A+2n compound nuclei and the unitary limit in nuclear physics

Panagioti E. Georgoudis

Research funded by the EU-H2020, Marie Sklodowska Curie Actions-Individual Fellowships, Grant Agreement No 793900 at *GANIL/CEA-DRF CNRS-IN2P3, Caen, France.*

Talk at Sant Angelo D' Ischia 2022, May 17, 2022

Summary of motivation

Long term goal: *To analyse the symmetries of nuclear structure in terms of the symmetries of fundamental interactions.*

Question: *Is there an explicit (algebraic) relation between the symmetries manifested in the structure of atomic nuclei and the symmetries that emerge in the classical limit of QCD, that is conformal symmetry?*

Introduce the unitary limit in low-lying collective nuclear states

A strong coupling problem
that manifests conformal symmetry.
M. Randeria, W. Zwerger, M. Zwierlein,
BCS to BEC Crossover and the Unitary Fermi Gas,
Springer-Verlag, Berlin (2012);
D.T. Son, Phys. Rev. D 78 (2008) 046003.

In the group theoretical framework of the Interacting Boson Model
F. Iachello, A Arima, "The Interacting Boson Model", Cambridge University Press (1987)

Unitary limit

Scattering problem at infinite scattering length that supports a bound state. Manifests Non-Relativistic Conformal symmetry, the BCS-BEC crossover and a quantum critical point.

Elaborated in light nuclei in the framework of EFTs:

- ***T. Mehen, I. W. Stewart and M. B. Wise, "Conformal Invariance in Non-Relativistic Field Theory", Phys. Lett. B, 474, 145 (2000).***
- ***I. Stetcu, J. Rotureau, B.R. Barrett, U. van Kolck, "Two and Three Nucleons in a Trap and the Continuum Limit" Ann. Phys. 325, 1644 (2010).***
- ***S Konig, H. W. Griesshammer, H.-W. Hammer and U.van Kolck "Nuclear Physics around the Unitarity limit", Phys. Rev. Lett. 118, 202501 (2017).***

Experimentally, the unitary limit is achieved in systems of trapped cold atoms:

$$\left(\sum_{i=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r_i^2} + \frac{1}{2}M\omega^2 R^2\right)\psi\left(r_1,\ldots r_N\right) = E\psi\left(r_1,\ldots r_N\right) \quad \left\{ \begin{array}{l} \omega = 0 \\ \qquad \dfrac{4\pi a\hbar^2}{M}\delta\left(r_{ij}\right) \to \lim\limits_{r_{ij}\to 0}\psi\left(R\right) = \dfrac{C}{r_{ij}} - \dfrac{1}{a} \\ E = 0 \end{array} \right.$$

F Werner and Y Castin, Phys Rev A, 74, 053604 (2006)

Unitary limit in trapped cold atoms: Symmetry-based approach

$$\left(\sum_{i=1}^{N} -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial r_i^2} + \frac{1}{2}M\omega^2 R^2\right)\psi\left(r_1,\ldots r_N\right) = E\psi\left(r_1,\ldots r_N\right)$$

$\omega = 0$

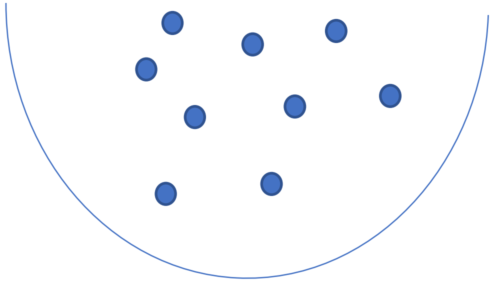
F Werner and Y Castin, Phys Rev A, 74, 053604 (2006)

E= 0

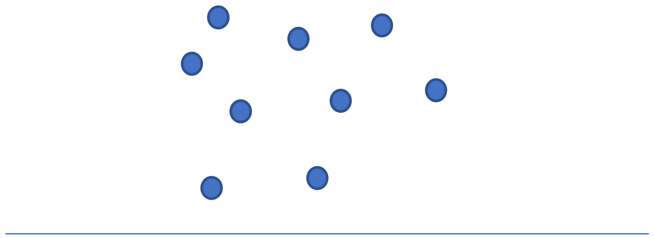
$$R^2 = \sum_{i=1}^{N} r_i^2$$

$$\frac{4\pi a\hbar^2}{M}\delta\left(r_{ij}\right) \rightarrow \lim_{r_{ij}\to 0}\psi\left(R\right) = \frac{C}{r_{ij}} - \frac{1}{a}$$

Unitary limit: $a \rightarrow \infty$ $\left\{\right.$ Zero energy state + boundary condition



$\omega = 0$ E= 0



$$\left|\psi_\nu^0\right\rangle = R^\nu \Phi\left(\Omega_k\right)$$

Take the "free" Hamiltonian $\boldsymbol{H = \sum_{i=1}^{N} \frac{p_i^2}{2M}}$ ***,***

the potential $\boldsymbol{K = \frac{1}{2}M\omega^2 R^2}$***, with*** $\boldsymbol{R^2 = \sum_{i=1}^{N} r_i^2}$.

Define the Operator $\boldsymbol{D = R\frac{\partial}{\partial R}}$

The operators H,K,D close under the one-dimensional conformal group. This is isomorphic to the SO(2,1) group.

V. de Alfaro, S. Fubini, G. Furlan, Nuovo Cim. 34A 569 (1976);
L.P. Pitaevskii, A. Rosch, Phys. Rev. A 55 (1997) R853;
T. Mehen, Phys. Rev. A 78, 023614 (2008).

Feshbach resonances in systems of cold atoms

$$a_{eff} = a\left[1 - \frac{\Delta B}{B - B_m}\right]$$

Experimentally the scattering length is tuned by an external magnetic field B:

The unitary limit is achieved by the appropriate Magnetic field:

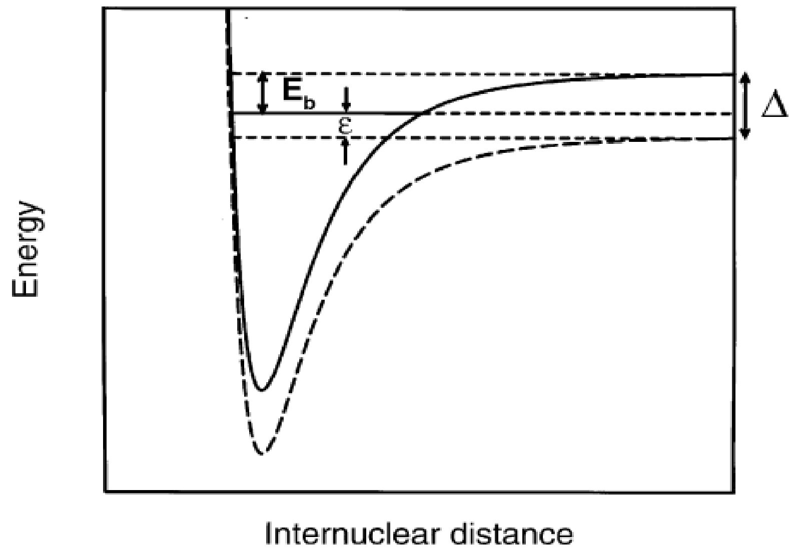


Fig. 1. Schematic representation of the molecular potentials of the incident and intermediate state channels. The energy difference of the continuum levels, Δ, is the sum of the binding energy, denoted here by E_b, of the quasi-bound state and the 'detuning' ε.

E. Timmermans, P. Tommasini, M. Hussein, A. Kerman, Physics Reports 315 (1999) 230.

Intermediate molecular states of the Feshbach Formalism in systems of cold atoms: Manifest the unitary limit.

Unitary limit for low-lying collective nuclear states?

Interacting Boson Model

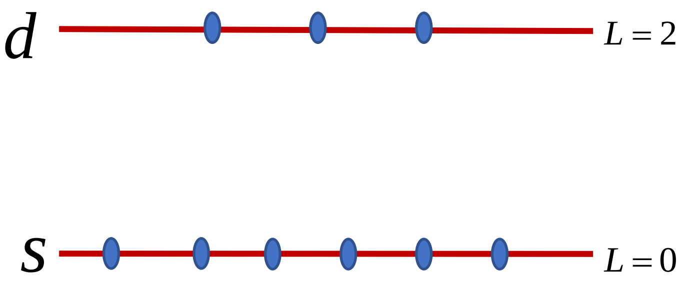
U(6)

F. Iachello, A. Arima, *The Interacting Boson Model,* Cambridge University Press, 1987.

Collective motion and Structural Evolution is mainly characterized by the valence nucleons.

