A Multiple Scattering model for the MuonE experiment

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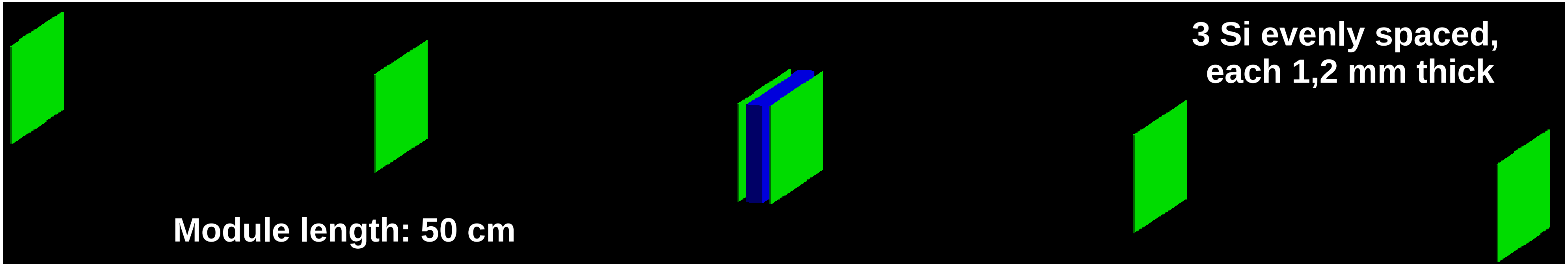
The main goal is to build the θ_R distribution

$$\frac{dN}{d\theta_R}(\theta_R) = \int \frac{dN}{d\theta_T}(\theta_T)\ g(\theta_R; \delta\theta, \theta_T) d\theta_T$$

$$\delta\theta = \sqrt{\delta\theta_X^2 + \delta\theta_Y^2}$$

- θ_T = true electron's angle
- θ_R = reconstructed electron's angle
- $\delta\theta_X$ = smearing in the plane XZ
- $\delta\theta_Y$ = smearing in the plane YZ

The function *g* models the smearing due to Multiple Scattering (MS) and tracking resolution effects. It depends on the smearing in the two planes XZ and YZ, and its shape depends on electron's energy, which is function of $\theta_{T.}$



In order to simulate the intrinsic tracking resolution, a gaussian smearing has been applied to the single hits, with σ_{CMS} = 26 µm:

