Zr isotopes as a case study for intertwined quantum phase transitions

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*with **A. Leviatan** and **F. Iachello***

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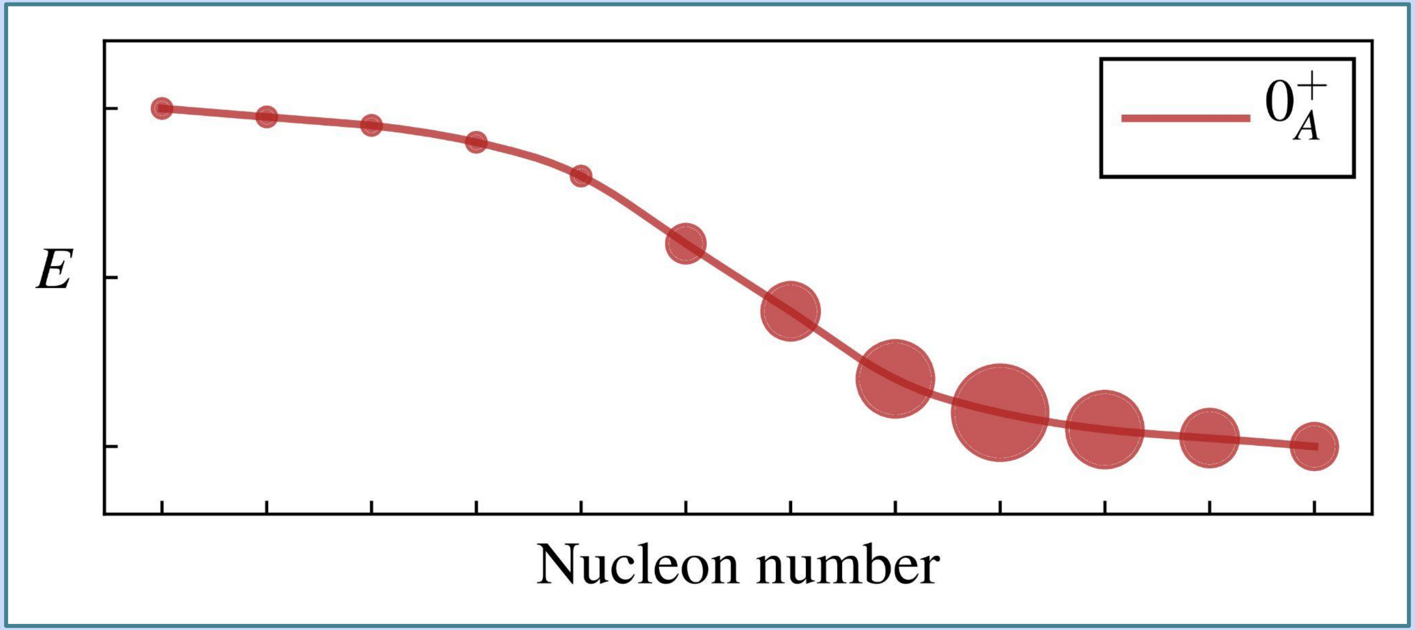
Introduction

Type I QPT: shape evolution (single configuration)



$$H = (1-\xi)H_1 + \xi H_2\,; \quad 0 \le \xi \le 1$$

Introduction

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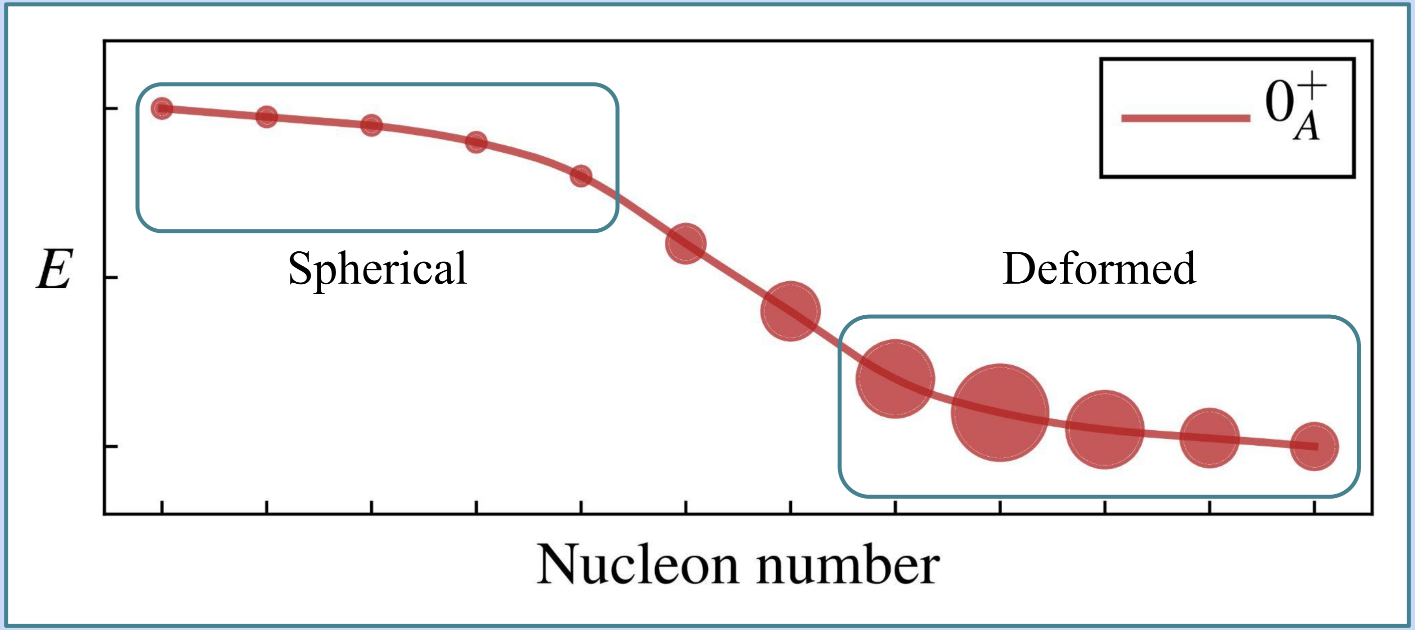
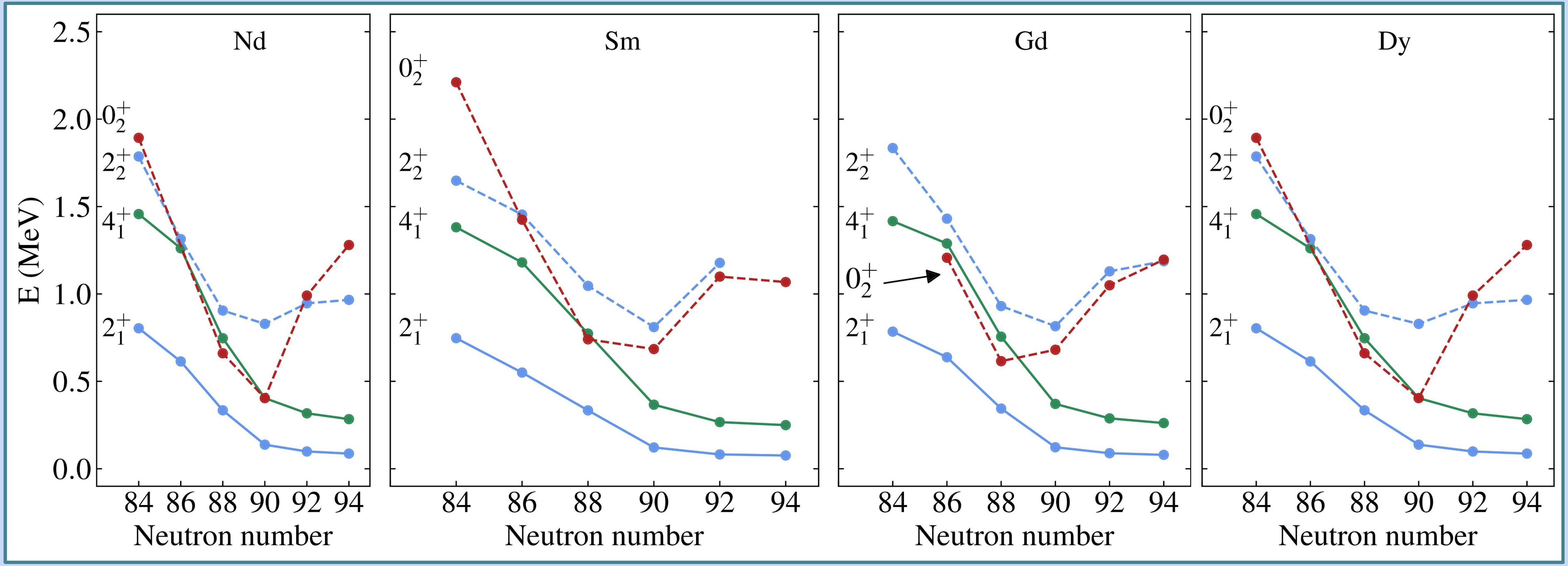