A Heavy even-even nucleus is represented by a fixed Boson number:

$\boldsymbol{N_b = N_d + N_s}$.



s and d bosons represent paired valence nucleons of angular momentum zero and two.

$$[d_\mu^\dagger, d_\nu] = \delta_{\mu\nu}$$

$$[s^\dagger, s] = 1$$

U(6)

The low-lying collective nuclear states sit in the representations of the U(6) symmetry group.

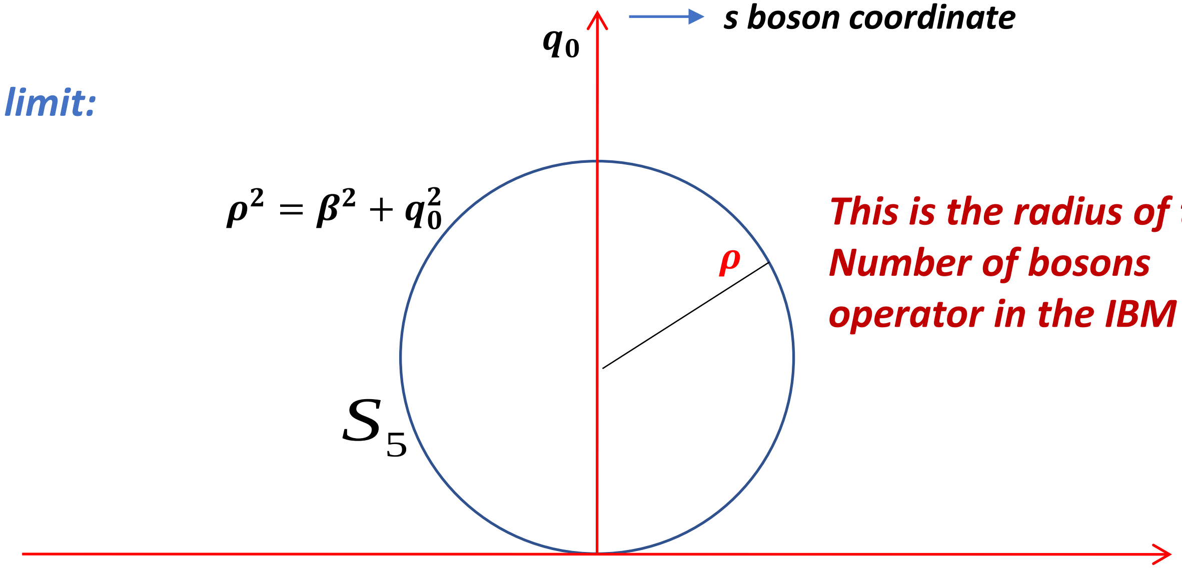
The Interacting Boson Model in the O(6) limit

A geometric representation of the d=6 harmonic Oscillator of the IBM in the O(6) limit:

$$\rho, \underbrace{\Omega_5 : (\Phi, \Theta, \Psi), \gamma, \chi}$$

3 Euler angles, gamma Unstable rotations and Deformation.

$$\rho^2 = \beta^2 + q_0^2$$



s boson coordinate

This is the radius of the Number of bosons operator in the IBM

$$R^5 \sim R_+ \times S_4$$

5-d manifold of the quadrupole Deformations of the nuclear surface

$$\frac{-\hbar^2}{2M}\left(\frac{1}{\rho^5}\frac{\partial}{\partial\rho}\rho^5\frac{\partial}{\partial\rho} - \frac{\sigma(\sigma+4)}{\rho^2}\right)\Phi(\rho) + \frac{1}{2}M\omega^2\rho^2\Phi(\rho) = \left(N_b + \frac{6}{2}\right)\hbar\omega\Phi(\rho)$$

$$N_b \rightarrow \sigma + 2J$$

O. Castanos, E. Chacon, A. Frank and M. Moshinsky, J. Math Phys 20, 35 (1979)

The Schrodinger equation of N=2 trapped cold atoms: Obeys the O(6) symmetry:

$$-\frac{\hbar^2}{2M}\left(\frac{1}{R^5}\frac{\partial}{\partial R}R^5\frac{\partial}{\partial R}-\frac{\lambda(\lambda+4)}{R^2}\right)\psi(R)+\frac{1}{2}M\omega^2R^2\psi(R)=E\psi(R)$$

Make an algebraic comparison of each radial term with the IBM equation in the O(6) limit:

$$\frac{-\hbar^2}{2M}\left(\frac{1}{\rho^5}\frac{\partial}{\partial\rho}\rho^5\frac{\partial}{\partial\rho}-\frac{\sigma(\sigma+4)}{\rho^2}\right)\Phi(\rho)+\frac{1}{2}M\omega^2\rho^2\Phi(\rho)=E\Phi(\rho)$$

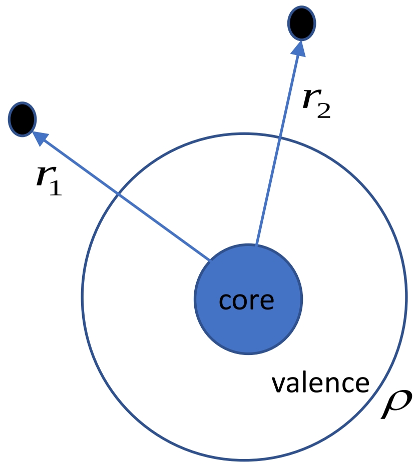
Define the IBM-compound Hamiltonian and its reaction channels:

$$\Psi(r,\rho)=\sum_n\Psi_n(r)\Phi_n(\rho)$$

$\lambda=0$: *s wave of the pair of neutrons (2n)*

$$H_c=H_{2N}+H_{IBM}+H_{2N/IBM}$$

$$H_c=H(R)+H(\rho)+H(R,\rho)$$



$R^2=r_1^2+r_2^2$

$r=R-\rho$

P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)

Model the A+2n compound nucleus at low temperature in analogy with cold and dilute atomic gases:

(a low-lying resonance)

The fermionic cold atoms form diatomic molecules of bosonic character around Feshbach resonances:

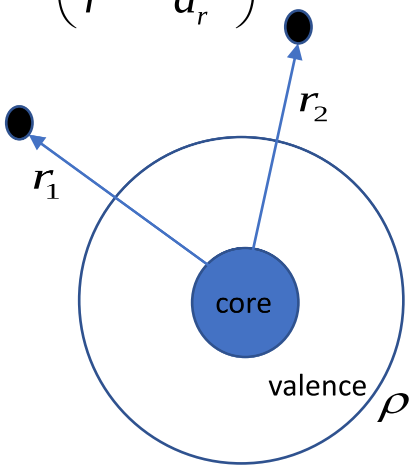
Unitary atom-atom interaction + unitary molecule-molecule interaction: ***D.S. Petrov, C. Salomon, G.V. Shlyapnikov, Phys. Rev. Lett. 93 (2004) 090404***

(nucleon-nucleon) *(boson-boson in the IBM)*

Unitary neutron-neutron interaction **+** ***Pair-Collective state unitary interaction***

$$\frac{4\pi\hbar^2 a}{M}\delta(r_1-r_2):\lim_{r_1\to r_2}\Psi_0(r)=\frac{C}{r_1-r_2}-\frac{1}{a}$$