$$\vec{x_i} \rightarrow \vec{x_i} + Gaus(0, \sigma_{CMS})$$

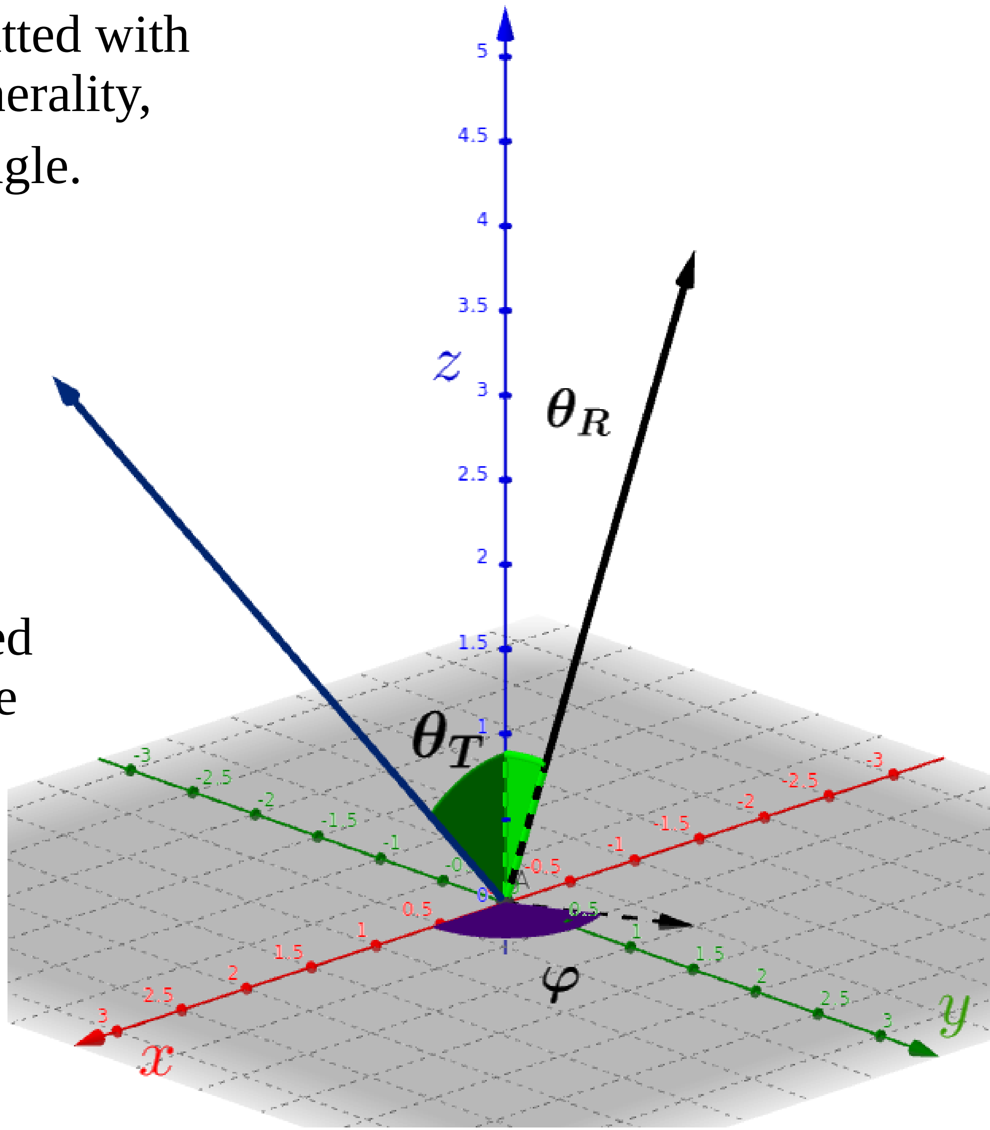
Angles definition and projections

- Let's assume that the scattered electron is emitted with $\varphi_T = 0$. This can be done without loss of generality, thanks to the isotropy of the azimuthal angle.

$$\theta_{XT} = \theta_T$$
$$\theta_{YT} = 0$$

- After MS effects electron's direction is defined by the angles θ_R and φ. It is related also to the total smearing in the two projections.

$$\theta_{XR} \approx \theta_R \cos\varphi = \delta\theta_X + \theta_T$$
$$\theta_{YR} \approx \theta_R \sin\varphi = \delta\theta_Y$$



Angles definition and projections

$\theta_{XT} = \theta_T$ $\theta_{YT} = 0$ $\theta_{XR} \approx \theta_R \cos\varphi = \delta\theta_X + \theta_T$ $\theta_{YR} \approx \theta_R \sin\varphi = \delta\theta_Y$

This means that θ_R is related to θ_T, $\delta\theta_X$ and $\delta\theta_Y$ by the relation

$$\theta_R = \sqrt{(\delta\theta_X + \theta_T)^2 + \delta\theta_Y^2}$$

Total smearing in the two projections are given by the sum of muon and electron contributions:

$$\delta\theta_X = \delta\theta_{\mu X} + \delta\theta_{eX}$$

$$\delta\theta_Y = \delta\theta_{\mu Y} + \delta\theta_{eY}$$

In order to model the distribution of $\delta\theta_X$ and $\delta\theta_Y$ we will start to analyze separately the two contributions

