Fundamental Physics with Nuclei

13th International Spring Seminar on Nuclear Physics: "Perspectives and Challenges in Nuclear Structure after 70 Years of Shell Model",
Sant'Angelo d'Ischia, May 20, 2022

Saori Pastore

https://physics.wustl.edu/quantum-monte-carlo-group

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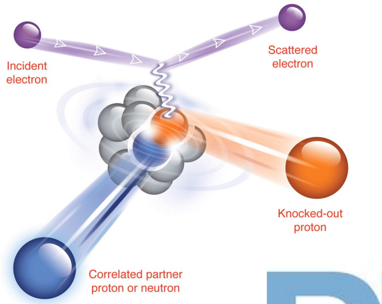
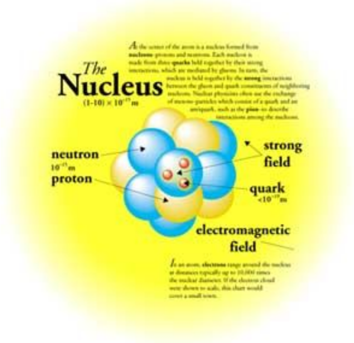
Computational Resources awarded by the DOE ALCC and INCITE programs

Fundamental Physics with Nuclei

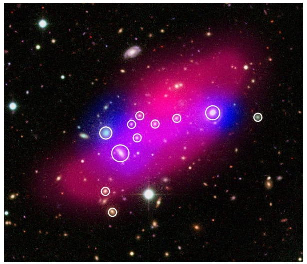
FRIB

Jefferson Lab



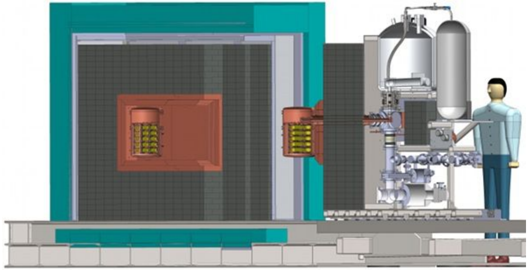


DUNE

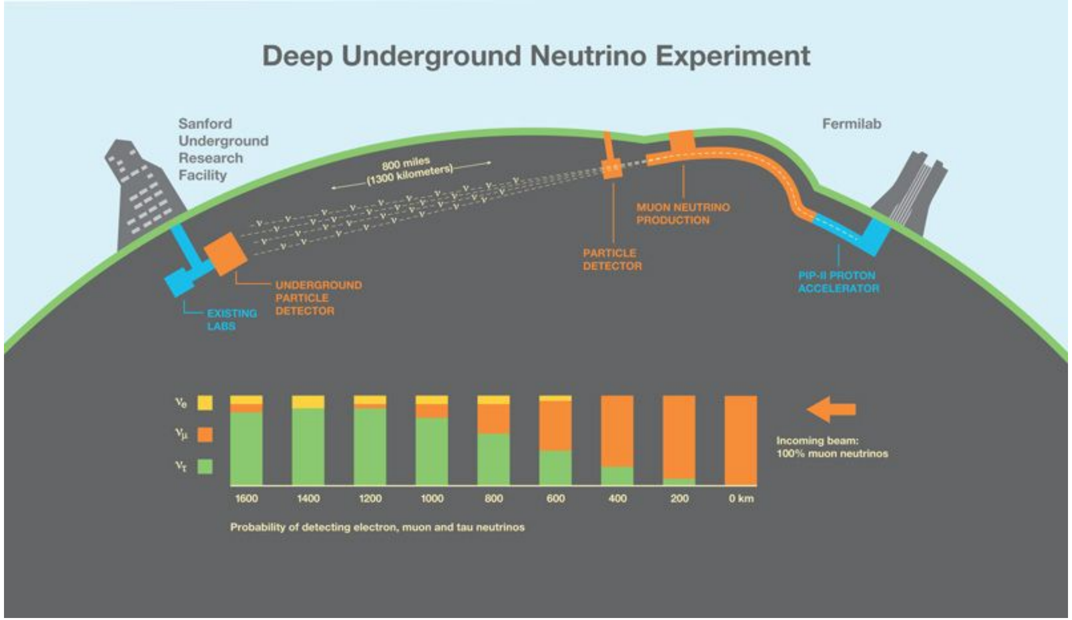


ESA, XMM-Newton, Gastaldello, CFHTL

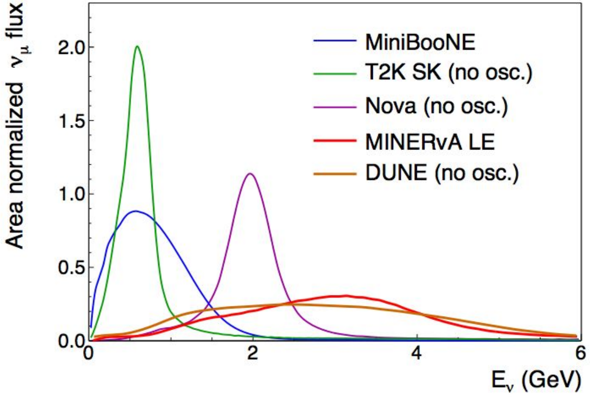
T2K



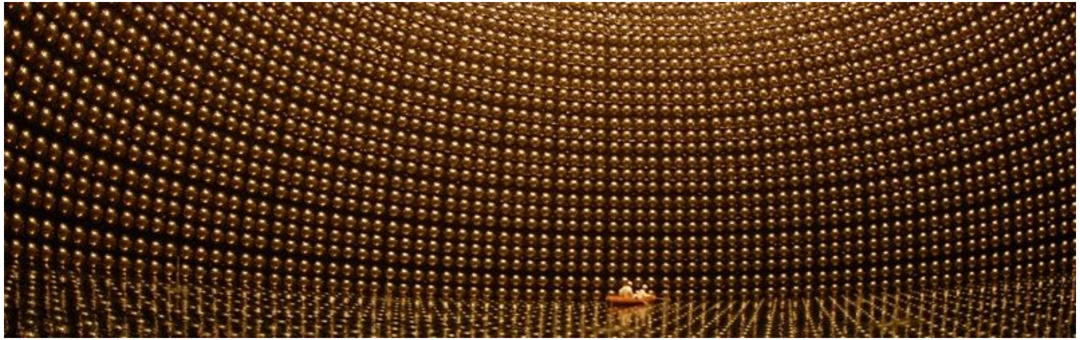
Accelerator Neutrinos’ Experiments



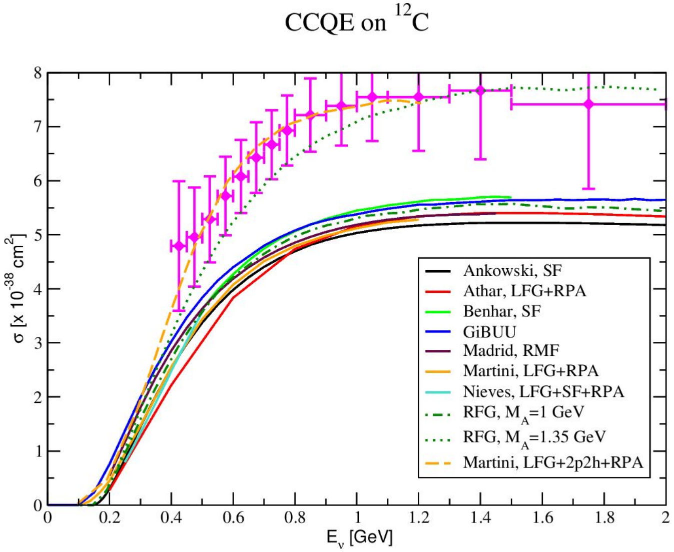
DUNE - Fermilab



Nuclei for Neutrino Oscillations’ Experiments

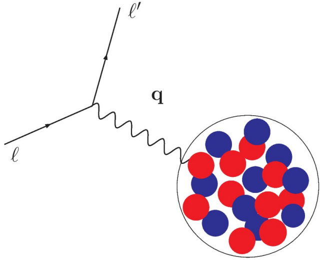


Neutrino-^{12}C cross section



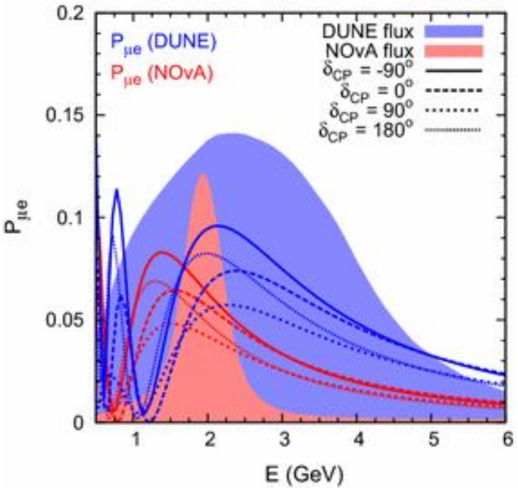
Alvarez-Ruso arXiv:1012.3871

$$P(\nu_\mu \rightarrow \nu_e) = sin^2 2\theta sin^2\left(\frac{\Delta m^2_{21} L}{2E_\nu}\right)$$

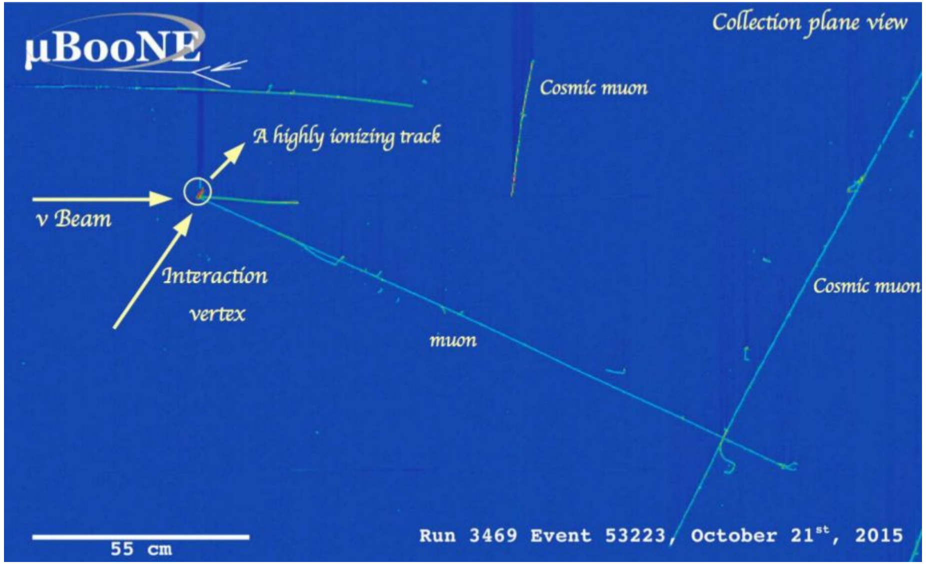


Nuclei are the active material in the detector. The energy of the incident neutrino is reconstructed from the observed final states using **neutrino event generators** that require **theoretical cross-sections**.

The needs of the experimental programs

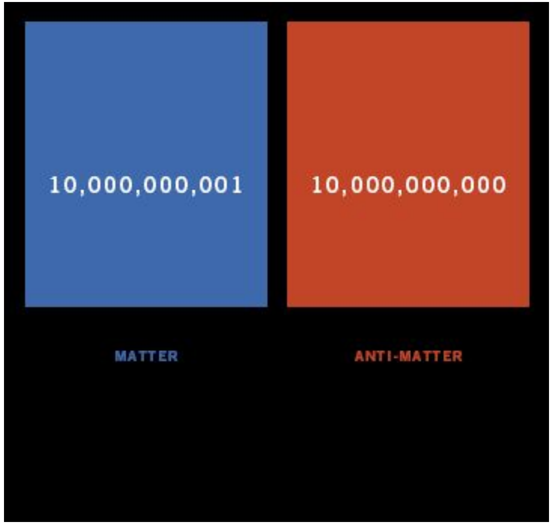


The experimental neutrino program is in need of accurate **theoretical calculations of neutrino-nucleus cross-sections with quantified theoretical errors** to ensure a robust implementation of interaction models in experiments

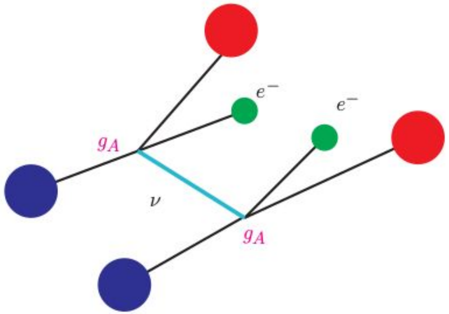


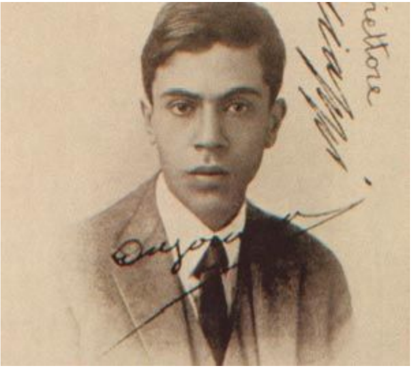
The range of challenges is extreme; ultimately we would like to be able to predict both inclusive and **exclusive cross sections across a wide range of kinematics**.

