Removal of the center of mass in nuclei within the Equation of Motion Phonon Method

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Motivation

Exact **separation** of **Center of Mass** (CM) admixtures in the **many-body** states very complicated task. We cannot use **Jacobi coordinates** as in **few-body** physics. **However, low-lying states in all nuclei (including heavy systems) are affected significantly by CM**. Important for study of e.g. **Pygmy** dipole resonances, low-energy **E0**, **E2** transitions etc.

Outline:

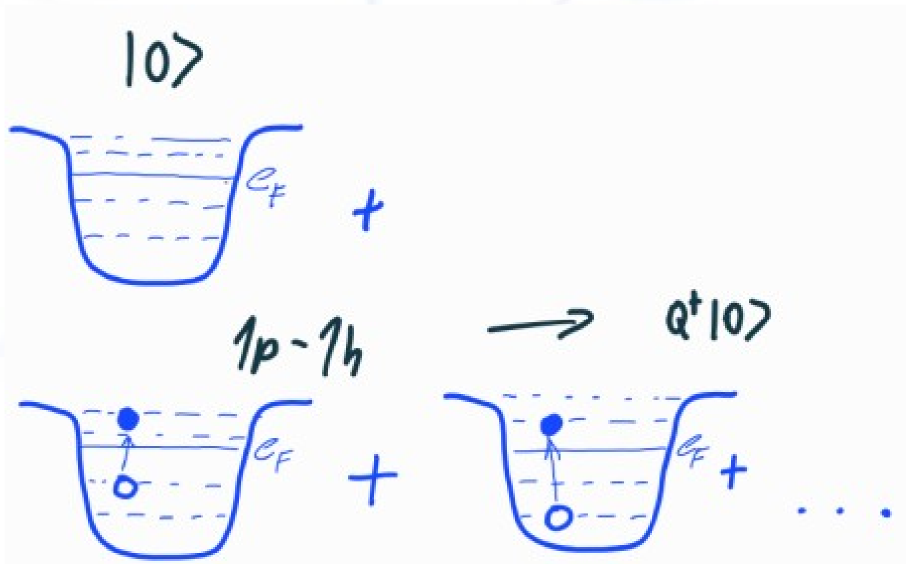
- **Equation of Motion Phonon Method** (EMPM)
- **Singular Value Decomposition Method** (SVD)
- **Center of Mass** (CM) Problem
- **Results** – demonstration of **SVD** on calculation of energy spectrum of ^{4}**He** within **EMPM**
- **Results** – **EMPM** calculations in ^{40}**Ca,** ^{208}**Pb** and their comparison with **STDA, SRPA** methods
- **Summary** and Future Plans

Equation of Motion Phonon Method

Intrinsic nuclear **Hamiltonian**

$$H = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A} V_{ij} = H_{int} + \frac{\boldsymbol{P}^2}{2Am}$$

In **Tamm-Dancoff Approximation** (TDA) phonons are linear superpositions of **1particle-1hole** excitation on top of mean-field **Slater determinant** (HF)



n=0 → **HF**

n=1 → **TDA**

$$[\boldsymbol{H}_{intr}, \boldsymbol{Q}_{\nu}^{\dagger}]|0\rangle \equiv \hbar\omega_{\nu}\boldsymbol{Q}_{\nu}^{\dagger}|0\rangle$$

$$|\nu\rangle = \boldsymbol{Q}_{\nu}^{\dagger}|0\rangle, \quad \boldsymbol{Q}_{\nu}|0\rangle = 0$$

$$O_{\nu}^{\dagger} = \sum_{ph} c_{ph}^{\nu} a_p^{\dagger} a_{\bar{h}}$$

Hilbert space – divided into subspaces

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

$$\begin{aligned}
\mathcal{H}_0 &= \{|HF>\} \\
\mathcal{H}_1 &= \{O_{\nu_1}^{\dagger}|HF>\} \\
\mathcal{H}_2 &= \{O_{\nu_1}^{\dagger}O_{\nu_2}^{\dagger}|HF>\} \\
&\;\;. \\
&\;\;. \\
\mathcal{H}_n &= \{O_{\nu_1}^{\dagger}O_{\nu_2}^{\dagger}\ldots O_{\nu_n}^{\dagger}|HF>\}
\end{aligned}$$

Equation of Motion Phonon Method

Equation of Motion (EoM) – recursive eq. to solve **eigen-energies** on each **n**-phonon subspace while knowing the (**n-1**)-phonon eigen-energies

$$\begin{aligned}\mathcal{H}_0 &= \{|HF>\}\\ \mathcal{H}_1 &= \{O^\dagger_{\nu_1}|HF>\}\\ \mathcal{H}_2 &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}|HF>\}\\ &\;\;\vdots\\ \mathcal{H}_n &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}...O^\dagger_{\nu_n}|HF>\}\end{aligned}$$

$$< n,\beta\,|\left\{\left[H,O^\dagger_\lambda\right]\times|\,n-1,\alpha>\right\}^\beta = \left(E^{(n)}_\beta - E^{(n-1)}_\alpha\right) < n,\beta\,|\left\{O^\dagger_\lambda\times|\,n-1,\alpha>\right\}^\beta$$