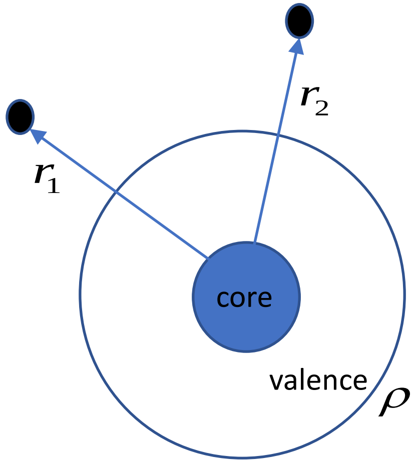
$$\frac{4\pi^3 a_r\hbar^2}{M}\delta(r):\quad \lim_{r\to 0}\Psi_0(r,\rho)=\Phi_0(\rho)\left(\frac{C}{r^4}-\frac{1}{a_r{}^4}\right)$$



$$R^2=r_1^2+r_2^2$$

$$r=R-\rho$$

$$H_c=\frac{-\hbar^2}{2M}\left(\frac{1}{r^5}\frac{\partial}{\partial r}r^5\frac{\partial}{\partial r}-\frac{\lambda(\lambda+4)}{r^2}\right)+H(\rho)+\underbrace{s^\dagger+s}_{\text{The analog of the magnetic field}}$$



Write down the pair-collective state scattering length

$$R^2 = r_1^2 + r_2^2$$

$$r = R - \rho$$

Pair scattering in O(6) symmetry:
6-d geometry for the scattering:

$$R = \sqrt{r_1^2 + r_2^2}\,, \Omega_5 : (\theta_1, \varphi_1), (\theta_2, \varphi_2), \alpha = \tan^{-1}\left(r_1 / r_2 \right)$$

$$\sigma = \frac{(4\pi)^3}{1/a_r^2(k_r) + k_r^2}, \quad k_r^2 = k_1^2 + k_2^2$$

Exhausts the unitarity bound when $\sigma \to \frac{(4\pi)^3}{k_r^2}$

That scattering is characterized by a pair-collective state scattering length:

$$k_r \cot\delta_0 = -\frac{1}{a_r(k_r)}, \quad a_r(k_r) = \frac{1}{a_r} - \frac{1}{2} k_r{}^2 r^* + \cdots$$

Resonance condition: $\frac{1}{a_r(k_r)} = 0$

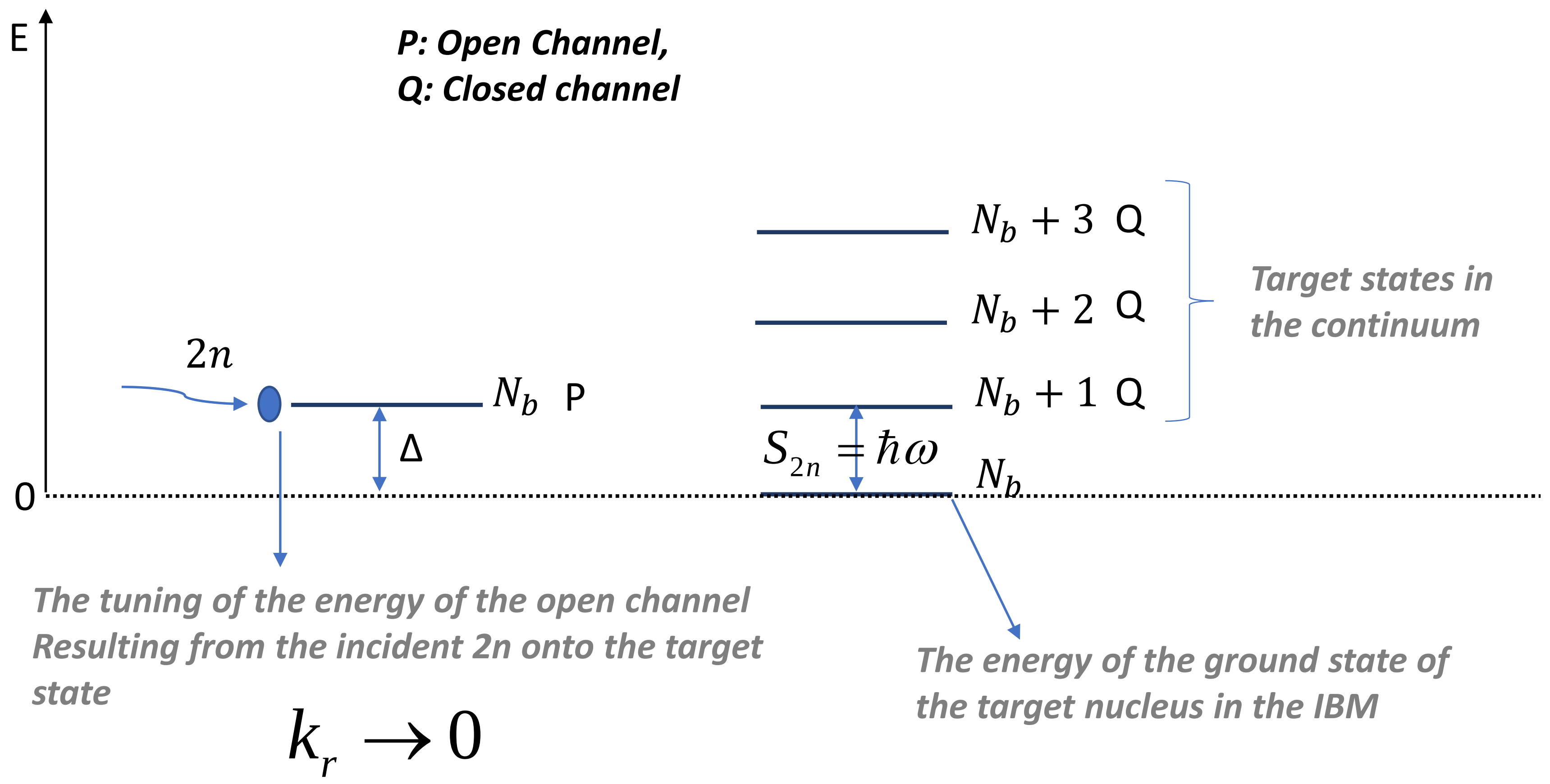
$$\frac{1}{a_r(E_m)} = 0$$

Effective range for the pair-collective state interaction:
Measurable via the width of the intermediate state:

$$\Gamma_m = \frac{\hbar^2 k_r}{M\, r^*}$$

P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)

Classify the channels in terms of the IBM boson number



Intermediate states of the IBM-compound Hamiltonian H_c:

H. Feshbach, A.K. Kerman, R.H. Lemmer, Ann. Phys. 41, 230 (1967)

Stationary states of the A+2n compound system (trapped IBM states in H_c)

$$\left(E - H_{PP}\right) P\Psi\left(r,\rho\right) = H_{PQ} Q\Psi\left(r,\rho\right)$$

$$\left(E - H_{QQ}\right) Q\Psi\left(r,\rho\right) = H_{QP} P\Psi\left(r,\rho\right)$$

Total energy

$$\xrightarrow[E \to \varepsilon_m]{H_{QP} = 0}$$

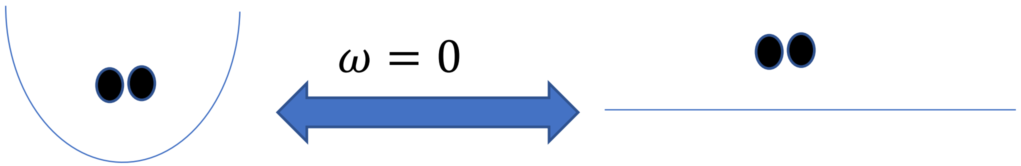
$$\left(T_r + E_m\right)\Psi_m\left(r\right) = \varepsilon_m \Psi_m\left(r\right)$$

energy of the intermediate state formed by the incident 2n with the target nucleus.