Neutrinoless double beta decay



Hitoshi Murayama



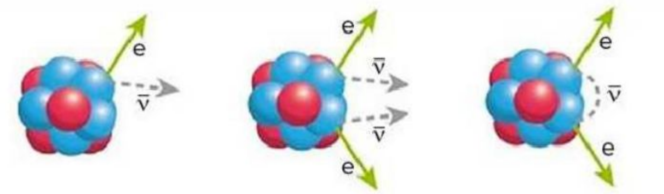


Ettore Majorana

$$(Z,N) \rightarrow (Z+2,N-2)+2e$$

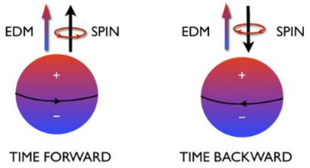
Lepton number is not conserved

Decay Rate $\propto$ (**nuclear matrix element**)^2 x $(m_{\beta\beta})^2$

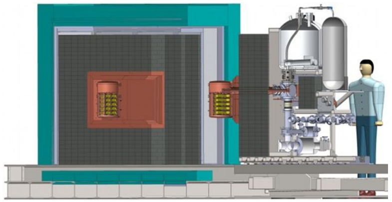


2015 Long Range Plan for Nuclear Physics

Ground States' Electroweak Moments, Form Factors, Radii



Neutrinoless Double Beta Decay, Muon-Capture



Accelerator Neutrino Experiments, Lepton-Nucleus XSecs

$(\omega,q)\sim0$ MeV

$\omega\sim$few MeVs
$q\sim0$ MeV

$\omega\sim$few MeVs
$q\sim10^2$ MeV

$\omega\sim$tens of MeVs

$\omega\sim10^2$ MeV

FRIB

Electromagnetic Decay, Beta Decay, Double Beta Decay & inverse processes

JINA-CEE

Nuclear Rates for Astrophysics

DUNE

Jefferson Lab

Strategy

Validate the Nuclear Model against available data for strong and electroweak observables

- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- Electron-Nucleus Scattering Cross Sections, ...

Use attained information to make (accurate) predictions for BSM searches and precision tests

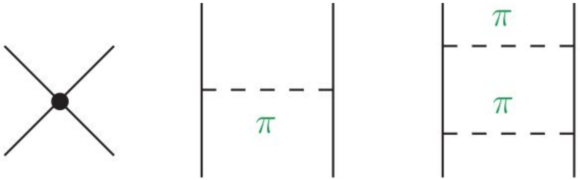
- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- Neutrino-Nucleus Scattering Cross Sections, ...
- ...

Many-body Nuclear Interactions

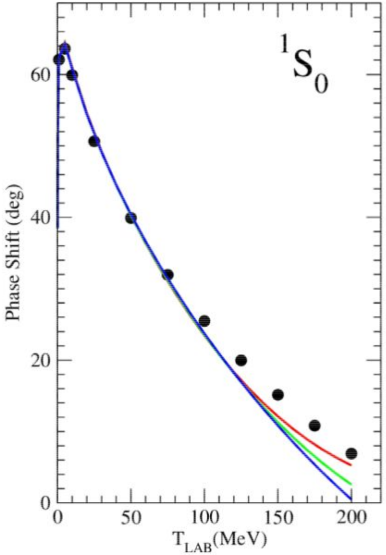
Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



Contact term: short-range
Two-pion range: intermediate-range $r \propto (2\, m_\pi)^{-1}$
One-pion range: long-range $r \propto m_\pi^{-1}$



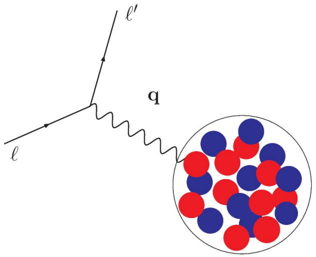
SP et al. PRC80(2009)034004

Hideki Yukawa

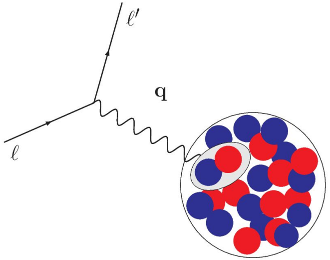
AV18+UIX; AV18+IL7
Wiringa, Schiavilla, Pieper *et al*.

chiral πNΔ
N3LO+N2LO Piarulli *et al*. **Norfolk Models**

Many-body Nuclear Electroweak Currents



one-body



two-body

- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

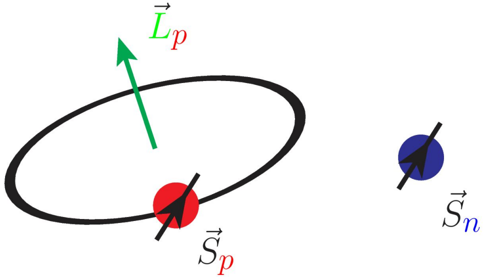
$$\mathbf{q}\cdot\mathbf{j} = [\,H,\,\rho\,] = [t_i + v_{ij} + V_{ijk},\,\rho\,]$$

Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$



Magnetic Moment: Single Particle Picture

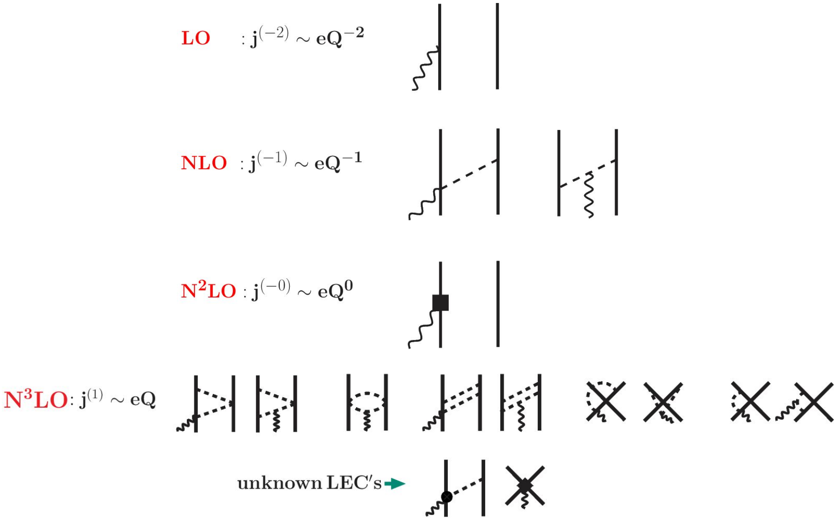
Many-body Currents

- **Meson Exchange Currents** (MEC)

Constrain the MEC current operators by imposing that the current conservation relation is satisfied with the given two-body potential

- **Chiral Effective Field Theory Currents**

Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (**LECs**), need to be **determined by either fits to experimental data or by Lattice QCD calculations**

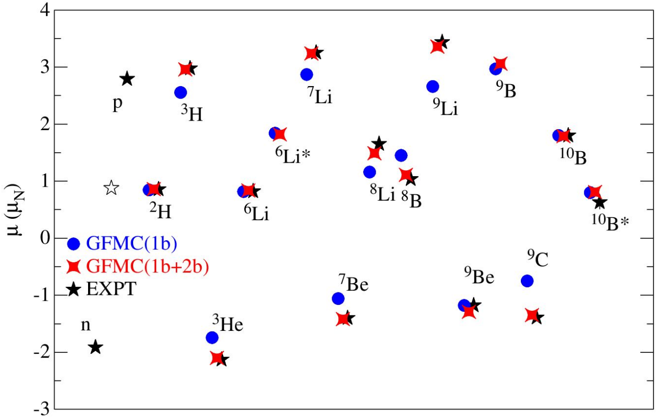


Electromagnetic Current Operator

SP *et al.* PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001, PRC87(2013)014006
Park *et al.* NPA596(1996)515, Phillips (2005)
Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

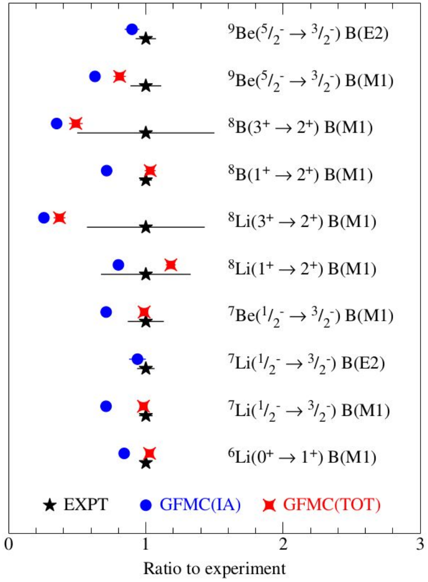
Electromagnetic Observables

Magnetic moments

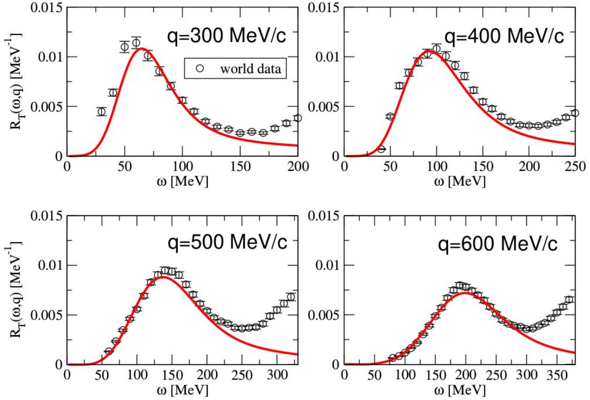


SP *et al*. PRC87(2013)035503, PRC101(2020)044612

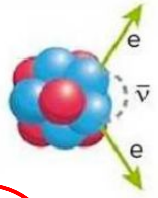
EM decay



e-^{4}He particle scattering

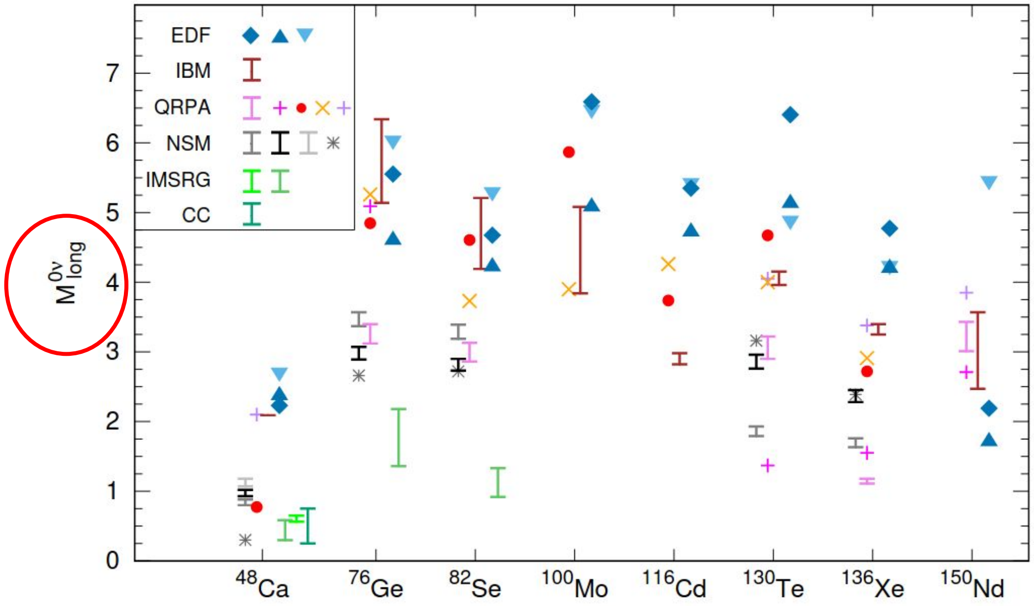


Neutrinoless Double Beta Decay

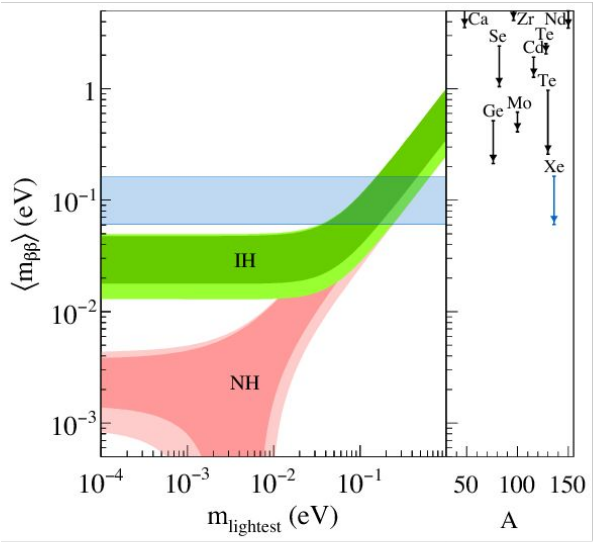


$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q,Z)|M_{0\nu}|^2 m_{\beta\beta}^2$$

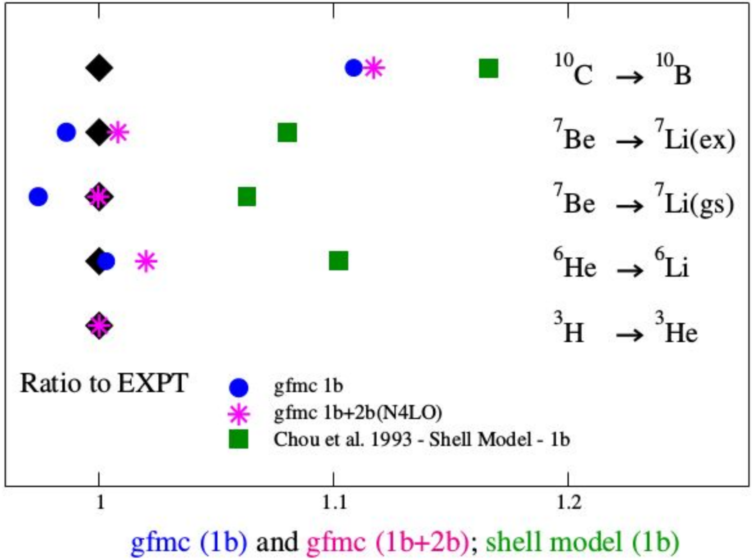
$$(Z,N) \to (Z+2,N-2)+2e$$



Agostini, Menendez et al, arXiv:2202.01787 (2022)