$$H = (1-\xi)H_1 + \xi H_2 \,; \quad 0 \leq \xi \leq 1$$

Introduction

Example

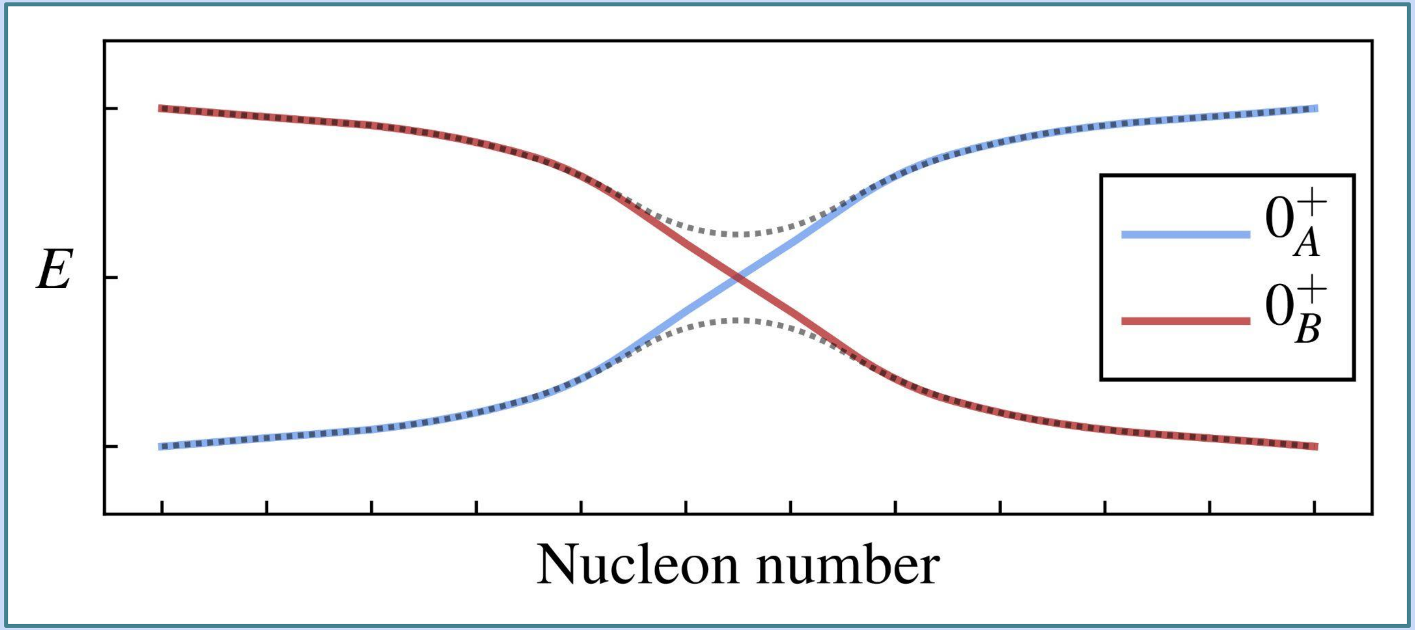
Z = 60 Z = 62 Z = 64 Z = 66



Spherical [U(5)] → Deformed [SU(3)]

Introduction

Type II QPT: configuration crossing (multiple configurations)

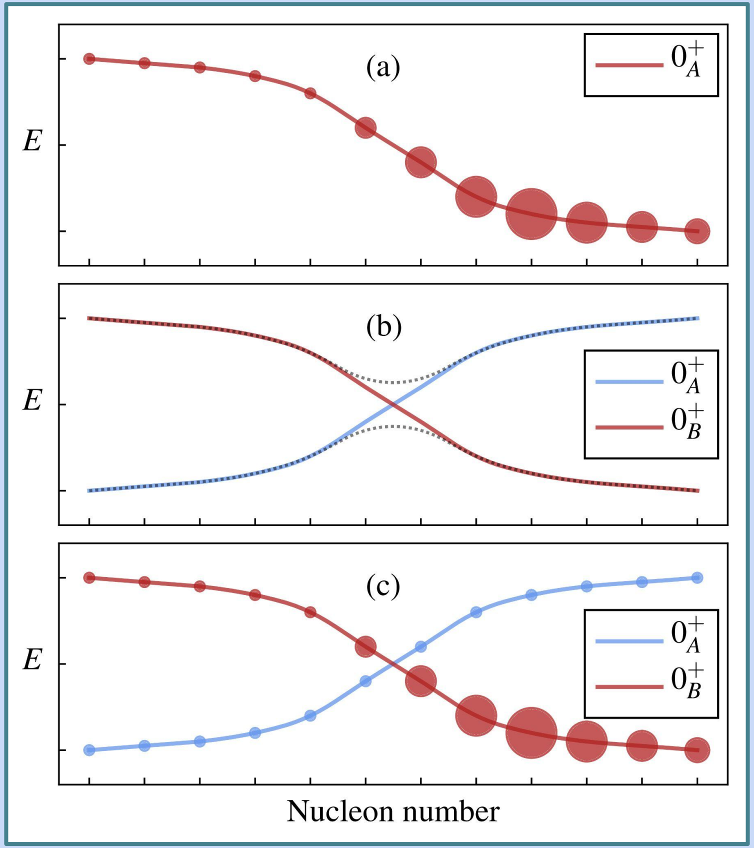


$$H = \begin{bmatrix} \hat{H}_A(\xi_A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi_B) \end{bmatrix}$$

Light Pb-Hg isotopes

Introduction

Intertwined quantum phase transitions (IQPT)



IQPT:

Type I and Type II simultaneously

(Type II with weak mixing)

N. Gavrielov, A. Leviathan and F. Iachello,
Phys. Rev. C **99**, 064324 (2019)
Phys. Scr. **95**, 024001 (2020)
Phys. Rev. C **105**, 014305 (2022)

Introduction

Boson counting

Boson number: $\hat{N} = \hat{n}_s + \hat{n}_d = s^\dagger s + \sum_\mu d^\dagger_\mu d_\mu$ $\quad N = N_\pi + N_\nu$

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Boson counting

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0p-0h, 2p-2h, 4p-4h, … → $[N] \oplus [N+2] \oplus [N+4] \oplus \ldots$

normal$\oplus$intruder states

Introduction

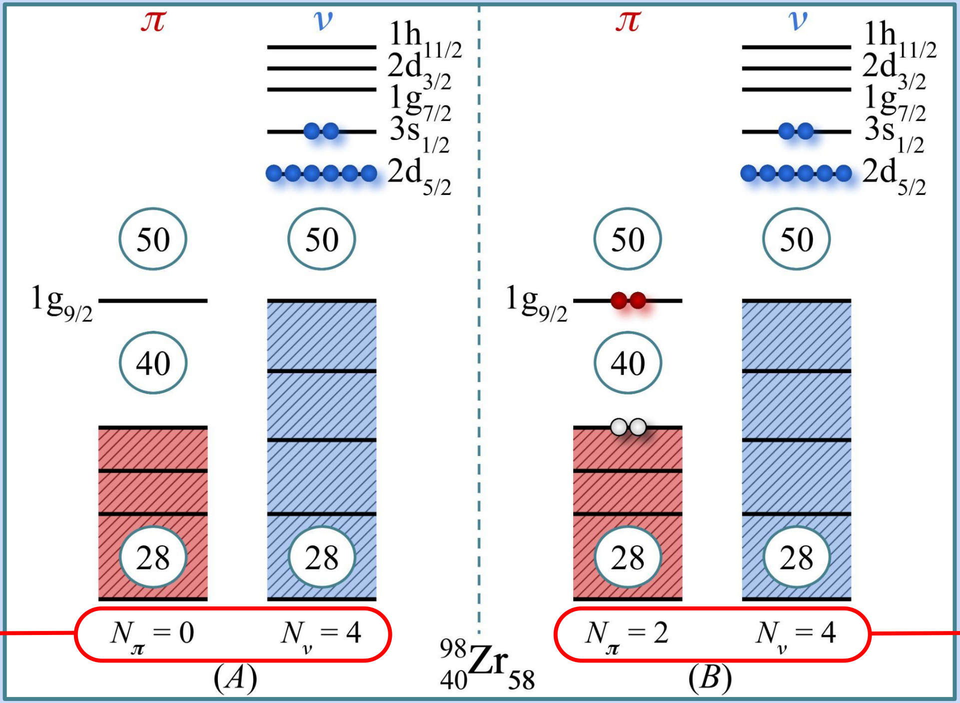
Boson counting: ^{98}Zr

Boson number: $\hat{N} = \hat{n}_s + \hat{n}_d = s^\dagger s + \sum_\mu d_\mu^\dagger d_\mu$ $N = N_\pi + N_\nu$

0p-0h protons

2p-2h protons

across Z=40



^{90}Zr core

Z=40 subshell closure

N=50

Configuration A:
Seniority-like neutron excitations.
P. Federman and S. Pittel,
Phys. Rev. C **20**, 820 (1979)

$N = N_\pi + N_\nu = 4$

$N = N_\pi + N_\nu = 6$

Introduction

IBM-1-CM Hamiltonian

Yale

$$H = H_A^{(N)} + H_B^{(N+2)} + W^{(N,N+2)}$$

Normal configuration (0p-0h)*:*

$$H_A = \varepsilon_d^{(A)} n_d + \kappa^{(A)} Q \cdot Q$$ [N] irrep.

Intruder configuration (2p-2h)*:*

$$H_B = \varepsilon_d^{(B)} n_d + \kappa^{(B)} Q \cdot Q + \kappa'^{(B)} L \cdot L + \Delta_{\mathrm{p}}$$ [N+2] irrep.

Coupling:

$$W^{(N,N+2)} = \omega \, [(d^\dagger d^\dagger)^{(0)} + (s^\dagger)^2] + h.c.$$ [N]⊕[N+2] irrep.