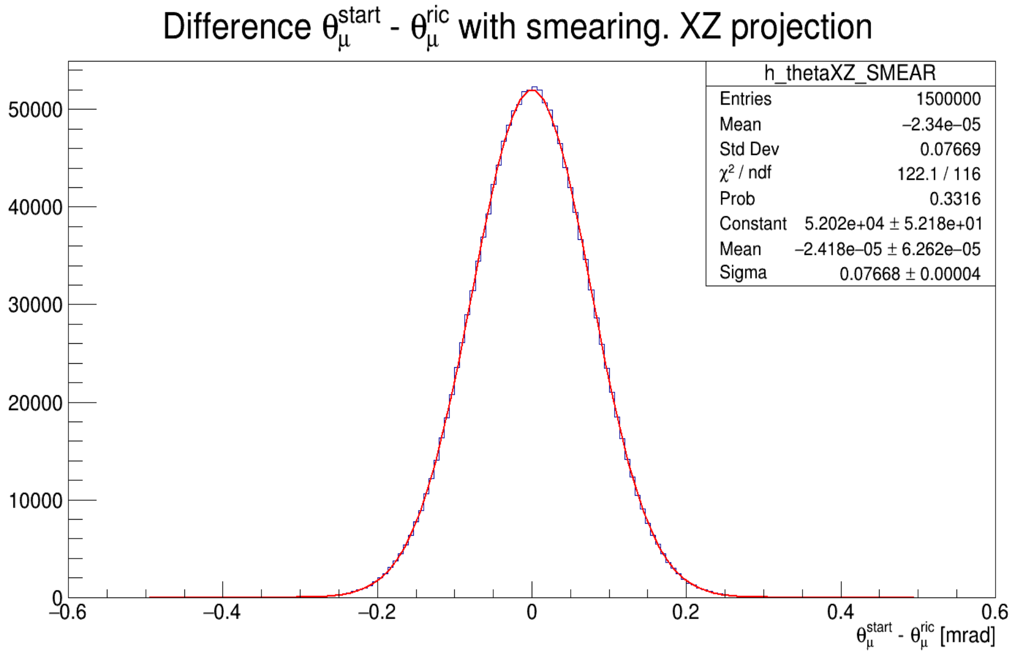
Determination of $f_\mu(\delta\theta_{\mu X})$

Muons with E = 150 GeV ± 3% generated along the the z axis, passing through the upstream detector

Intrinsic resolution of the tracker is the most relevant effect

$$\delta\theta_{\mu X,Y} = \theta^{R}_{\mu\ X,Y}$$

$$f_\mu(\delta\theta_{\mu X}) = \frac{1}{\sqrt{2\pi}\sigma_\mu} e^{-\frac{(\delta\theta_{\mu X}-\mu_\mu)^2}{2\sigma_\mu^2}}$$



Determination of $f_e(\delta\theta_{eX})$

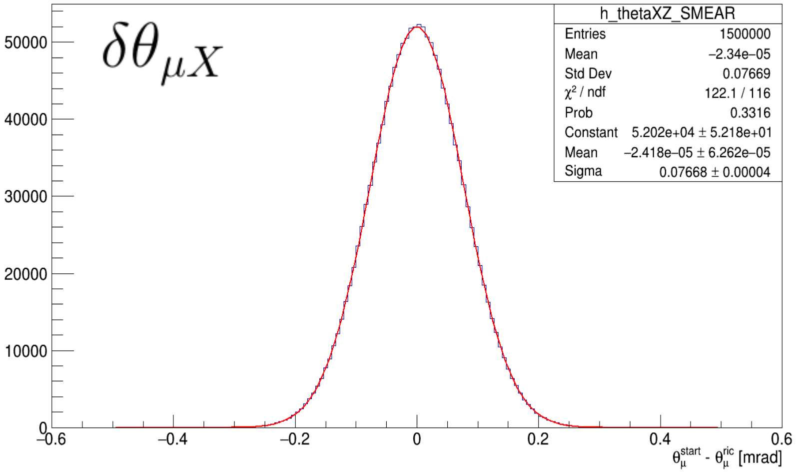
Generation of electrons at fixed energy/angle in the center of Be target, passing through the downstream detector