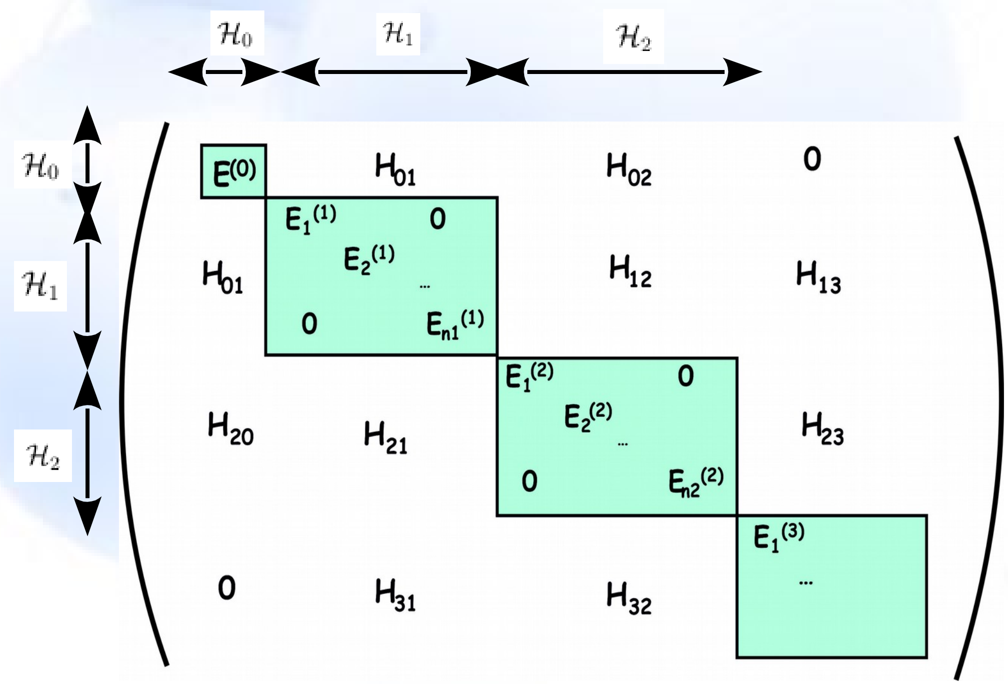
n-phon state | (**n-1**)-phon state | **n**-phon state | (**n-1**)-phon state

At the end we **diagonalize** nuclear **Hamiltonian** in **(0+1+2+...+n)** - phonon **basis**

$$\boldsymbol{H} = \sum_{i=1}^{A}\frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A}\boldsymbol{V}_{ij} = \boldsymbol{H}_{int} + \frac{\boldsymbol{P}^2}{2Am}$$

Hamiltonian represented in multiphonon basis

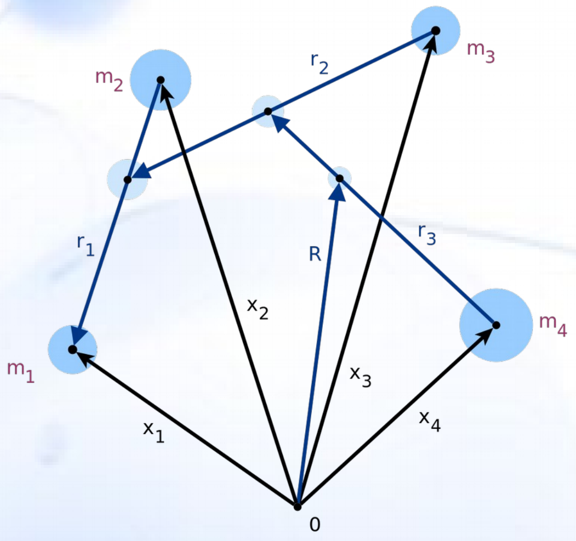
$$\boldsymbol{H}_{intr} = \sum_{n,\alpha} E^n_\alpha|n,\alpha\rangle\langle n,\alpha| + \sum_{nn',\alpha\alpha'}|n',\alpha'\rangle\langle n',\alpha'|\boldsymbol{H}_{intr}|n,\alpha\rangle\langle n,\alpha|$$



Center of Mass Problem

Factorization of CM in Hamiltonian $$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j}^{A} V_{ij} = H_{int} + \frac{P^2}{2Am}$$

Factorization of CM in wave function $$\psi(R, r) = \varphi_{CM}(R)\chi_{int}(r)$$



For exact separation of the CM in w.f. - **Jacobi coordinates**
However, in **many-body** methods it is **not achievable** to work with these coordinates.

