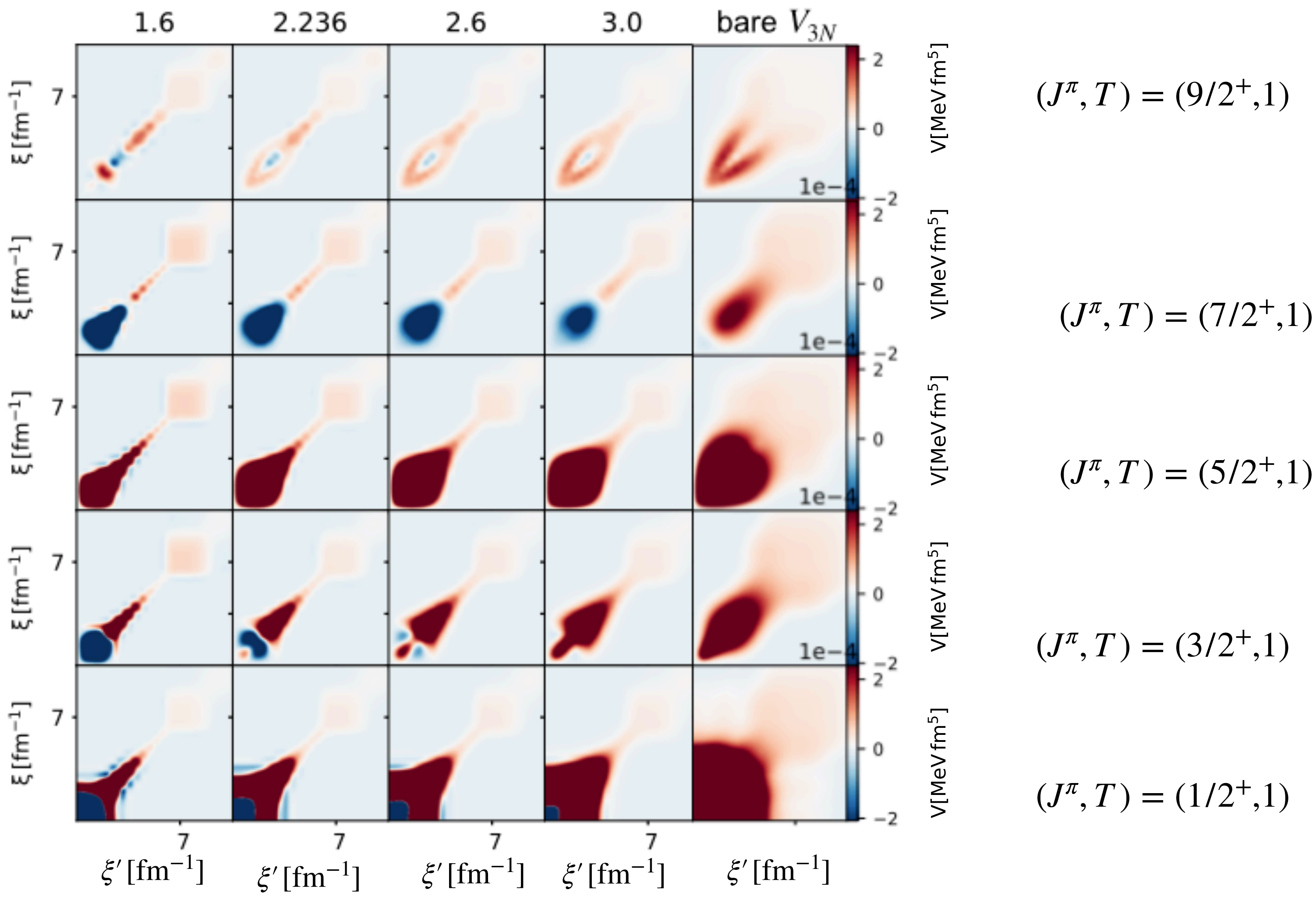


YN: χNLO19(650)

(HL et al EPJA 8 (2020))

