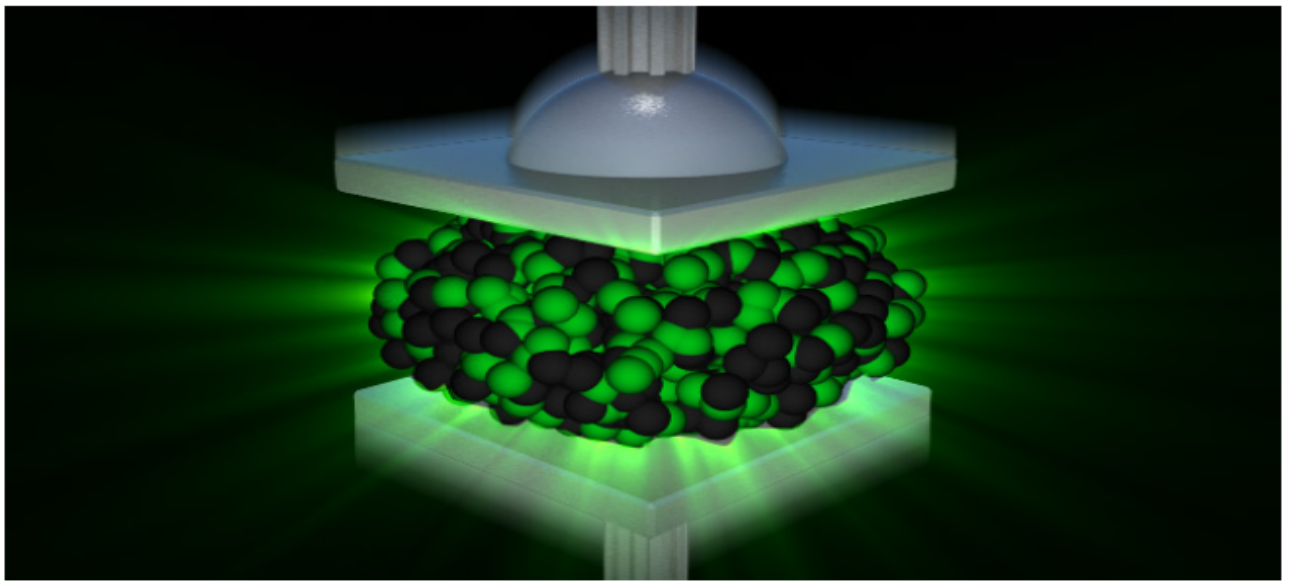




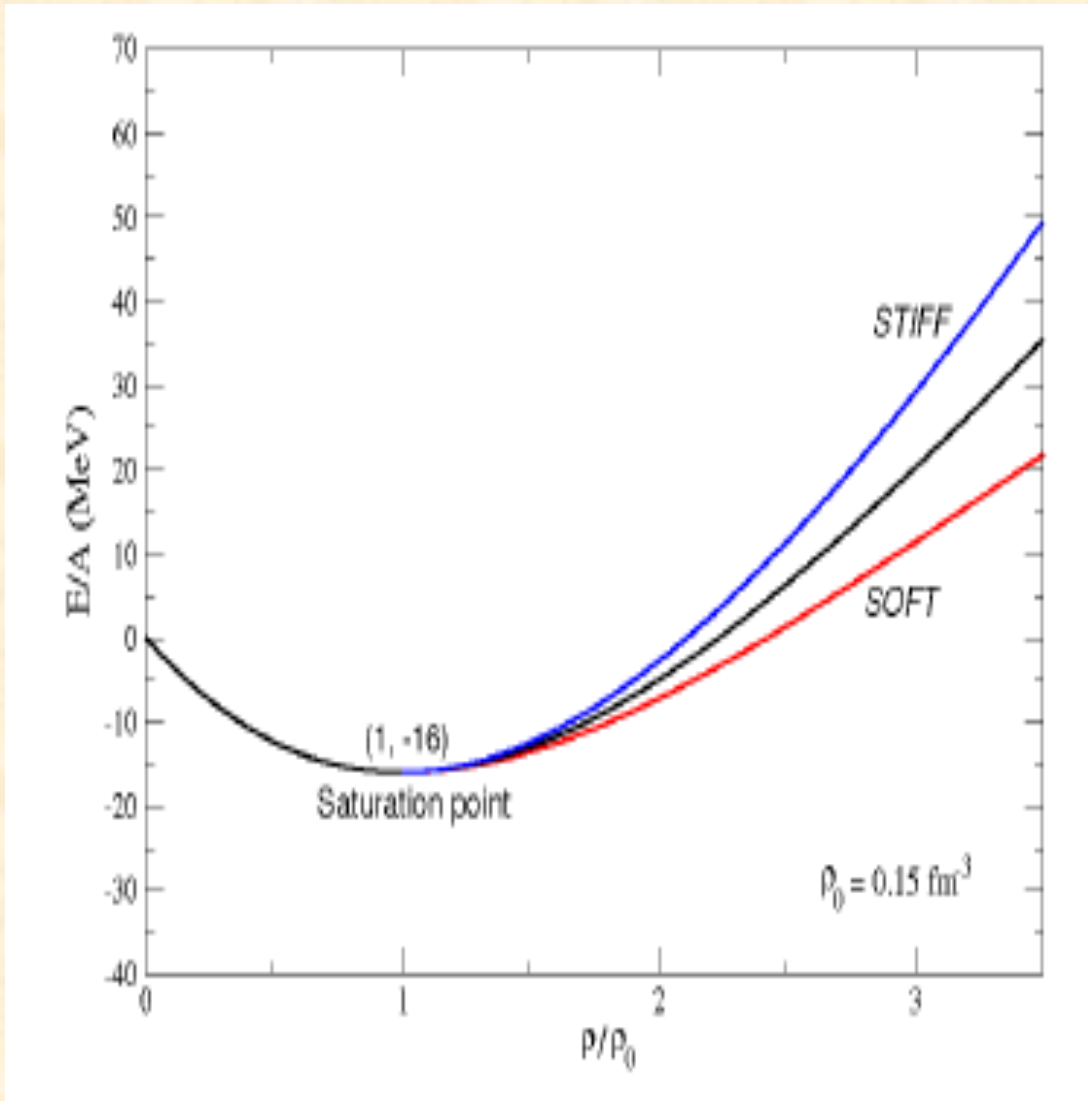
Nuclear Incompressibility: Does it depend on nuclear structure?



U. Garg
University of Notre Dame

Supported in part by the National Science Foundation

Ischia, May 17, 2022



$$K_\infty = K_F^2 \frac{\partial^2}{\partial K_F^2} \left(\frac{E_B}{A} \right) \quad K_F = \left(\frac{3\pi^2}{2} \rho_0 \right)^{1/3}$$



Nuclear Incompressibility

$$K_{\infty}$$

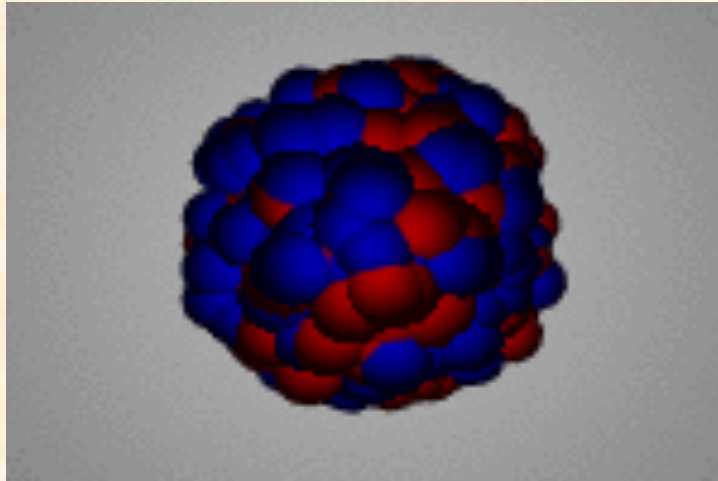
- Nuclear Equation of State
 - Neutron Star
 - Stellar Collapse
 - Supernovae
- High-energy Heavy Ion Collisions





The Isoscalar Giant Monopole Resonance

$$I = 0$$



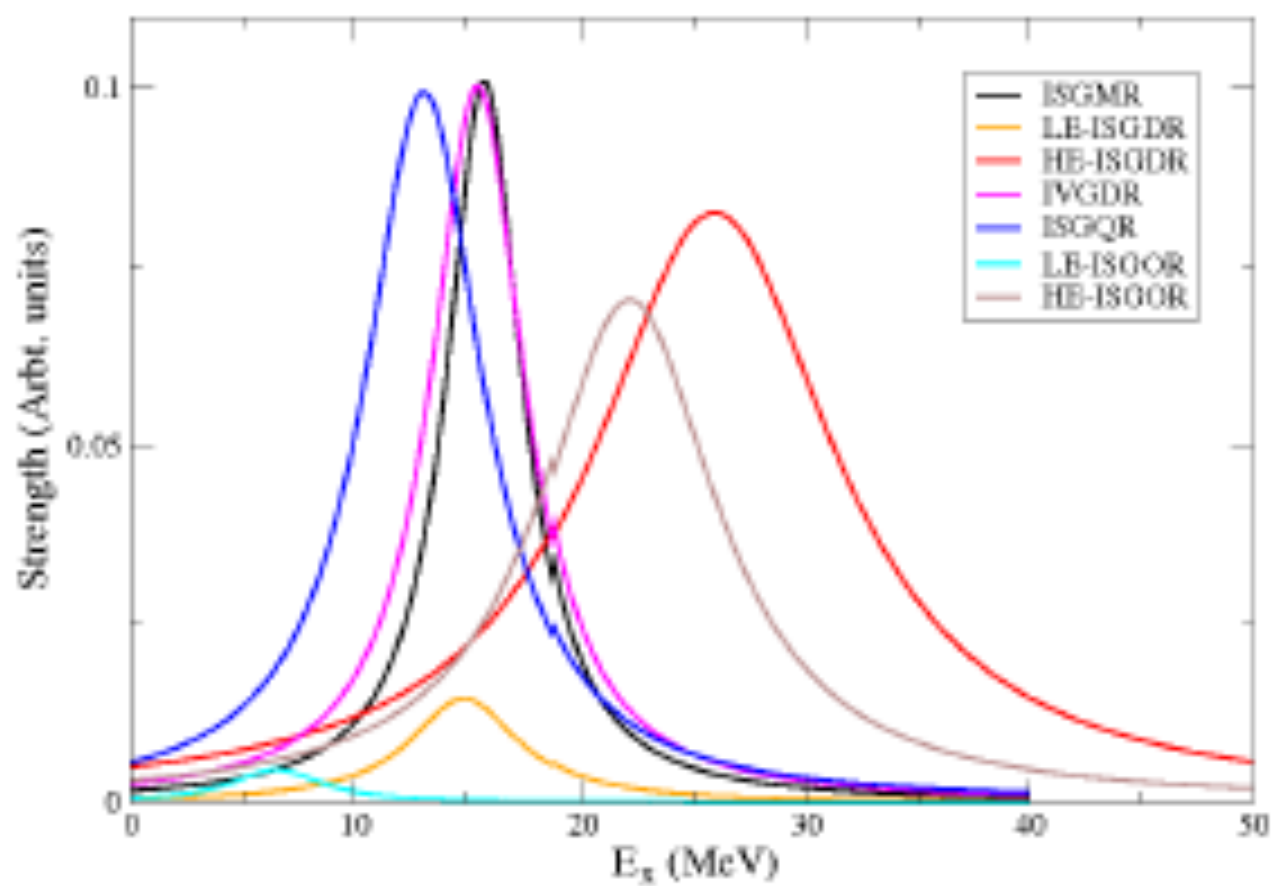
"Breathing Mode"