In methods which are based on the **mean field**:

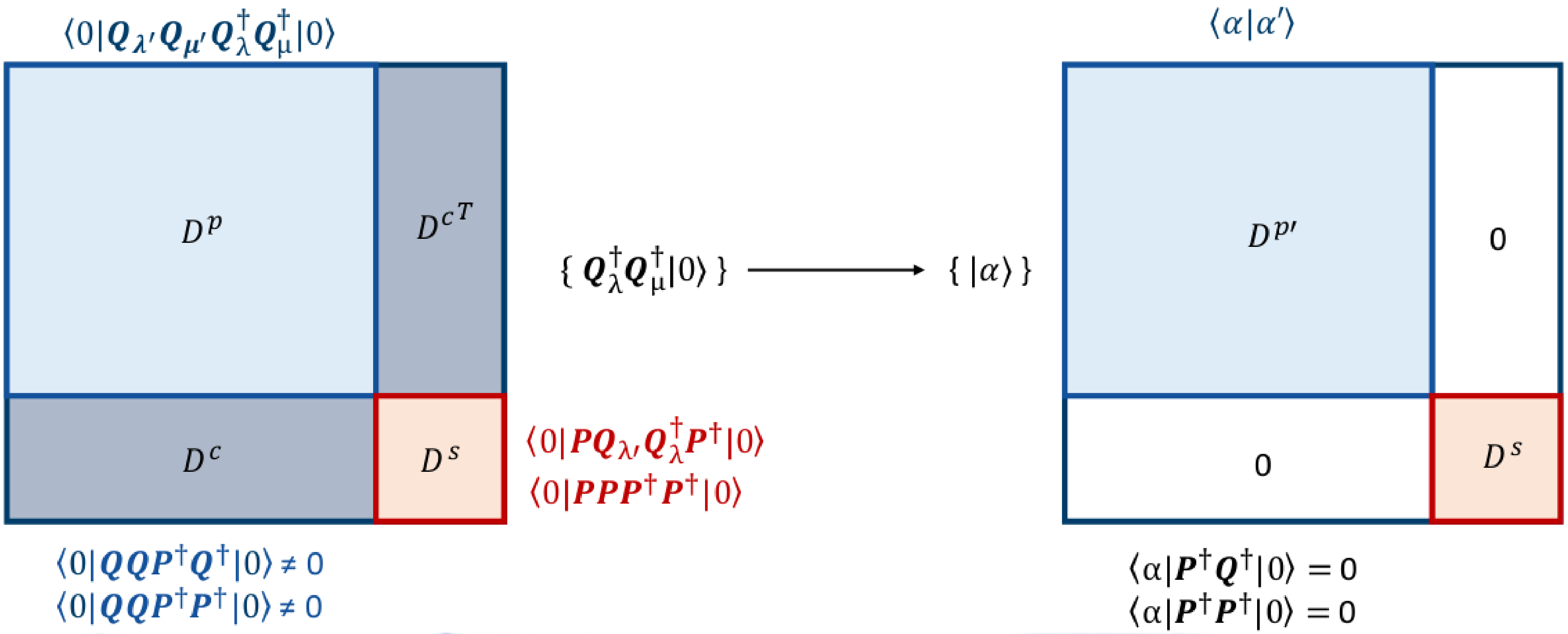
- **central potential** with fixed origin to which particle motion is reffered → **Slater determinants**
- **exact factorization** of CM is **not possible** (finite model spaces)
- **wave functions** and **effective Hamiltonians** are not translationally invariant
- **CM spurious** states appear in the **energy spectra**
- **CM problem** is inherently present in all models based on the **mean field**

Singular Value Decomposition

G. De Gregorio, F. Knapp, N. Lo Iudice, P. Vesely, **Phys. Lett. B 821**, 136636 (2021)

Demonstration of extraction of **CM spuriosity** in **2-phonon** space:
Overlap matrix D among all 2-phonon states. There is a subspace of 2-phonon configurations affected by spuriosity.

SVD acts on the **D^c** submatrix – transformation to **separate** spurious and spurious-free **subspaces**.