Introduction

IBM-1-CM Hamiltonian

$$H = H_A^{(N)} + H_B^{(N+2)} + W^{(N,N+2)}$$

Pairing

Normal configuration (0p-0h):

Quadrupole

$$H_A = \varepsilon_d^{(A)}\, n_d + \kappa^{(A)} Q \cdot Q$$ [*N*] irrep.

Angular momentum

Intruder configuration (2p-2h):

$$H_B = \varepsilon_d^{(B)}\, n_d + \kappa^{(B)} Q \cdot Q + \kappa'^{(B)} L \cdot L + \Delta_p$$ [*N*+2] irrep.

Off-set energy

Coupling:

$$W^{(N,N+2)} = \omega\, [(d^\dagger d^\dagger)^{(0)} + (s^\dagger)^2] + h.c.$$ [*N*]⊕[*N*+2] irrep.

Introduction

Wave function structure and decomposition

Yale

$$|\boldsymbol{\psi}; L\rangle = a\,|\boldsymbol{\psi}; [N], L\rangle + b\,|\boldsymbol{\psi}; [N{+}2], L\rangle\,; \;\; a^2 + b^2 = 1$$

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Wave function structure and decomposition

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$$|\boldsymbol{\psi}; [N], L\rangle = \sum_{n_d,\,\boldsymbol{\tau},\,n_\Delta} C^{(N,L)}_{n_d,\,\boldsymbol{\tau},\,n_\Delta}\,|N, n_d, \boldsymbol{\tau}, n_\Delta, L\rangle$$

U(5)

$$|\boldsymbol{\psi}; [N], L\rangle = \sum_{(\boldsymbol{\lambda},\boldsymbol{\mu}),K} C^{(N,L)}_{(\boldsymbol{\lambda},\boldsymbol{\mu}),K}\,|N, (\boldsymbol{\lambda},\boldsymbol{\mu}), K, L\rangle$$

SU(3)

$$|\boldsymbol{\psi}; [N], L\rangle = \sum_{\boldsymbol{\sigma},\,\boldsymbol{\tau},\,n_\Delta} C^{(N,L)}_{\boldsymbol{\sigma},\,\boldsymbol{\tau},\,n_\Delta}\,|N, \boldsymbol{\sigma}, \boldsymbol{\tau}, n_\Delta, L\rangle$$

SO(6)

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$$|\boldsymbol{\psi}; [N], L\rangle = \sum_{n_d,\boldsymbol{\tau},n_\Delta} C^{(N,L)}_{n_d,\boldsymbol{\tau},n_\Delta}\,|N, n_d, \boldsymbol{\tau}, n_\Delta, L\rangle \rightarrow P^{(N,L)}_{n_d} = \sum_{\boldsymbol{\tau},n_\Delta} [C^{(N,L)}_{n_d,\boldsymbol{\tau},n_\Delta}]^2$$

U(5) decomposition

$$|\boldsymbol{\psi}; [N], L\rangle = \sum_{(\boldsymbol{\lambda},\boldsymbol{\mu}),K} C^{(N,L)}_{(\boldsymbol{\lambda},\boldsymbol{\mu}),K}\,|N, (\boldsymbol{\lambda},\boldsymbol{\mu}), K, L\rangle \rightarrow P^{(N,L)}_{(\boldsymbol{\lambda},\boldsymbol{\mu})} = \sum_{K} [C^{(N,L)}_{(\boldsymbol{\lambda},\boldsymbol{\mu}),K}]^2$$

SU(3) decomposition

$$|\boldsymbol{\psi}; [N], L\rangle = \sum_{\boldsymbol{\sigma},\boldsymbol{\tau},n_\Delta} C^{(N,L)}_{\boldsymbol{\sigma},\boldsymbol{\tau},n_\Delta}\,|N, \boldsymbol{\sigma}, \boldsymbol{\tau}, n_\Delta, L\rangle \rightarrow P^{(N,L)}_{\boldsymbol{\sigma}} = \sum_{\boldsymbol{\tau},n_\Delta} [C^{(N,L)}_{\boldsymbol{\sigma},\boldsymbol{\tau},n_\Delta}]^2$$

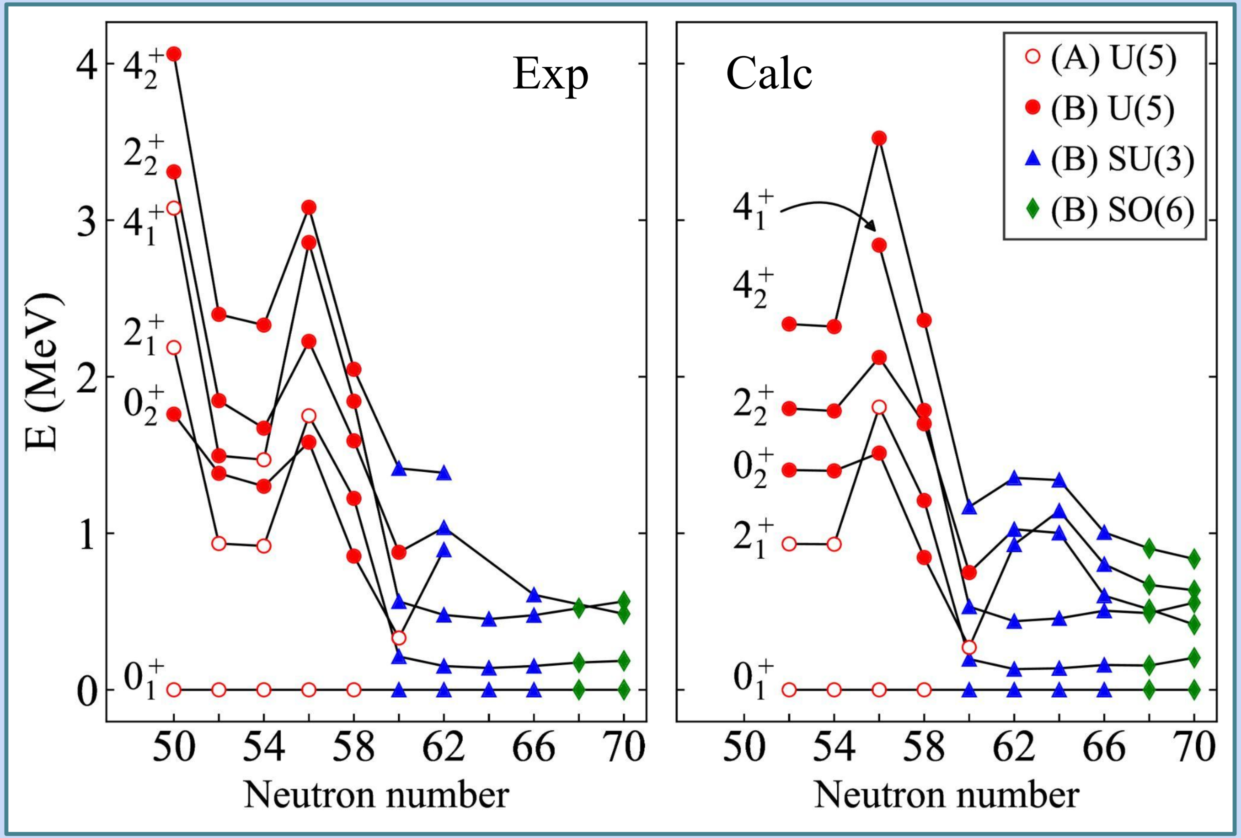
SO(6) decomposition

$$\rightarrow P^{(N,L)}_{\boldsymbol{\tau}} = \sum_{n_d,n_\Delta} [C^{(N,L)}_{n_d,\boldsymbol{\tau},n_\Delta}]^2$$