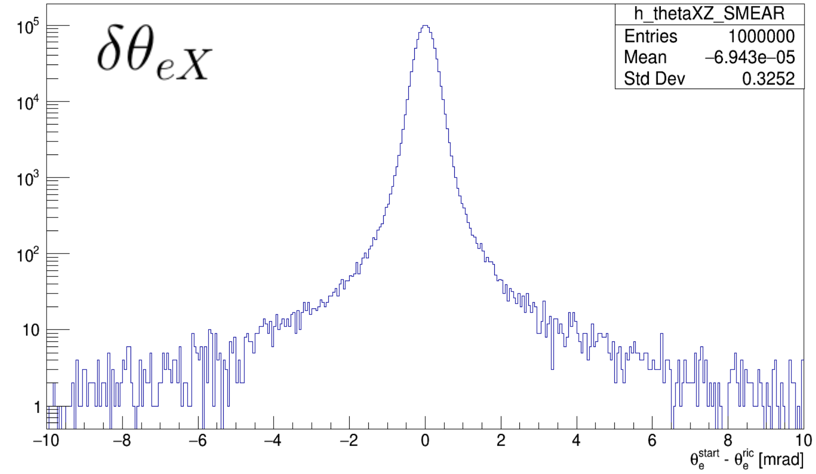
$$\delta\theta_{eX,Y} = \theta_e^T{}_{X,Y} - \theta_e^R{}_{X,Y}$$

Several samples generated, with E_e ϵ [1, 135] GeV, in order to obtain a parameterization of MS af a function of energy

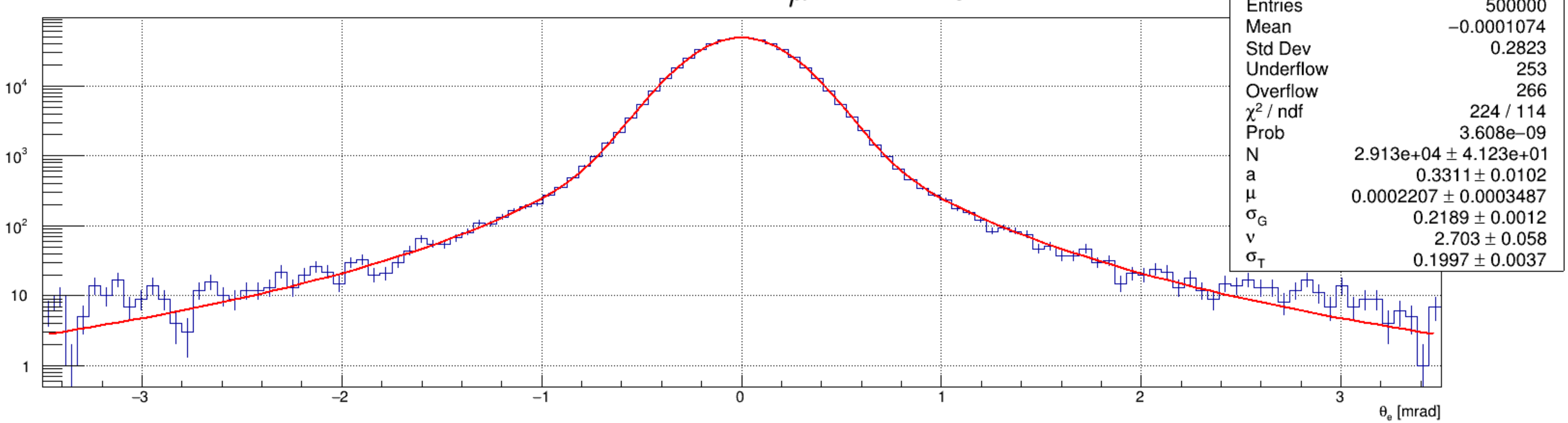
$$f_e(\delta\theta_{eX}, \vec{p}) = N\left[(1-a)\frac{1}{\sqrt{2\pi}\sigma_G}e^{-\frac{(\delta\theta_{eX}-\mu)^2}{2\sigma_G^2}} + a\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\sigma_T\Gamma(\frac{\nu}{2})}\left(1+\frac{(\delta\theta_{eX}-\mu)^2}{\nu\sigma_T^2}\right)^{-\frac{\nu+1}{2}}\right]$$

In this way it is possible to determine the distribution of the total smearing in the planes XZ and YZ