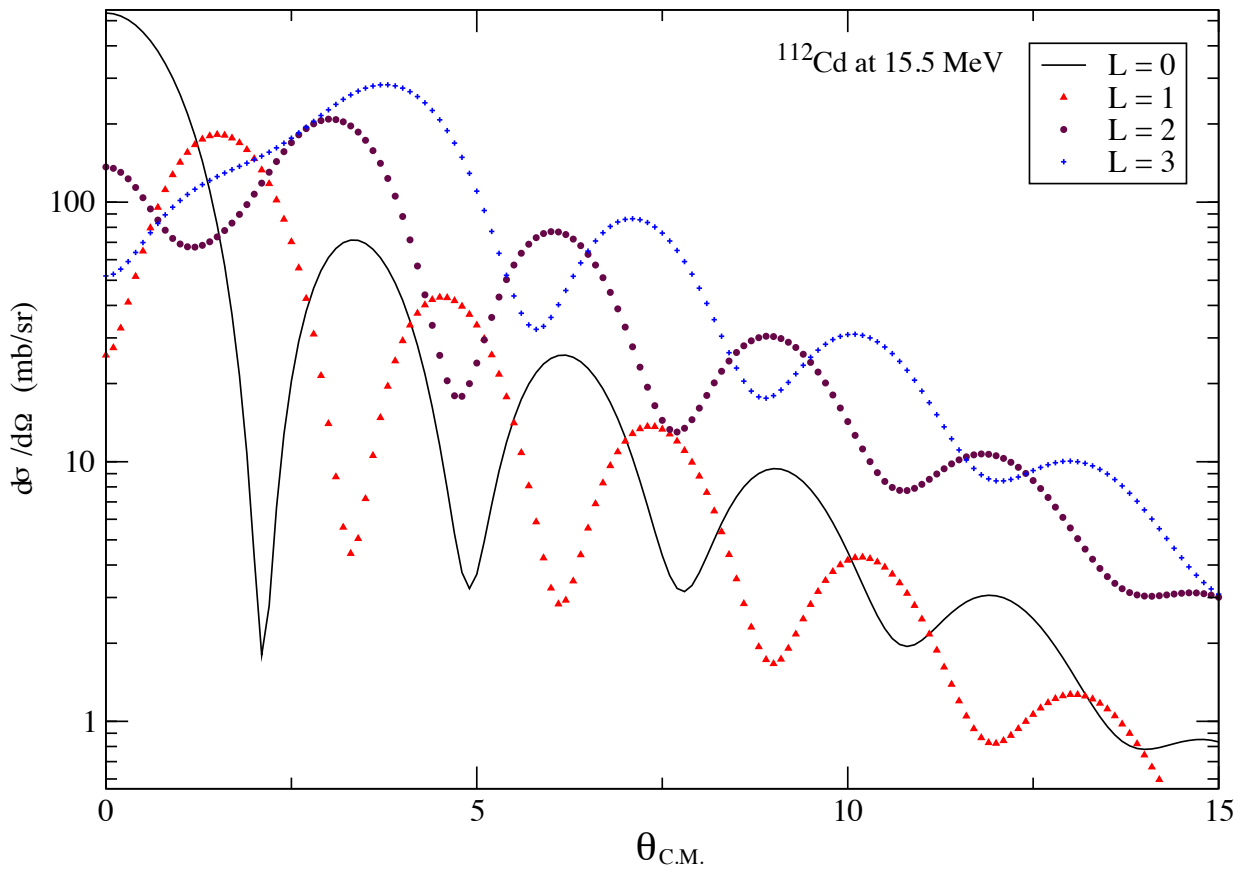
$$\frac{\sum r_i^2}{2\hbar\omega}$$

The energy of ISGMR may be directly related to the nuclear incompressibility

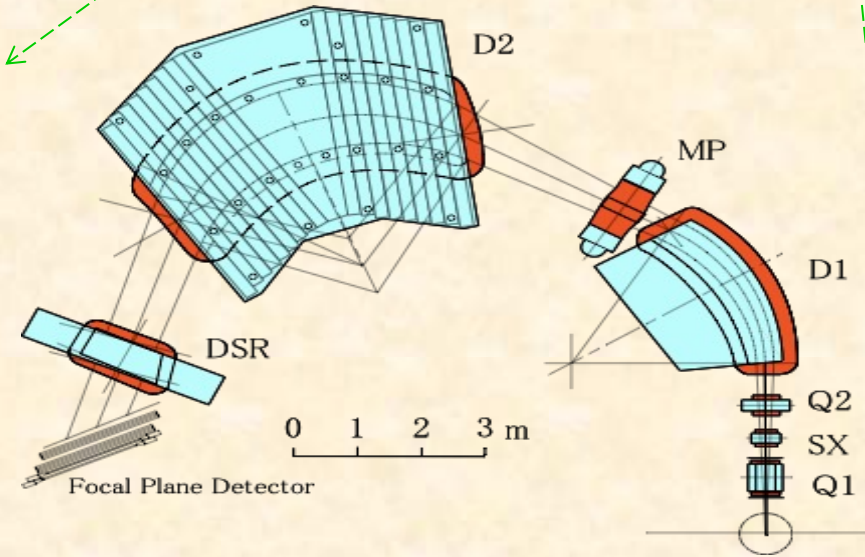
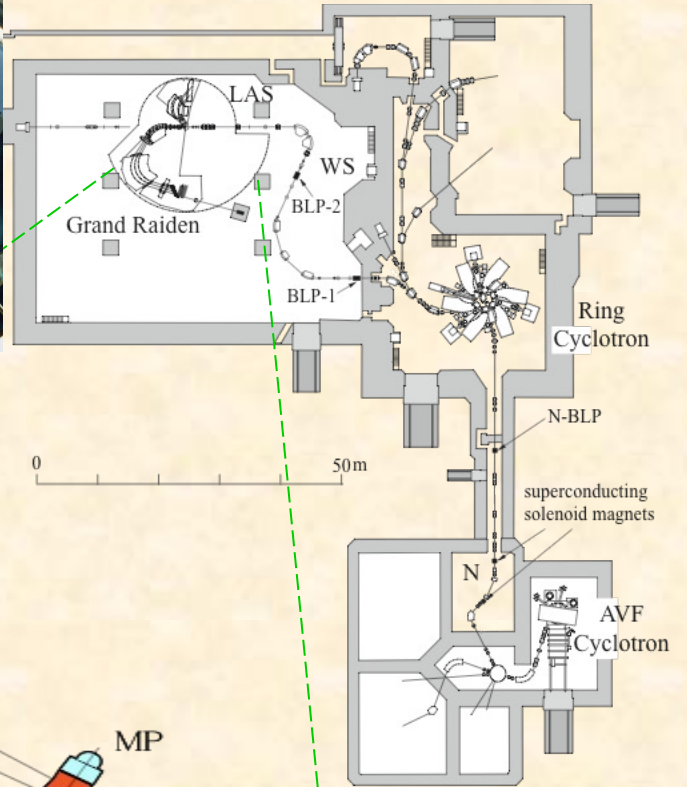
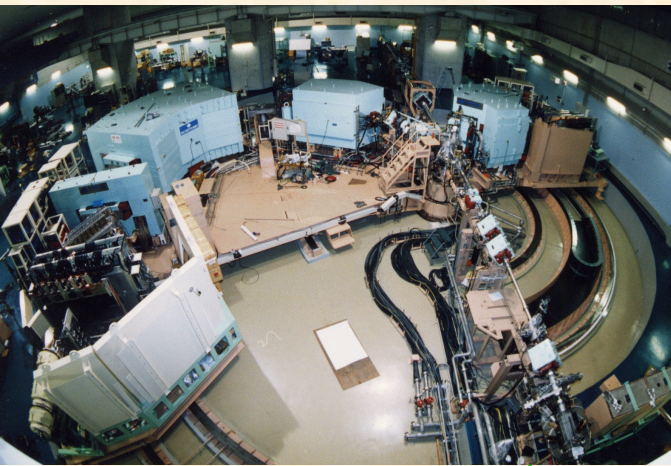
$$E_{GMR} = \hbar \sqrt{\frac{K_A}{m\langle r^2 \rangle}}$$

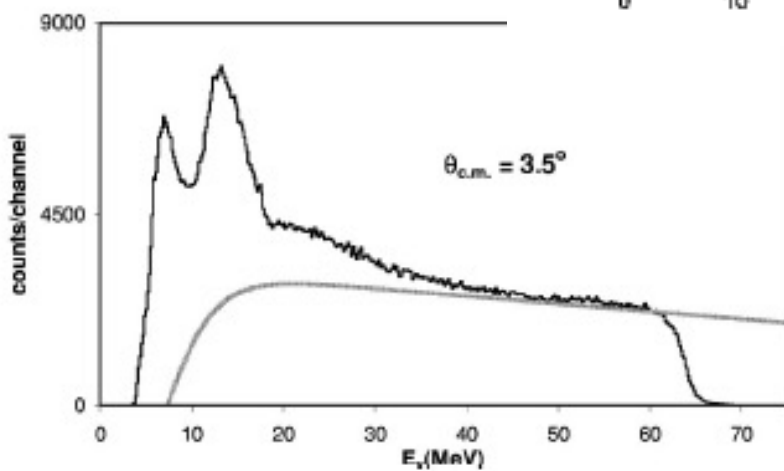
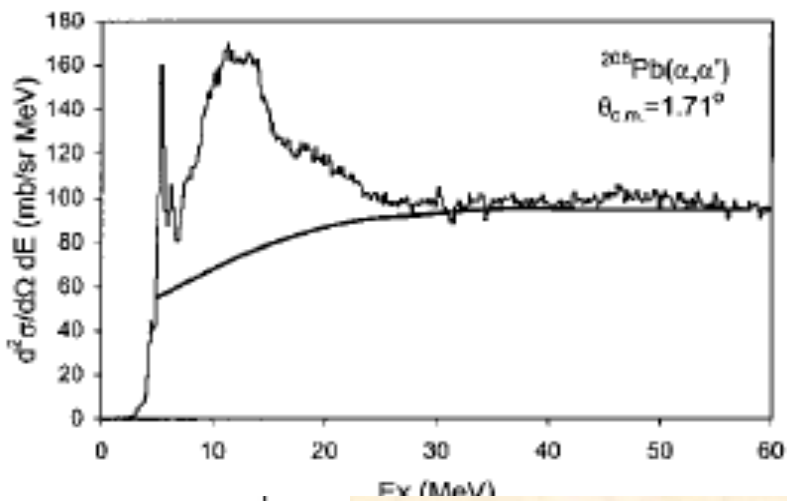
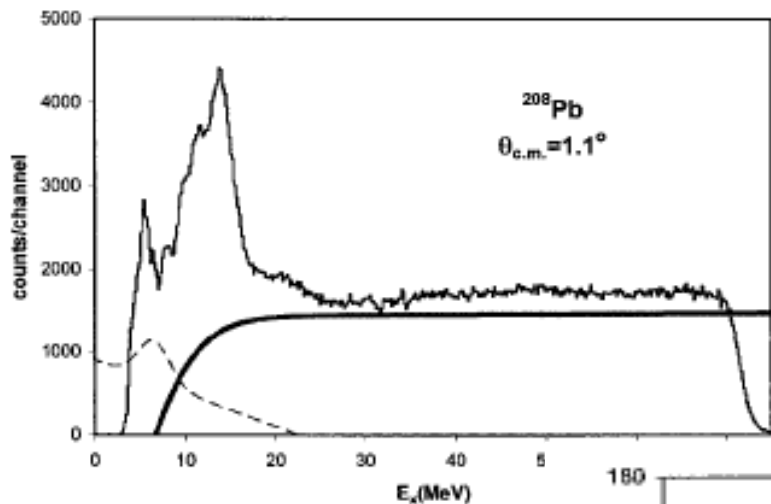
animation courtesy T. Aumann

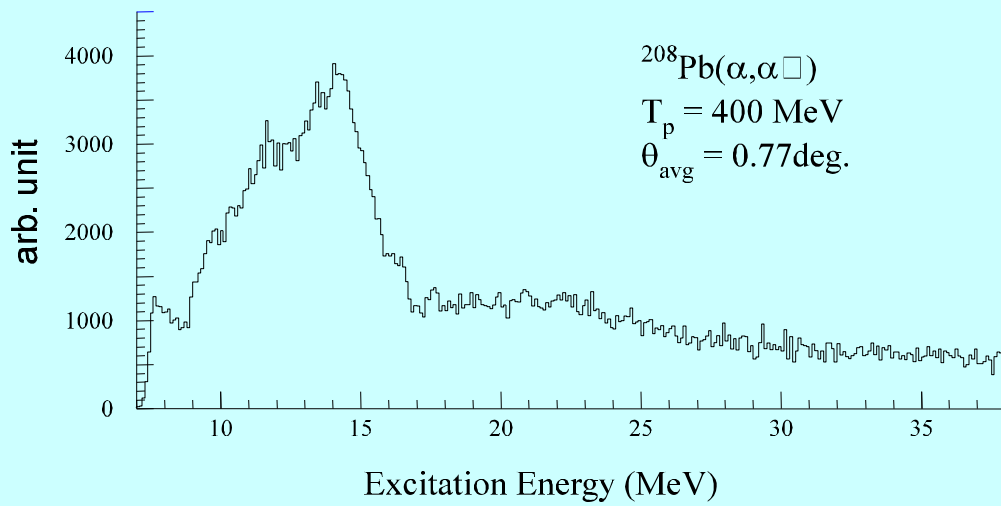
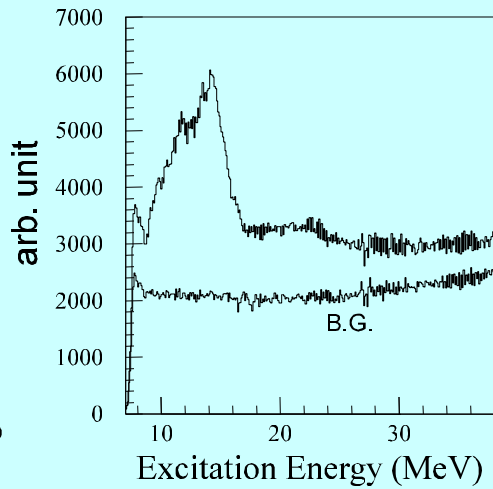
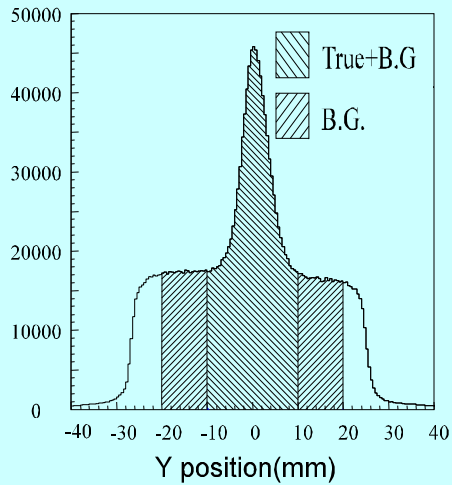