SO(5) decomposition

Results

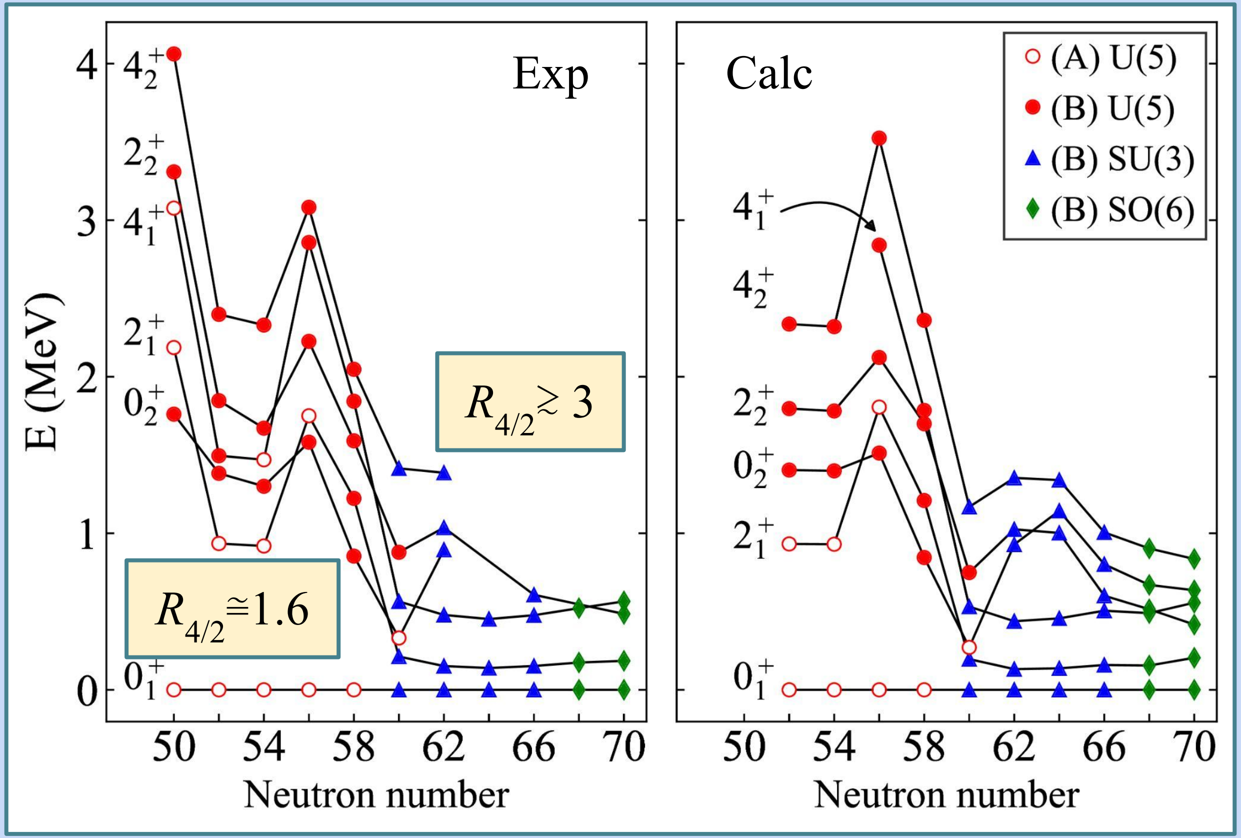
Energy levels



Results

Energy levels

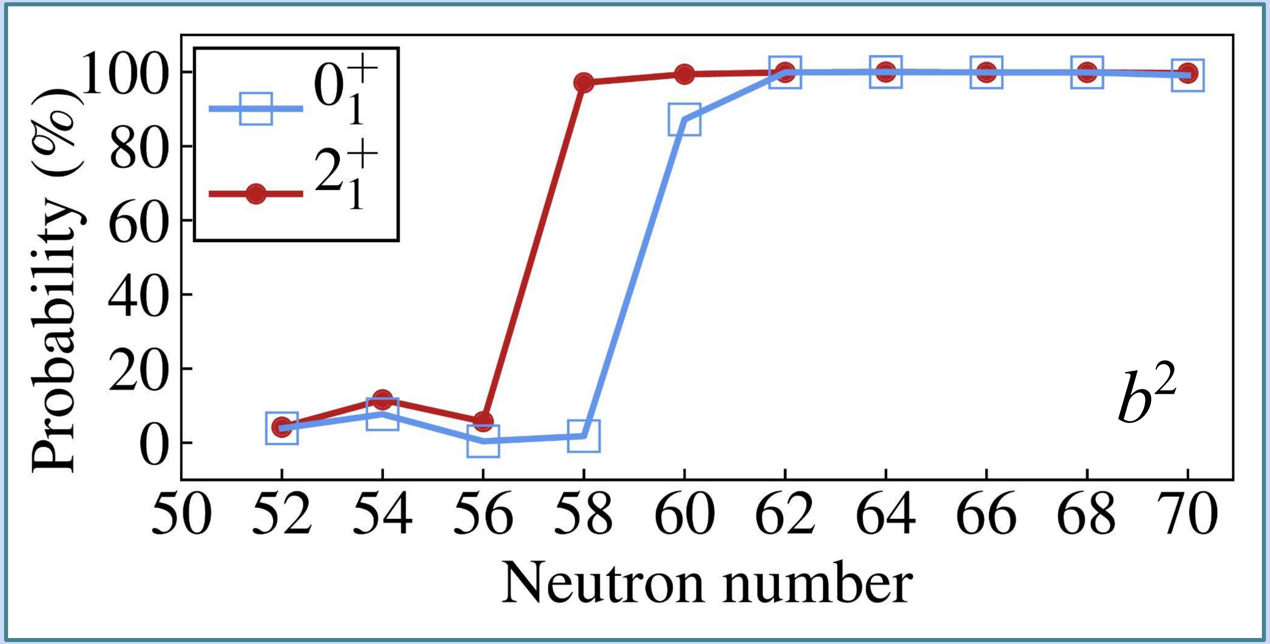
$R_{4/2} = E(4_1^+)/E(2_1^+)$



Results: Type II QPT

Evolution of configuration content: b^2

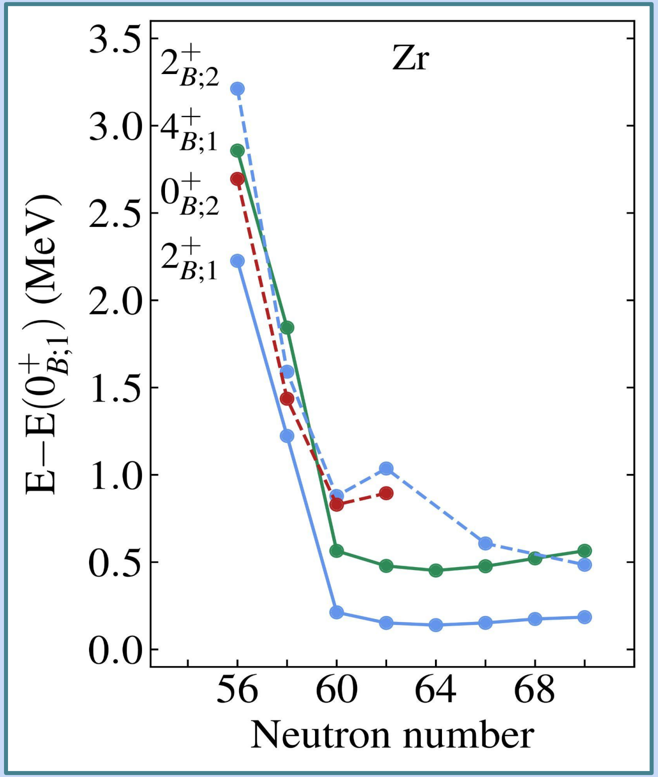
$$|\boldsymbol{\psi}; L\rangle = a|\boldsymbol{\psi}; [N], L\rangle + b\,|\boldsymbol{\psi}; [N+2], L\rangle$$