Singular Value Decomposition

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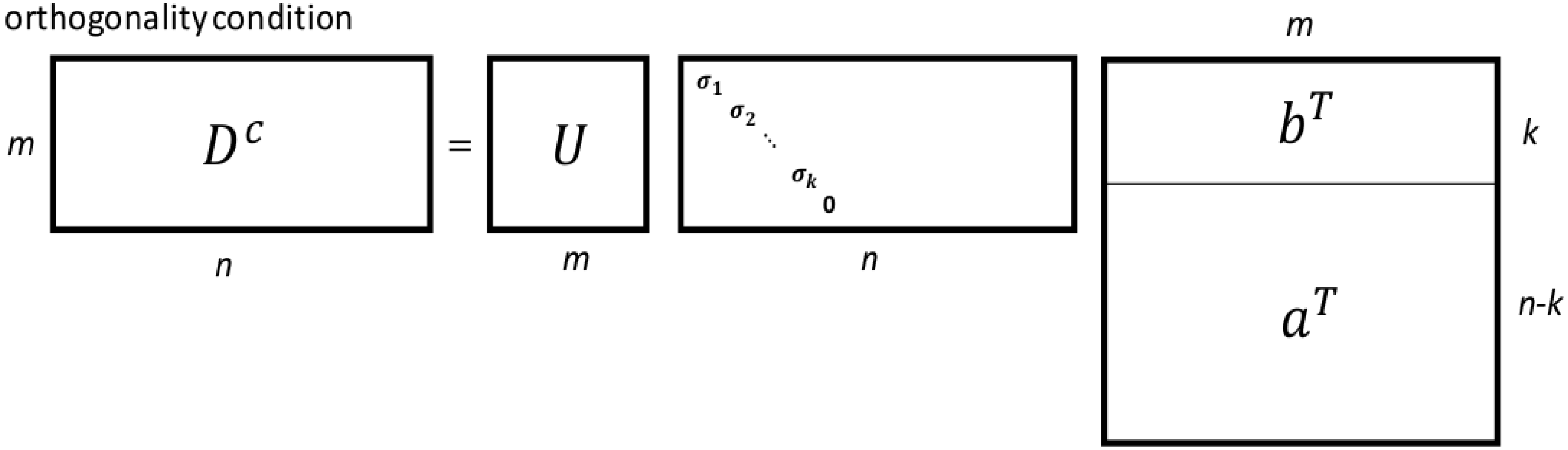
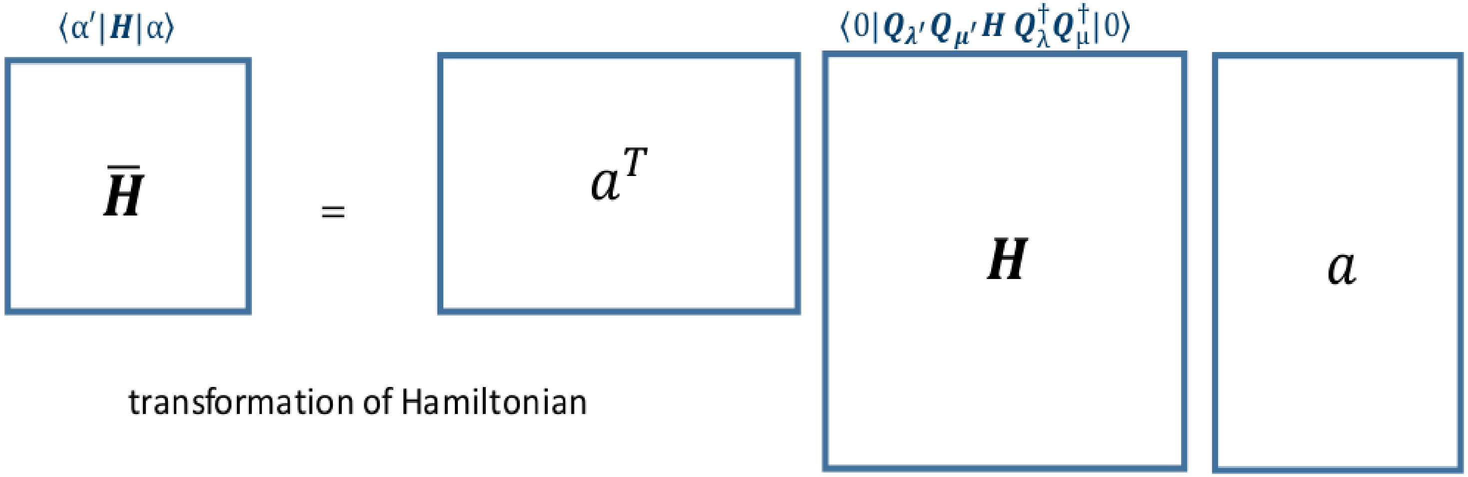
Singular value decomposition of real *m* x *n* matrix $\boldsymbol{D^c}$: $\boldsymbol{D^c} = \boldsymbol{U\Sigma V^T}$, where (*m* x *m*) $\boldsymbol{U}$ and (*n* x *n*) $\boldsymbol{V}$ are orthogonal matrices
right singular vectors $\boldsymbol{V}$ corresponding to zero singular values span the null space (kernel) of $\boldsymbol{D^c}$
$\boldsymbol{D^c} a = 0 \leftrightarrow$ orthogonality condition