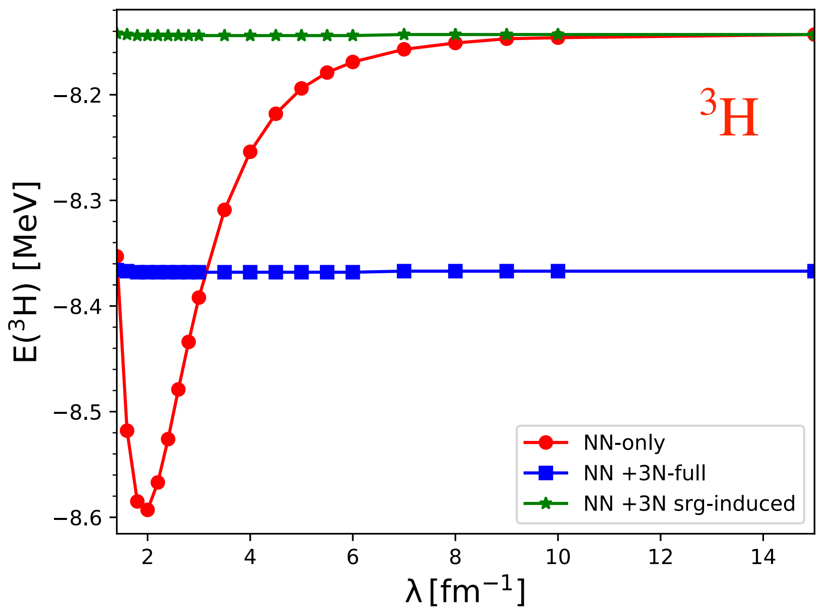
SRG evolution of $V_{123}(pq\alpha, p'q'\alpha')$

- hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2;$ $\quad tan\theta = \frac{2p}{\sqrt{3}q},$ $\quad \theta = \frac{\pi}{12};$ $\quad \alpha = \alpha' = 1 \Rightarrow V_{123} = V_{123}(\xi', \xi)$



3N: χN^{2}LO(550)

$E(^3H)$ with $\chi N^2LO(500)$



3N: $\chi N^2LO(500)$

$c_D = -1.28,\ c_E = -0.38$

- SRG is approximately unitary if higher-body forces are omitted
- contributions from the SRG-induced and (bare) chiral 3N forces are comparable