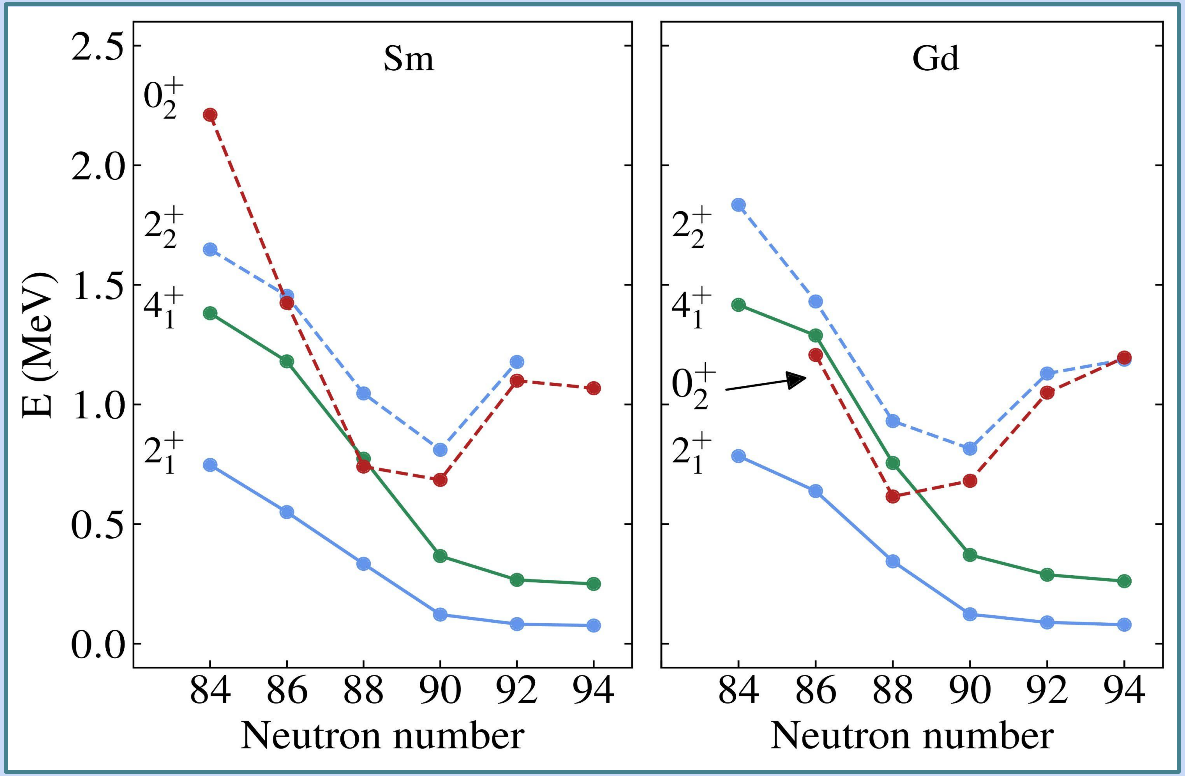


W. Witt *et al.*, Phys. Rev. C **98**, 041302(R) (2018)

Results

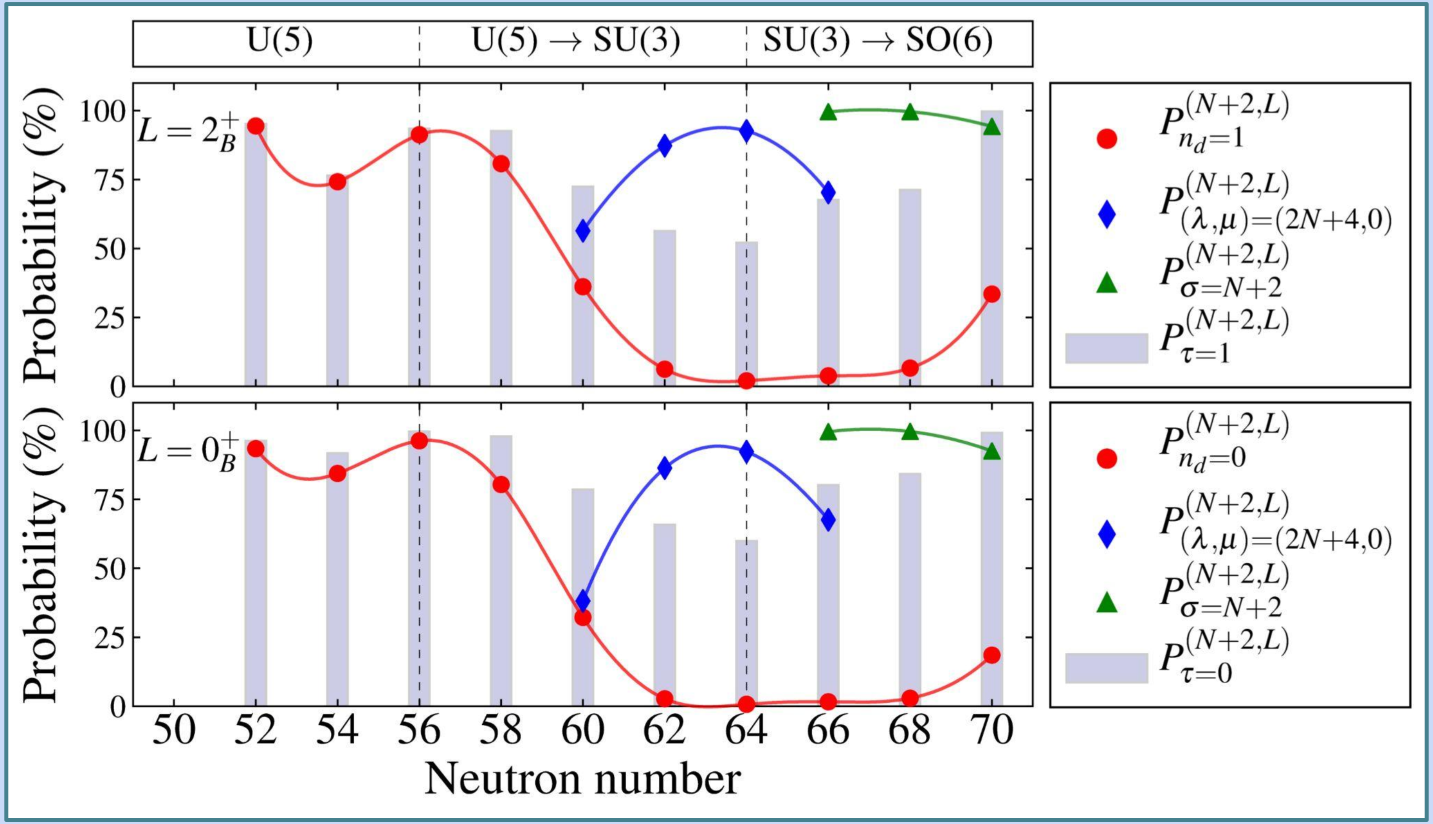
Comparison with Sm and Gd isotopes





Results: Type I QPT

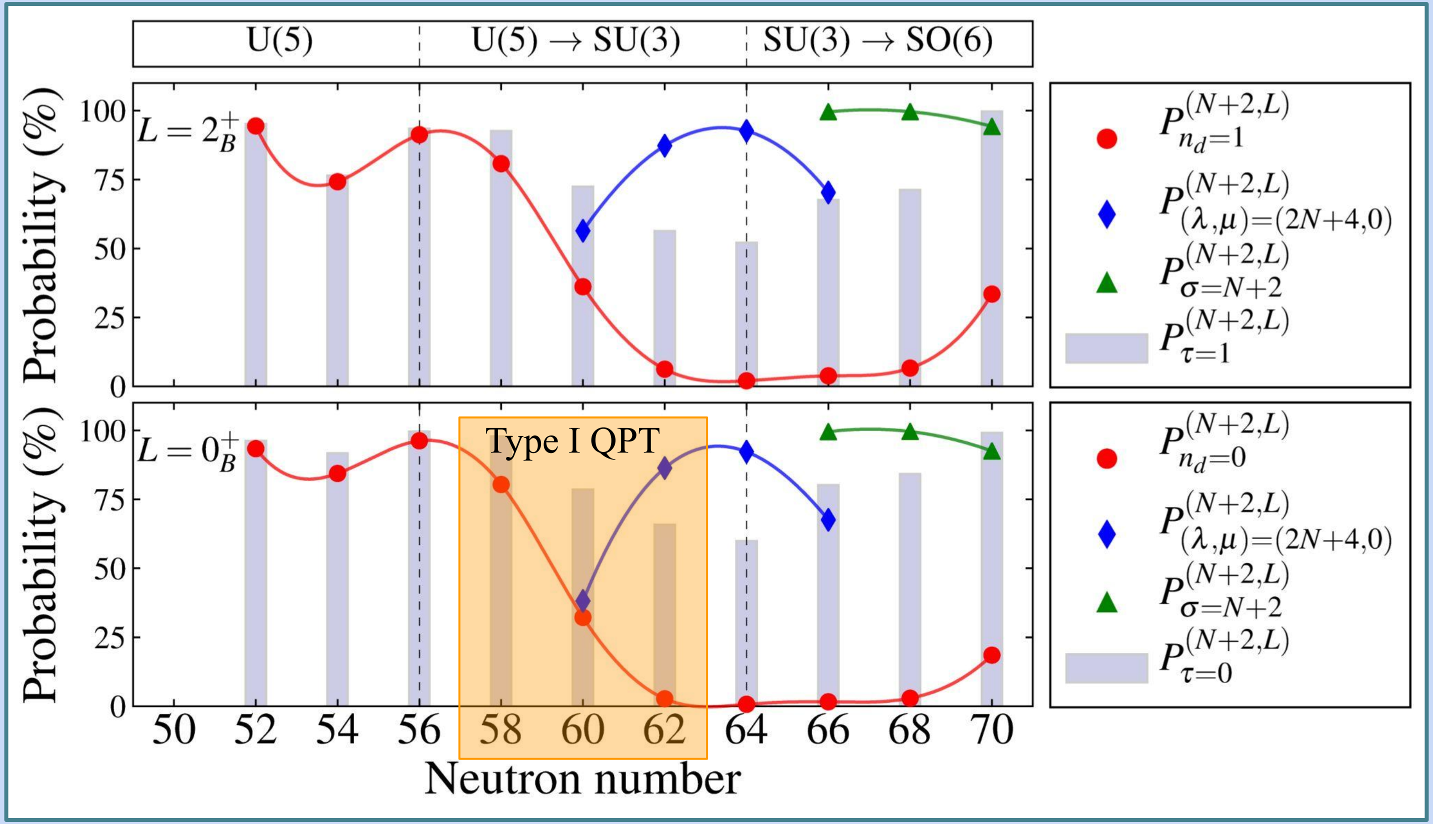
Evolution of symmetry content



Results: Type I QPT

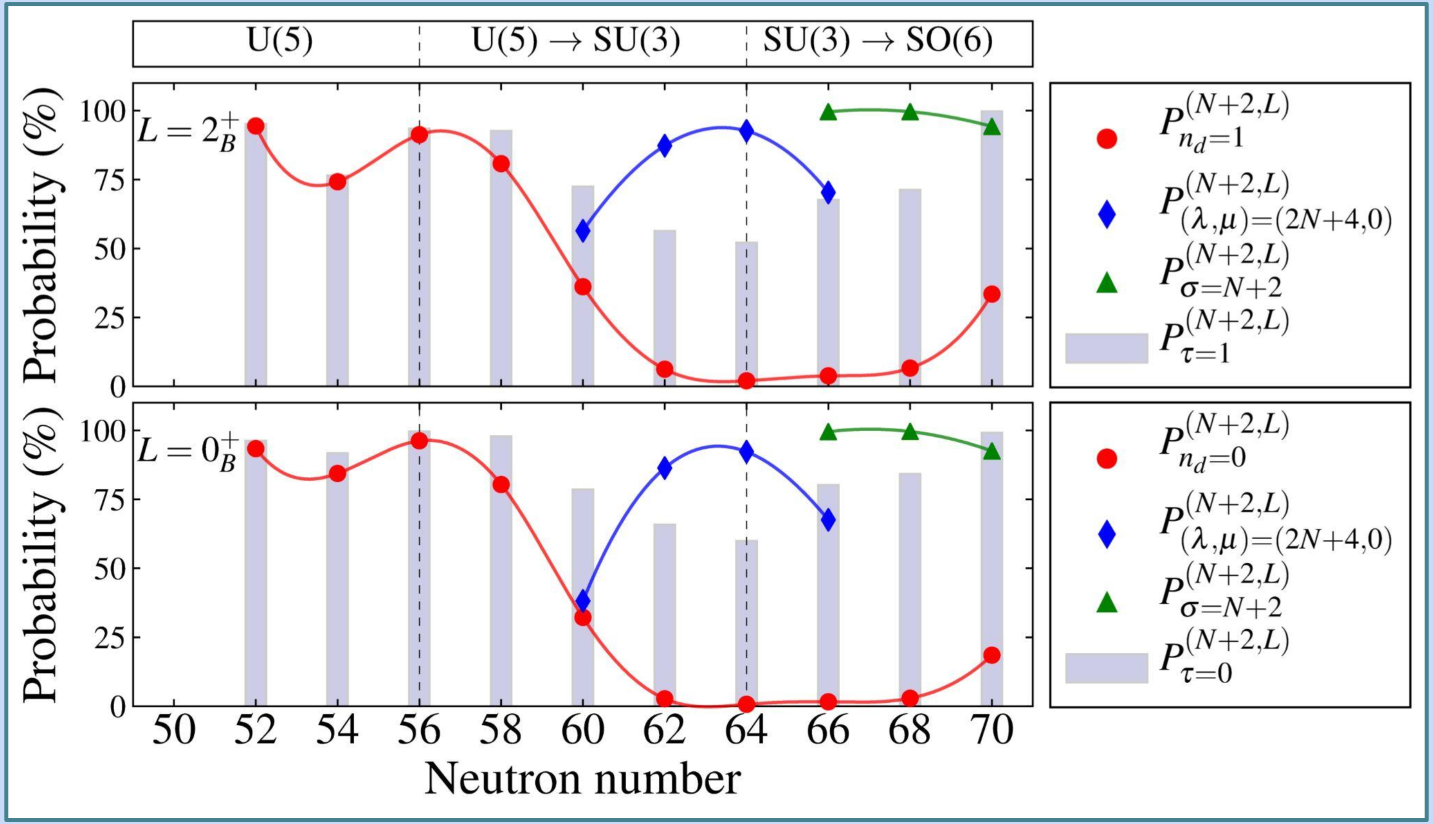
Evolution of symmetry content

Yale



Results: Type I QPT

Evolution of symmetry content

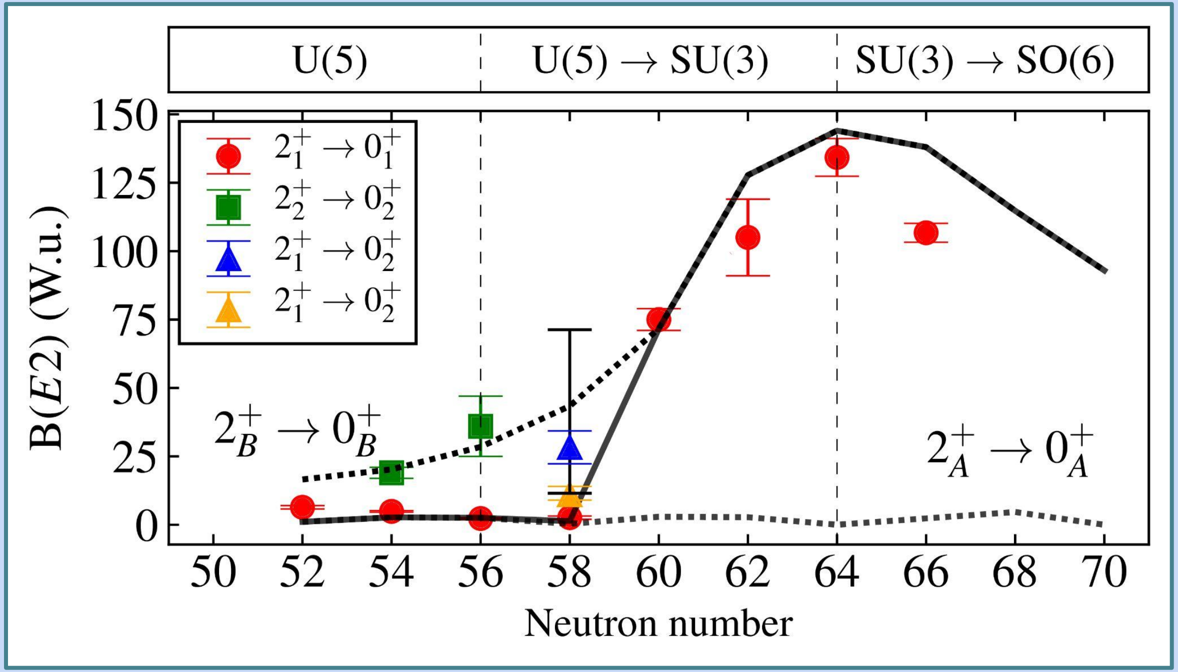