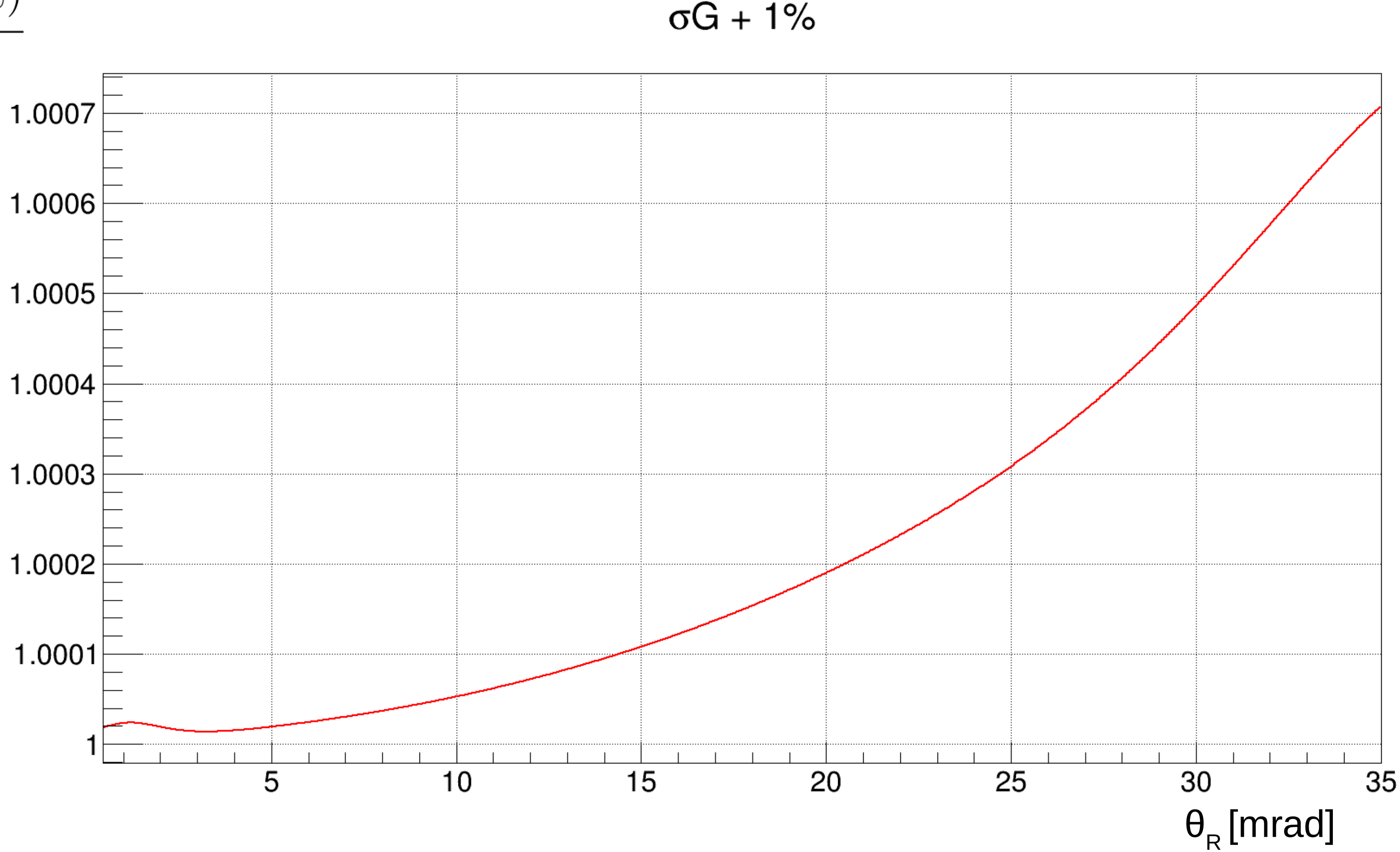
$$\delta\theta_X = \delta\theta_{\mu X} + \delta\theta_{eX}$$