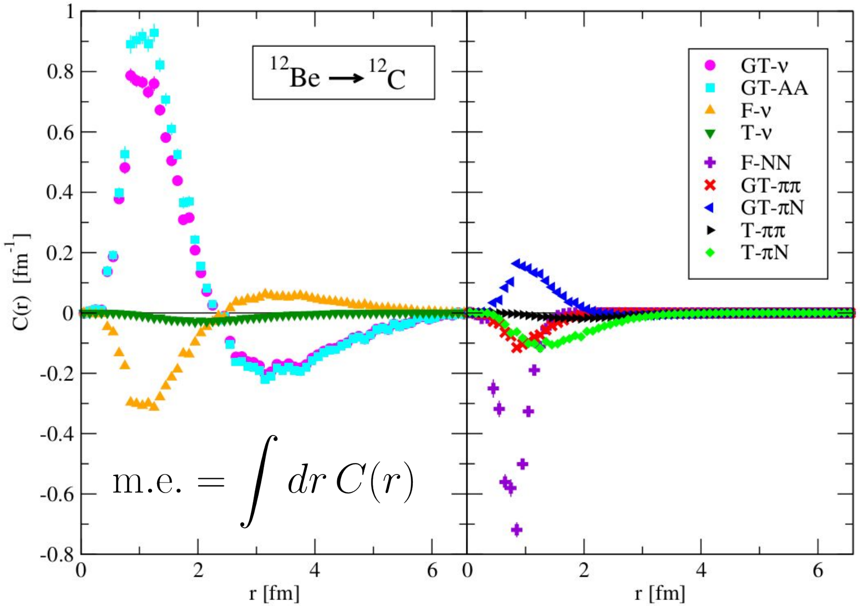


Beta decay



SP *et al*. PRC97(2018)022501

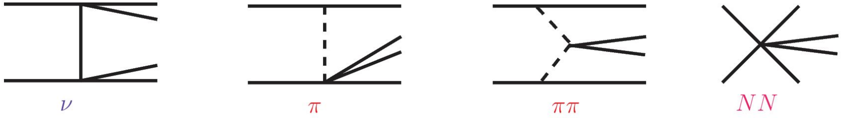
Neutrinoless Double Beta Decay Matrix Elements



Long-range

One-pion & short range

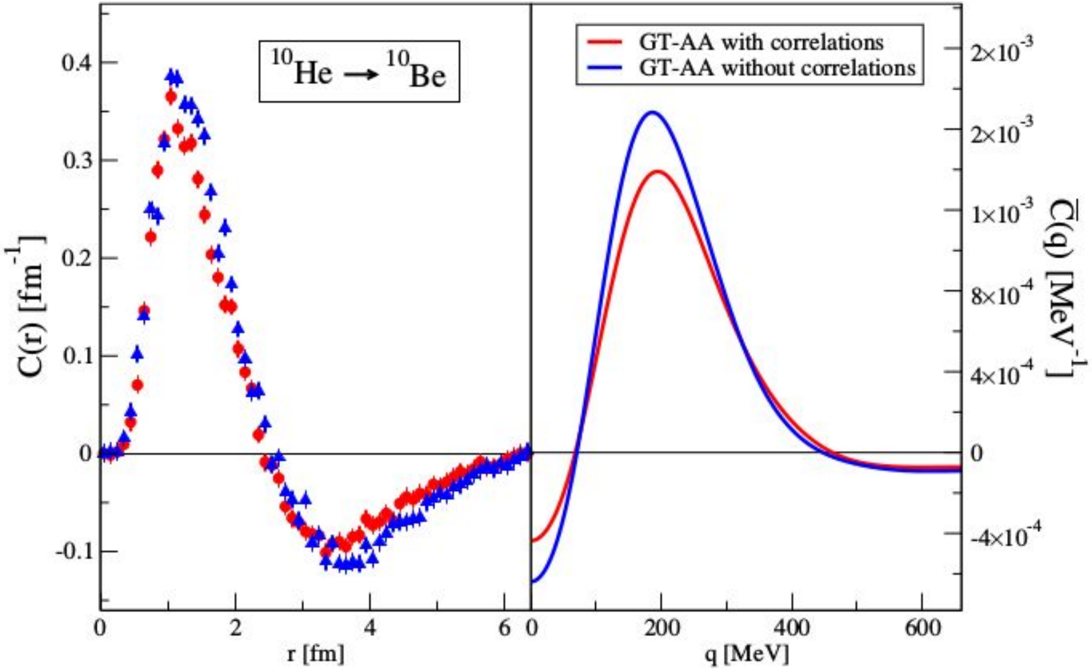
SP *et al.* PRC97(2018)014606



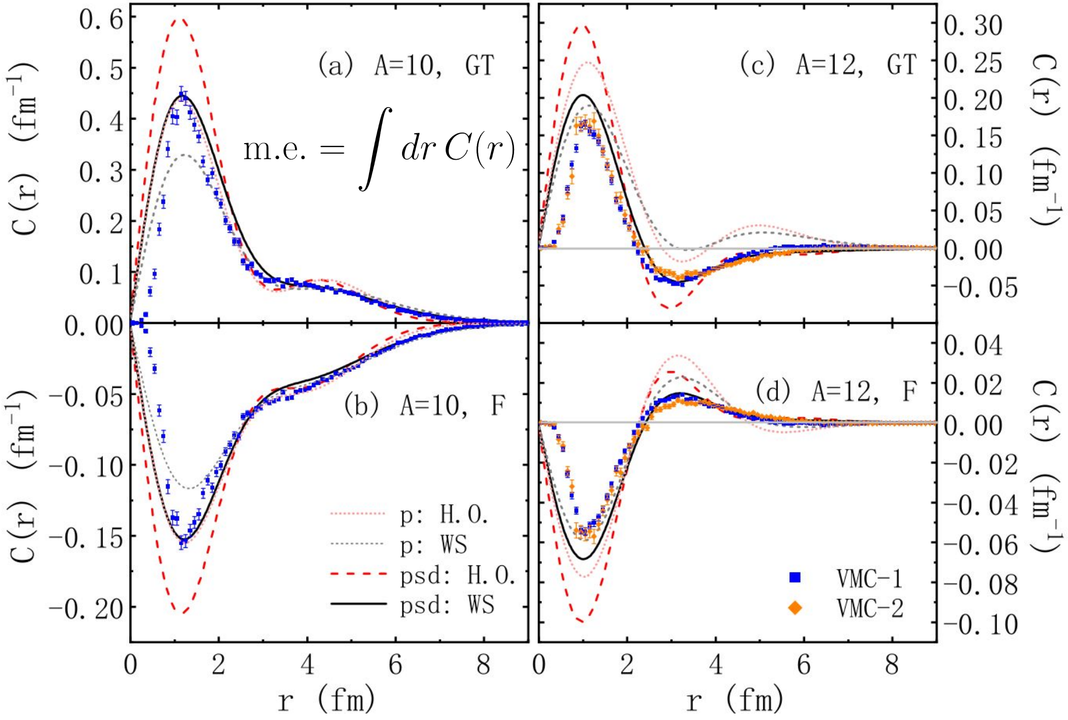
Cirigliano Dekens DeVries Graesser Mereghetti *et al.*
PLB769(2017)460, JHEP12(2017)082, PRC97(2018)065501

- Leading operators in neutrinoless double beta decay are two-body operators
- These observables are particularly sensitive to short-range and two-body physics
- Transition densities calculated in momentum space indicate that the momentum transfer in this process is of the order of **q ~ 200 MeV**

Correlations in neutrinoless double beta decay ME



Comparison with Shell-Model Calculations



$$\text{m.e.} = \int dr\, C(r)$$

X. Wang *et al.* PLB798(2019)134974

Closer agreement between Shell-Model calculations with Variational Monte Carlo results is reached by

- Increasing the size of the model space
- Wood-Saxon single particle wave functions are superion in describing the tails of the densities wrt harmonic oscillator wave functions
- Phenomenological Short-Range-Correlations functions further improve the agreement

Partial muon capture rates: VMC calculations

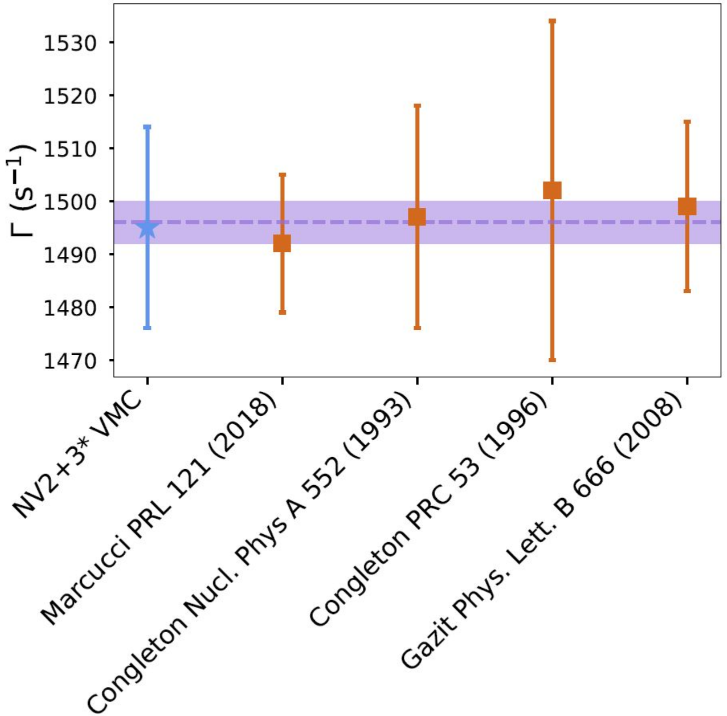
Γ_{VMC}(avg.) = 1495 s^{-1} ± 19 s^{-1}
Γ_{expt} = 1496.0 s^{-1} ± 4.0 s^{-1}
Ackerbauer *et al.* PLB417, 224(1998)

Momentum transfer **q~ 100 MeV**

Two-body correction is ~8% of total rate on average for A=3

Garrett King *et al.* PRC2022

$${}^{3}\mathrm{He}(1/2^{+};1/2) \rightarrow {}^{3}\,\mathrm{H}(1/2^{+};1/2)$$



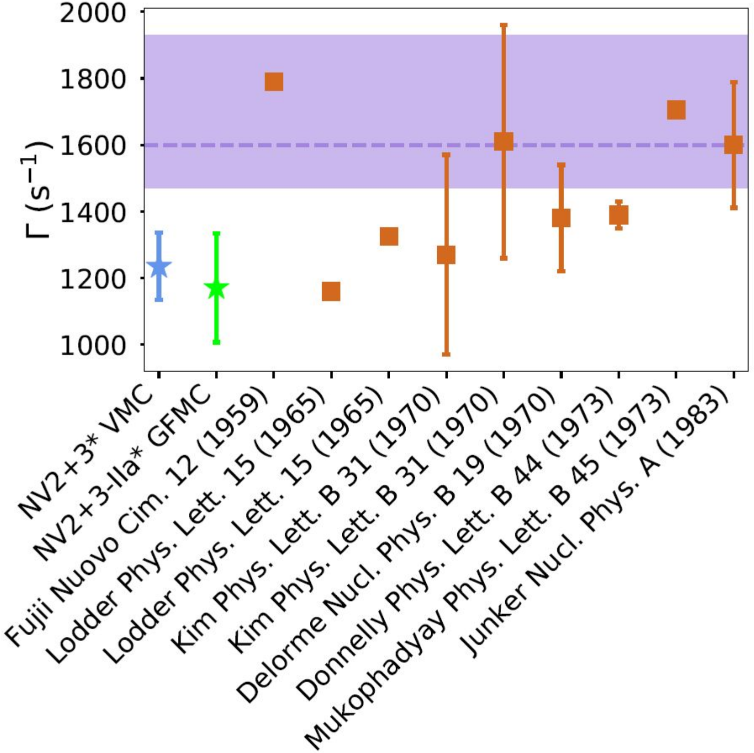
Partial muon capture rates: VMC calculations

$\Gamma_{VMC}(\text{avg.}) = 1235\ s^{-1} \pm 101\ s^{-1}$
$\Gamma_{GFMC}(\text{IIa*}) = 1171\ s^{-1} \pm 164\ s^{-1}$
$\Gamma_{expt} = 1600\ s^{-1}\ +330/-129\ s^{-1}$
Deutsch *et al.* PLB26(1968)315