Results

E2 transitions

Yale

$$T(E2) = e^{(A)}Q_{\chi}^{(N)} + e^{(B)}Q_{\chi}^{(N+2)}$$



$e^{(A)}$=0.9 √W.u.

$e^{(B)}$=2.24 √W.u.

52-58: $2_A^+ \longrightarrow 0_A^+ = 2_1^+ \longrightarrow 0_1^+$

54-56: $2_B^+ \longrightarrow 0_B^+ = 2_2^+ \longrightarrow 0_2^+$

58: $2_B^+ \longrightarrow 0_B^+ = 2_1^+ \longrightarrow 0_2^+$

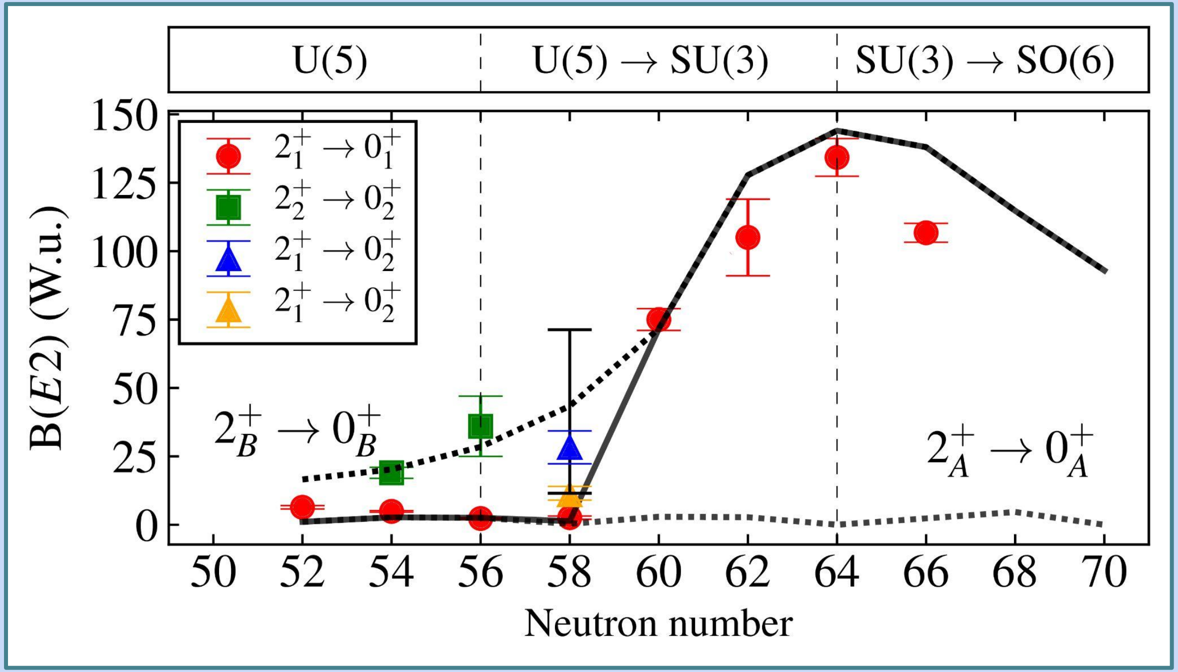
60-70: $2_B^+ \longrightarrow 0_B^+ = 2_1^+ \longrightarrow 0_1^+$