Assuming $\delta\theta_Y = 0$, $\theta_R = \delta\theta_X + \theta_T$

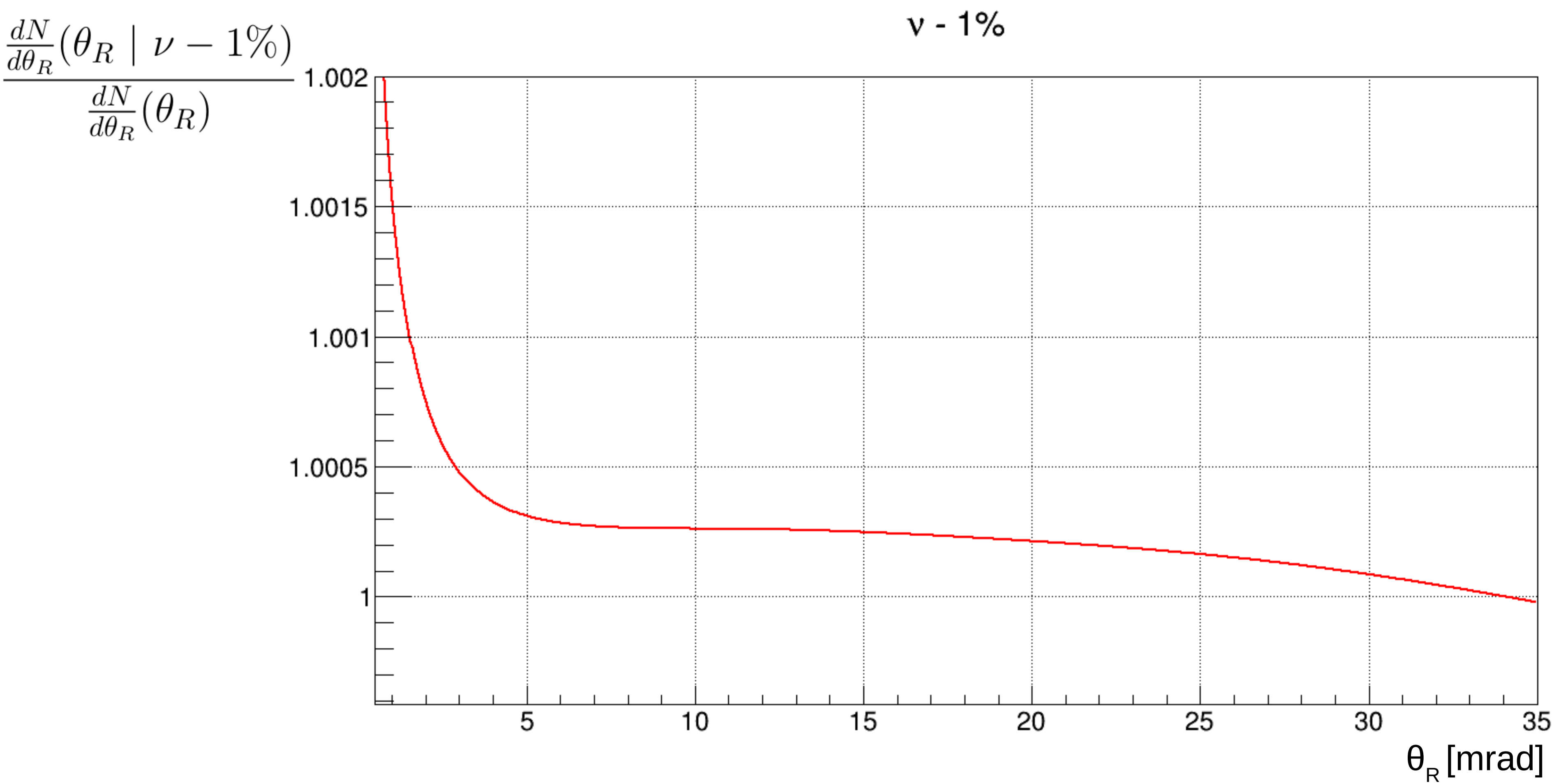
What happens in this case to the distribution of θ_R, if I increase the value of σ_G by 1%?

$$\frac{\frac{dN}{d\theta_R}(\theta_R \mid \sigma_G + 1\%)}{\frac{dN}{d\theta_R}(\theta_R)}$$



Assuming $\delta\theta_Y = 0$, $\theta_R = \delta\theta_X + \theta_T$

What happens in this case to the distribution of θ_R, if I increase the effect of the tails by 1%?