Results for $A = 3 - 6$

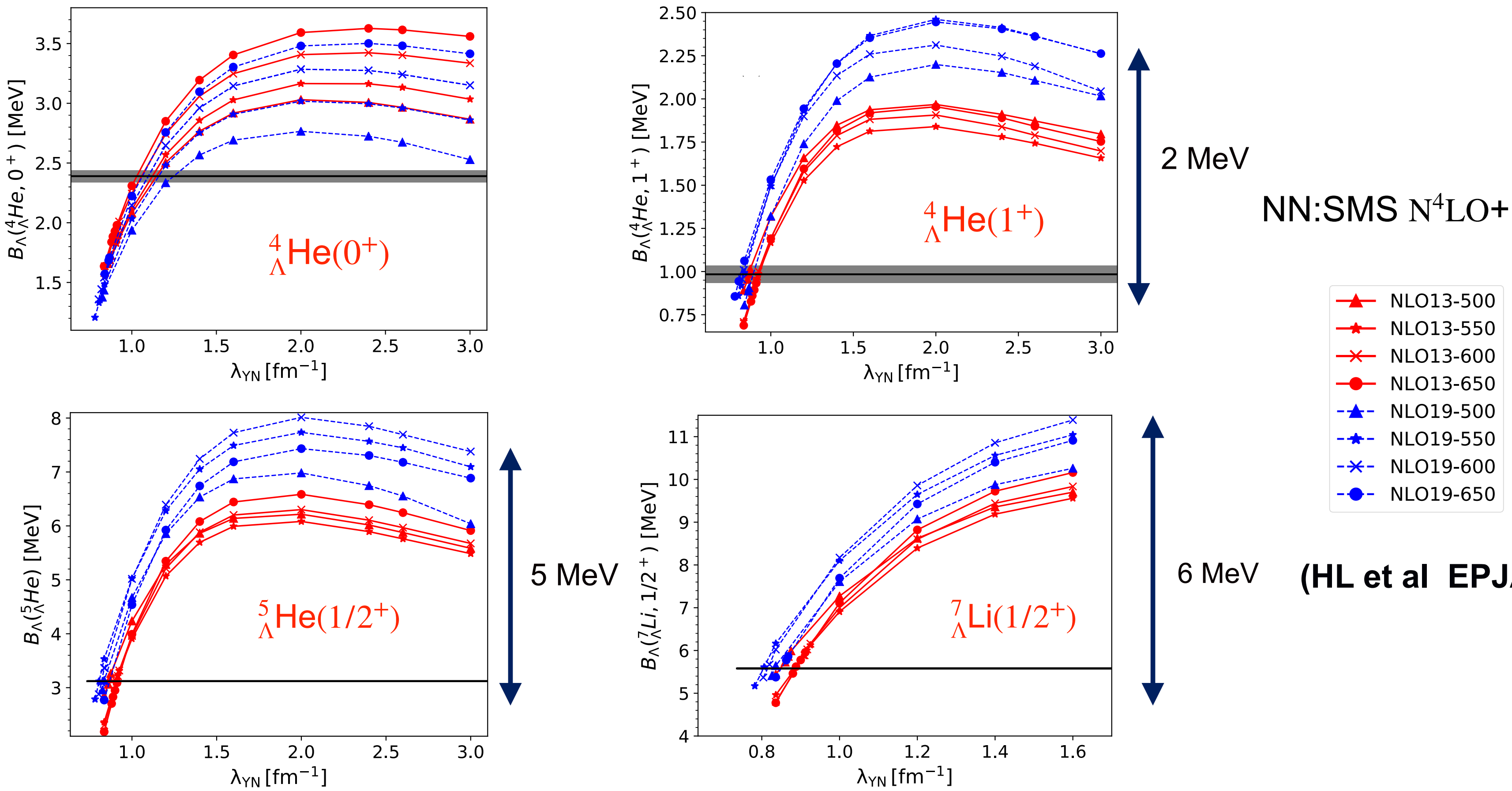
3N + 2N : χN^{2}LO(500)

	$N^2LO(500)$	$N^2LO(500) + 3N$ J−NCSM	$N^2LO(500) + 3N$ F−Y*	Exp.
^3H	-7.92	-8.477	-8.482	-8.482
^4He	-25.85	-28.57	-28.72	-28.296
$^6Li(1^+)$	$-28.77(5)$	$-32.19(20)$		-31.99
$E_x(^6Li, 3^+)$	2.81(7)	2.16(7)		2.19

* P. Maris et. al., PRC 103. 054001

Impact of YN interactions on B_Λ

- NLO13 and NLO19 are almost phase equivalent (**J.Haidenbauer et al NPA 915 2019)**)
- NLO13 leads to a stronger $\Lambda N - \Sigma N$ transition ⟶ manifest in higher-body observables



NN:SMS N^{4}LO+(450)

(HL et al EPJA (2020))

- B_Λ(NLO19) > B_Λ(NLO13) ⟶ possible contribution of chiral YNN force
- B_Λ is strongly dependent on λ_{YN} ⟶ contribution of SRG-induced YNN force is significant

(R. Wirth et al PRL (2014,2016), PRC(2018))

Impact of an increased $B_\Lambda({}^3_\Lambda\text{H})$ on ${}^7_\Lambda$Li spectrum