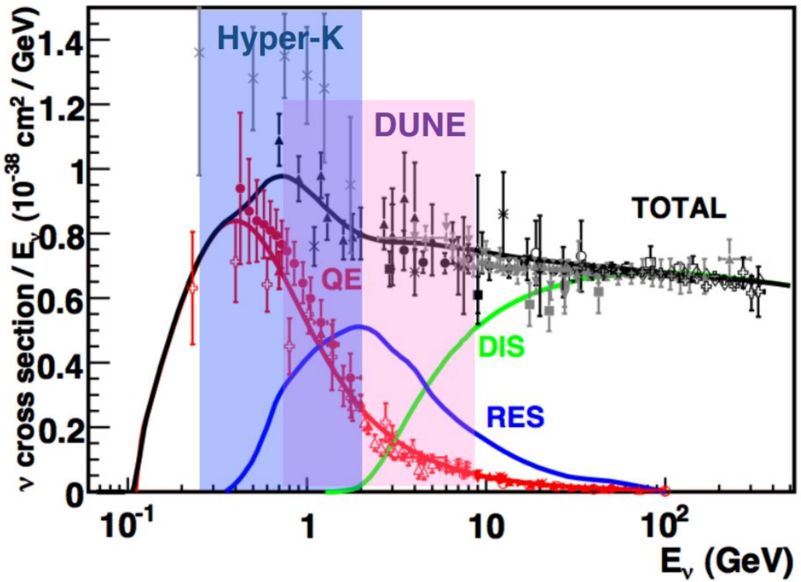
Garrett King *et al.* PRC2022

Outlook at FRIB: extraction of the Gamow-Teller strength A=11, A=12

$${}^{6}\text{Li}(1^+;0) \rightarrow {}^{6}\text{He}(0^+;1)$$



Neutrino cross section anatomy



Formaggio & Zeller

Quasi-elastic: dominated by single-nucleon knockout

Resonance: excitation to nucleonic resonant states which decay into mesons

Deep-inelastic scattering: where the neutrino resolves the nucleonic quark content

Each of these regimes requires knowledge of both the **nuclear ground state** and the **electroweak coupling** and **propagation of the struck nucleons, hadrons, or partons**

A challenge for achieving precise neutrino-nucleus cross-section is **reliably bridging the transition regions which use different degrees of freedom**

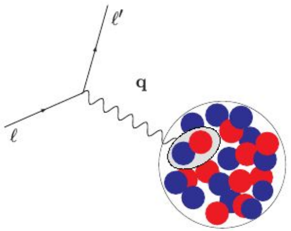
Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_\alpha(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) |\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$
Transverse response induced by the current operator $O_T = \boldsymbol{j}$
5 Responses in neutrino-nucleus scattering

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$



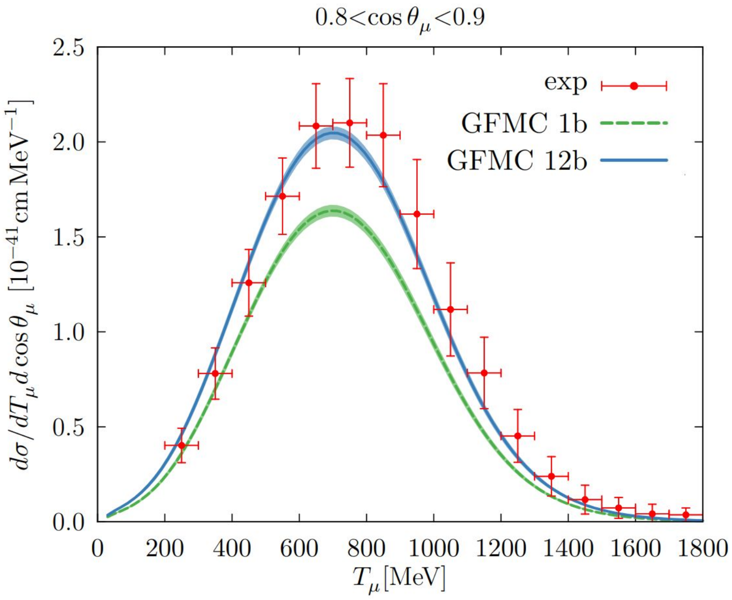
For a recent review on QMC, SF methods see
Rocco *Front. In Phys.*8 (2020)116

Inclusive Cross Sections with Integral Transforms

Exploit integral properties of the response functions and closure to avoid explicit calculation of the final states (Lorentz Integral Transform **LIT**, **Euclidean**, …)

$$\int_0^\infty \mathrm{d}\omega\, \mathrm{e}^{-\tau\omega}\, R_{\alpha\beta}(q,\omega) = \langle i|\, j_\alpha^\dagger(\mathbf{q})\, \mathrm{e}^{-\tau(H-E_i)}\, j_\beta(\mathbf{q})\, |i\rangle$$

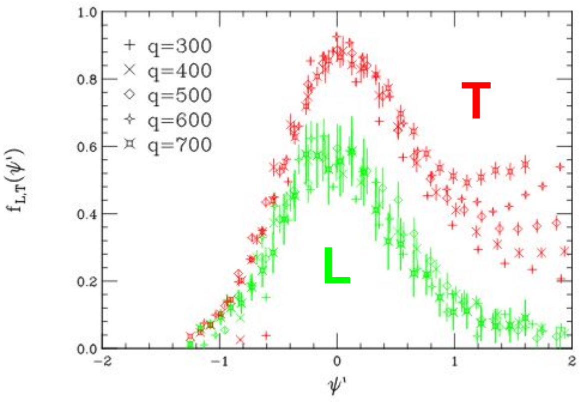
Lovato et al. PRX10 (2020)



Lepton-Nucleus scattering: Data

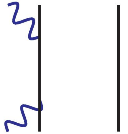
Transverse Sum Rule

$$S_T(q) \propto \langle 0|\mathbf{j}^\dagger\ \mathbf{j}|0\rangle \propto \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{1b}|0\rangle + \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{2b}|0\rangle + \dots$$



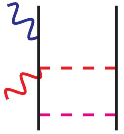
^{4}He Electromagnetic Data
Carlson *et al.* PRC65(2002)024002

Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term



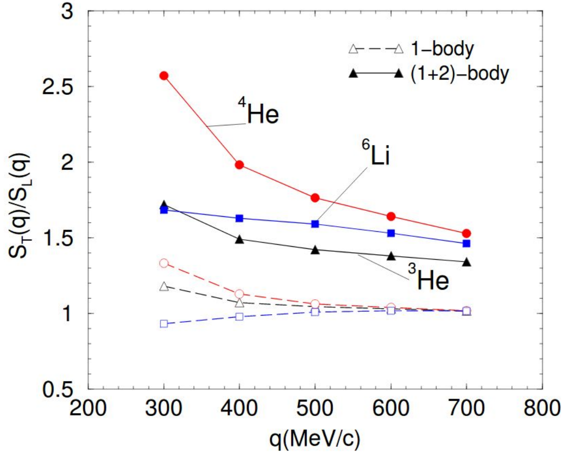
$$\langle \mathbf{j}_{1b}^\dagger\ \ \mathbf{j}_{1b}\rangle\ > 0$$

Leading one-body term



$$\langle \mathbf{j}_{1b}^\dagger\ \mathbf{j}_{2b}\ v_\pi\rangle \propto \langle v_\pi^2\rangle\ > 0$$

Interference term

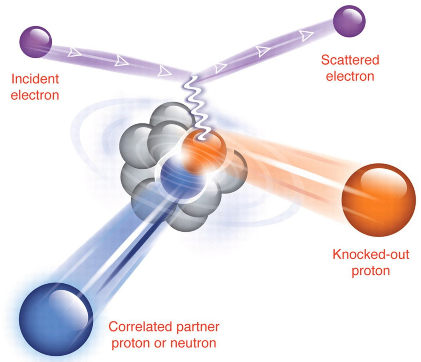


Transverse/Longitudinal Sum Rule
Carlson *et al.* PRC65(2002)024002

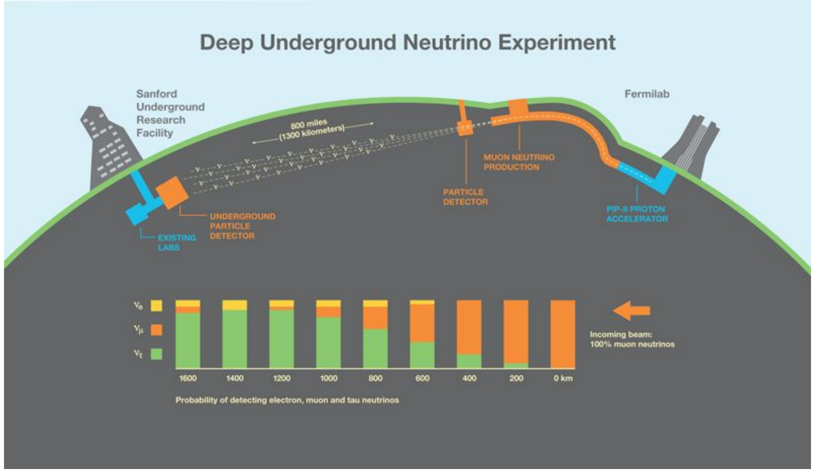
Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from **A > 12** without losing two-body physics
- Account for **exclusive processes**
- Incorporate **relativistic effects**



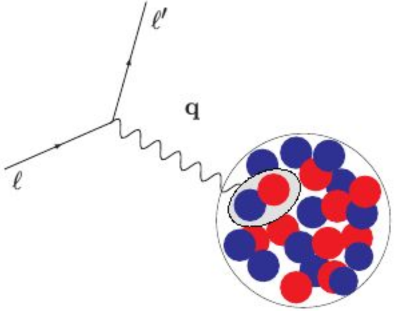
Subedi et al. Science320(2008)1475



[Stanford Lab article]

[e4u collaboration]

Short-Time-Approximation



Short-Time-Approximation:

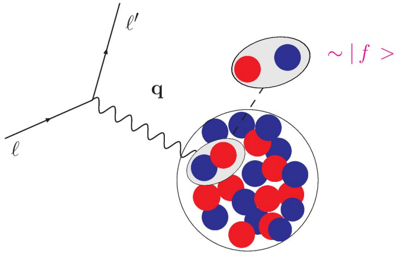
- Based on Factorization
- Retain two-body physics
- Correctly accounts for interference

$$R(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi}\, \mathrm{e}^{i(\omega+E_0)t} \langle 0|O^{\dagger}\, \mathrm{e}^{-iHt}\, O|0\rangle$$

$$O_i^{\dagger}\, \mathrm{e}^{-iHt}\, O_i + O_i^{\dagger}\, \mathrm{e}^{-iHt}\, O_j + O_i^{\dagger}\, \mathrm{e}^{-iHt}\, O_{ij} + O_{ij}^{\dagger}\, \mathrm{e}^{-iHt}\, O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i<j} v_{ij}$$

Factorization Schemes



Short-Time-Approximation:

- Based on Factorization
- **Retains two-body physics**
- Response functions are given by the **scattering from pairs of fully interacting nucleons** that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides “more” **exclusive information in terms of nucleon-pair kinematics** via the Response Densities

Response Functions ∝ Cross Sections

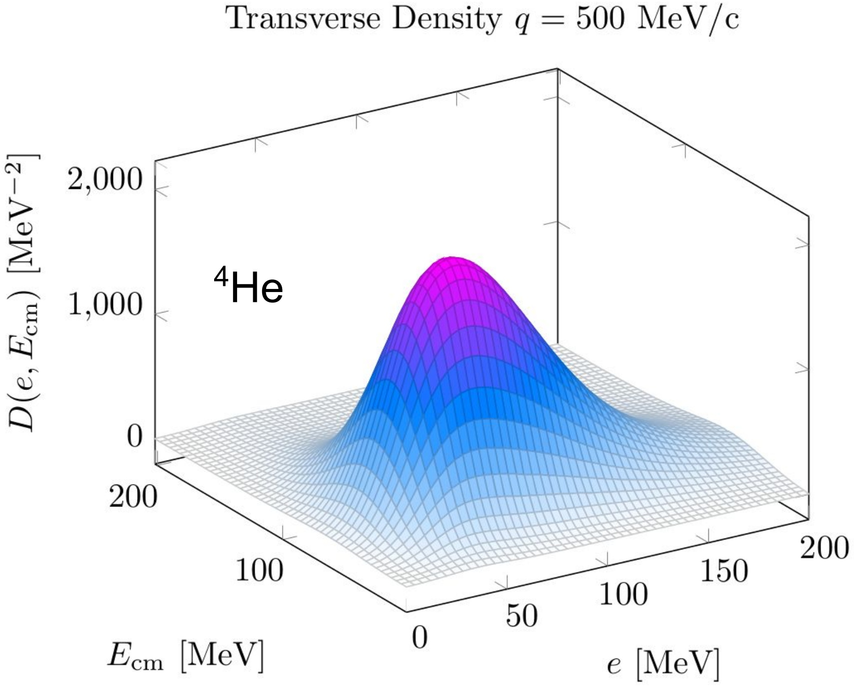
$$R_\alpha(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) |\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$

Response ***Densities***

$$R(q,\omega) \sim \int \delta\left(\omega + E_0 - E_f\right)\, dP'\, dp' \mathcal{D}(p', P'; q)$$