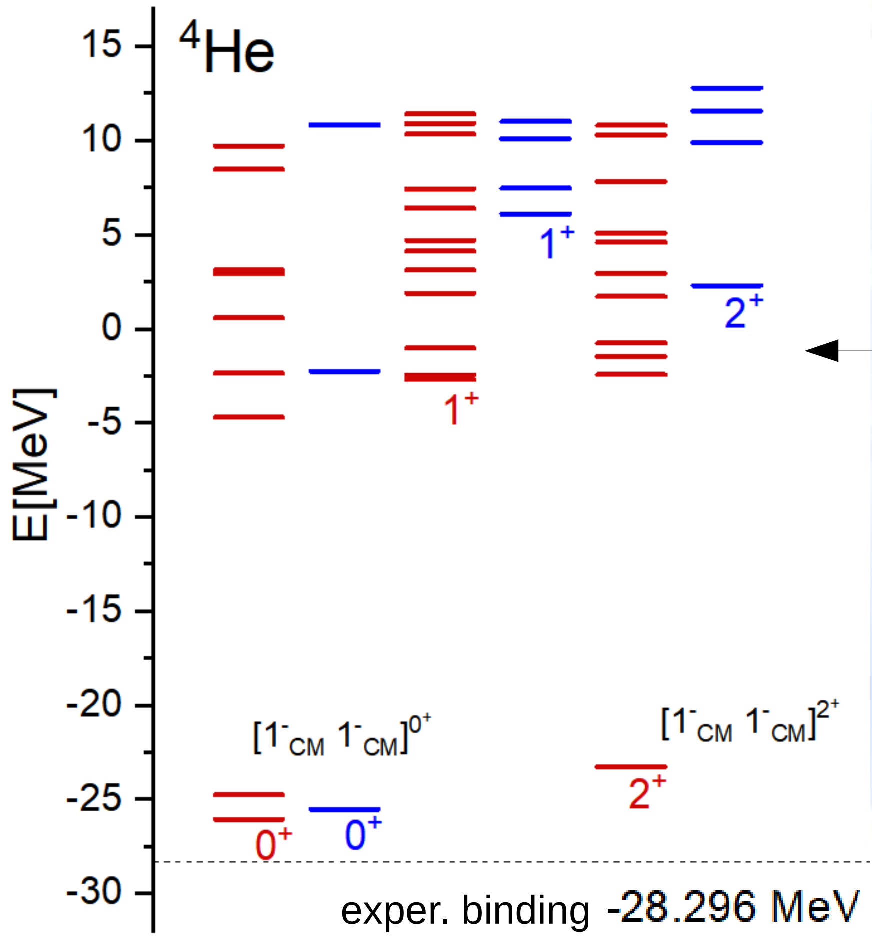


transformation of Hamiltonian

Results – CM removal in ^{4}He

G. De Gregorio, F. Knapp, N. Lo Iudice, P. Vesely, **Phys. Lett. B 821**, 136636 (2021),
Phys. Rev. C 105, 024326 (2022).

Calculation of ^{4}**He** in **0+1+2+3** phonon space.



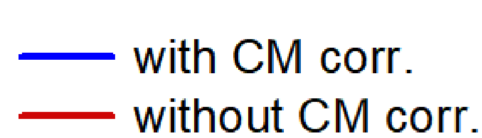
NN+NNN interaction – chiral **NNLO**$_{sat}$.

Demonstration of **SVD** method to separate CM on the **EMPM** calculation of ^{4}**He**. Without CM correction there are states affected by CM spuriousity.

Positive parity states **0$^+$, 1$^+$, 2$^+$**.

Full up to 3-phonon space calculation with **N**$_{max}$ = 4

$\hbar\omega$ =20 MeV



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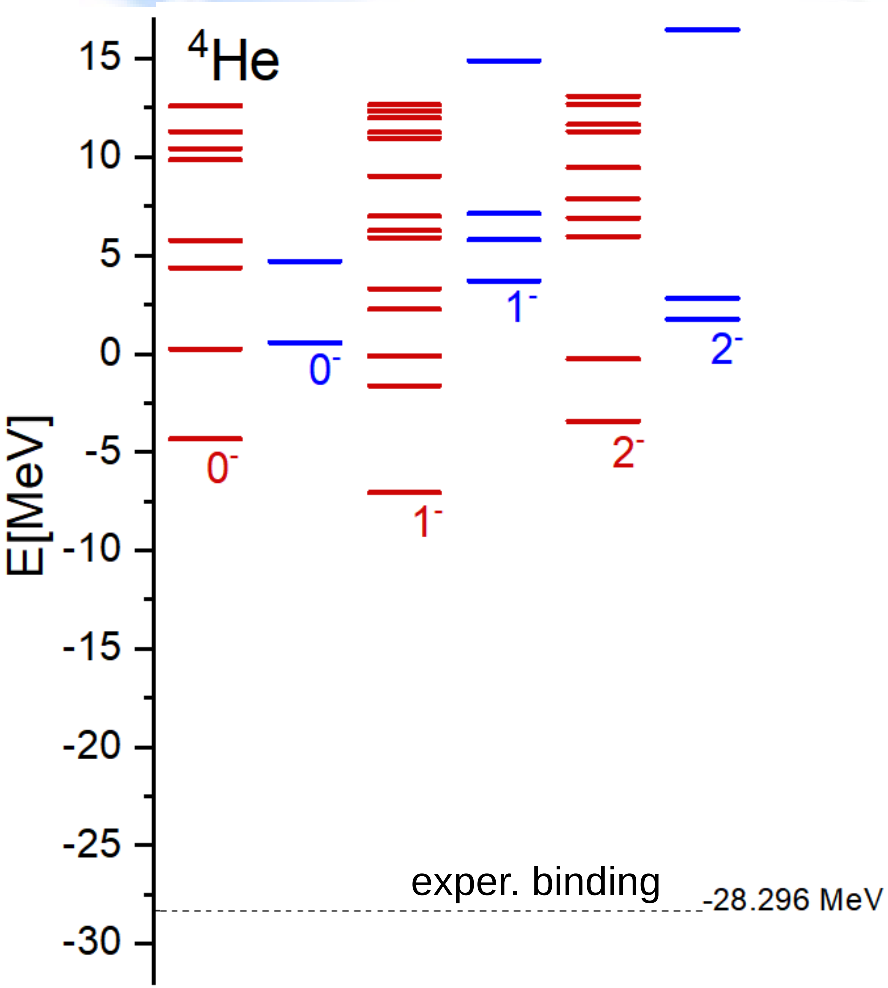