IBM (target) state: $\langle m | H(\rho) | m \rangle$

Solve the closed channel equation

A trapped state in the IBM Harmonic Oscillator



$$E_m = \varepsilon_m$$

The zero-energy solution in the equation of the intermediate state

Intermediate states of the IBM-compound Hamiltonian H_c:

H. Feshbach, A.K. Kerman, R.H. Lemmer, Ann. Phys. 41, 230 (1967)

Stationary states of the A+2n compound system (trapped IBM states in H_c)

$$\left(E-H_{PP}\right)P\Psi\left(r,\rho\right)=H_{PQ}Q\Psi\left(r,\rho\right)$$

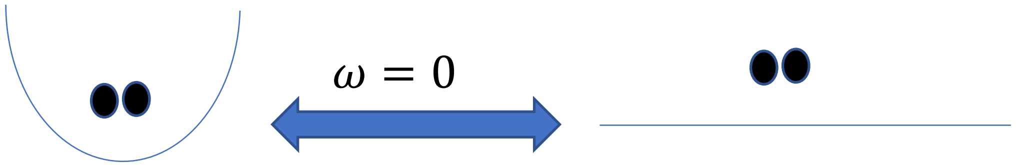
$$\left(E-H_{QQ}\right)Q\Psi\left(r,\rho\right)=H_{QP}P\Psi\left(r,\rho\right)$$

Total energy

$$\xrightarrow[E\to\varepsilon_m]{H_{QP}=0}$$

$$\left(T_r+E_m\right)\Psi_m\left(r\right)=\varepsilon_m\Psi_m\left(r\right)$$

energy of the intermediate state formed by the incident 2n with the target nucleus.



$$E_m=\varepsilon_m$$

$$T_r\Psi_m\left(r\right)=0\Psi_m\left(r\right)$$

Unitarity

A zero-energy solution for the intermediate state that satisfies the boundary condition of the unitary interactions:

$$\frac{4\pi^3 a_r\hbar^2}{M}\delta\left(r\right)\to\lim_{r\to 0}\Psi_0\left(r,\rho\right)=\Phi_0\left(\rho\right)\left(\frac{C}{r^4}-\frac{1}{a_r{}^4}\right)$$

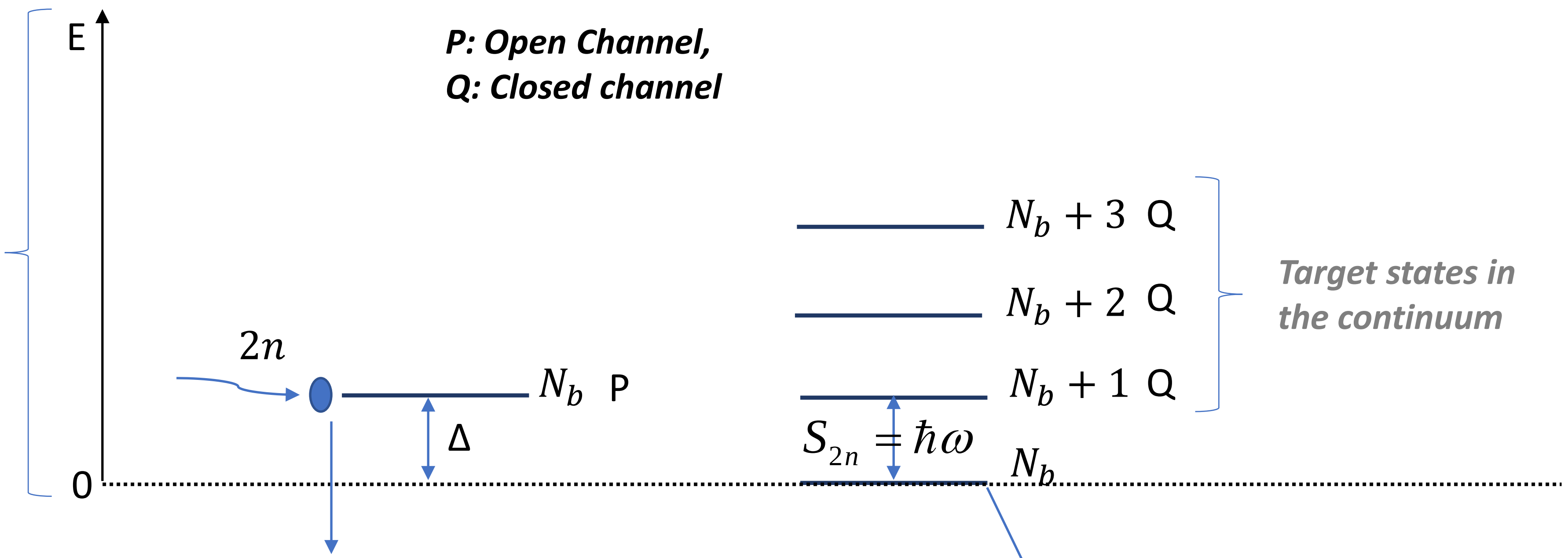
The intermediate state behaves as a pure IBM state in the continuum

Crossing conditions between the open and the closed channels

$$H(R,\rho)=\frac{4\pi^3 a_r \hbar^2}{M}\delta(r)+F, \quad F=s^\dagger+s$$

$$\lim_{r\to 0}\Psi_0(r,\rho)=\Phi_0(\rho)\left(\frac{C}{r^4}-\frac{1}{a_r^{\,4}}\right)$$

$r=R-\rho$



The tuning of the energy of the open channel Resulting from the incident 2n onto the target state

The energy of the ground state of the target nucleus in the IBM

The crossing of the energy of the $2n+N_b$ open channel with the energy of the N_b+1 closed channel manifests the unitary limit: A low-lying resonance resulting from the capture of the 2n as an s boson.

Tuning of the scattering length via the fluctuation of the cross section

$$\frac{\exp(-ik_r r)}{r^{5/2}} - S_0 \frac{\exp(ik_r r)}{r^{5/2}}, \qquad S_0 = \exp(2i\delta_0) = \exp(-2ik_r a_r)$$

Intermediate States of the Feshbach Formalism:

Application for IBM states in the continuum:

A+2n Compound nucleus

Originally introduced in nuclear Physics end examined thoroughly in the 70's as bound states in the Continuum: Doorway states

G.E. Mitchell, A. Richter, H.A. Weidenmuller, Rev. Mod. Phys. 82 (2010) 2845; H. Feshbach, Rev. Mod. Phys. 46 (1974) 1

Fluctuation

$$S_0 \to S_0^{'} = \exp(-2ik_r a_r)\underbrace{\left(1 - \frac{i\Gamma_1}{E - E_1 + i\Gamma_1/2}\right)}$$

Fluctuation: Tunes the scattering length

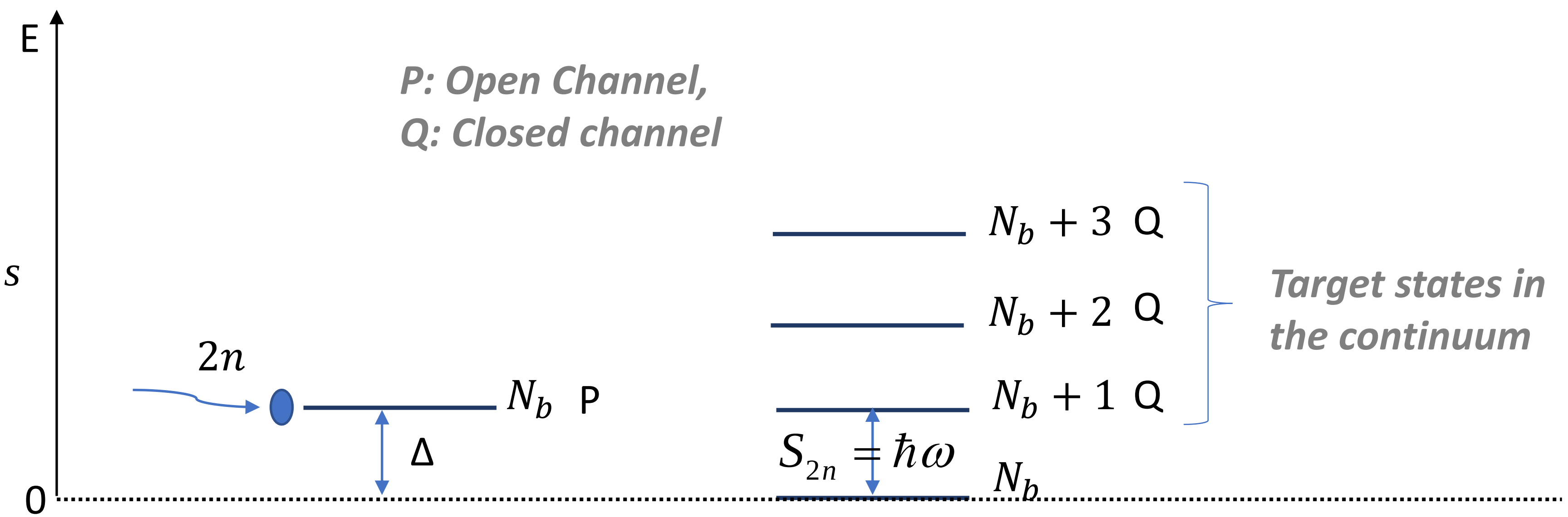
$$a_{reff} = a_r + a_r', \quad a_r' = \frac{1}{2k_r}\tan^{-1}\left[\frac{\Gamma_1(E-E_1)}{(E-E_1)^2 + \Gamma_1^2/4}\right]$$

$$\sigma_{ce} = \frac{(4\pi)^3}{k_r^2}\left(\left\langle \left|S_0^{'}\right|^2 \right\rangle - \left|\left\langle S_0^{'}\right\rangle\right|^2\right)$$