(α, α') at 400 MeV

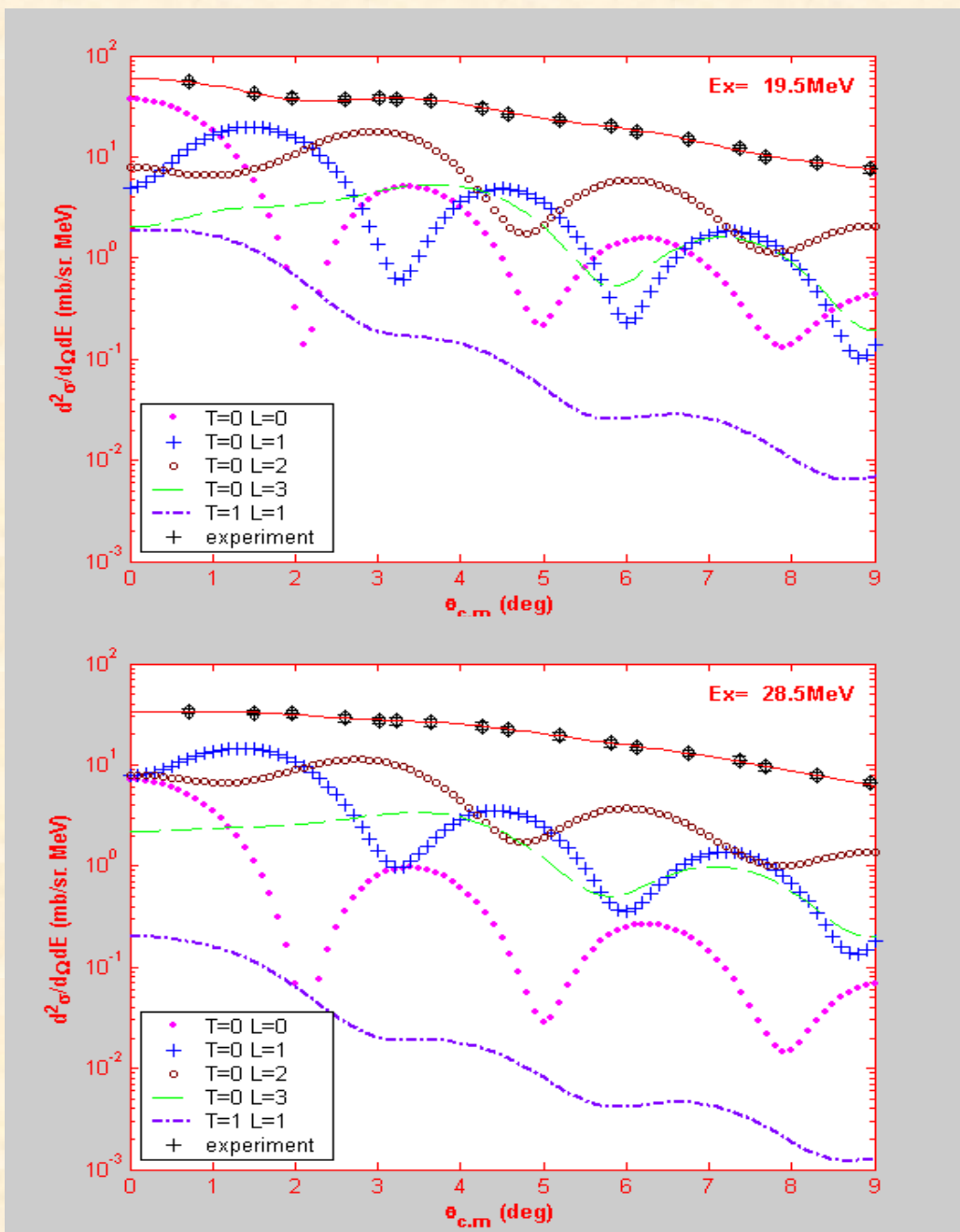


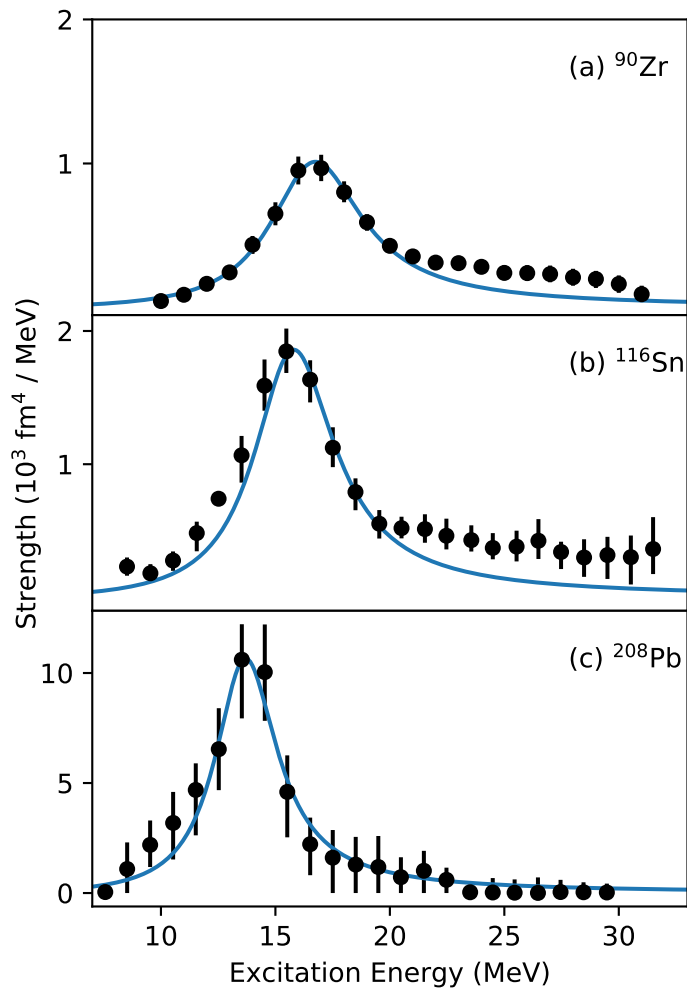






^{112}Sn





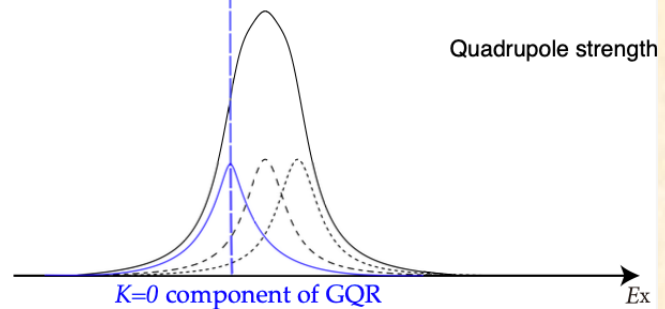
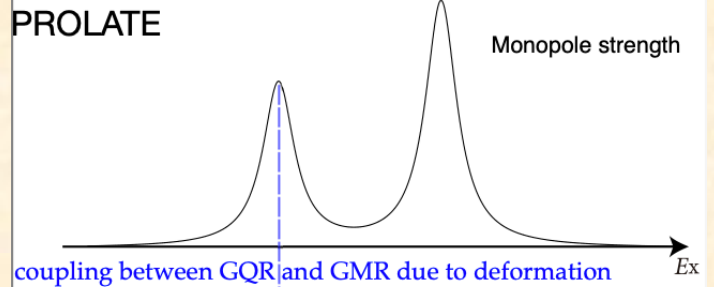
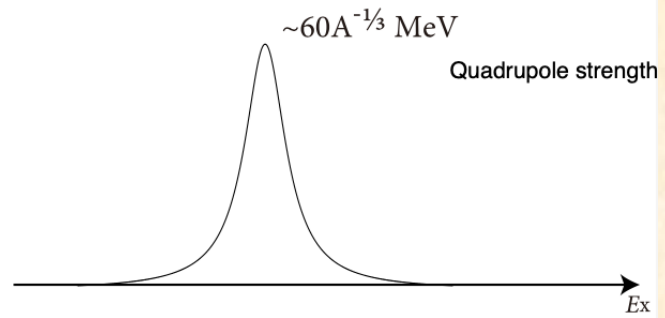
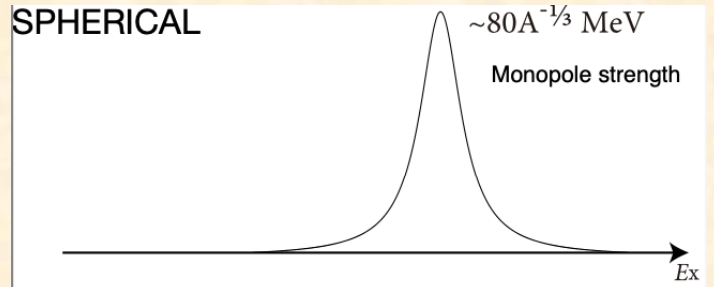
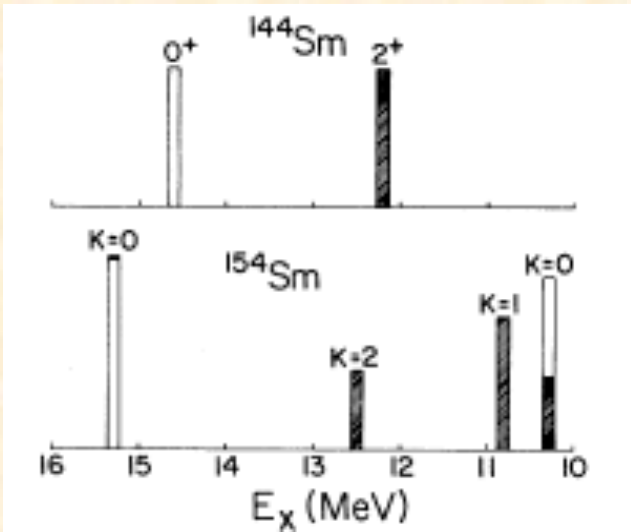


From GMR data on ^{208}Pb and ^{90}Zr
 $K_{\infty} = 240 \pm 20 \text{ MeV}$

This number is consistent with both GMR and ISGDR data and with non-relativistic and relativistic calculations