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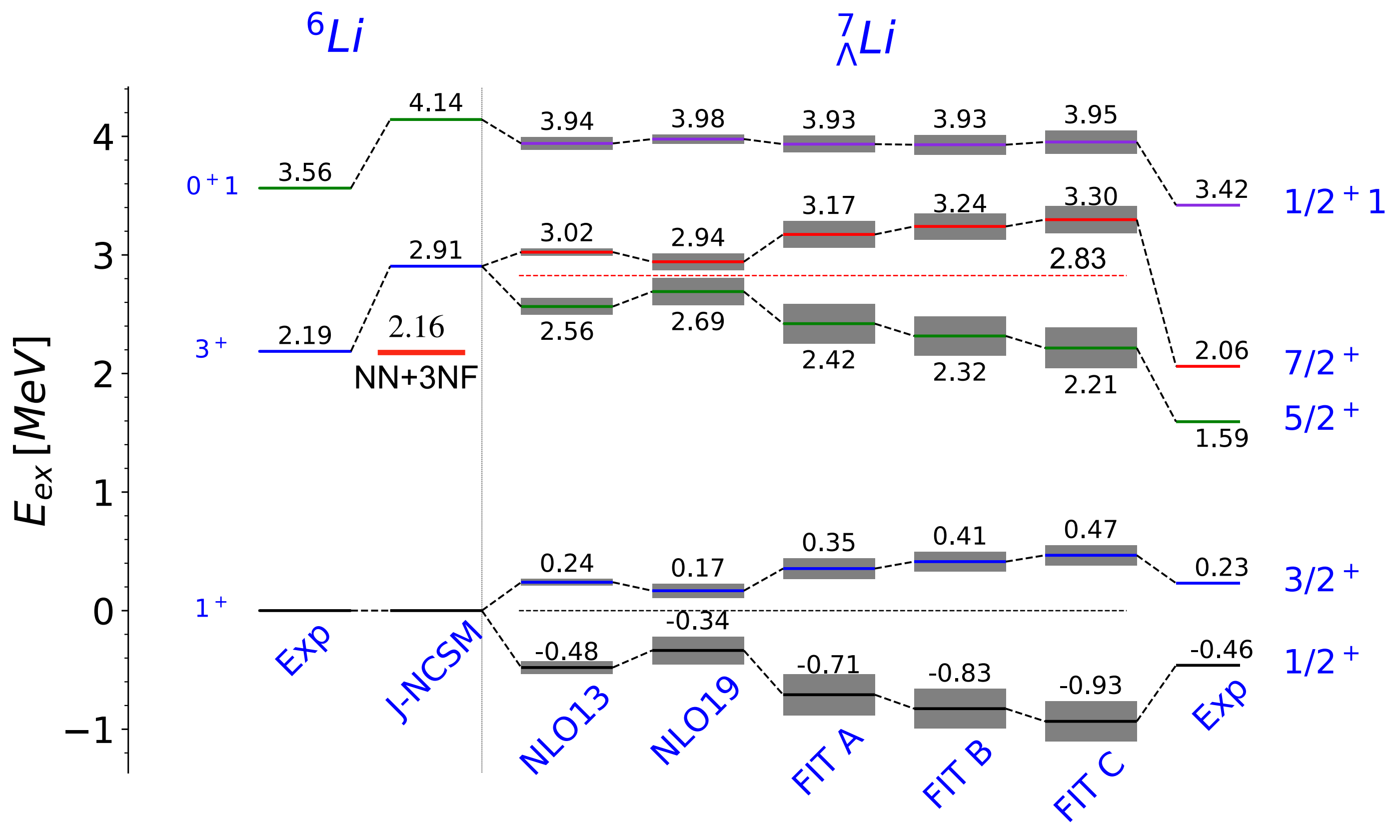
- Choose λ_{YN} to reproduce: $B_\Lambda({}^5_\Lambda\text{He}) = 3.12 \pm 0.02$ MeV

$B_\Lambda({}^3_\Lambda\text{H})$ is used to fix relative strength of spin **singlet/triplet** ΛN interaction

$B_\Lambda({}^3_\Lambda\text{H}) = 0.13 \pm 0.05$ MeV (up to 2019: NLO13, NLO19) **(J.Haidenbauer et al NPA 915 2019))**

$= 0.41 \pm 0.12$ MeV (STAR 2019: FITA, FITB, FITC) **(HL et al PLB (2020)**



→ overall effect of an increased $B_\Lambda({}^3_\Lambda\text{H})$ on ${}^7_\Lambda$Li spectrum is small

Summary

- **Develop *ab initio* Jacobi NCSM for light nuclei & hypernuclei**
  - study $A = 3 - 6$ nuclei using chiral NN & 3N at $\mathrm{N^2LO}$