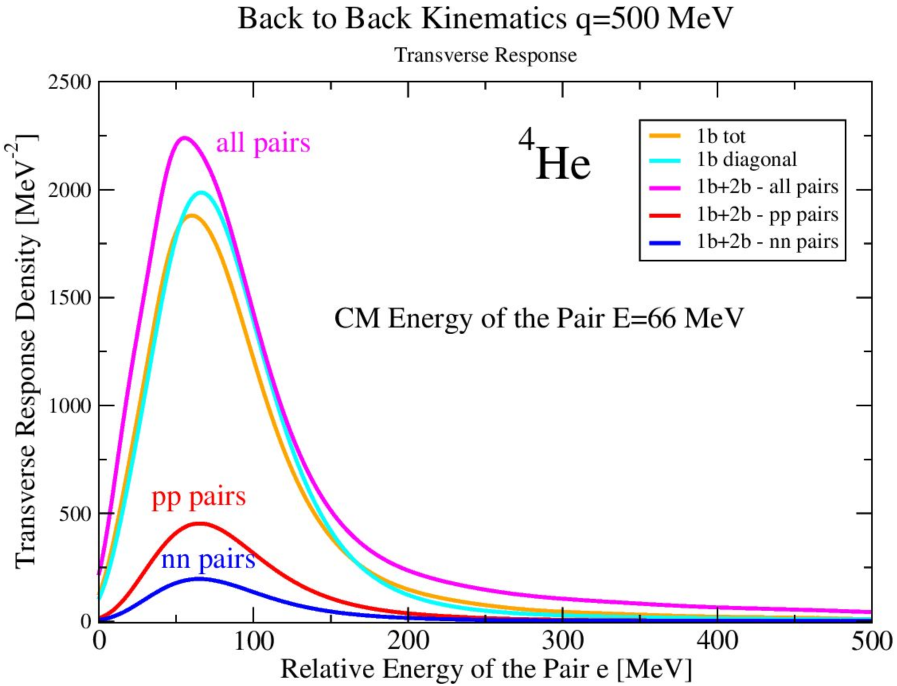
P’ and *p’* are the CM and relative momenta of the struck nucleon pair

Transverse Response Density: *e*-^{4}He scattering



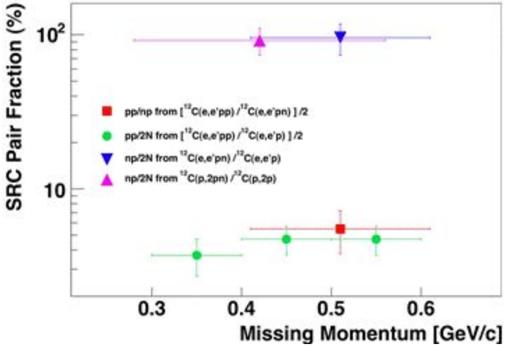
SP *et al.* PRC101(2020)044612

e-^{4}He scattering in the back-to-back kinematic



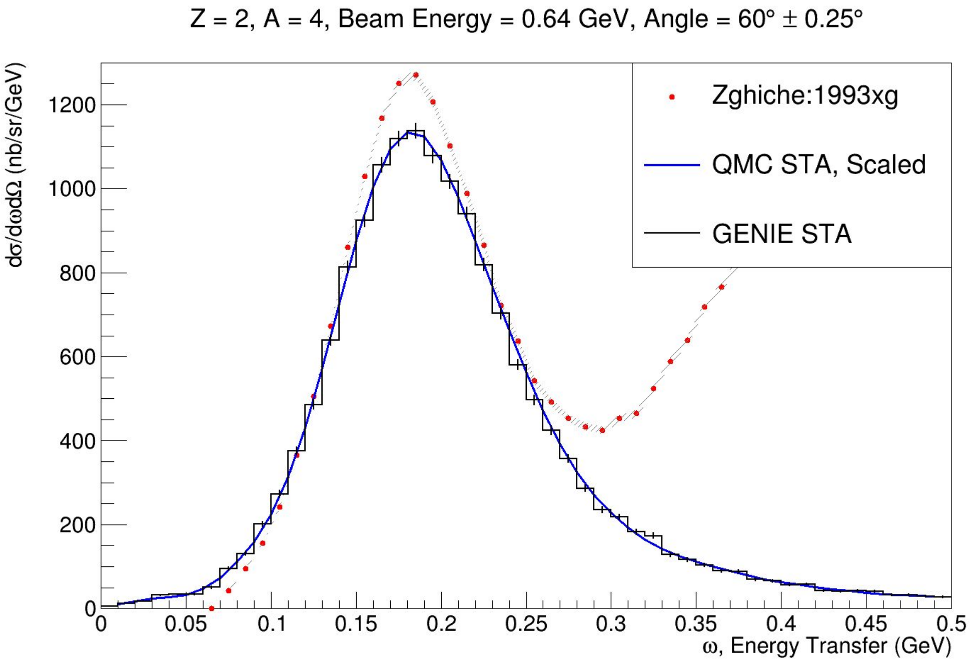
SP *et al.* PRC101(2020)044612

- pp pairs
- nn pairs
- all pairs 1body
- all pairs tot



Subedi et al. Science320(2008)1475

GENIE validation using e-scattering



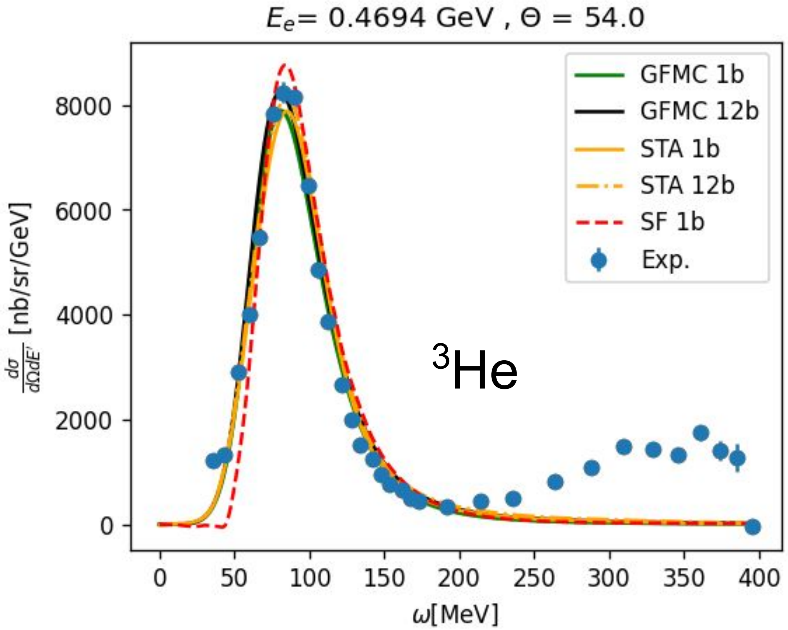
Barrow Gardiner Betancourt SP *et al.*
PRD 103 (2021) 5, 052001

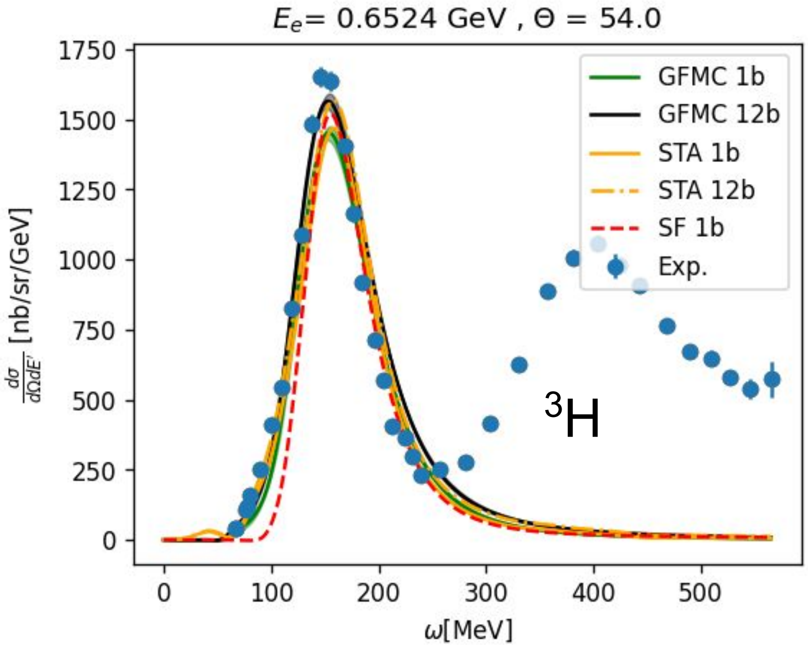
Ongoing work

- Implementation of moment-morphin interpolation techniques
- Implementations of response ***Densities*** in GENIE
- ^{12}C response densities with Lorenzo Andreoli

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega) \right]$$

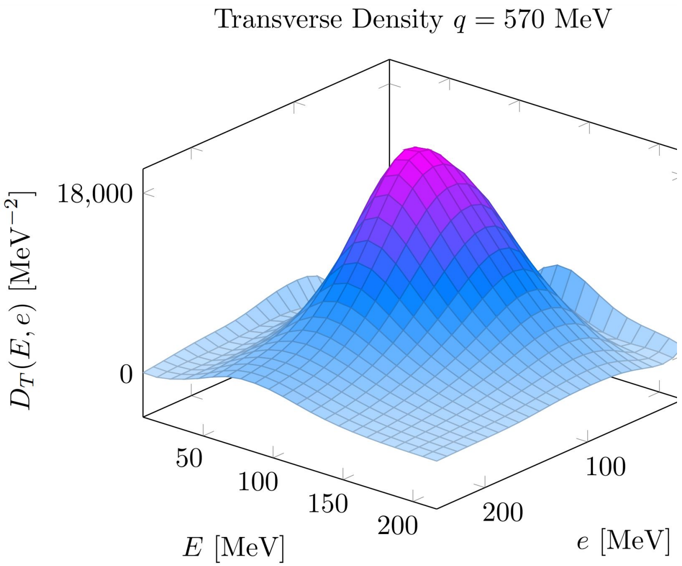
GFMC SF STA: Benchmark & error estimate

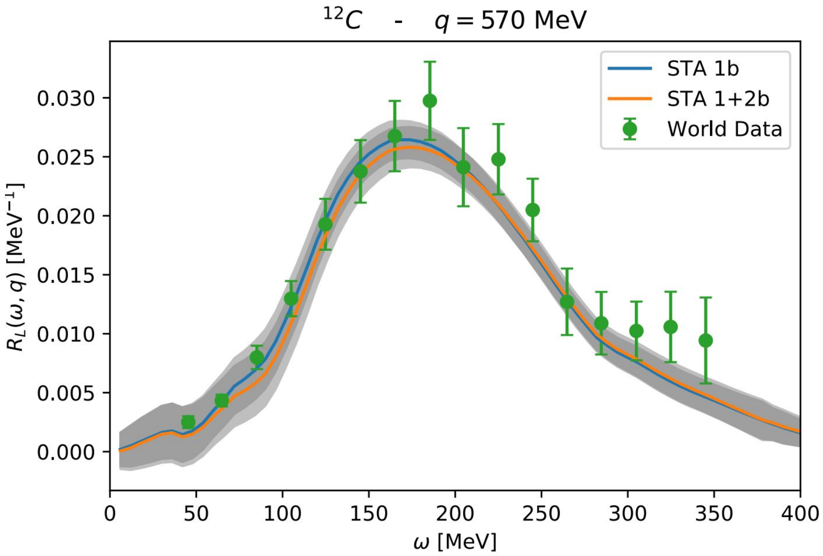




Andreoli, Rocco *et al.* submitted to PRC

STA for Carbon 12: Preliminary results

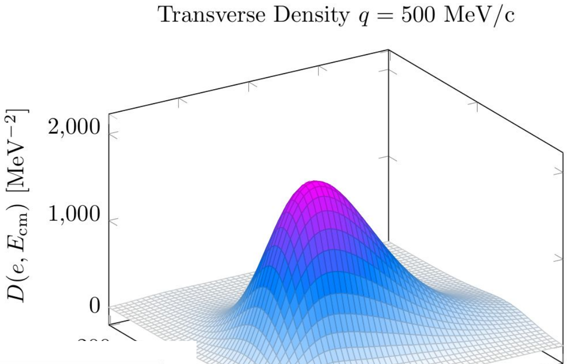


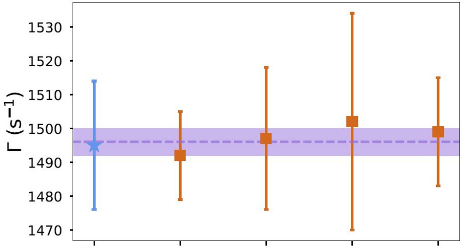


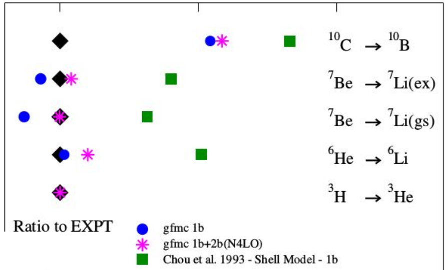
Lorenzo Andreoli *et al.* in preparation

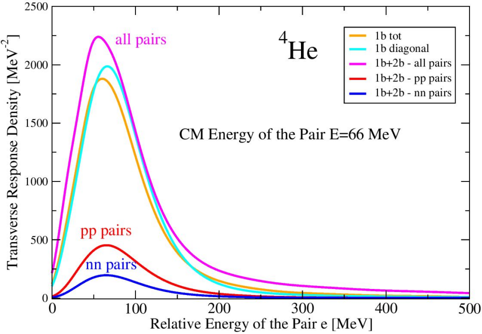
Summary

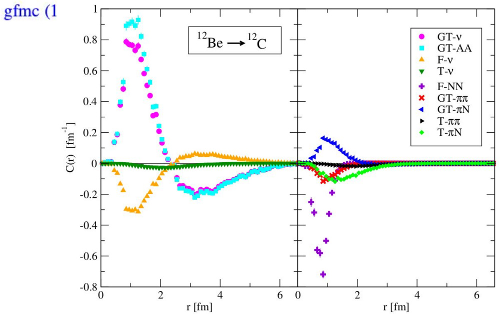
Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where many-body effects play an essential role to explain available data.











Close **collaborations** between **NP**, **LQCD**, **Pheno**, **Hep**, **Comp**, **Expt**, …
are required to progress
e.g., NP is represented in the Snowmass process

It's a very exciting time!