Effect of deformation on ISGMR





β_2

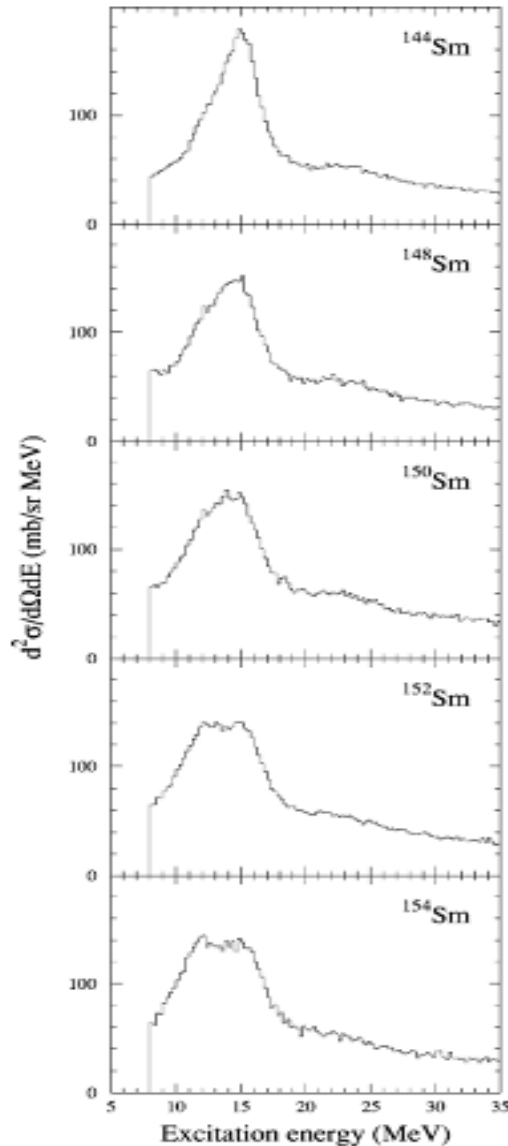
0.0881

0.142

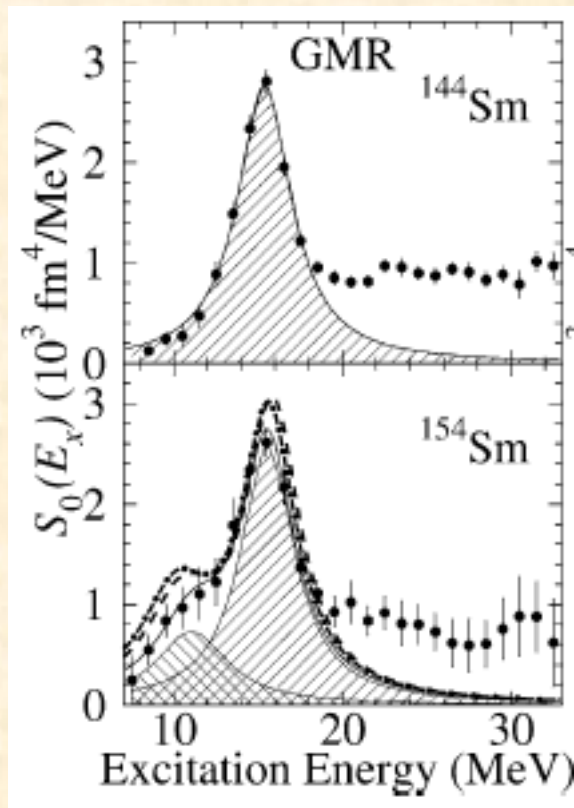
0.193

0.308

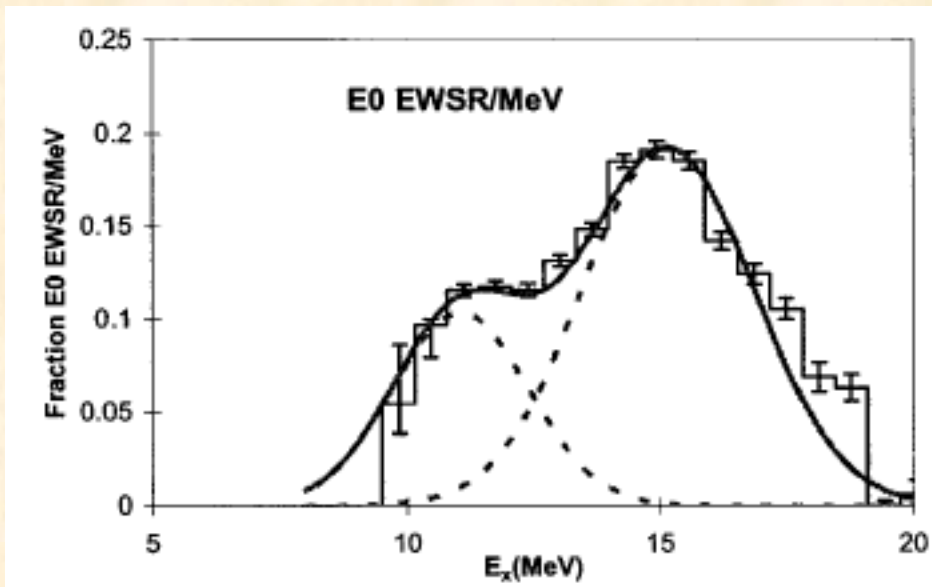
0.339



Well understood
in terms of
K=0 mixing of
ISGMR and ISGQR



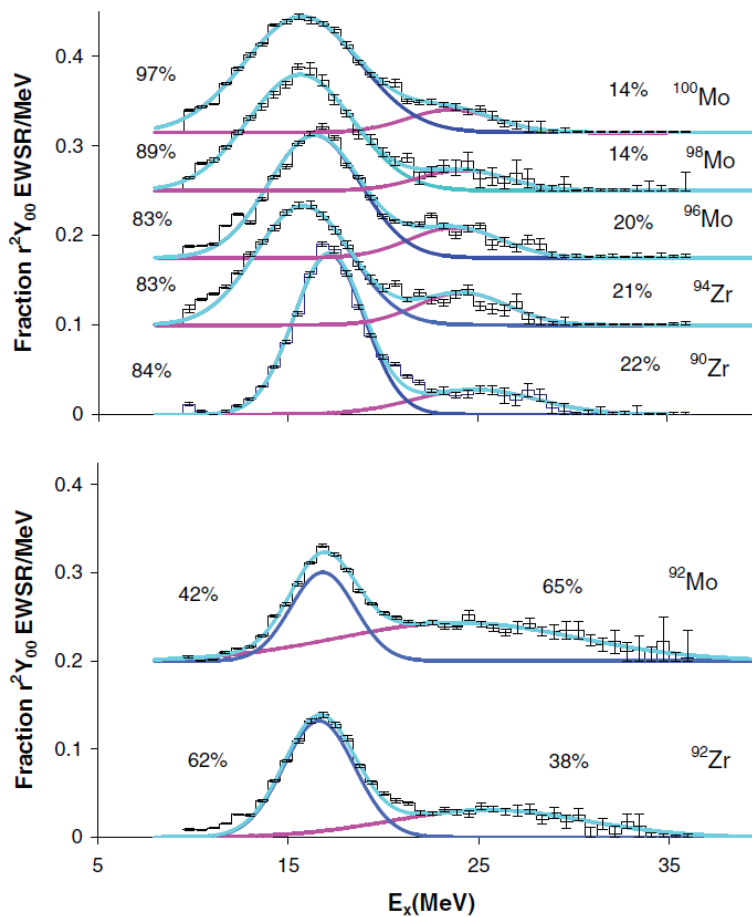
RCNP Data

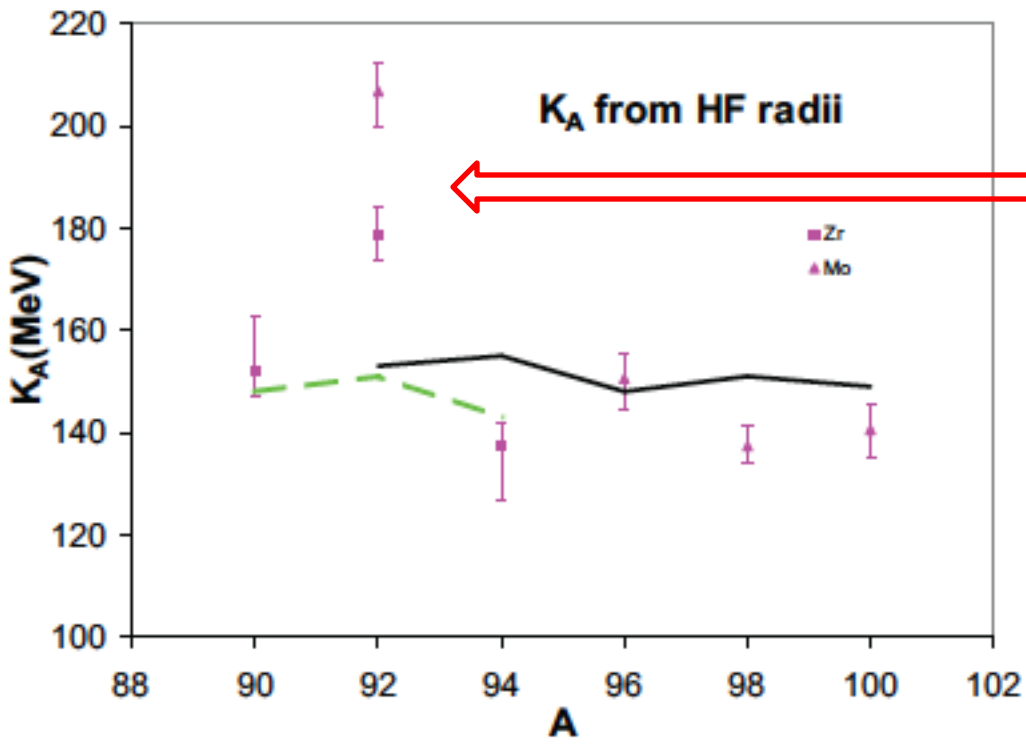


A&M Data

PHYSICAL REVIEW C **88**, 021301(R) (2013)

Unexpected characteristics of the isoscalar monopole resonance in the $A \approx 90$ region: Implications for nuclear incompressibility

D. H. Youngblood,¹ Y.-W. Lui,¹ Krishichayan,^{1,2} J. Button,¹ M. R. Anders,¹ M. L. Gorelik,³ M. H. Urin,³ and S. Shlomo¹¹Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA



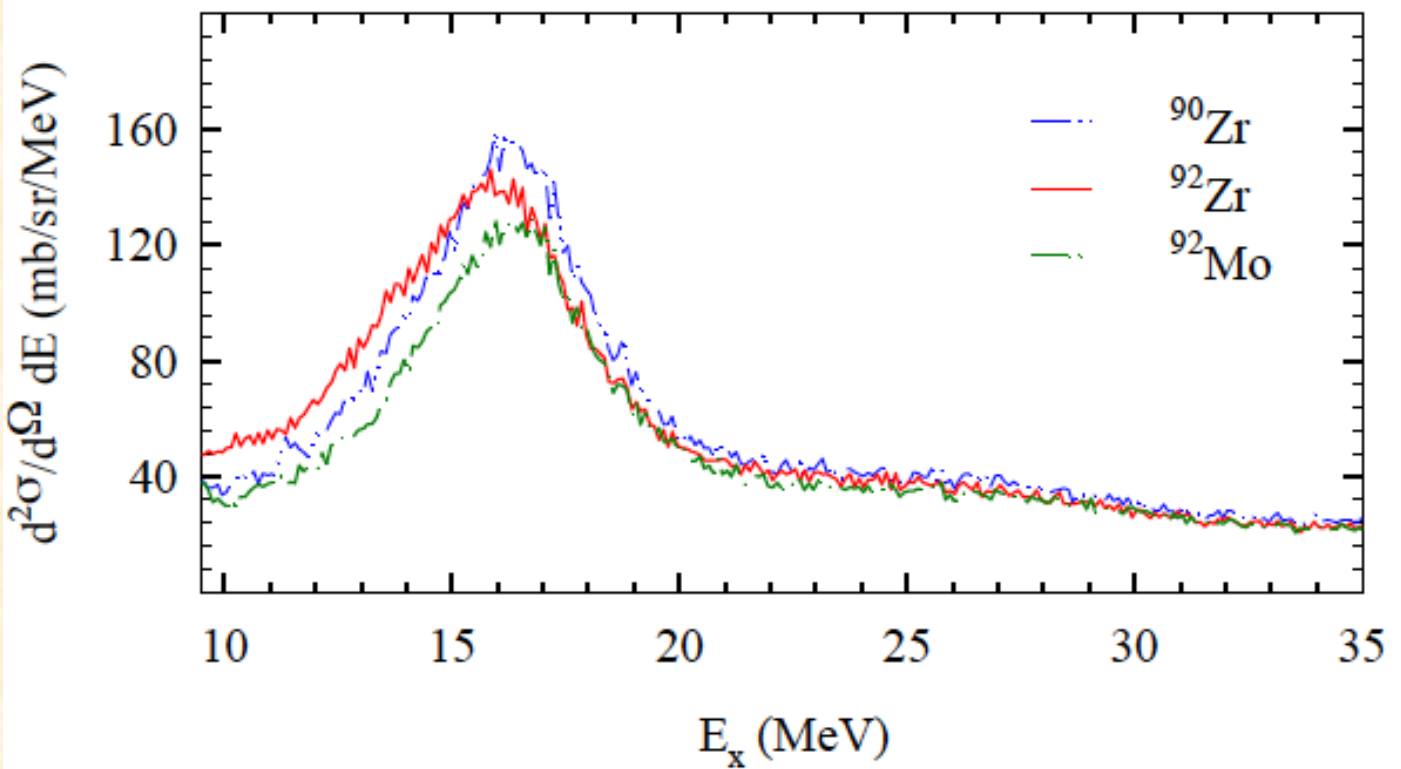


^{92}Zr and ^{92}Mo

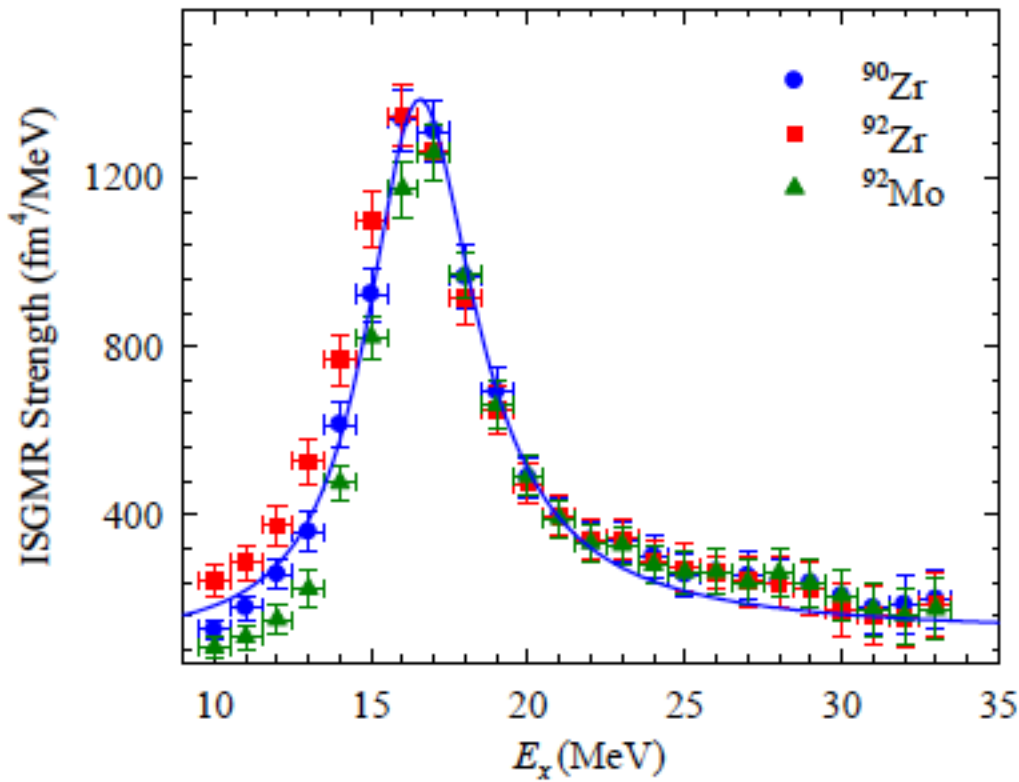
E_{ISGMR} : Centroids 1.2 MeV and 2.8 MeV
higher than ^{90}Zr

EWSR in second peak: 62% and 42%

- K_A for ^{92}Zr and ^{92}Mo is higher than that for ^{90}Zr by 27 and 56 MeV, respectively!
- Significant nuclear structure contributions to the nuclear compressibility !
- These are the first observations of this kind since first identification of ISGMR in 1970s !

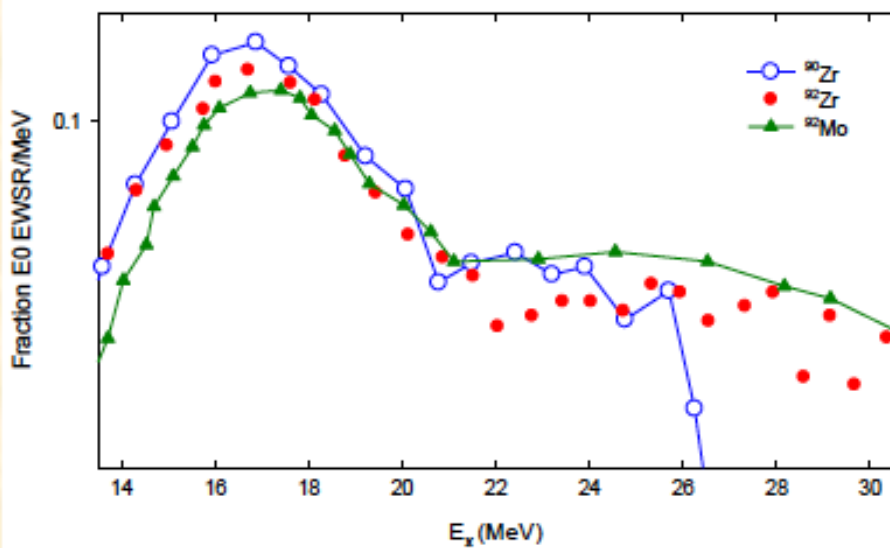
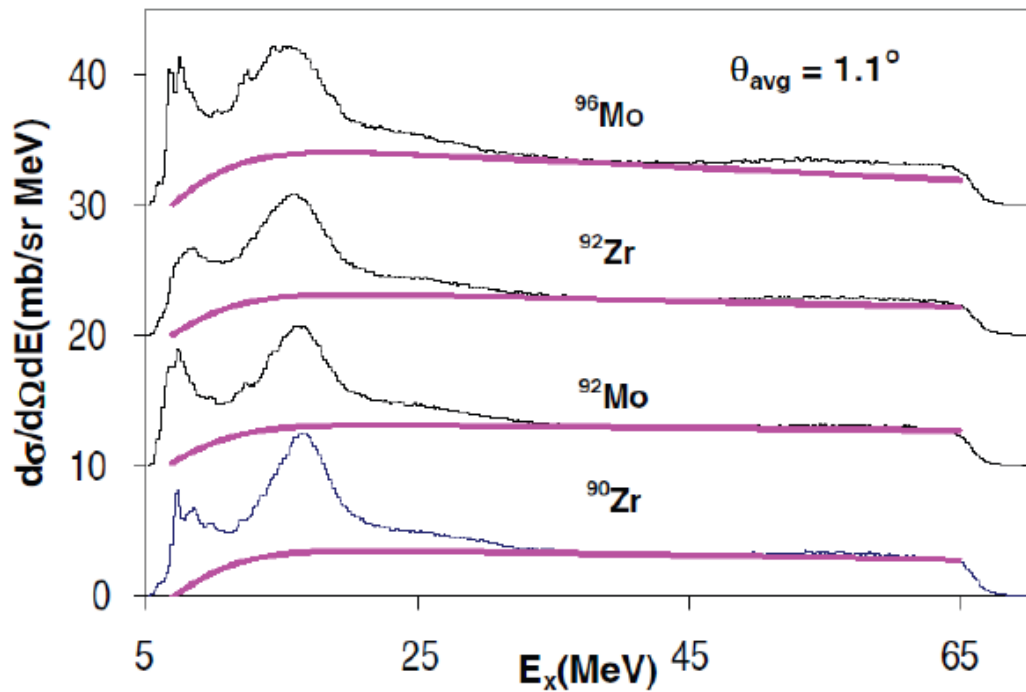


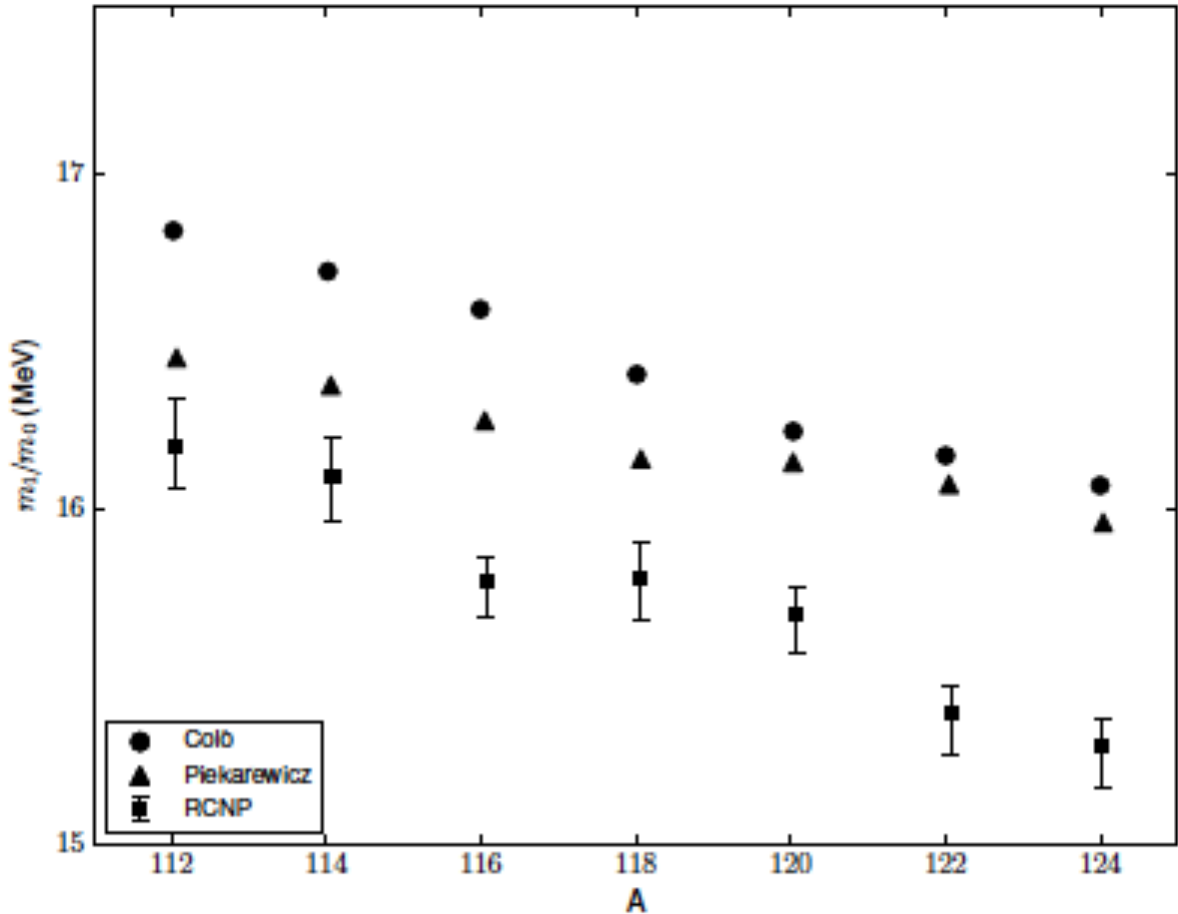
Zero Deg Spectra



Nucleus	E_m (MeV)	Γ (MeV)
⁹⁰ Zr	16.55 ± 0.08	4.2 ± 0.3
⁹² Zr	16.12 ± 0.04	4.5 ± 0.2
⁹² Mo	16.79 ± 0.11	4.2 ± 0.4

Y. K. Gupta *et al.*, Phys. Lett. B 760, 482 (2016)





Why are tins so “Fluffy”?