E. Timmermans, P. Tommasini, M. Hussein, A. Kerman, Physics Reports 315, 230 (1999); P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)

Connection with the experiment

$$H_{QP} = \langle N_b^{'} | F | N_b \rangle, \quad F = s^\dagger + s$$



$$\Gamma_1 = b^2 \left(\frac{4M}{\hbar^2} \right) k_r, \quad b = \sqrt{N_b + 1} \underbrace{\int dr \Psi_m (r) u_0^+ (r)}$$

Spectroscopic factor of the intermediate state

$$\lim_{E \to E_1} a_{reff} = a_r - \frac{2Mb^2}{\hbar^2 (E - E_1)}$$

$$\sigma_{ce} = \frac{(4\pi)^3}{k_r^2} \frac{\Gamma_1^2}{(E - E_1)^2 + \Gamma_1^2 / 4}$$

Exhausts the unitarity bound

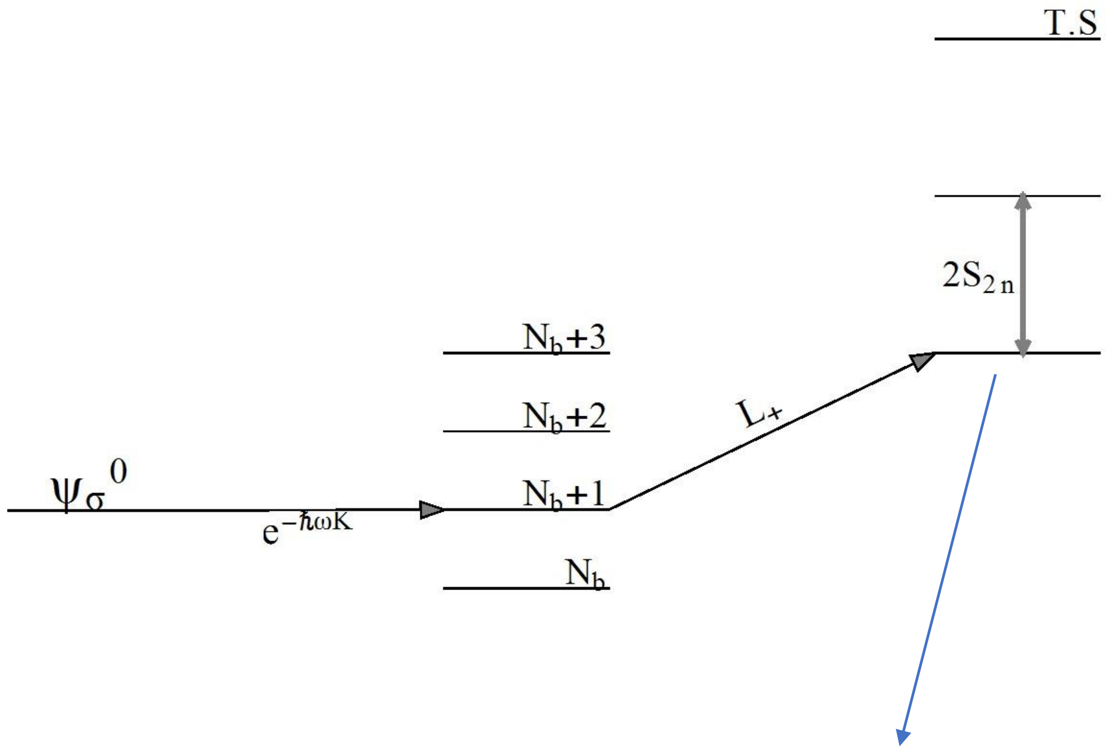
for $E \to E_1$: $\sigma_{ce} \to \frac{(4\pi)^3}{k_r^2}$

Consequence: Conformal Symmetry in the intermediate states of the A+2n compound nucleus

$$L_{\pm} = \pm iD + \frac{1}{\hbar\omega}\left(H - K\right): \quad \left\{ \begin{array}{l} L_{+} = -(d^{\dagger}d^{\dagger} + s^{\dagger}s^{\dagger}) \\ L_{0} = \frac{1}{2}\left(d^{\dagger}d + s^{\dagger}s + \frac{6}{2}\right) \\ L_{-} = -(dd + ss) \end{array} \right\}$$

$$\left[H, D\right] = -2iH, \quad \left[K, D\right] = 2iK, \quad \left[K, H\right] = i\hbar^{2}\omega^{2}D$$

Conformal group in one dimension.



<u>One expects a regularity pattern of fluctuations of the cross section with determined energies and widths.</u>

<u>A Tower of equally spaced states represents the SO(2,1) group.</u>

P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)

Conclusions

- *In nuclear physics, the tuning of the scattering length is imprinted on the fluctuations of the cross-sections.*
- *The unitary limit manifests itself in a heavy even-even A+2n compound nucleus when the resonance formed by the incident neutron pair on a heavy even-even target has the energy of one boson more with respect to the energy of the ground state of the target. This energy difference is the two-neutron separation energy.*
- *For the compound elastic case, the width of the resonance is provided for connection with the experiment.*

The unitary limit of the neutron pair with the collective ground state of the even-even target, hosts the representations of the one-dimensional conformal group.

$\Downarrow$

A regularity pattern of the fluctuations of the cross section emerges at the unitary limit of the A+2n compound nucleus in contrast with their usual random appearance in A+1n compound nuclei.

THE END

Backup Slides

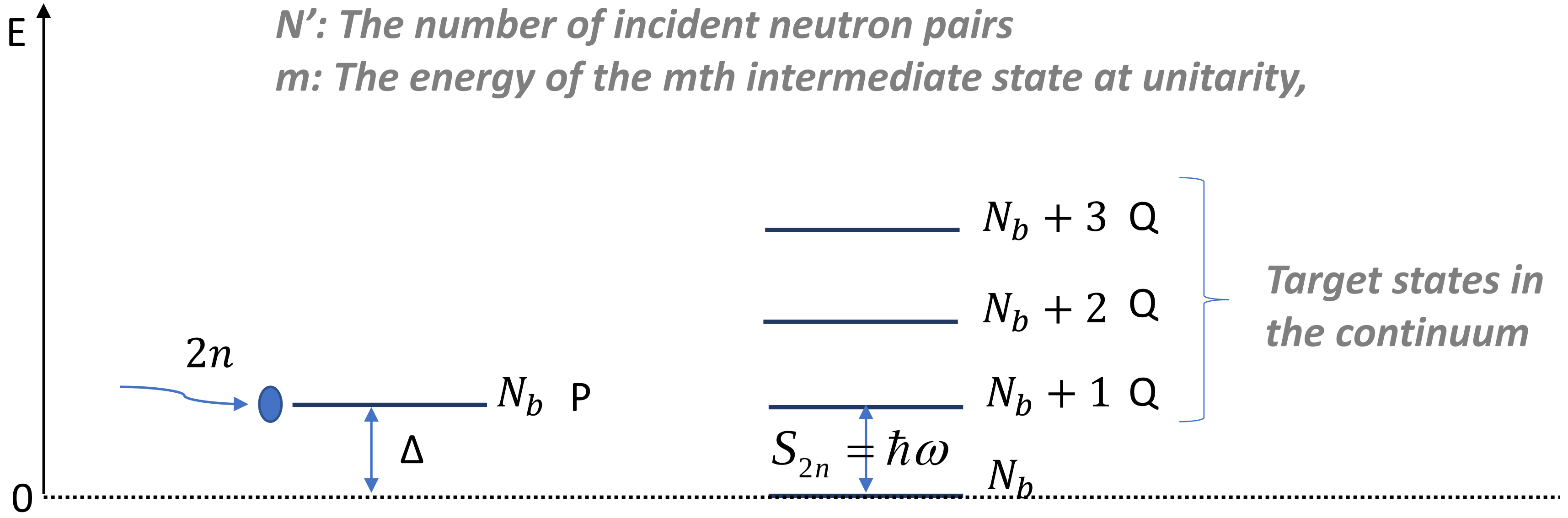
Analogy of the magnetic field with the boson number:

$$a_{eff} = a\left[1 - \frac{\Delta B}{B - B_m}\right] \Longrightarrow a_{reff} = a_r\left[1 - \frac{\Delta N'}{N' - m}\right]$$

$$\Delta = E_m + \frac{\partial \Delta}{\partial B}\left(B - B_m\right) \to N_b + m + \frac{\partial \Delta}{\partial N'}\left(N' - m\right)$$