Collaborators

WashU: **Andreoli Bub King Piarulli**
LANL: Baroni Carlson Cirigliano Gandolfi Hayes Mereghetti
JLab+ODU: Schiavilla
ANL: Lovato Rocco Wiringa
UCSD/UW: Dekens
Pisa U/INFN: Kievsky Marcucci Viviani
Salento U: Girlanda
Huzhou U: Dong Wang
Fermilab: Gardiner Betancourt
MIT: Barrow

Washington University in St. Louis
Arts & Sciences

Theory Alliance
FACILITY FOR RARE ISOTOPE BEAMS

U.S. DEPARTMENT OF ENERGY
INCITE
LEADERSHIP COMPUTING

Los Alamos
NATIONAL LABORATORY
EST. 1943

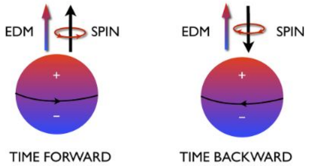
Argonne
NATIONAL LABORATORY

U.S. DEPARTMENT OF
ENERGY
Office of Science

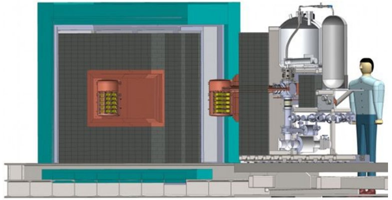
McDonnell Center
for the Space Sciences

Fermilab

Ground States' Electroweak Moments, Form Factors, Radii



Neutrinoless Double Beta Decay, Muon-Capture



Accelerator Neutrino Experiments, Lepton-Nucleus XSecs

(ω,q)~0 MeV

ω~few MeVs
q~0 MeV

ω~few MeVs
q~10^2 MeV

ω~tens of MeVs

ω~10^2 MeV

FRIB

Electromagnetic Decay, Beta Decay, Double Beta Decay & inverse processes

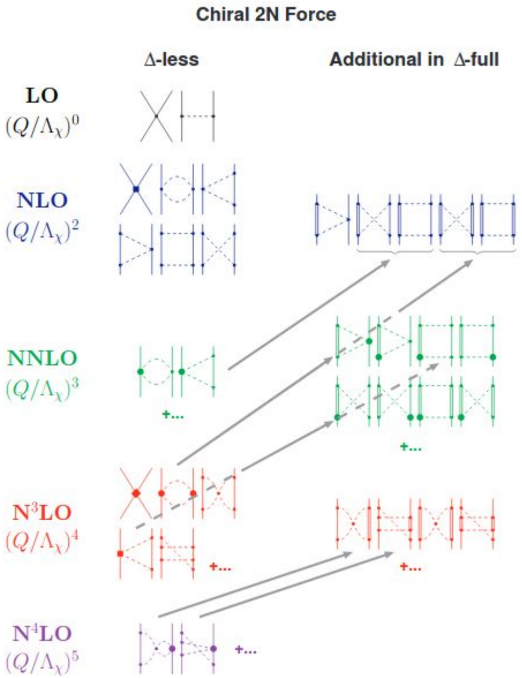
JINA-CEE

Nuclear Rates for Astrophysics

DUNE

Jefferson Lab

Norfolk Two- and Three-body Potentials

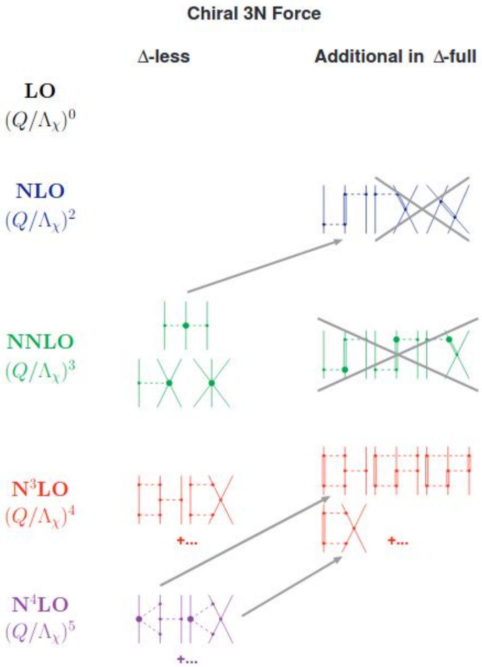


Norfolk Chiral Potentials

NV2+3

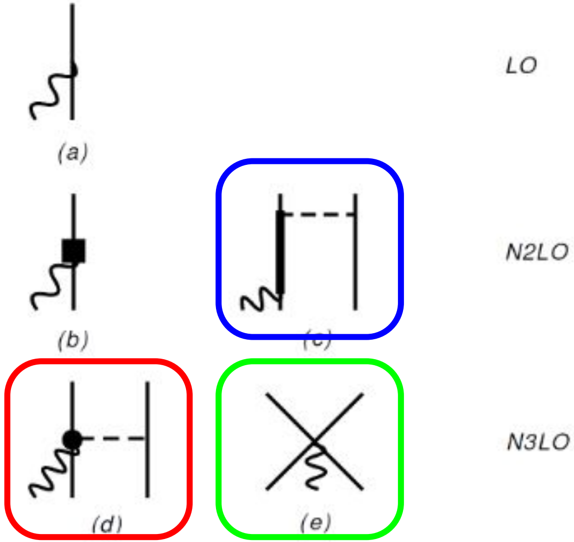
developed in Piarulli *et al*.
PRC91(2015)024003
PRC94(2016)054007

26 LECs fitted to np and pp Granada database (2700-3700 data points; 125-200 MeV) with a chi-square/datum ~1



Figs. credit Entem and Machleidt Phys.Rept.503(2011)1

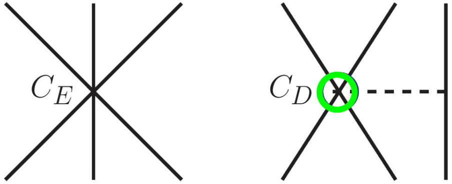
Axial currents with Δ at tree-level



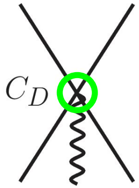
Two body currents of one pion range (red and blue) with c_3 c_4 from Krebs *et al.* Eur.Phys.J.(2007)A32

Contact current involves the LEC $\boldsymbol{c_D}$

Three-body Force and the Axial Contact Current



Three-body force



Axial two-body contact current

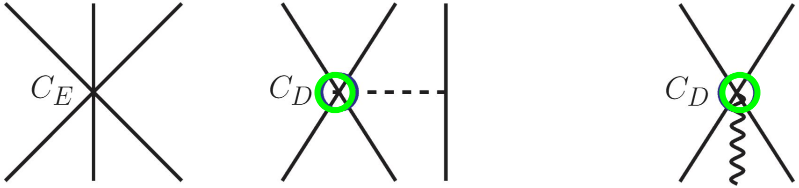
LECs c_D and c_E are fitted to:

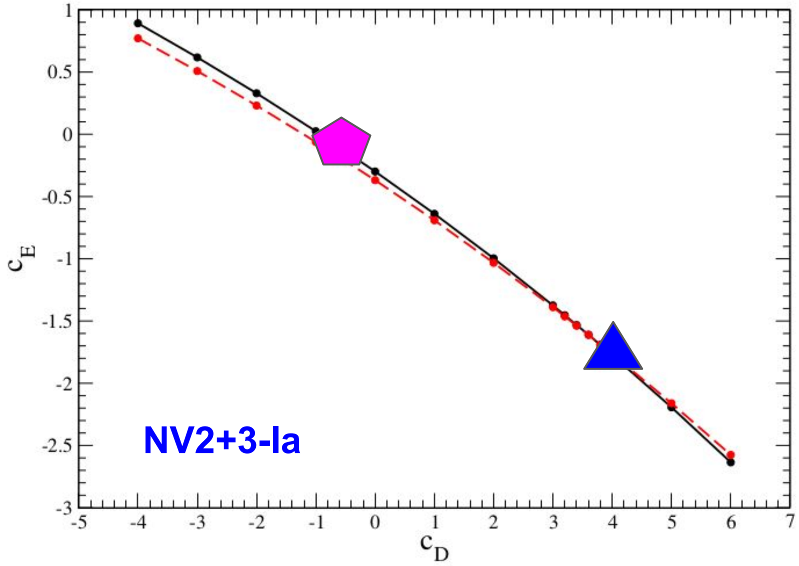
- **trinucleon B.E.** and ***nd* doublet scattering length** in **NV2+3-Ia**
- **trinucleon B.E.** and **Gamow-Teller matrix element of tritium** **NV2+3-Ia***

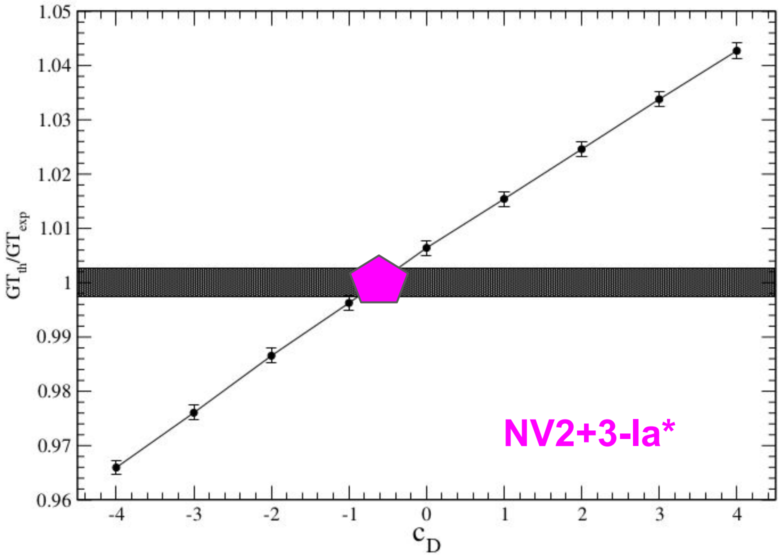
Baroni *et al.* PRC98(2018)044003

Energies A=8-10 slightly better with non-starred models

Fitting strategies

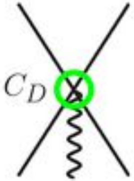






Baroni *et al.* PRC98(2018)044003

Contact Current

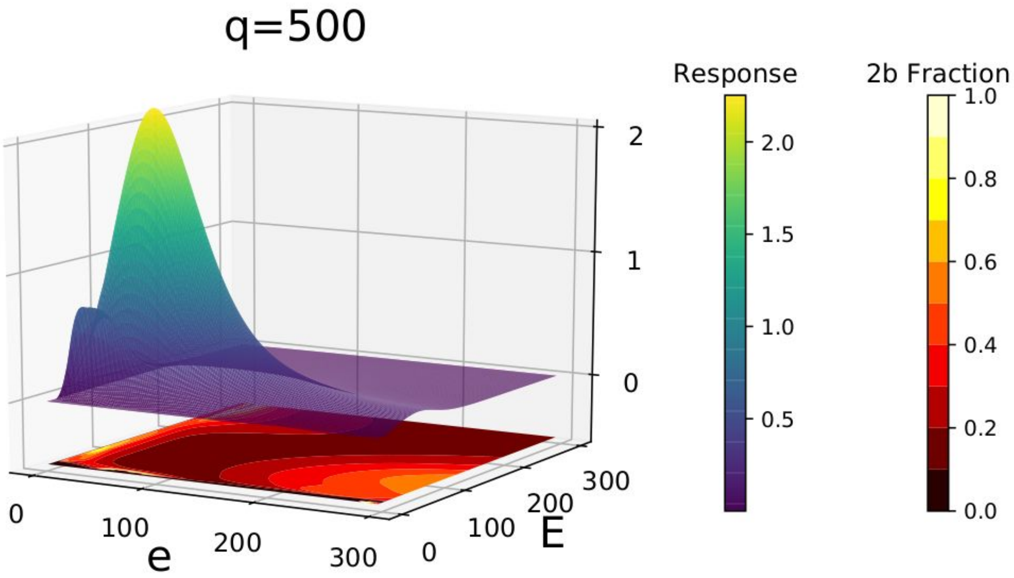


	NV2+3-Ia	NV2+3-Ia*
c_D	3.666	-0.635
c_E	-1.638	-0.090
z_0	0.090	1.035

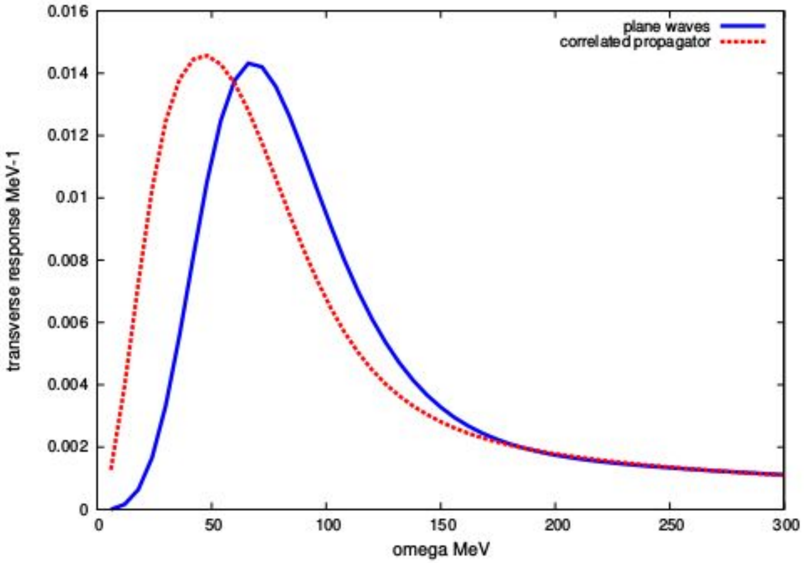
$$\mathbf{j}^{\rm N3LO}_{5,a}(\mathbf{q};{\rm CT}) = z_0\, {\rm e}^{i\,\mathbf{q}\cdot\mathbf{R}_{ij}}\, \frac{{\rm e}^{-\tilde{r}_{ij}^2}}{\pi^{3/2}}\, \left(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j\right)_a \left(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j\right)$$

$$z_0 \quad = \quad \frac{g_A}{2}\,\frac{m_\pi^2}{f_\pi^2}\,\frac{1}{(m_\pi R_{\rm S})^3}\left[-\frac{m_\pi}{4\,g_A\,\Lambda_\chi}\,c_D+\frac{m_\pi}{3}\,(c_3+2\,c_4)+\frac{m_\pi}{6m}\right]$$