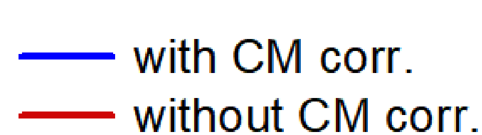
NN+NNN interaction – chiral **NNLO**$_{sat}$.

Demonstration of **SVD** method to separate CM on the **EMPM** calculation of ^{4}**He**. Without CM correction there are states affected by CM spuriousity.

Negative parity states **0^- , 1^- , 2^-**.

Full up to 3-phonon space calculation with **N_{max}** = 4

$\hbar\omega$ =20 MeV



Results – comparison EMPM, STDA, SRPA

iv E1 dipole strength in **^{40}Ca**

N_{max} = 6,   NN interaction: **UCOM**

Preliminary

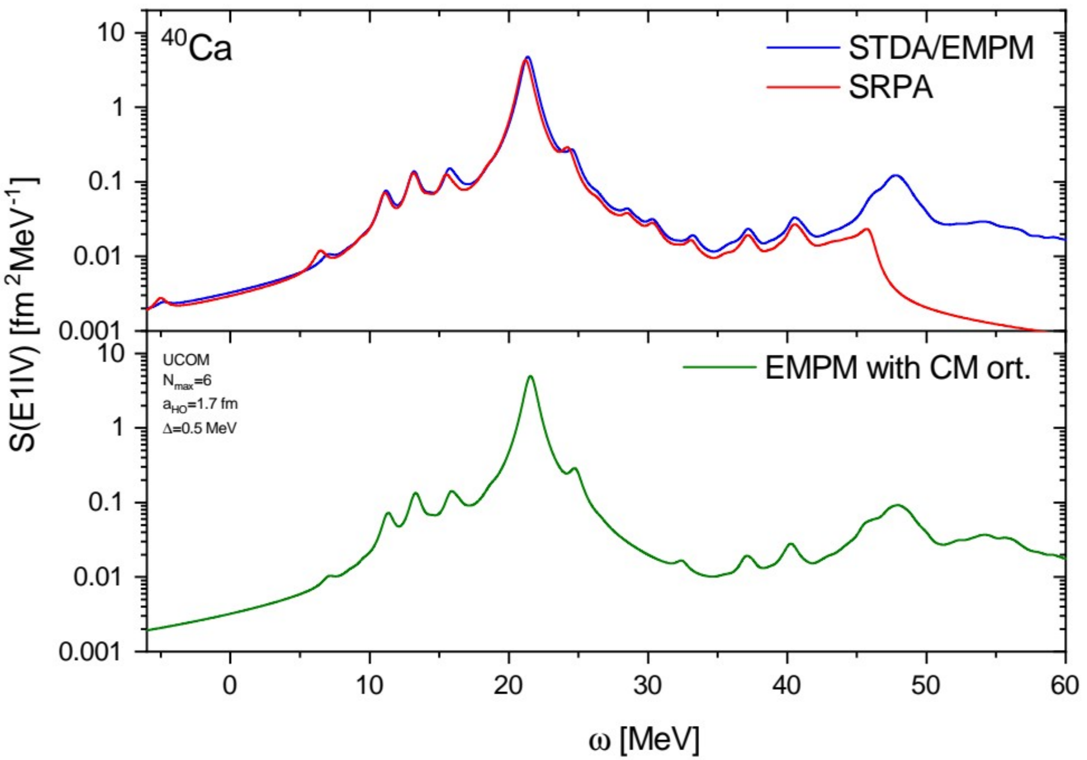
SRPA results not far from **STDA**
(STDA/SRPA results by P. Papakonstantinou)

In **EMPM** the **CM correction** can be done → usually removes some of low lying states (they have **spurious** character)

EMPM in **1+2** phonon space **without CM correction**

fully **equivalent** to **STDA**

Advantage of **EMPM** to STDA/SRPA is that in EMPM **can be done CM correction**.