N': The number of incident neutron pairs

m: The energy of the mth intermediate state at unitarity,



For the IBM-compound Hamiltonian H_c solve the coupled channel equations in the Feshbach Formalism:

$$\left(E - H_{PP}\right) P\Psi(r,\rho) = H_{PQ} Q\Psi(r,\rho)$$

$$\left(E - H_{QQ}\right) Q\Psi(r,\rho) = H_{QP} P\Psi(r,\rho)$$

H. Feshbach, Ann. Phys. 5 (1958) 357;
E. Timmermans, P. Tommasini, M. Husein,
A. Kerman, Physics Reports 315, 230 (1999).

$$P = \left|\Phi_0(\rho)\right\rangle\left\langle\Phi_0(\rho)\right|$$ ***Choose one open channel for the ground state of the target with N_b bosons***

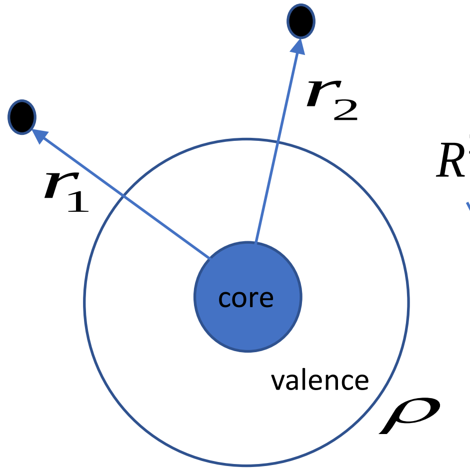
$$Q = \sum_{n>0}\left|\Phi_n(\rho)\right\rangle\left\langle\Phi_n(\rho)\right|$$ ***Closed channels with target states of $N_b + 1, N_b + 2$ … bosons***

$$\Psi(r,\rho) = \sum_n \Psi_n(r)\Phi_n(\rho)$$ ***The channel wavefunction***

Pair wavefunction for the open channel: $P\Psi(\mathbf{r},\boldsymbol{\rho}) = \Psi_0(r)$.
Switch off the couplings and solve the first equation after the transformation $\Psi_0(r) = (r^{-2})\chi(r)$:

$$-\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{(\lambda+2)^2}{r^2}\right)\chi(r) = k_r^2\chi(r),\ k_r^2 = \frac{2ME}{\hbar^2}$$

Solutions: Bessel functions of order λ+2

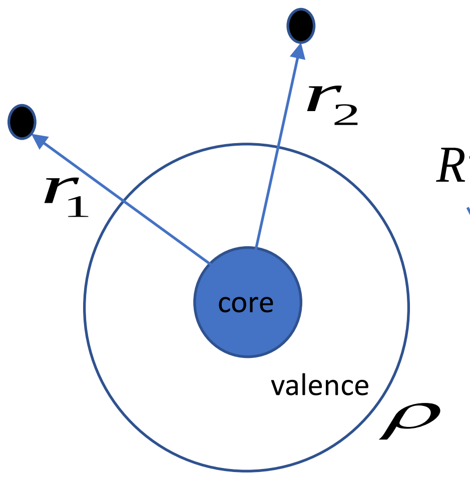


$$R^2 = r_1^2 + r_2^2$$

$$r = R - \rho$$

P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)

The open channel solution (n=0)



$$R^2 = r_1^2 + r_2^2$$

$$r = R - \rho$$

Pair scattering in O(6) symmetry:
6-d geometry for the scattering

$$R = \sqrt{r_1^2 + r_2^2},\ \Omega_5 : (\theta_1, \varphi_1), (\theta_2, \varphi_2),\ \alpha = \tan^{-1}\left(r_1 / r_2 \right)$$

Boundary condition of the pair wavefunction at infinity in d=6:

$$\lim_{r\to\infty} \Psi_0(r) = \frac{\exp(-ik_r r)}{r^{5/2}} - S\frac{\exp(ik_r r)}{r^{5/2}}, \qquad S = \exp(2i\delta_0) = \exp(-2ik_r a_r)$$

Boundary condition near the reaction center: Unitary interaction for the pair-collective state interaction in d=6:

$$\frac{4\pi^3 a_r \hbar^2}{M}\delta(r) \to \lim_{r\to 0}\Psi_0(r,\rho) = \Phi_0(\rho)\left(\frac{C}{r^4} - \frac{1}{a_r^{\,4}}\right)$$

Solution for the s wave
In terms of Bessel functions
Of order λ+2

$$\Psi_0(r) = 8C\left(\frac{N_{0+2}(k_r r)}{r^2} + \frac{\left(e^{2i\delta_0} - 1\right)}{2ik_r}\frac{H^{(1)}_{0+2}(k_r r)}{r^2}\right), \quad C = \sqrt{\frac{\pi}{2k_r}}e^{i5\pi/4}$$

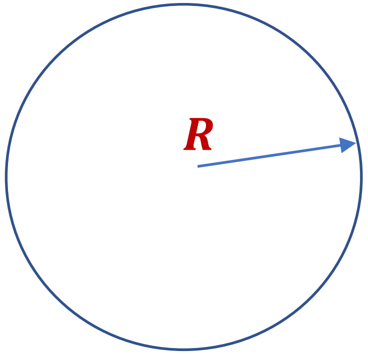
1. The hyperspherical equation of N particles confined in the harmonic Oscillator trap

$$\frac{-\hbar^2}{2M}\left(\frac{1}{R^{3N-1}}\frac{\partial}{\partial R}R^{3N-1}\frac{\partial}{\partial R}-\frac{\Lambda}{R^2}\right)\Phi(R)+\frac{1}{2}M\omega^2R^2\Phi(R)=E\Phi(R)$$

Schrodinger Equation of N trapped particles in hyperspherical coordinates

F Werner and Y Castin, Phys Rev A, 74, 053604 (2006); J.L. Bohn, B.D. Esry, C.H. Greene, Phys. Rev. A 58, 584 (1998).

d=6: $N=2, R=\sqrt{r_1^2+r_2^2}, \Omega_5:(\theta_1,\varphi_1),(\theta_2,\varphi_2), \alpha=\tan^{-1}\left({}^{r_1}\!/_{r_2}\right)$



$$-\frac{\hbar^2}{2M}\left(\frac{1}{R^5}\frac{\partial}{\partial R}R^5\frac{\partial}{\partial R}-\frac{\lambda(\lambda+4)}{R^2}\right)\Phi(R)+\frac{1}{2}M\omega^2R^2\Phi(R)=E\Phi(R)$$

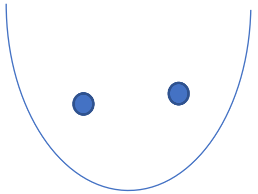
$\lambda=l_1+l_2:$ *The total angular momentum of the **N=2 particles** - the eigenvalue of the dilatation operator* $\boldsymbol{D}=\boldsymbol{R}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial R}}$

$$\left|\psi_\lambda^0\right\rangle=R^\lambda\Phi\left(\Omega_k\right)$$

P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)