Transverse Response Density: two-body physics

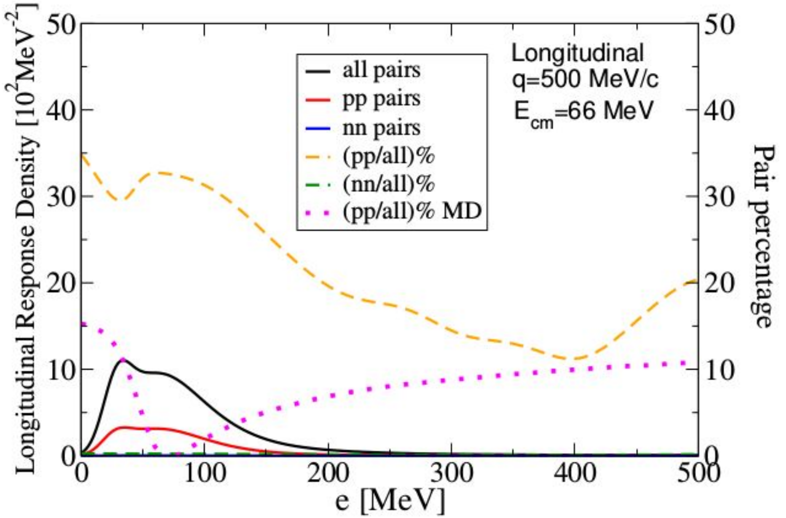


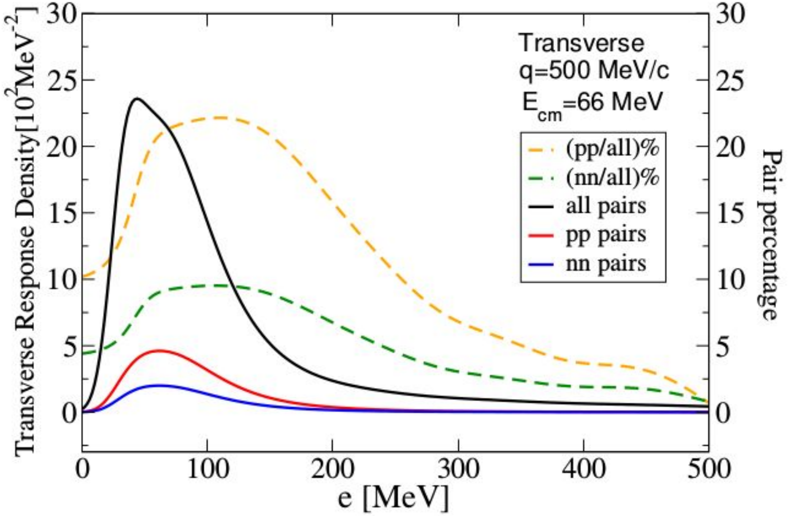
Correlated pairs vs uncorrelated pairs



Scattering from uncorrelated vs correlated nucleon pairs

Back to back scattering and particle identity





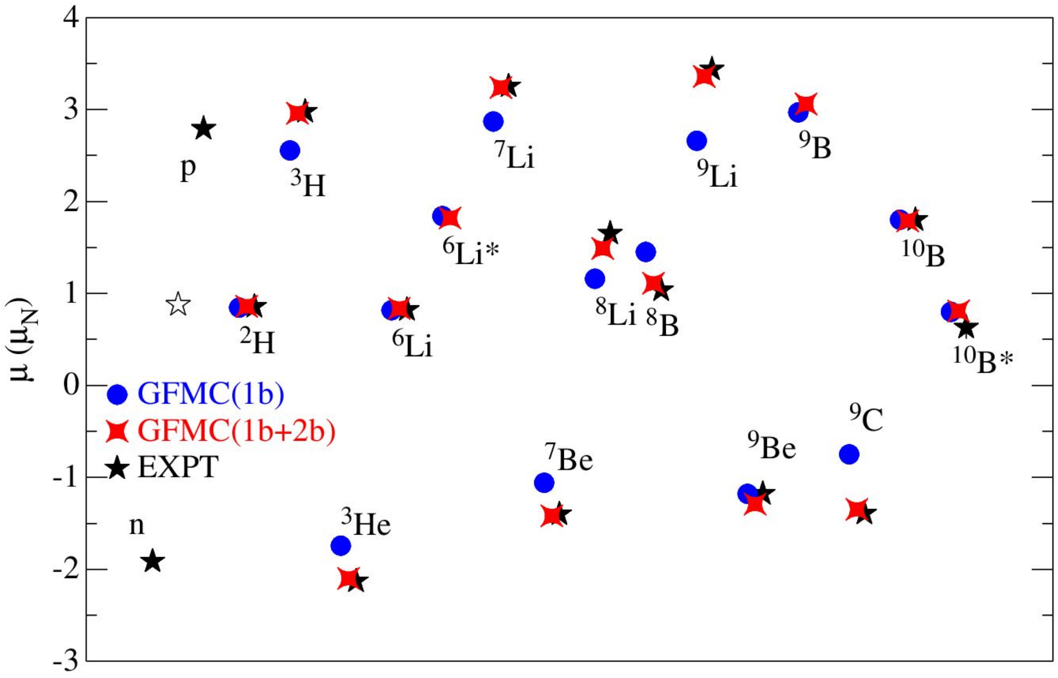
tot

pp nn

pp/all % pp/all % from momentum distributions

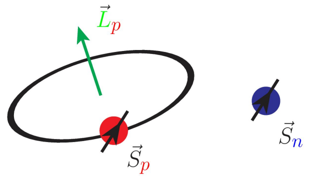
nn/all %

Magnetic Moments of Light Nuclei



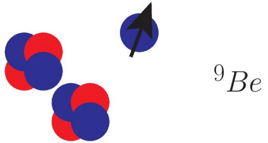
SP *et al*. PRC87(2013)035503

Single particle picture

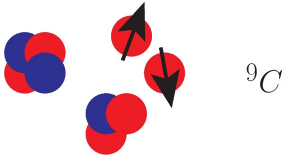


$$\mu_N(1\mathrm{b}) = \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i(1-\tau_{i,z})/2]$$

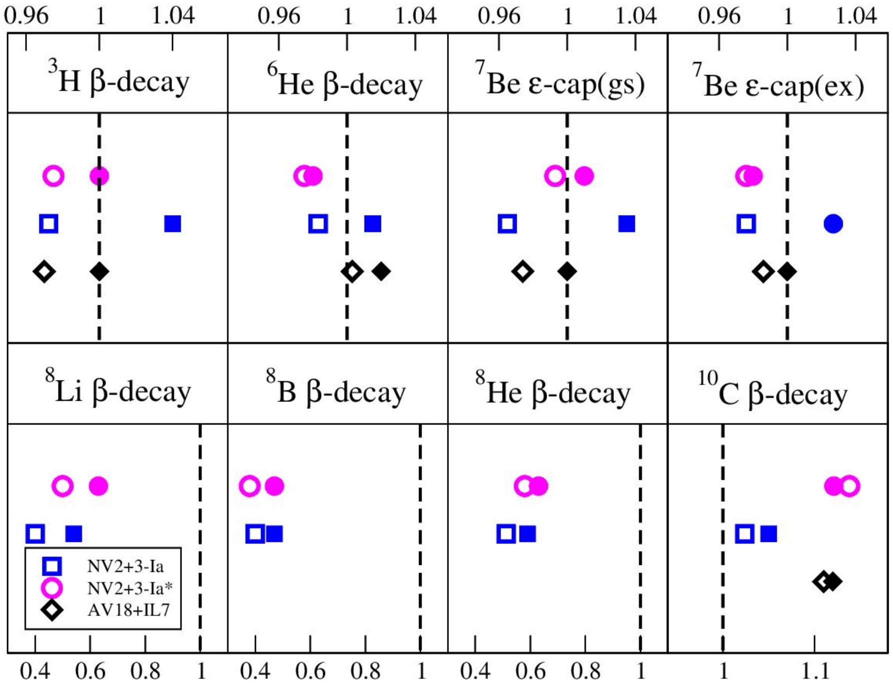
Small two-body current effects



Large two-body current effects



Beta Decay and Electron Capture in Light Nuclei



Garrett King *et al.* PRC102(2020)025501

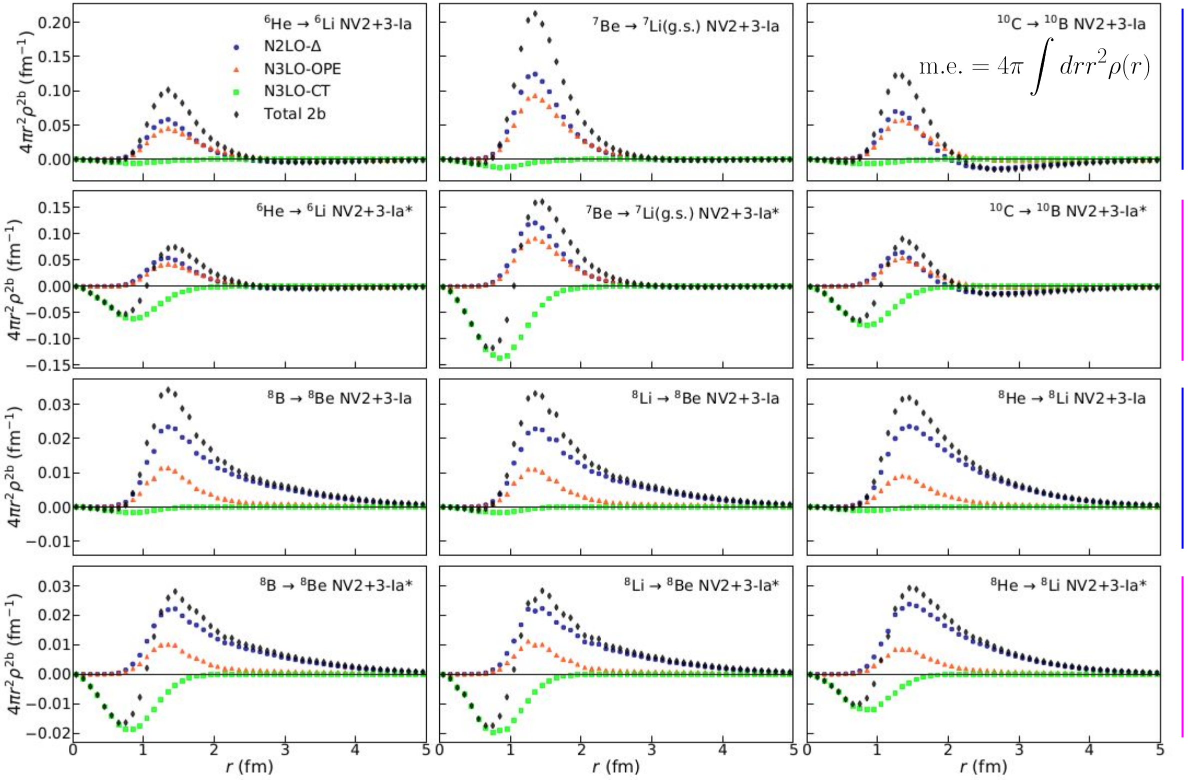
Calculations based on

- chiral interactions and currents
  NV2+3-Ia Norfolk unstarred
  NV2+3-Ia* Norfolk* starred
  Piarulli *et al.* PRL120(2018)052503
  Baroni *et al.* PRC98(2018)044003

- phenomenological **AV18+IL7** potential and chiral axial currents (hybrid calculation)

Two-body currents are small/negligible;
Results for A=6-7 are within 2% of data;
Results for A=8 are off by a 30-40%;
Results for A=10 are affected by the second $J^\pi=(1^+)$ state in ^{10}B