    ⟶ accurate prediction for the binding and first excitation energies
  - investigate $A = 4 - 7\ \Lambda$ hypernuclei using chiral YN NLO13 & NLO19 potentials

    ⟶ SRG-YN evolution strongly affects the Λ-separation energies

    difference in NLO13 & NLO19 predictions is attributed to contribution of 3BFs

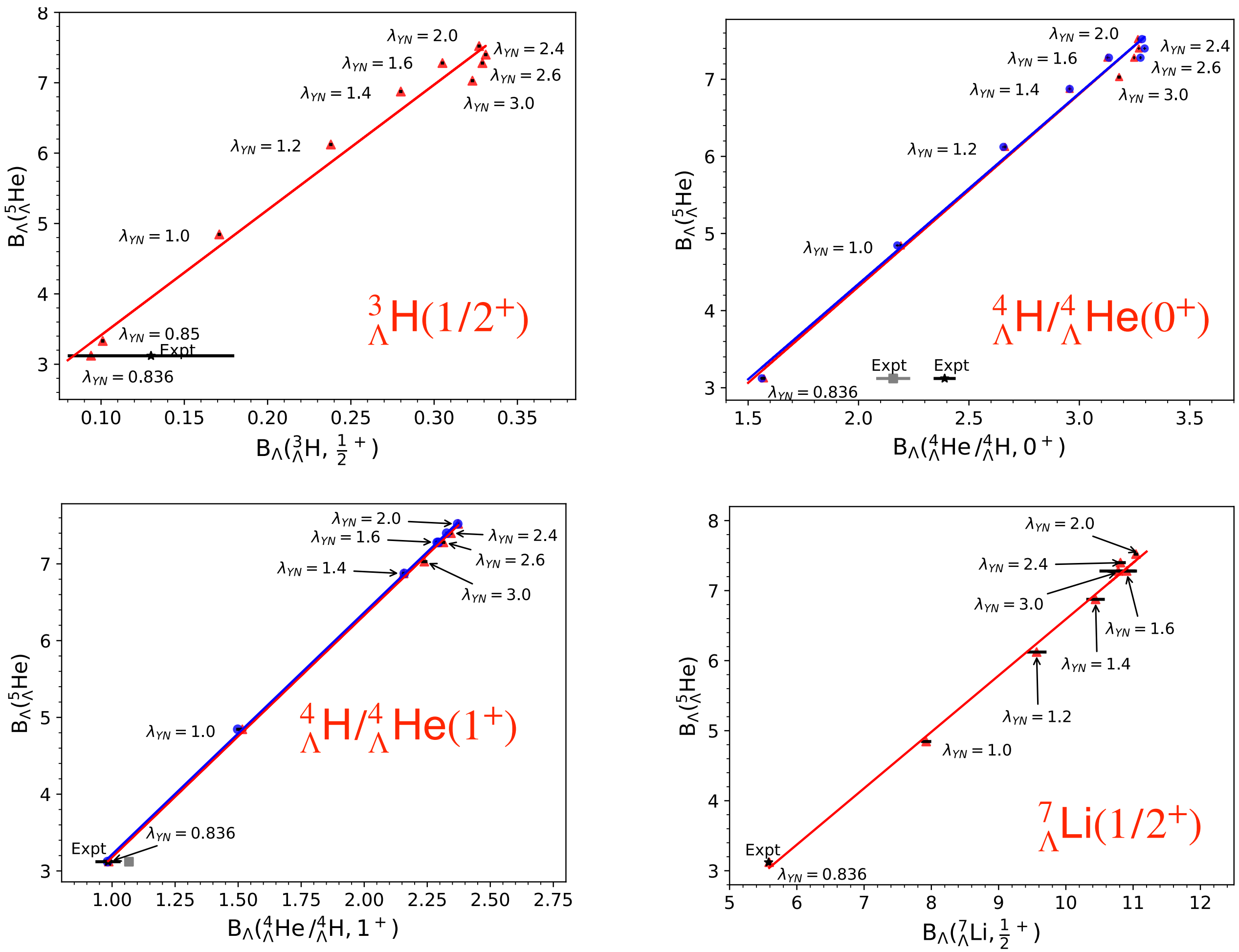
- **On going project: inclusion of SRG-induced (chiral) YNN forces**

  ⟶ using information on light hypernuclei to constrain YN interactions

Thank you for the attention!