Results – comparison EMPM, STDA, SRPA

E0 dipole strength in **^{40}Ca**

N_{max} = 6, NN interaction: **UCOM**

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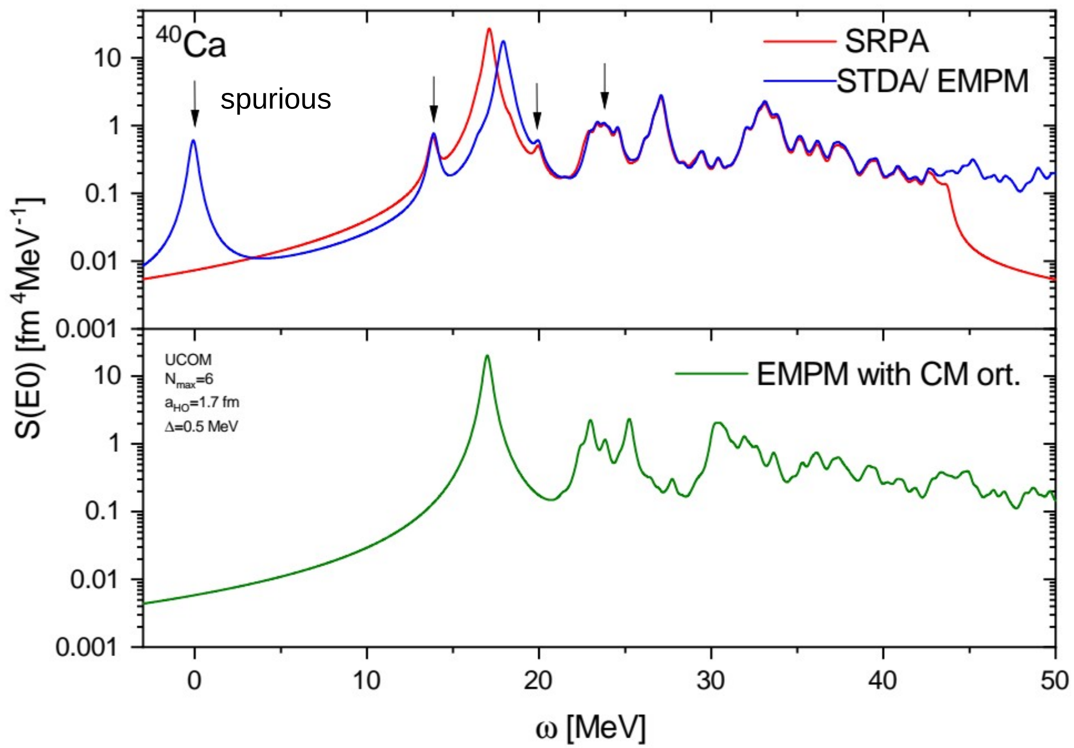

Results – comparison EMPM, STDA, SRPA

E2 dipole strength in ^{40}**Ca**

N_{max} **= 6**,   NN interaction: **UCOM**

Preliminary

SRPA results not far from **STDA**
(STDA/SRPA results by P. Papakonstantinou)

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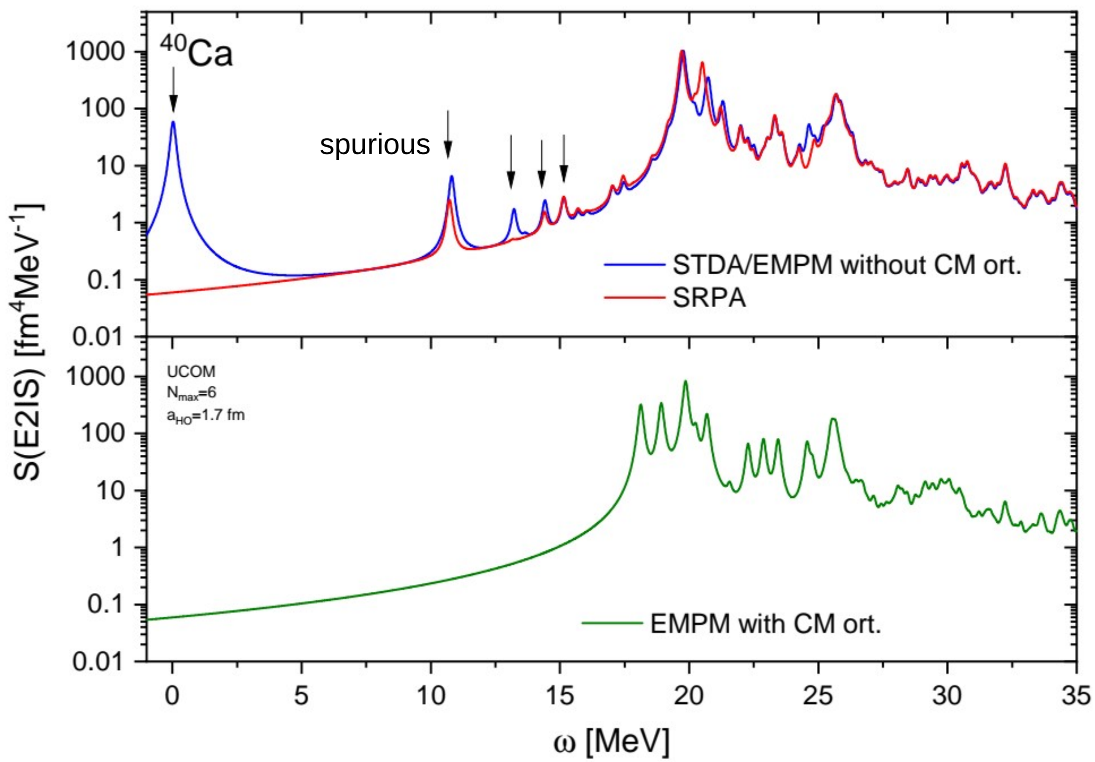