T. Li *et al.*, Phys. Rev. Lett. **99**, 162503 (2007)

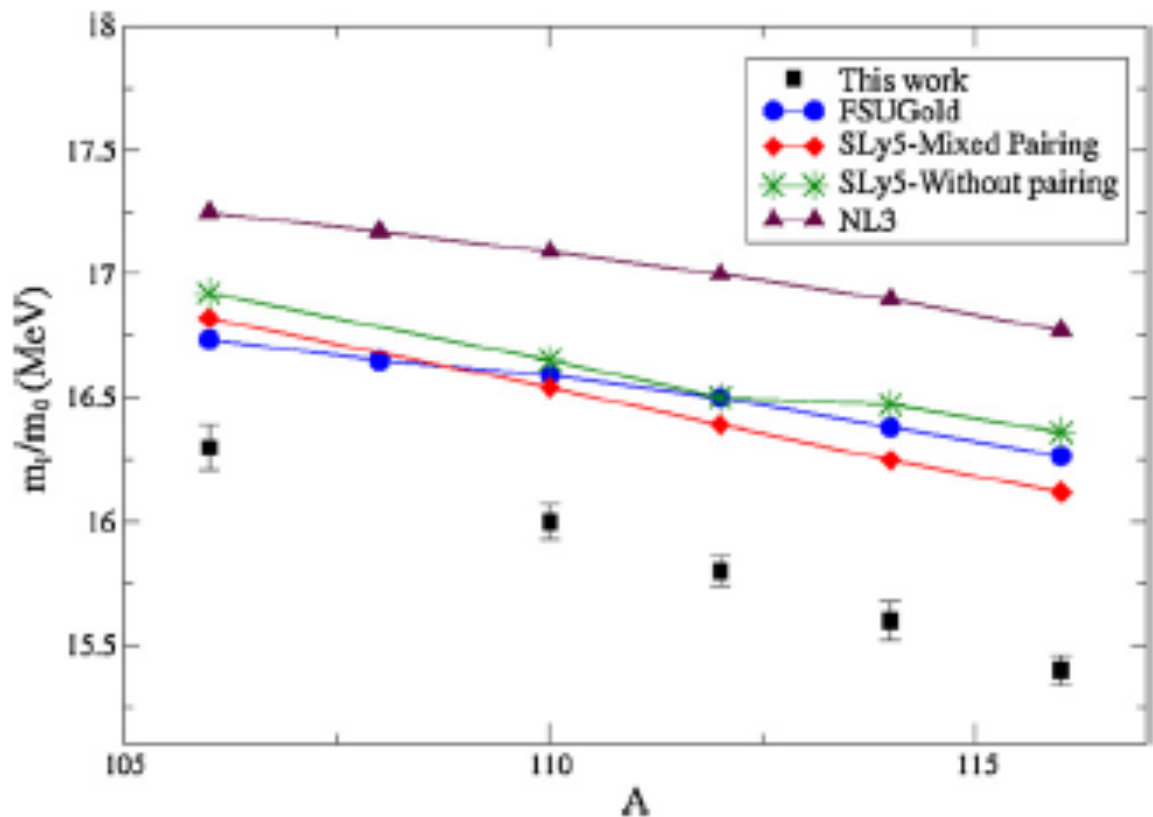
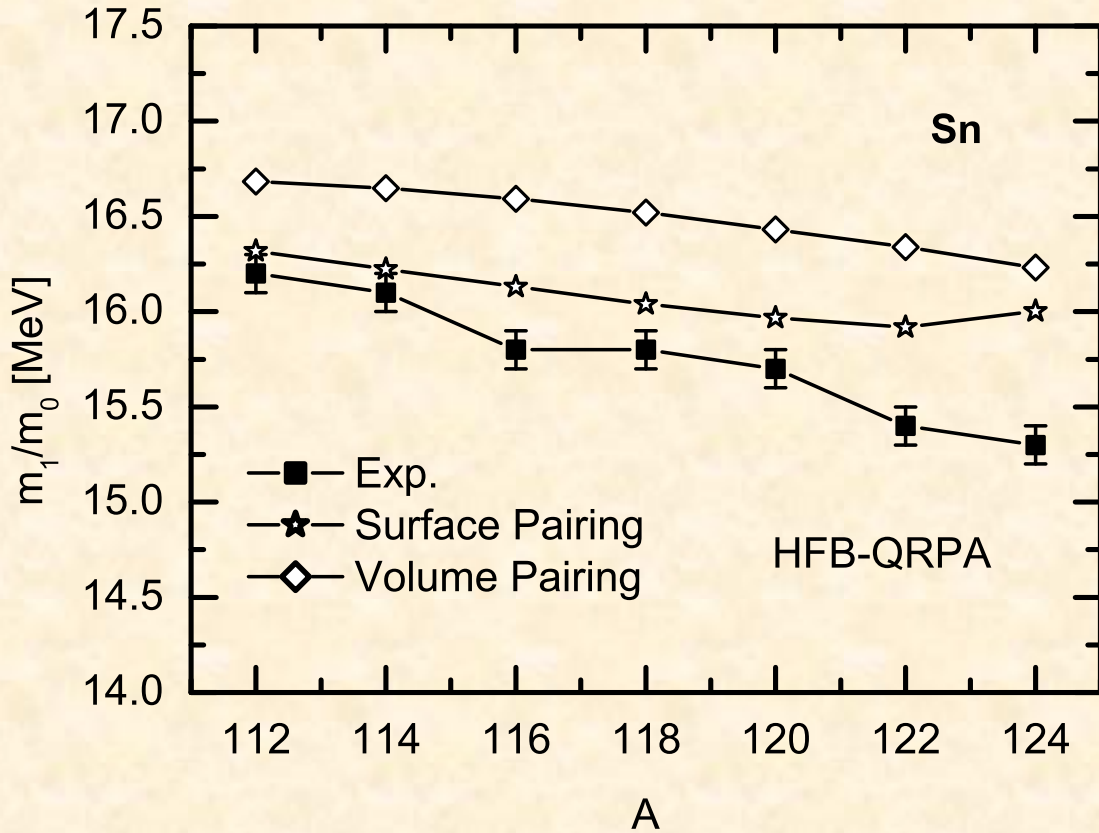
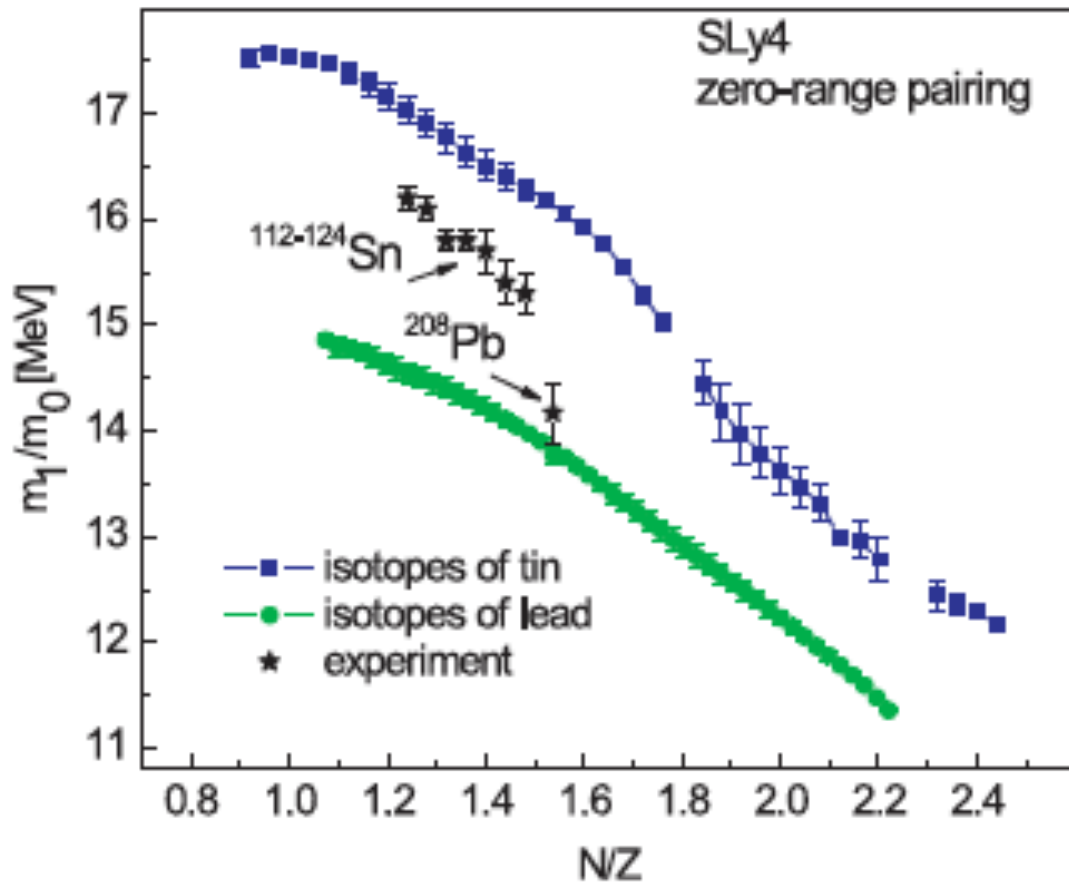
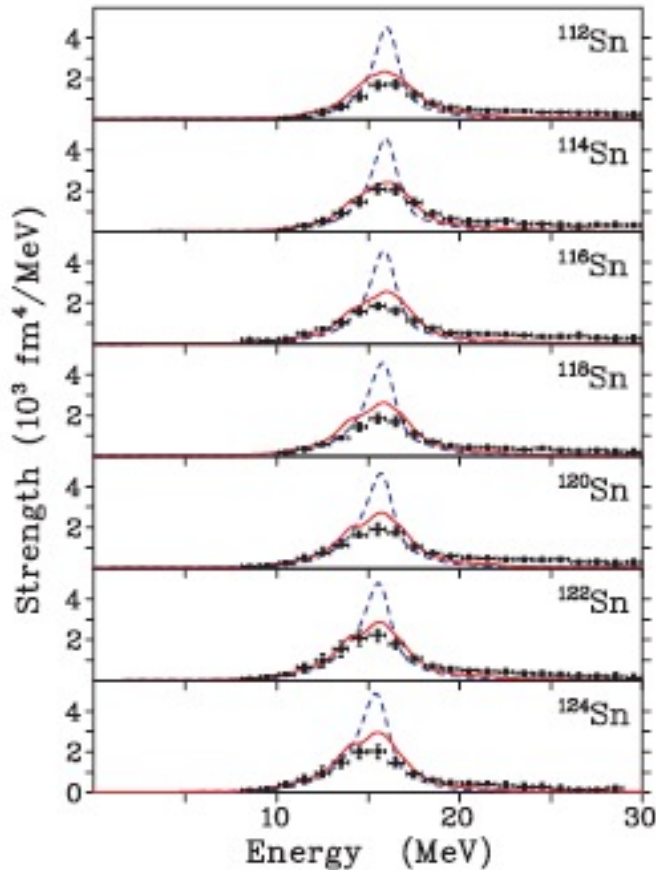


Fig. 3. (Color online.) Systematics of the moment ratio, m_1/m_0 for the ISGMR strength distributions in the Cd isotopes investigated in this work. The experimental results (squares) are compared with relativistic calculations performed using the FSUGold (circles) and NL3 (triangles) effective interactions. Also presented are results from non-relativistic calculations performed using the Sly5 parameter set in the HF-BCS + QRPA formalism with and without the mixed pairing interaction (diamonds and stars, respectively) [36]. The solid lines are to guide the eye.



SkM* ($K_\infty \sim 215$ MeV)

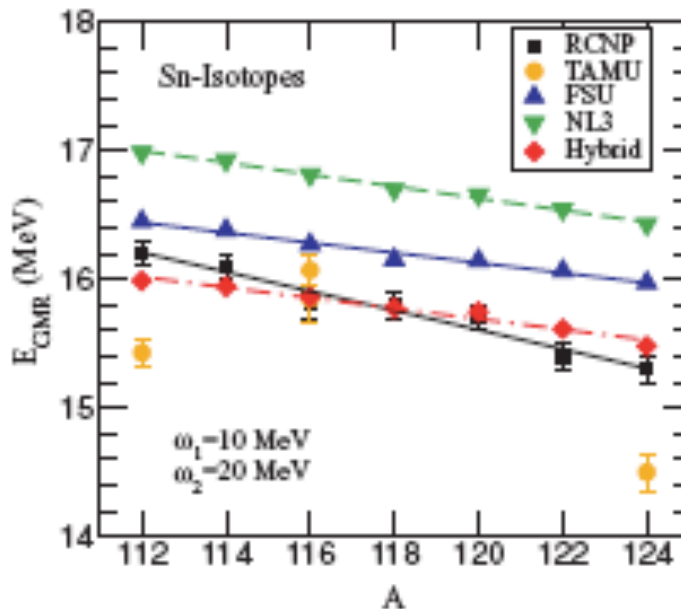




V. Tselyaev *et al.*,
Phys. Rev. C **79**, 034309 (2009)

Self-consistent HF+BCS with
T5 Skyrme Interaction;
 $K_\infty = 202$ MeV

the experiment. On the whole, these results do not allow us to decrease the ambiguity in the value of K_∞ as compared with the previous known estimates. Note, however, that the main goal of our work is not to solve the problem of the nuclear matter incompressibility but to find under which conditions one can obtain reasonable description of the experimental data for the considered tin isotopes within the framework of the self-consistent approach including correlations beyond the QRPA.



$$K_{\infty} \sim 230 \text{ MeV}; K_{\tau} = -532 \text{ MeV}$$

although the improvement in the case of the Sn isotopes is significant and unquestionable, an important problem remains: the hybrid model underestimates the GMR centroid energy in ^{208}Pb —the heaviest doubly magic nucleus—by almost 1 MeV. This suggests that the rapid softening with neutron excess predicted by the hybrid model may be unrealistic.