Correlation of the Λ-separation energies

- B_Λ of different hypernuclei computed for a same range of λ_{YN} are strongly correlated



Idaho-N^{3}LO(500)

YN-NLO19(600)

- B_Λ of $^3_\Lambda$H, $^4_\Lambda$He(1$^+$), $^5_\Lambda$He and $^7_\Lambda$Li are well reproduced at $\lambda_{YN} = 0.84\ \text{fm}^{-1}$

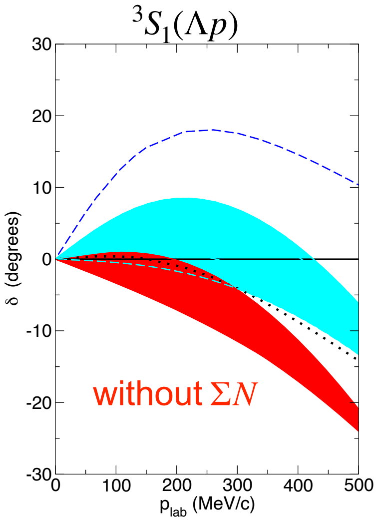
→ minimize effect of YNN forces **by tuning λ_{YN} so that a particular hypernucleus ($^5_\Lambda$He) is properly described**

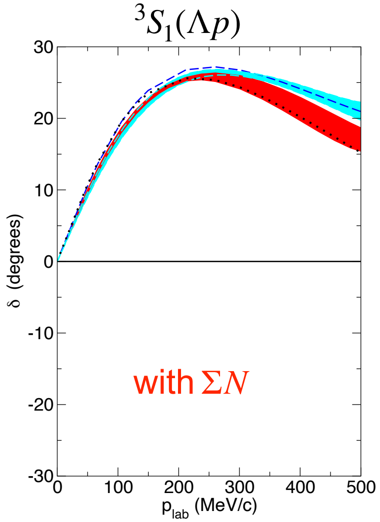
YN interactions at NLO

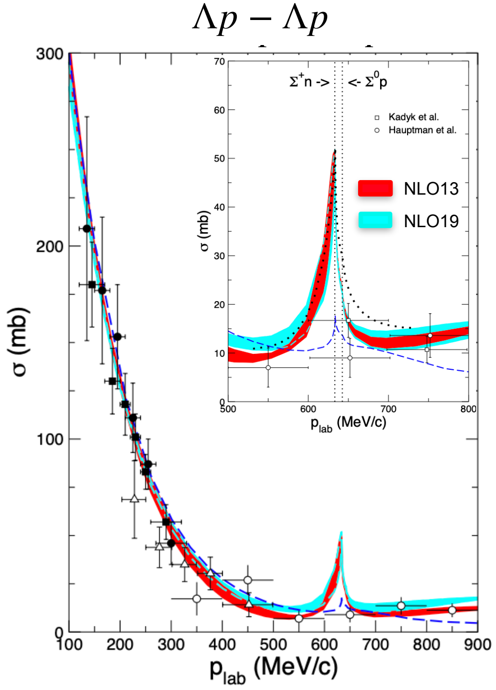
NLO13: J. Haidenbauer et al., NPA 915 (2013), NLO19: EPJ A 56 (2019) 91

- most of YN LECs are fitted to 36 YN data points $(\Lambda p \to \Lambda p,\ \Sigma N \to \Sigma N,\ \Sigma N \to \Lambda N)$
- two realisations at NLO: NLO13 and NLO19
  - almost phase equivalent
  - NLO13 leads to a larger transition potential $V_{\Lambda N - \Sigma N}$

not a two-body observable







(Haidenbauer 2019)

$\Lambda_{YN} = 500,..,650$ MeV

→ estimate theoretical uncertainty

→ NLO13 and NLO19 as a tool to estimate contribution of YNN force

(J. Haidenbauer et al. EPJA 56 (2019))