Axial Two-body Transition Density



Garrett King *et al.* PRC102(2020)025501

NV2+3-Ia ; **NV2+3-Ia***

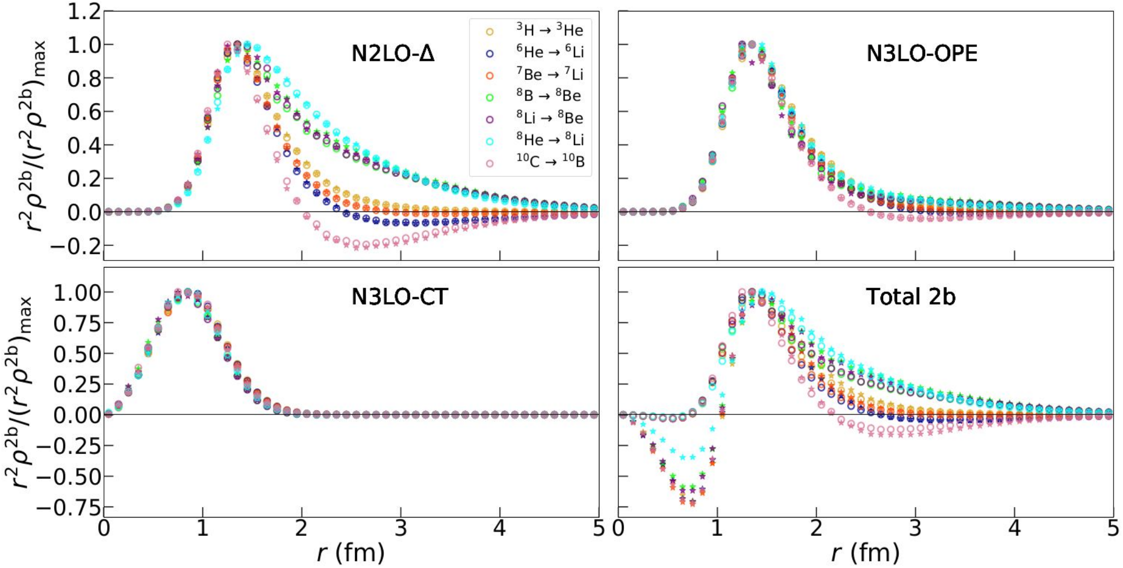
enhanced contribution from contact current in the starred model gives rise to nodes in the two-body transition density

Two-body axial currents

long-range at N2LO and N3LO

contact current at N3LO

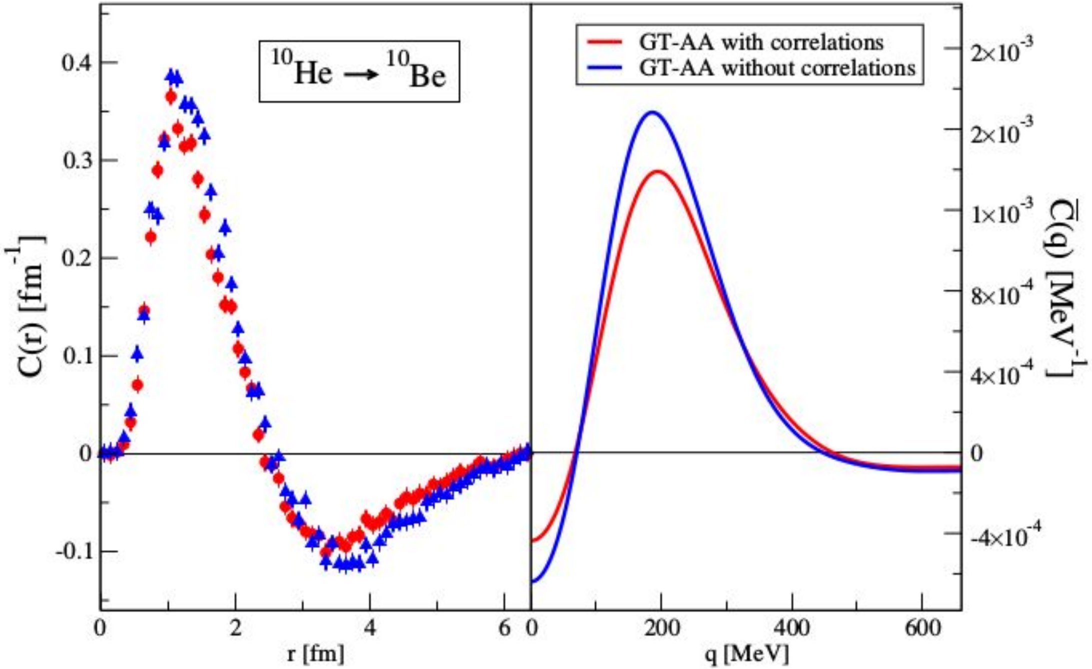
Scaling & Universality of Short-Range Dynamics



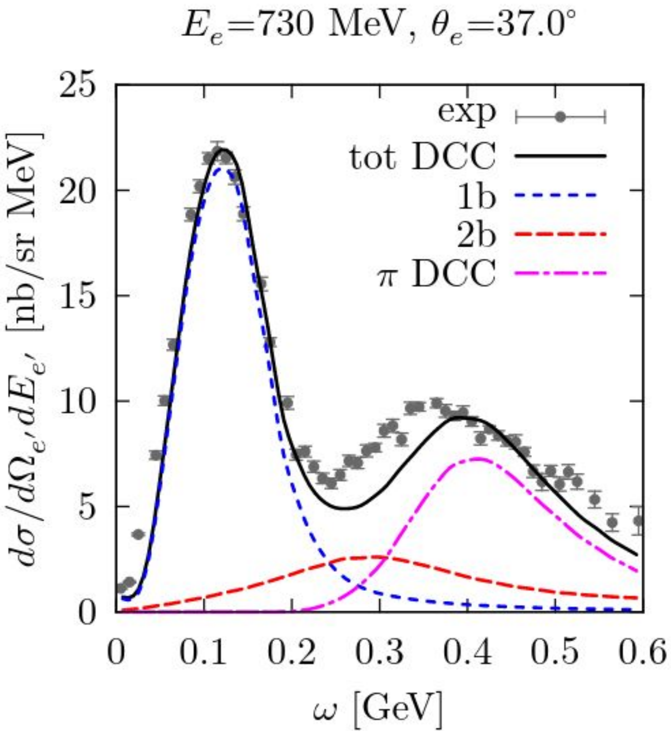
Garrett King *et al.* PRC102(2020)025501

NV2+3-Ia empty circles; NV2+3-Ia* stars
Different colors refer to different transitions

Correlations in neutrinoless double beta decay ME



Factorization for pion-production



Rocco et al. PRC100(2019)

Methods based on factorization of the final states can accommodated meson-production and relativistic effects.

Calculations based on the **Spectral Function formalism** (Rocco et al.) supplemented by the Dynamical Coupled-Channel (Sato, Nakamura, et al.) model for meson production

The Shallow Inelastic Scattering (SIS) remains to be investigated

Quantum Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: H = T + υ_{ij} + V_{ijk}

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using the trial wave function:

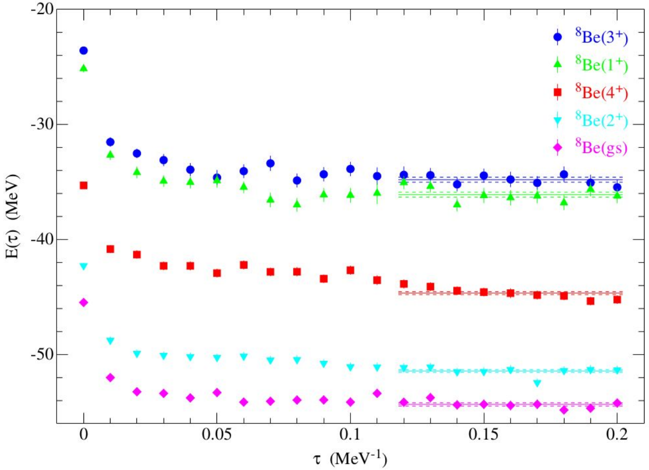
$$|\Psi_V\rangle = \left[\mathcal{S}\prod_{i<j}(1 + U_{ij} + \sum_{k\neq i,j} U_{ijk})\right]\left[\prod_{i<j} f_c(r_{ij})\right]|\Phi_A(JMTT_3)\rangle$$

Further improve the trial wave function by eliminating spurious contaminations via a Green's Function Monte Carlo propagation in imaginary time

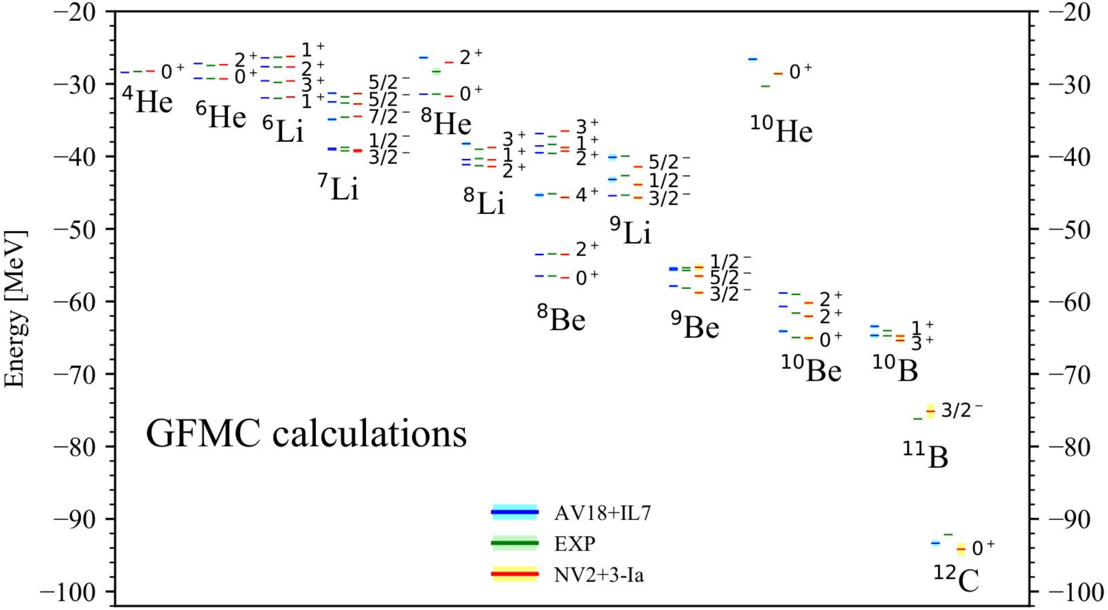
$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \to \infty) = a_0 \psi_0$$

Carlson, Wiringa, Pieper *et al.*

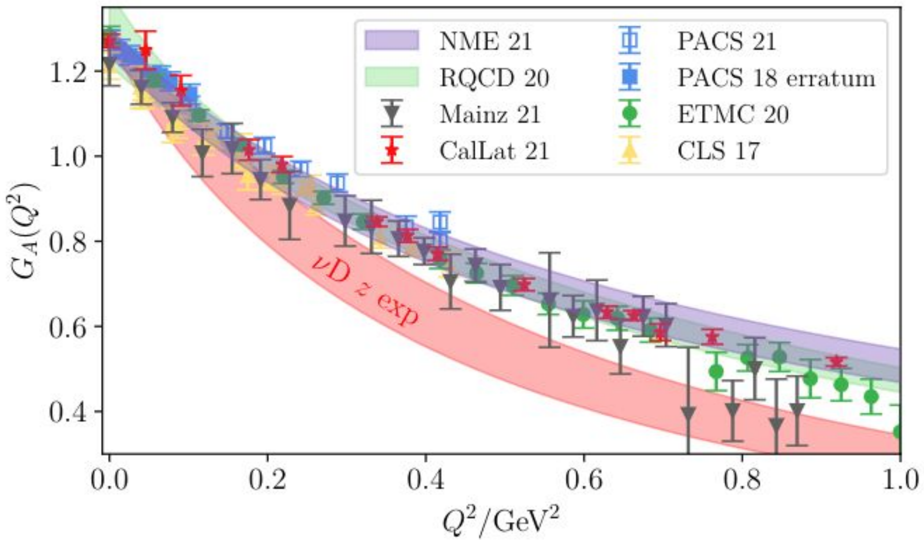


Energies



Piarulli *et al.* PRL120(2018)052503

LCQD inputs for neutrino-nucleus scattering



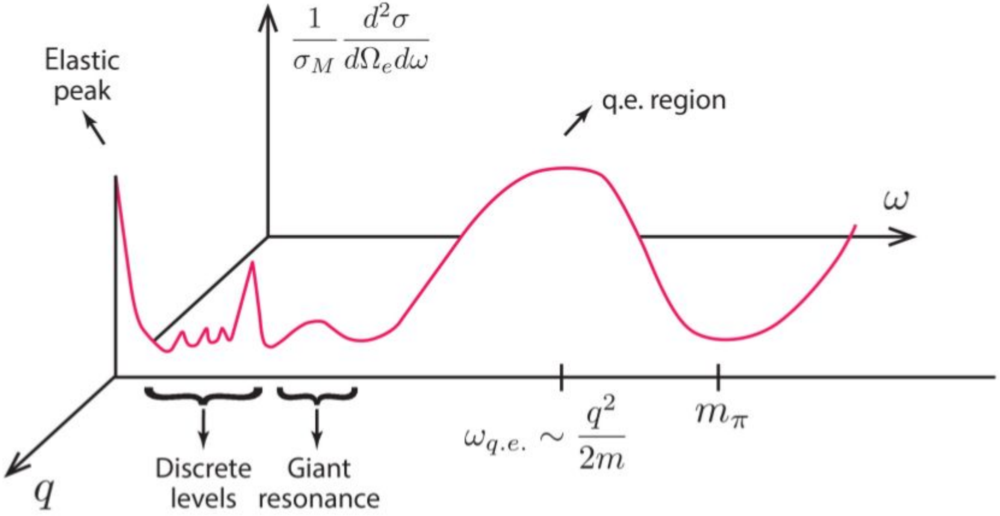
Snowmass WP: Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators; arXiv:2203.09030

Building blocks of ab initio nuclear approaches:

Nucleonic form factors
Transition form factors
Pion production amplitudes
Two-nucleon couplings (strong and EW)
…

Taken from data where available, or from theory

Electron-Nucleus Scattering Cross Section



Energy and momentum transferred (ω,q)

Current and planned experimental programs rely on theoretical calculations at different kinematics