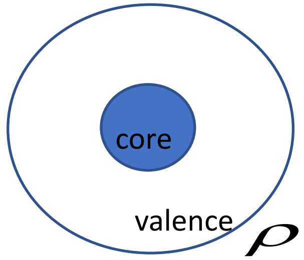
The Schrodinger equation of N=2 trapped cold atoms: Obeys the O(6) symmetry:

$$-\frac{\hbar^2}{2M}\left(\frac{1}{R^5}\frac{\partial}{\partial R}R^5\frac{\partial}{\partial R}-\frac{\lambda(\lambda+4)}{R^2}\right)\psi(R)+\left(\frac{1}{2}M\omega^2R^2\right)\psi(R)=E\psi(R)$$



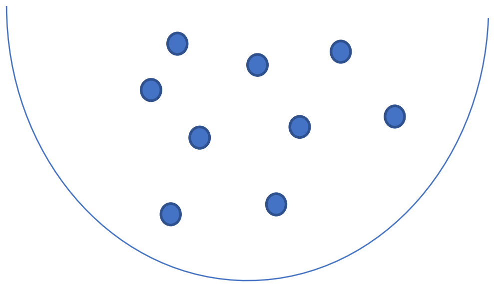
Make an algebraic comparison of each radial term with the IBM equation in the O(6) limit:

$$\frac{-\hbar^2}{2M}\left(\frac{1}{\rho^5}\frac{\partial}{\partial\rho}\rho^5\frac{\partial}{\partial\rho}-\frac{\sigma(\sigma+4)}{\rho^2}\right)\Phi(\rho)+\frac{1}{2}M\omega^2\rho^2\Phi(\rho)=E\Phi(\rho)$$

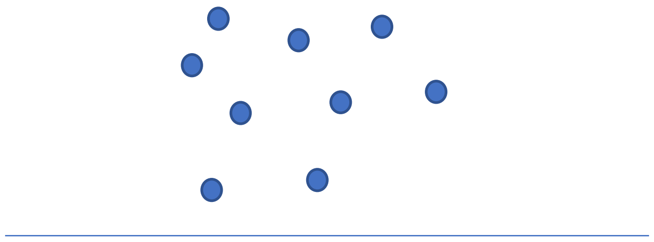


1. Apply the symmetry-based approach to the zero-energy state in the IBM equation :

$\lambda \to \sigma :$ *the eigenvalue of the dilatation operator* $\boldsymbol{D} = \boldsymbol{\rho}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial\rho}}$

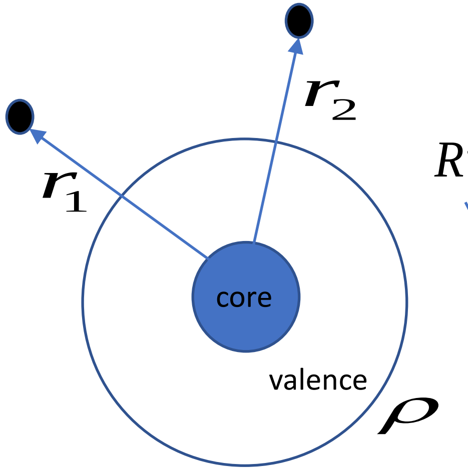


$$\omega = 0, \quad E = 0 < \frac{6}{2}\hbar\omega$$



$$E_{\sigma,J}=\left(N_b+\frac{6}{2}\right)\hbar\omega,\ N_b=\sigma+2J$$

$$\left|\psi_\sigma^0\right\rangle=\rho^\sigma\Upsilon(\Omega)$$



$$R^2 = r_1^2 + r_2^2$$

$$r = R - \rho$$

The choice of the core's position as the origin is an application of the more general case of the interaction of two similar particles with a fixed potential field represented by a heavy third body

Pair scattering in O(6) symmetry: 6-d geometry for the scattering and the trapped states.

P.M. Morse, H. Feshbach, Methods of Theoretical Physics, McGraw-Hill, Boston, 1999, pp. 1709–1745;
U. Fano, Rep. Prog. Phys. 46, 97 (1983)

$$H_c = H(r) + H(\rho) + H(r,\rho)$$

$$T_r + \frac{4\pi\hbar^2 a}{M}\delta(r_1 - r_2)$$

$$\frac{-\hbar^2}{2M}\left(\frac{1}{r^5}\frac{\partial}{\partial r}r^5\frac{\partial}{\partial r} - \frac{\lambda(\lambda+4)}{r^2}\right)$$

$$\langle N_b | H_{IBM} | N_b \rangle$$

What is this coupling Term? Examine the case That leads to a resonance between the two neutrons and the Collective state: A+2n compound nucleus in the Feshbach formalism

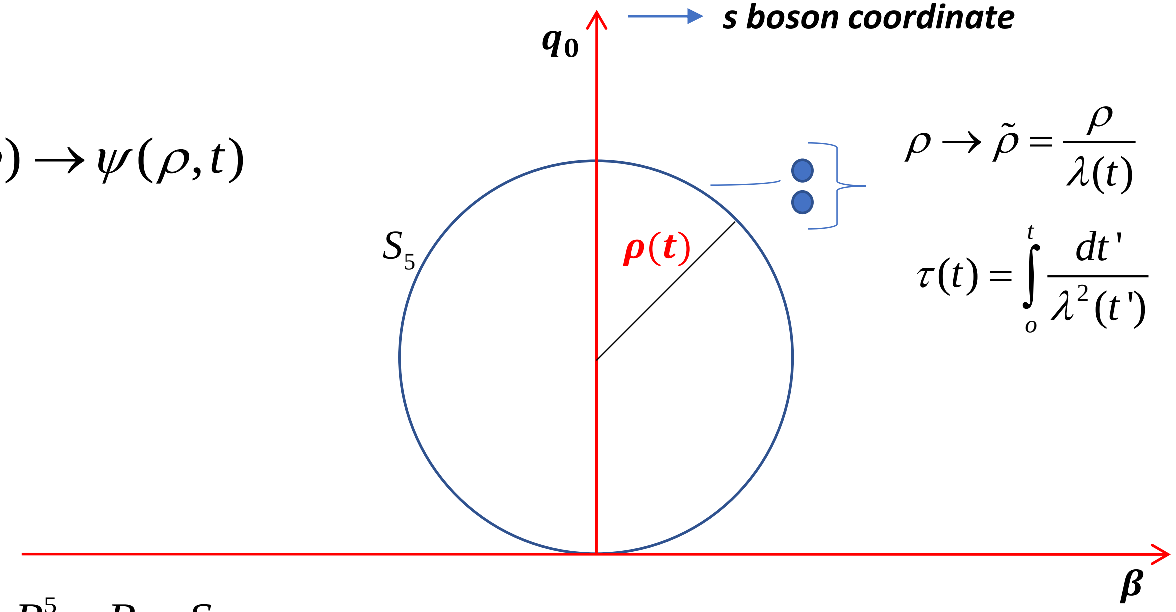
Solve the scattering problem for the neutron pair in terms of partial waves that are characterized by the O(6) angular momentum λ: Solutions in termσ of Bessel Functions of order λ+2.

Replace the Woods-Saxon term By appropriately defined unitary Interactions between the incident Neutrons and the collective nuclear state

Conformal transformation in one dimension:

Comparison with tower states in elongated Bose-Einstein condensates:

$$\psi(\rho) \rightarrow \psi(\rho, t)$$



$$\rho \rightarrow \tilde{\rho} = \frac{\rho}{\lambda(t)}$$

$$\tau(t) = \int_o^t \frac{dt'}{\lambda^2(t')}$$

$$R^5 \sim R_+ \times S_4$$

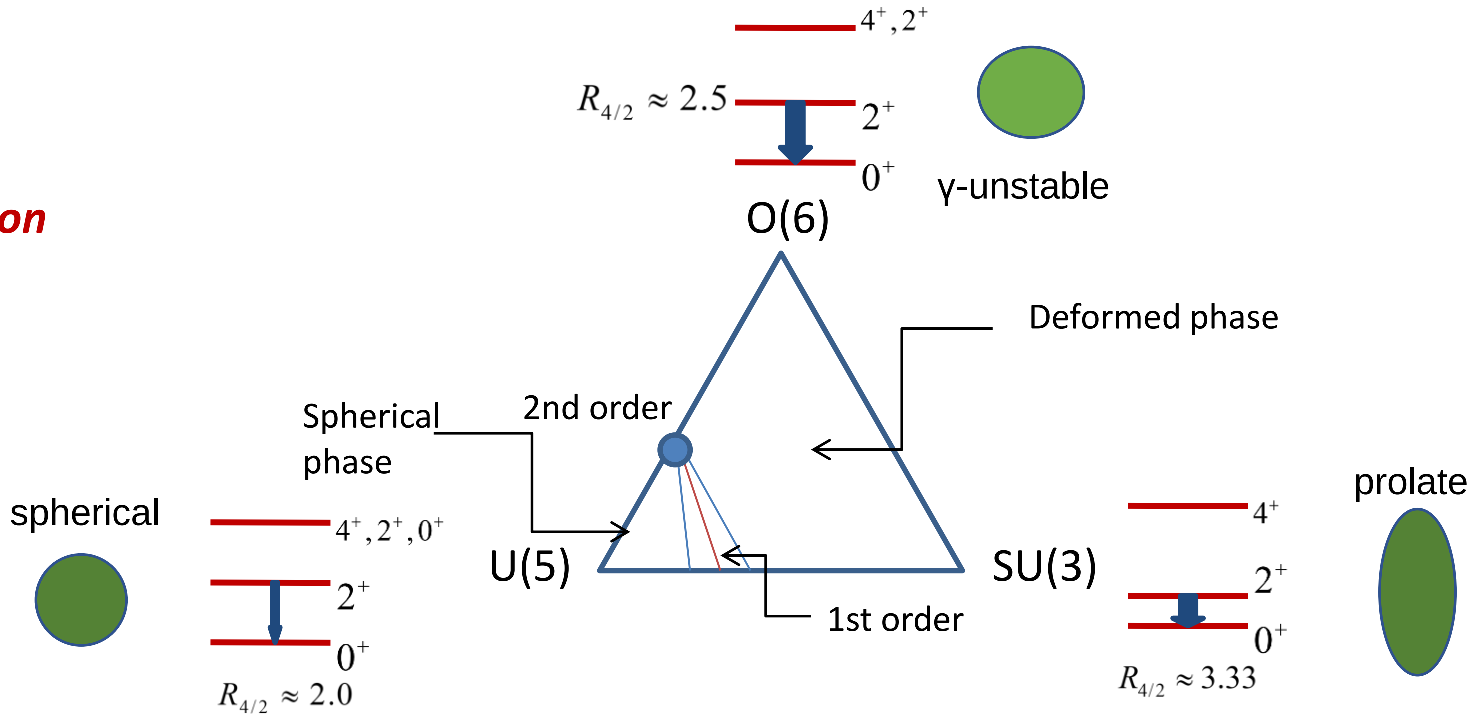
- ***Consequence:*** *Tower of equally spaced states (T.S) appear as a regularity pattern of a sequence Of fluctuations of the cross section*

P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)

Quantum Phase Transitions in the IBM

Critical Point Symmetries:
E(5) for the critical point of the
U(5)-O(6) Quantum Phase Transition

F. Iachello, Phys. Rev. Lett. 85 (2000) 3580



Implies conformal invariance that is manifested at the critical point of a 2nd order Phase Transition.

P. Cejnar, J. Jolie, R.F. Casten, Rev. Mod. Phys. 82, 2155, (2010) and references therein.

An Algebraic Comparison

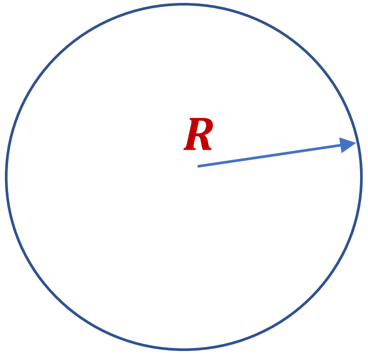
<u>*1. The hyperspherical equation of N particles confined in an harmonic Oscillator trap*</u>

$$\frac{-\hbar^2}{2M}\left(\frac{1}{R^{3N-1}}\frac{\partial}{\partial R}R^{3N-1}\frac{\partial}{\partial R}\right)\Phi(R)+\left(\frac{\hbar^2\Lambda}{2MR^2}+\frac{1}{2}M\omega^2R^2\right)\Phi(R)=E\Phi(R)$$

Schrodinger Equation of N trapped particles: E=0 and ω=0 Unitarity

F Werner and Y Castin, Phys Rev A, 74, 053604 (2006)

$$N=2$$



$$-\frac{\hbar^2}{2M}\left(\frac{1}{R^5}\frac{\partial}{\partial R}R^5\frac{\partial}{\partial R}\right)\Phi(R)+\left(\frac{\hbar^2}{2M}\frac{\lambda(\lambda+4)}{R^2}+\frac{1}{2}M\omega^2R^2\right)\Phi(R)=E\Phi(R)$$

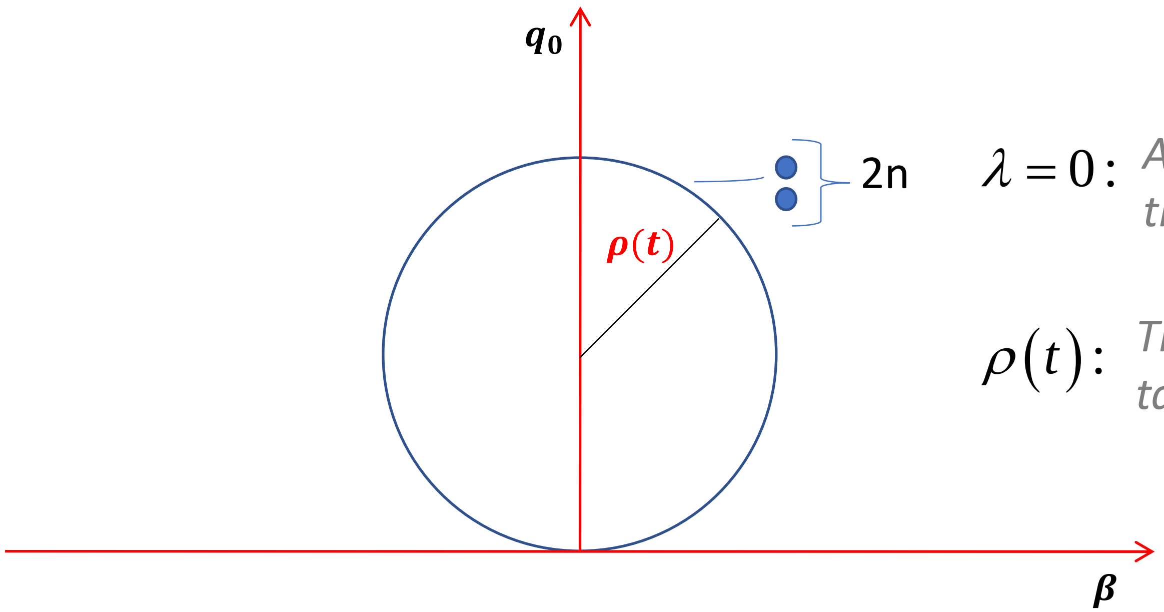
The Schrodinger equation of N=2 trapped cold atoms: Obeys the O(6) symmetry:

$$-\frac{\hbar^2}{2M}\left(\frac{1}{R^5}\frac{\partial}{\partial R}R^5\frac{\partial}{\partial R}-\frac{\lambda(\lambda+4)}{R^2}\right)\psi(R)+\left(\frac{1}{2}M\omega^2R^2\right)\psi(R)=E\psi(R)$$

Make an algebraic comparison of each radial term with the IBM equation in the O(6) limit:

$$\frac{-\hbar^2}{2M}\left(\frac{1}{\rho^5}\frac{\partial}{\partial\rho}\rho^5\frac{\partial}{\partial\rho}-\frac{\sigma(\sigma+4)}{\rho^2}\right)\Phi(\rho)+\frac{1}{2}M\omega^2\rho^2\Phi(\rho)=E\Phi(\rho)$$

2. Define the reaction channel that consists of two incident neutrons (solution of the hyperspherical equation) and the collective state of the target nucleus (solution of the IBM equation)



$\lambda=0$: *An incident s wave of two neutrons (2n) onto the collective nuclear state*

$\rho(t)$: *The boson number radius is the collective coordinate of the target nucleus*

P.E. Georgoudis, Nucl. Phys. A, 1015, 122297 (2021)