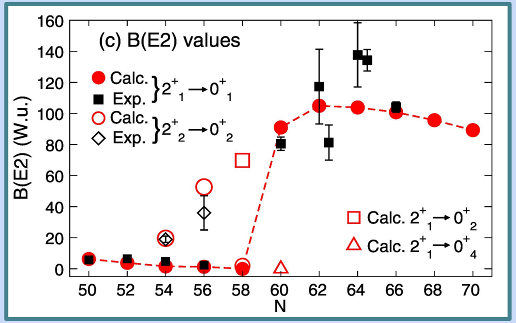
Results

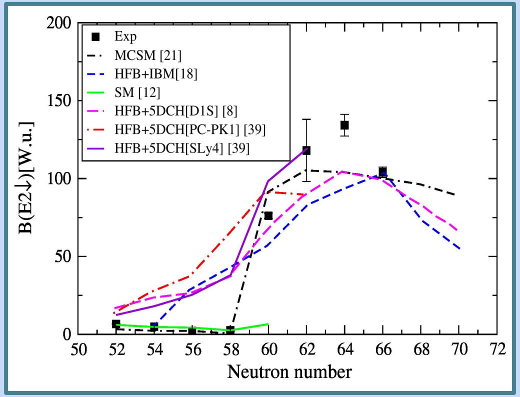
E2 transitions

Yale

T. Togashi *et al.*, Phys. Rev. Lett. **117**, 172502 (2016)







P. Singh *et al.*, Phys. Rev. Lett. **121**, 192501 (2018)

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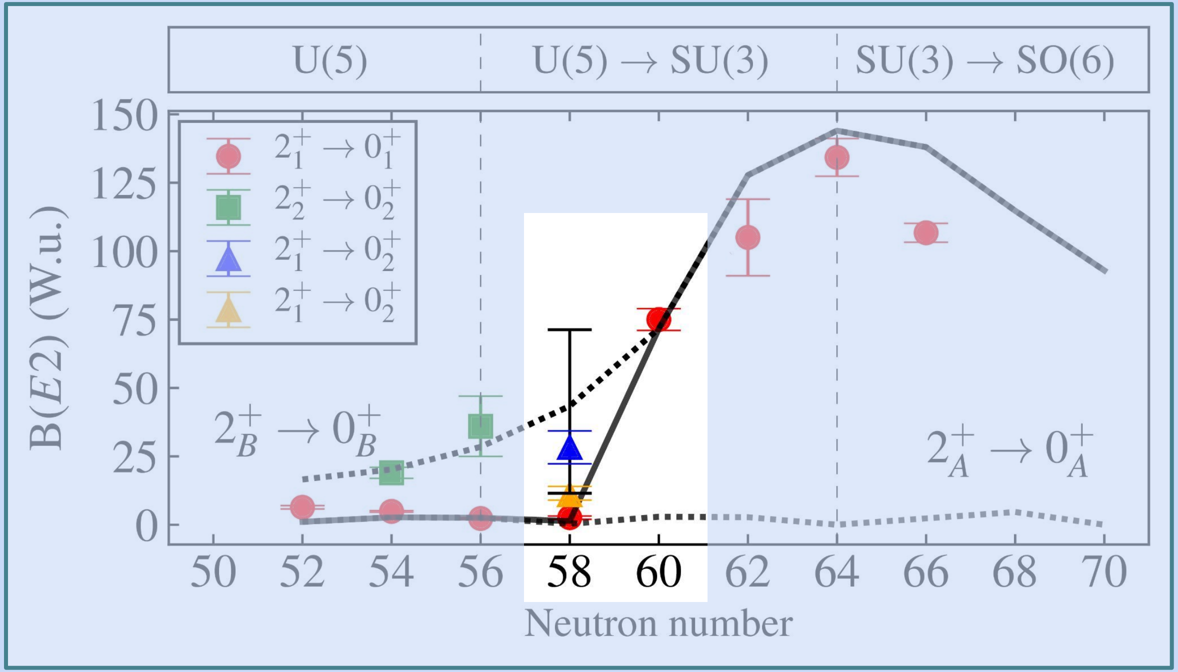

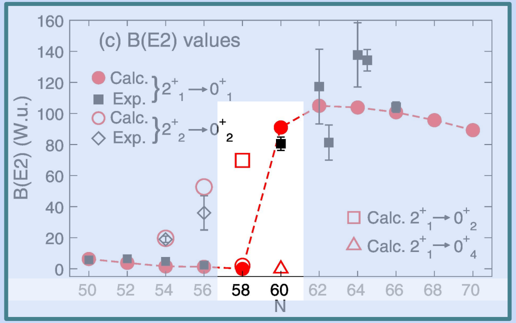

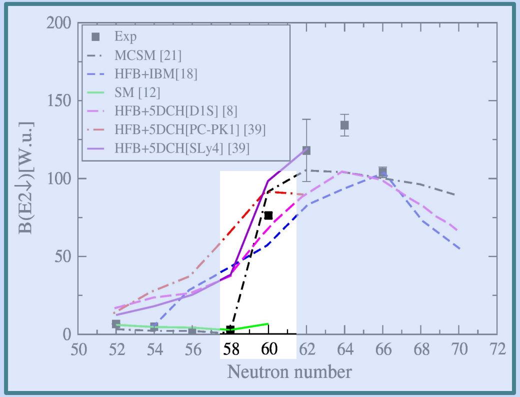

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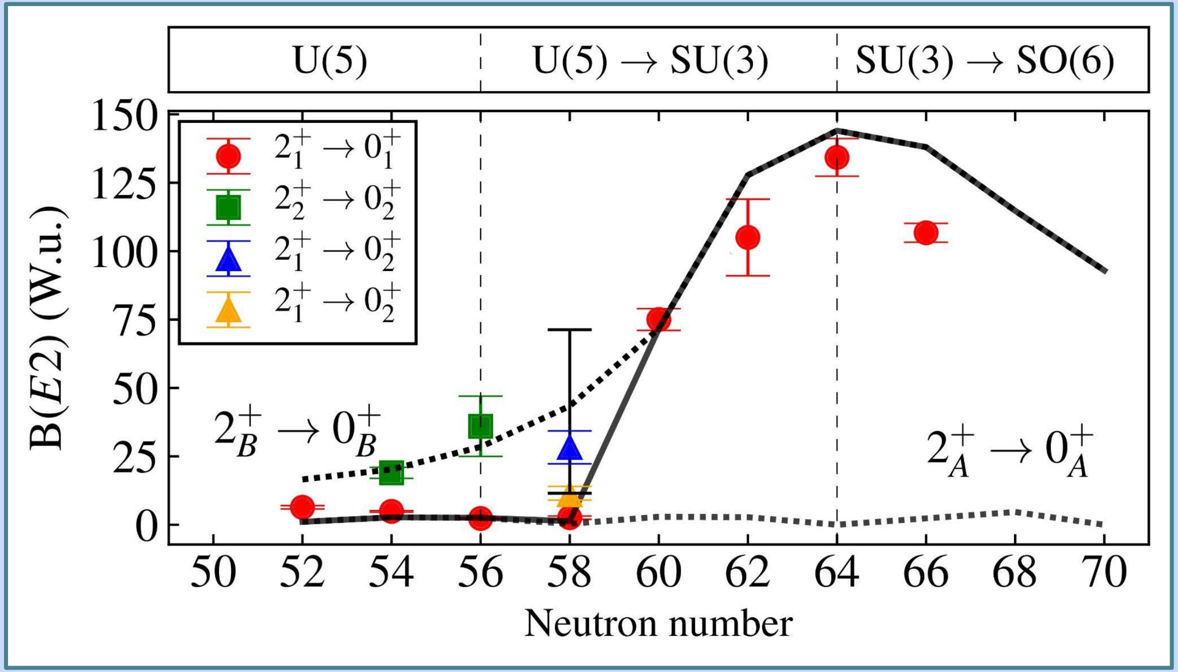
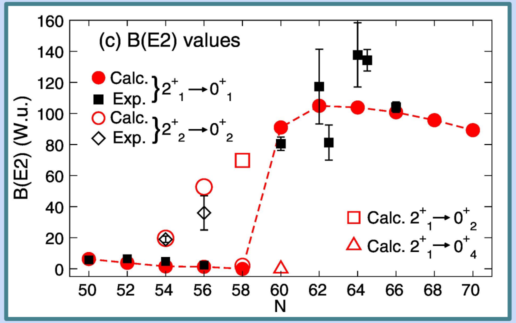
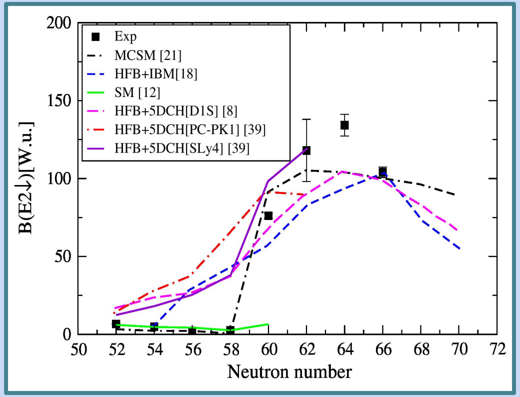
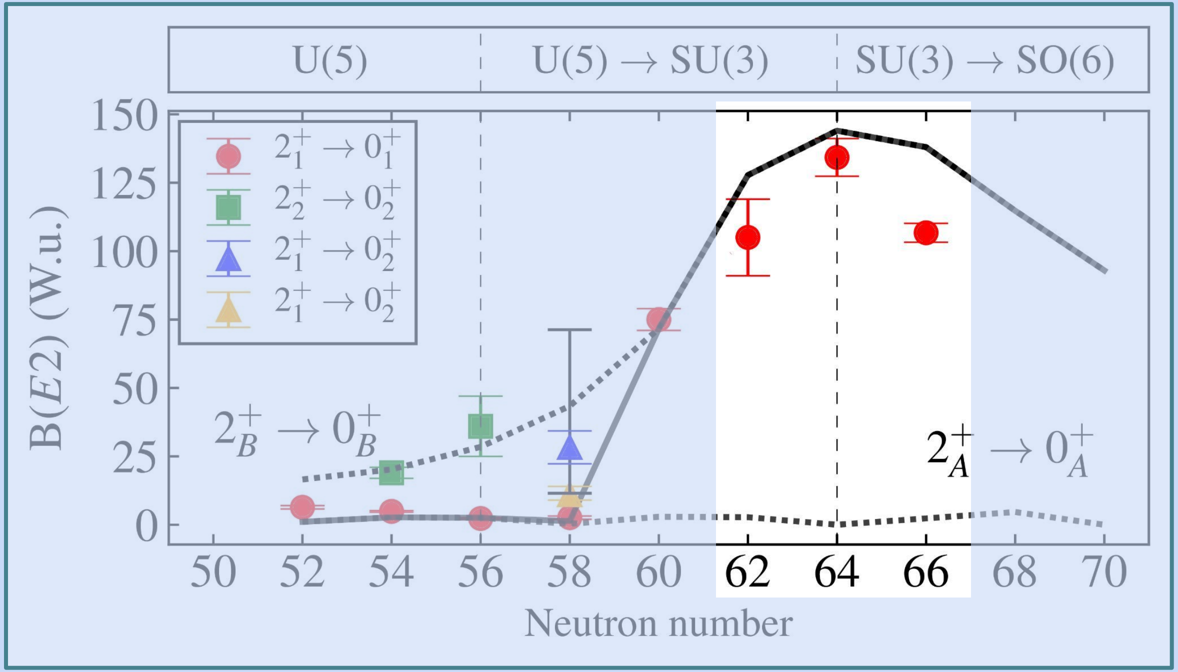