Thus, where does theory stand with respect to experiment? One possibility, given that FSUGold reproduces the centroid energy in both ^{90}Zr (with $\alpha = 0.11$) and ^{208}Pb (with $\alpha = 0.21$), is that its predictions for K_0 and K_{τ} are reliable, but that its failure to reproduce the GMR energies in tin is due to missing physics unrelated to the incompressibility of neutron-rich matter. We feel inclined to favor this possibility for two



Nuclear Physics **A399** (1983) 11-50
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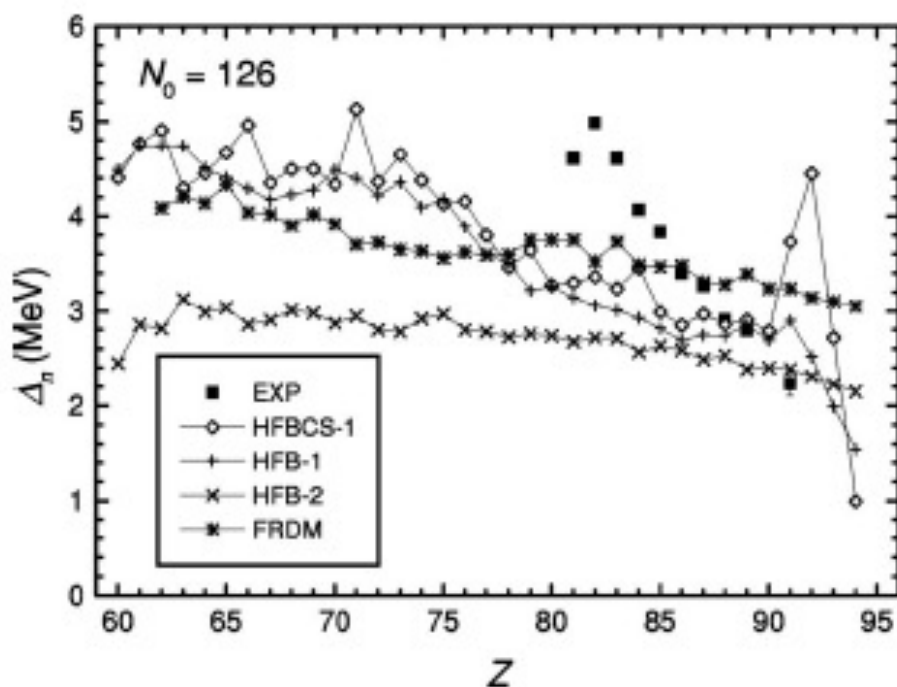
MUTUAL SUPPORT OF MAGICITIES AND RESIDUAL EFFECTIVE INTERACTIONS NEAR ^{208}Pb

N. ZELDES[†], T. S. DUMITRESCU^{††} and H. S. KÖHLER^{†††}

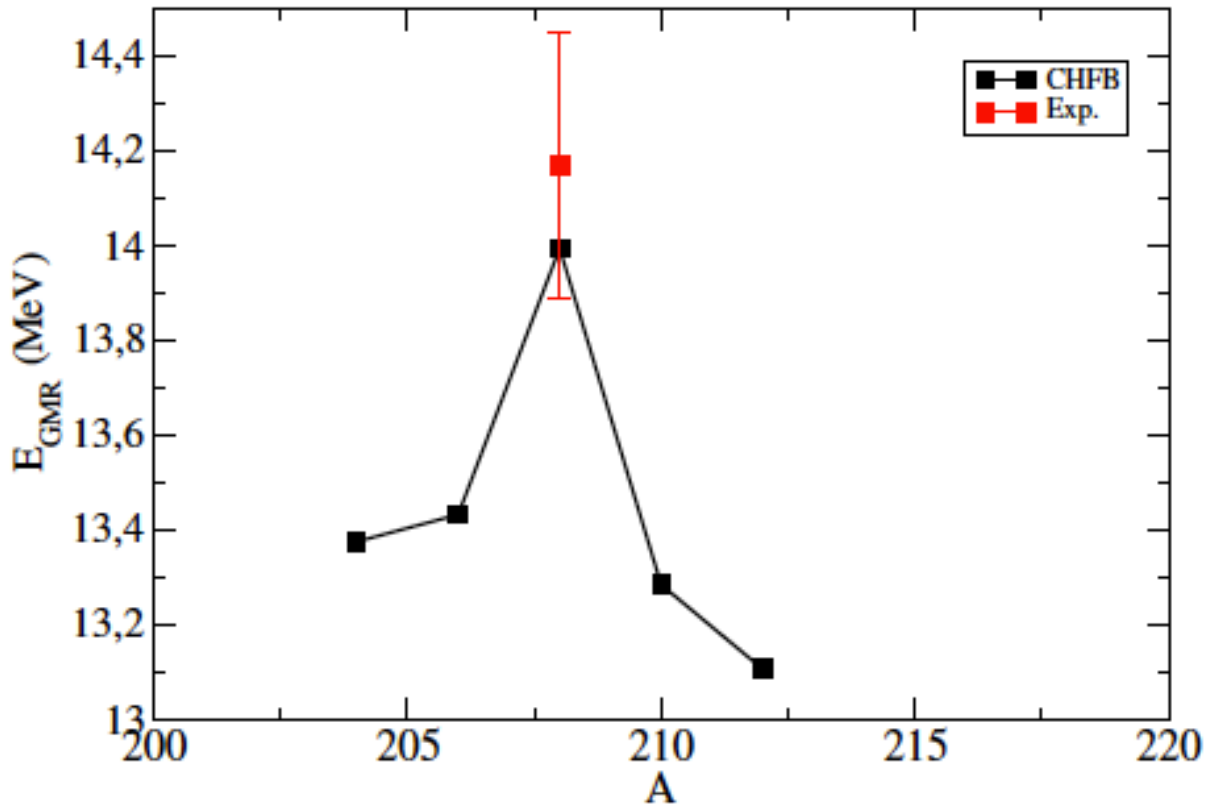
Niels Bohr Institute and Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 1 November 1982

Abstract: We summarize experimental evidence in the lead region on the increased stability associated with neutron magic number when the proton number is magic, and vice versa. The effect is interpreted in the framework of the nuclear shell model with empirical effective interactions. Its relation to spherical Hartree-Fock calculations is pointed out and used to test Skyrme-type forces. None of the considered Skyrme interactions reproduce the effect.



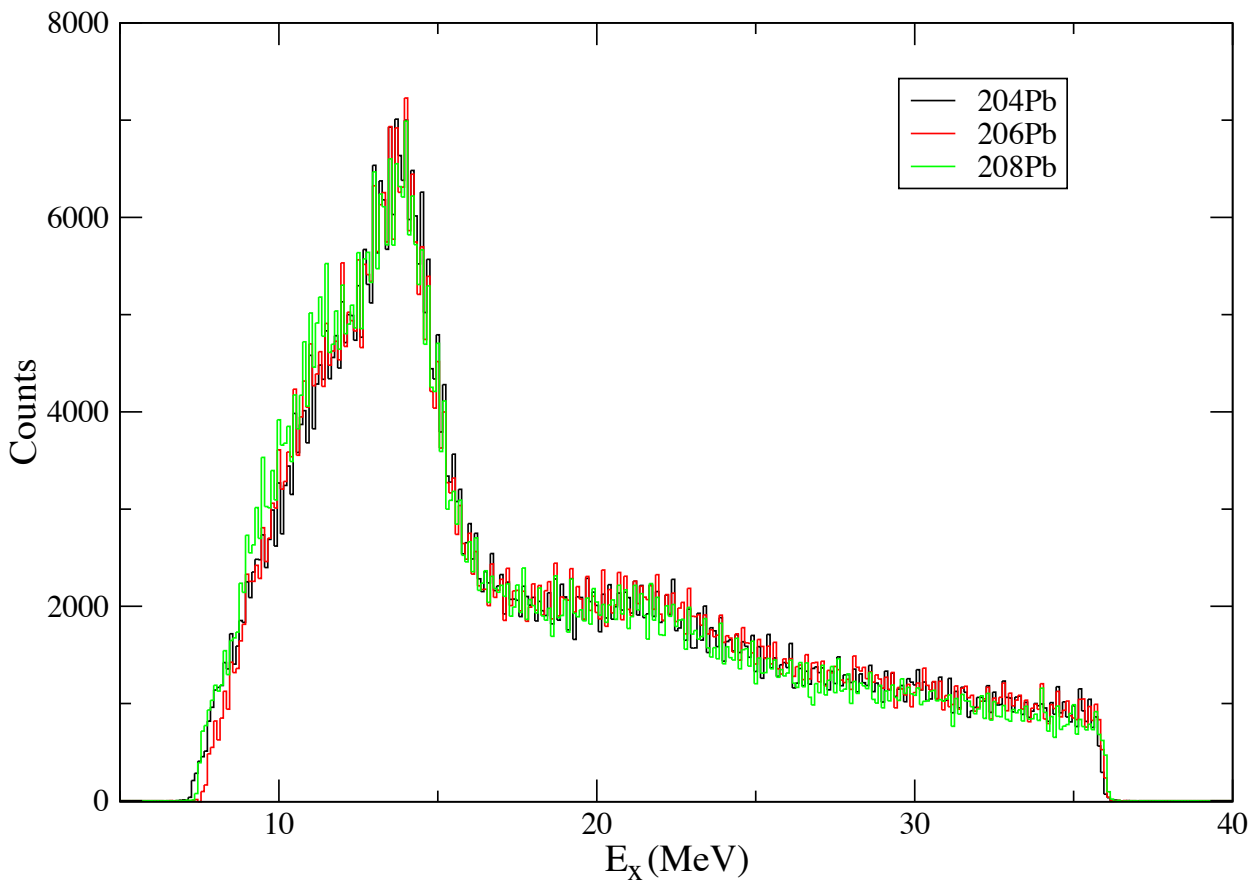
term). There are 27 such nuclei, and in the case of both HFB formulas their mean error (experiment – calculated) is -1.31 MeV, as compared to 0.040 MeV for the complete set of 1768 data points in the case of HFB-1 and 0.000 MeV (to three decimals) for the 2135 data points in the case of HFB-2. For HFBCS-1 the effect is smaller but still significant, the mean error for the 27 nuclei being -0.731 MeV, to be compared with 0.102 MeV for the complete fit.