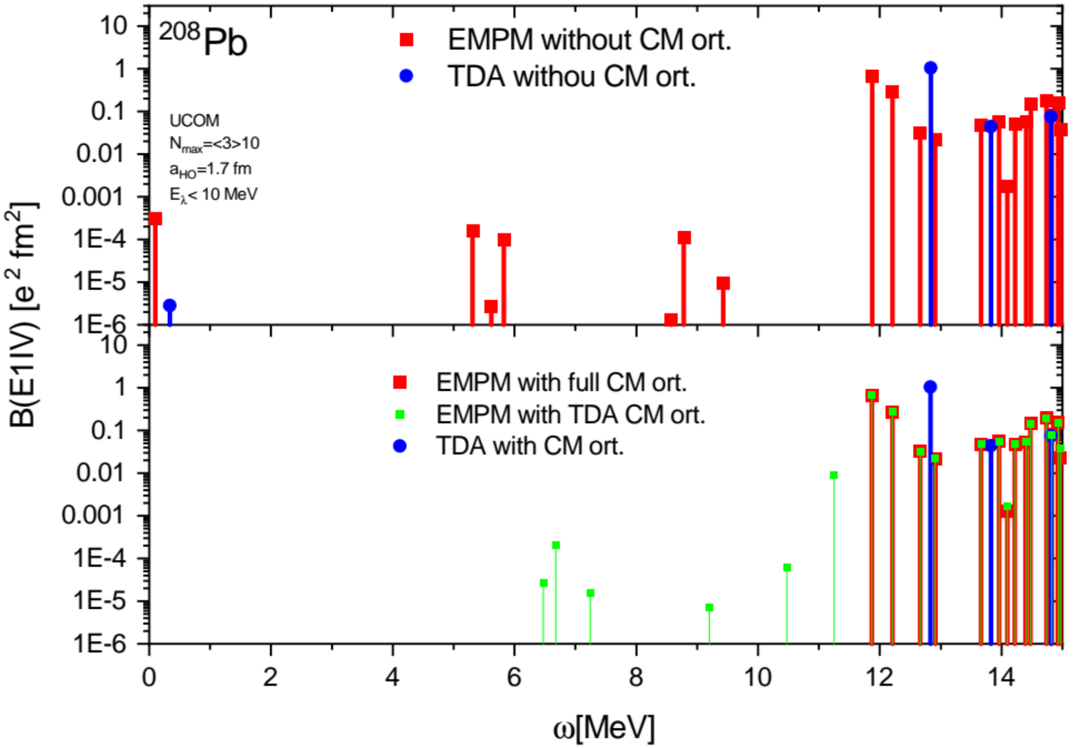
Results – ivE1 in ^{208}Pb

iv E1 dipole transitions in **^{208}Pb**, NN interaction: **UCOM**

Preliminary

EMPM – up to 2-phonon

In low-energy spectrum **Pygmy dipole** resonance - **CM correction** crucial.
3 modes in calc.: - **without CM** correction
- **TDA with CM** correction, **EMPM without CM** correction
- **TDA+EMPM with CM** correction (SVD method applied)



Summary

- Discussion of **Equation of Motion Phonon Method**
- Discussion of **Center of Mass** problem in **many-body** method
- Introduction of **Singular Value Decomposition** Method for **CM factorization**
- Demonstration of **SVD** within **EMPM** on the calculation of energy spectrum in ^{4}**He**
- **EMPM** calculations of ^{40}**Ca,** ^{208}**Pb** and comparison with **STDA, SRPA** methods
- Full **equivalence** of **EMPM** and **STDA** demonstrated
- **Our plans**:
    - Extension of **SVD** method for **quasiparticle** variant of **EMPM**
    - Removal of **spurious CM** and particle number **N** modes from energy spectra
    - Study of structure of **open shell** nuclei ^{18,20}**O,** ^{42,44,46}**Ca**

Thank you for your attention!!