P. Singh *et al.*, Phys. Rev. Lett. **121**, 192501 (2018)

Results
E2 transitions



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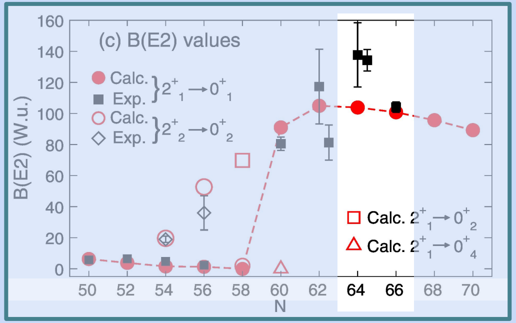

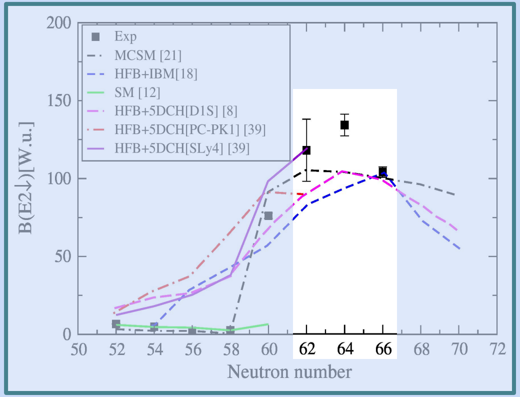

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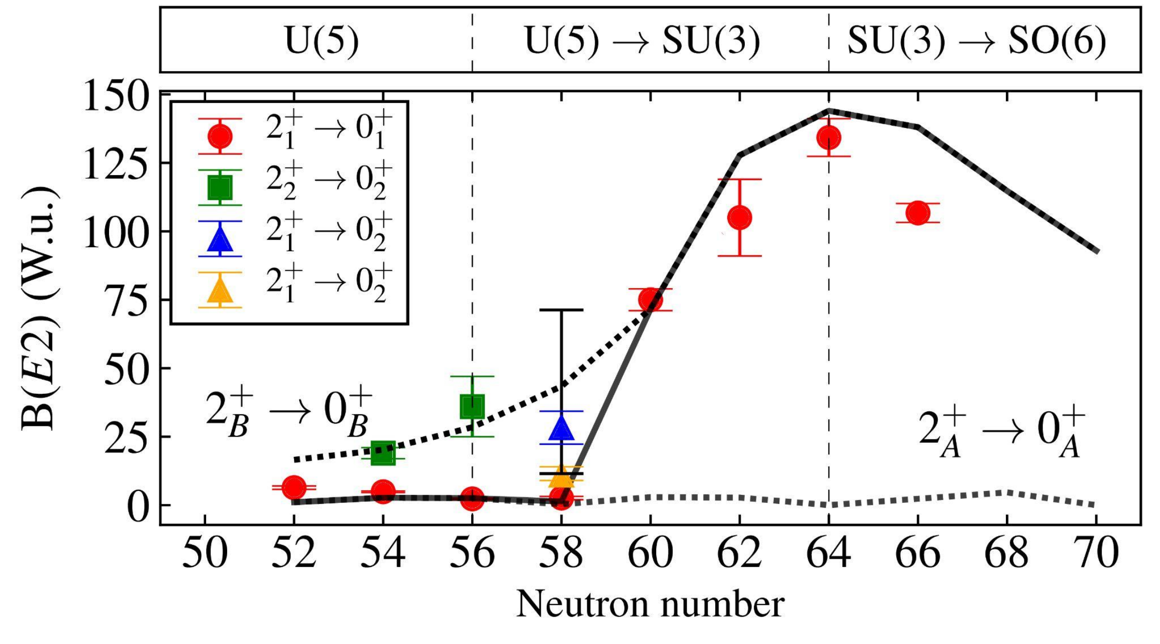
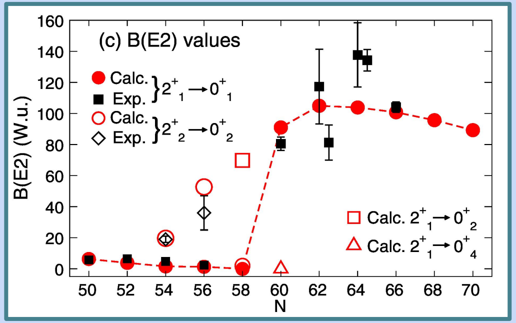
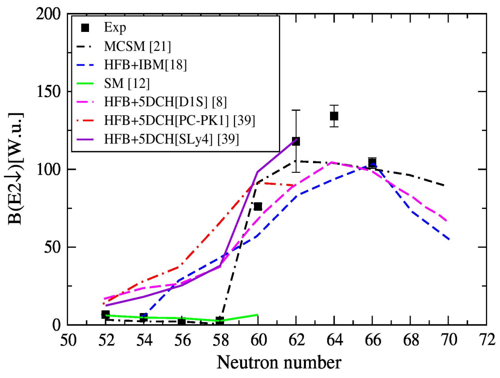




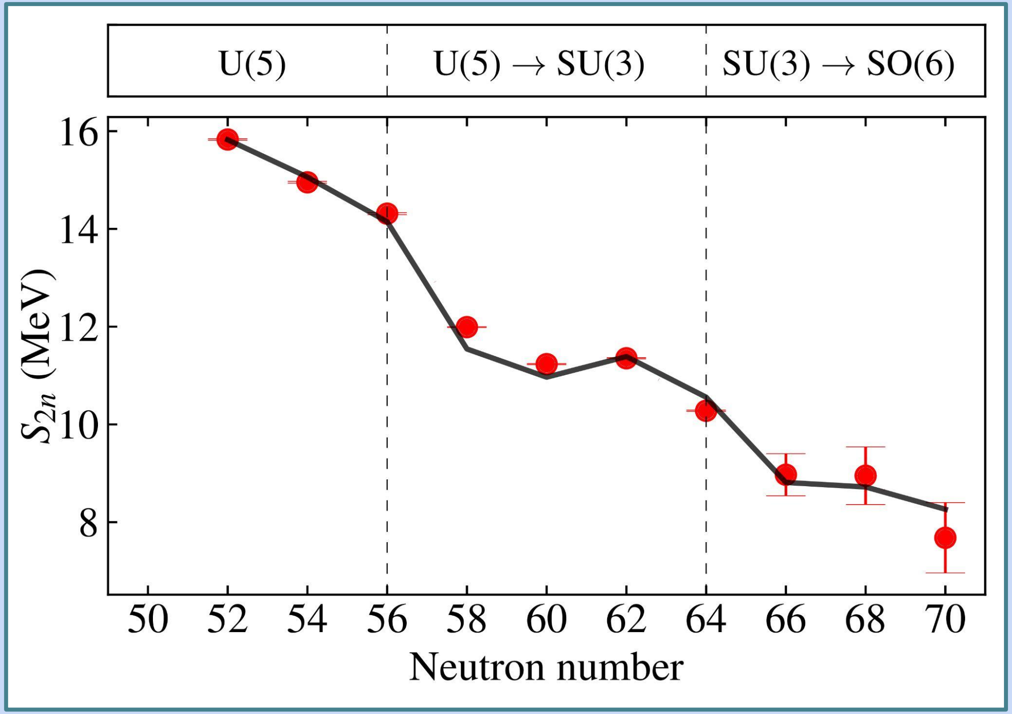


P. Singh *et al.*, Phys. Rev. Lett. **121**, 192501 (2018)

Results

Two neutron separation energy

$$S_{2n} = -\tilde{A} - \tilde{B}N_\nu \pm S_{2n}^{\text{def}} - \Delta_n$$



$\tilde{A} = -16.5$ MeV
$\tilde{B} = -0.758$ MeV

$S_{2n}^{\text{def}} = \langle H \rangle_{GS}$
+ for particles
− for holes

$$\Delta_n = \begin{cases} 0 \text{ MeV } 50\text{-}56 \\ 2 \text{ MeV } 58\text{-}70 \end{cases}$$

Conclusions

Yale

- Quantum analysis of the evolution of energy levels and other observables (two-neutron separation energies, *E2* and *E0* transition rates, isotope shifts and magnetic moments).
- Calculated change in the configuration content and symmetry-content of wave functions.
- Classical analysis based on coherent states, examined individual shapes and their evolution with neutron number.

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All point toward the occurrence of ***IQPTs****:*

- Type II QPT between two configurations (normal and intruder) at N=60.
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- Crossover [SU(3)-SO(6)] of the intruder B configuration for N=66-70.

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Other **IQPTs**?

Thank you