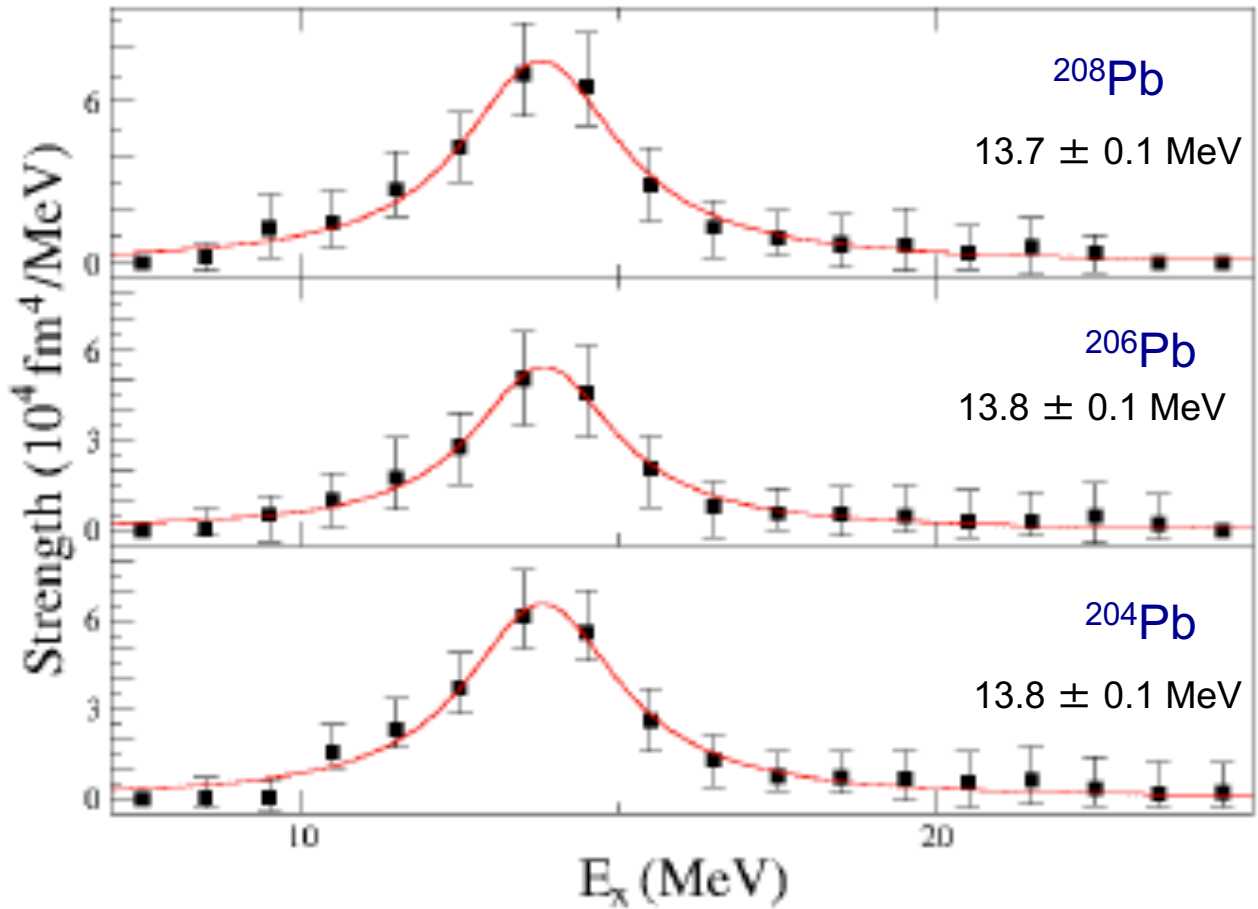


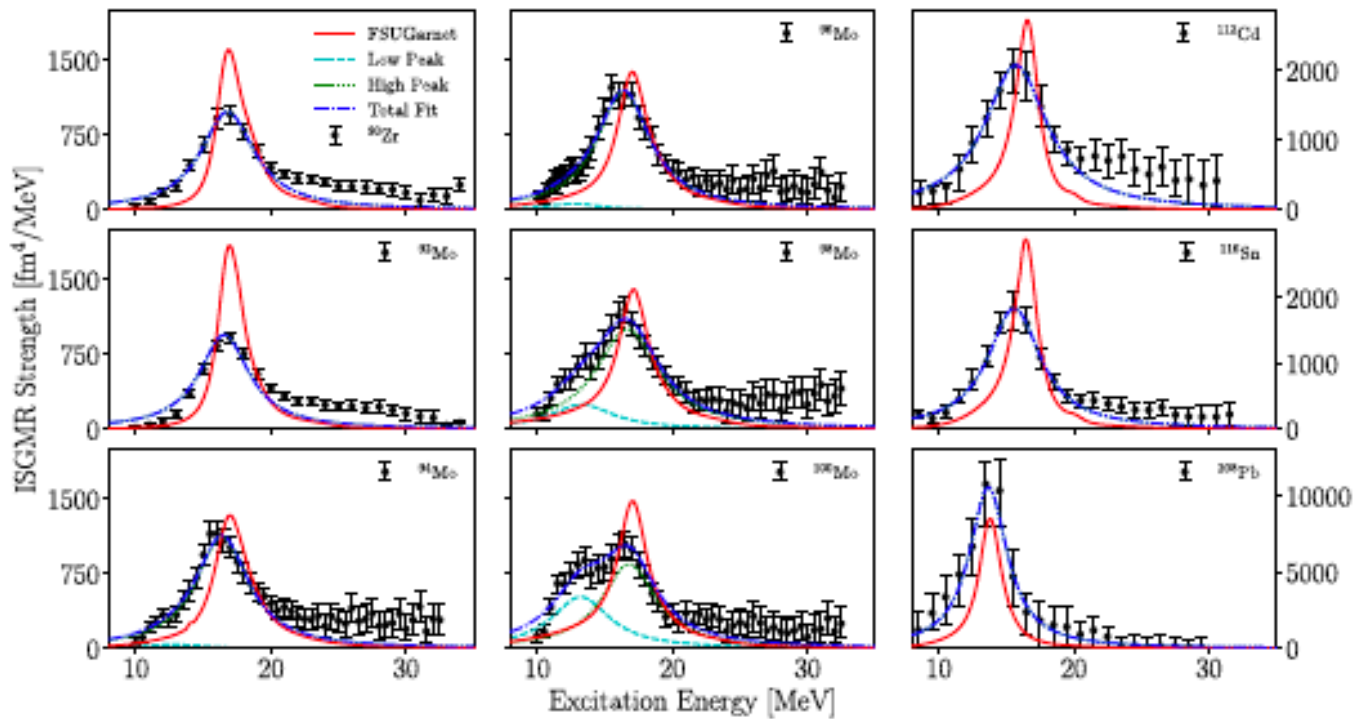
E. Khan, Phys. Rev. C 80, 053702 (2009)



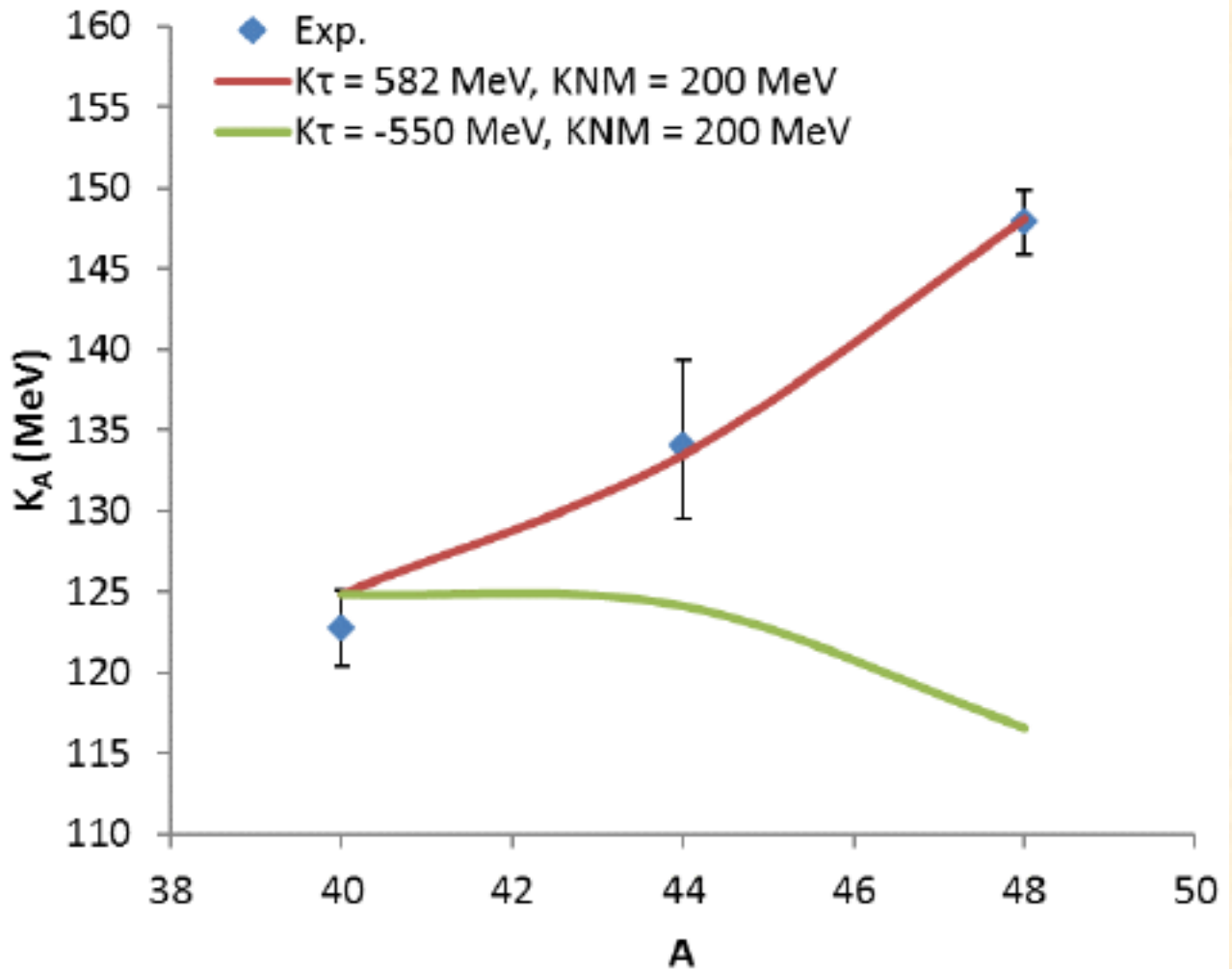
0^0 spectra







K.B. Howard *et al.*, Phys. Lett. B **807**, 135608 (2020)



J. Button et al., Phys. Rev. C **96**, 054330 (2017)

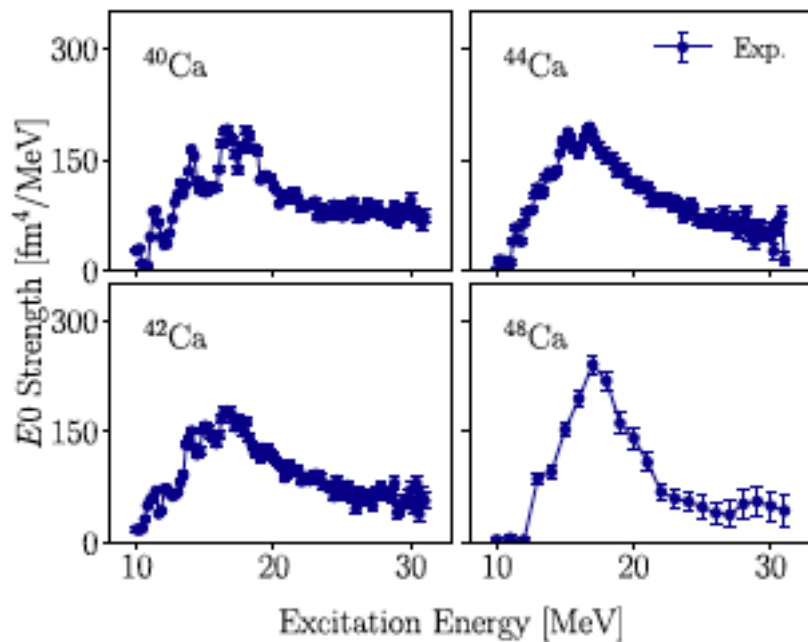
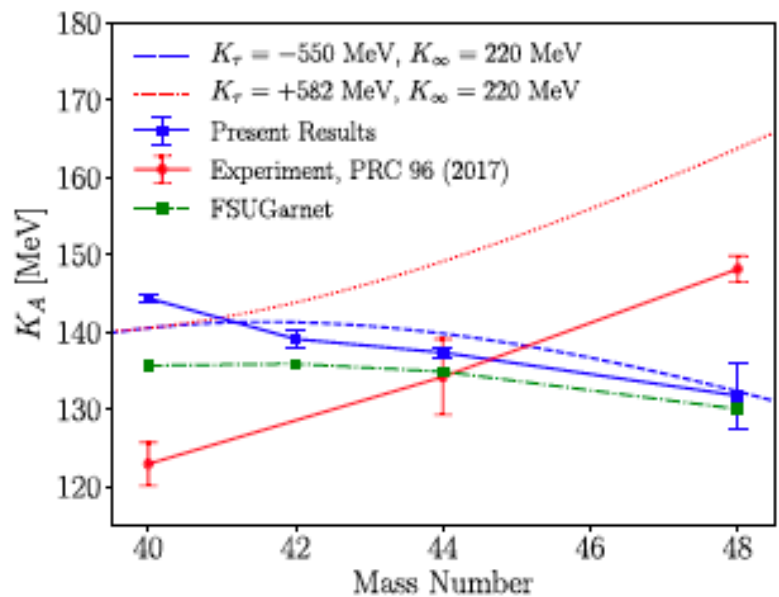


Fig. 3. Extracted isoscalar monopole strength distributions for $^{40,42,44,48}\text{Ca}$.





- ◆ We have investigated “structure effects” on the ISGMR (the “breathing mode”) : deformation, open-shell nuclei, direct shell structure effects, via inelastic scattering of 386-MeV α particles at extremely forward angles (including 0°).
- ◆ In deformed nuclei, the ISGMR shows a splitting of strength into two components. This splitting is attributed to coupling with the $K=0$ component of the ISGQR.
- ◆ In the Cd and Sn isotopes, the ISGMR energy is significantly lower than that expected from the accepted value of K_∞ . As of now, there is no satisfactory theoretical explanation of this “fuzziness” of open-shell nuclei. This effect appears to start manifesting in Mo nuclei even.
- ◆ The ISGMR response of $^{90,92}\text{Zr}$ and ^{92}Mo is practically identical, in contrast with a recent report claiming significant shell structure contribution to nuclear incompressibility.



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Grazie!

धन्यवाद

Thanks!





The Question Kitten