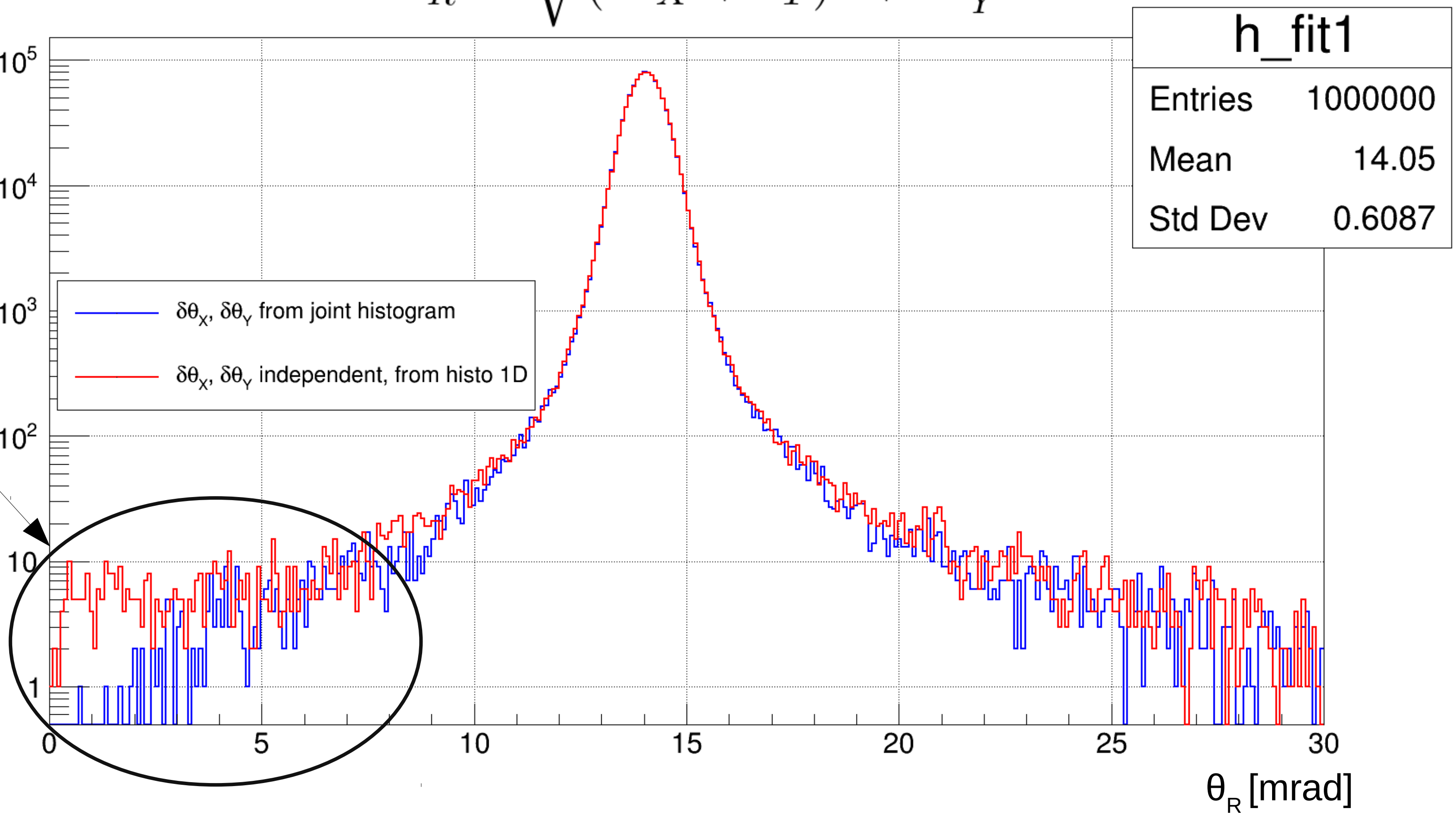


The two projections $\delta\theta_X$ and $\delta\theta_Y$ have the same distribution, but

are they independent?

$$\theta_R = \sqrt{(\delta\theta_X + \theta_T)^2 + \delta\theta_Y^2}$$

Assuming independency brings to overestimate the effect of the tails



A parameterization of the joint distribution ($\delta\theta_X$, $\delta\theta_Y$) is necessary to construct properly the θ_R distribution

Assuming a rotational simmetry, it is convenient move to polar coordinates:

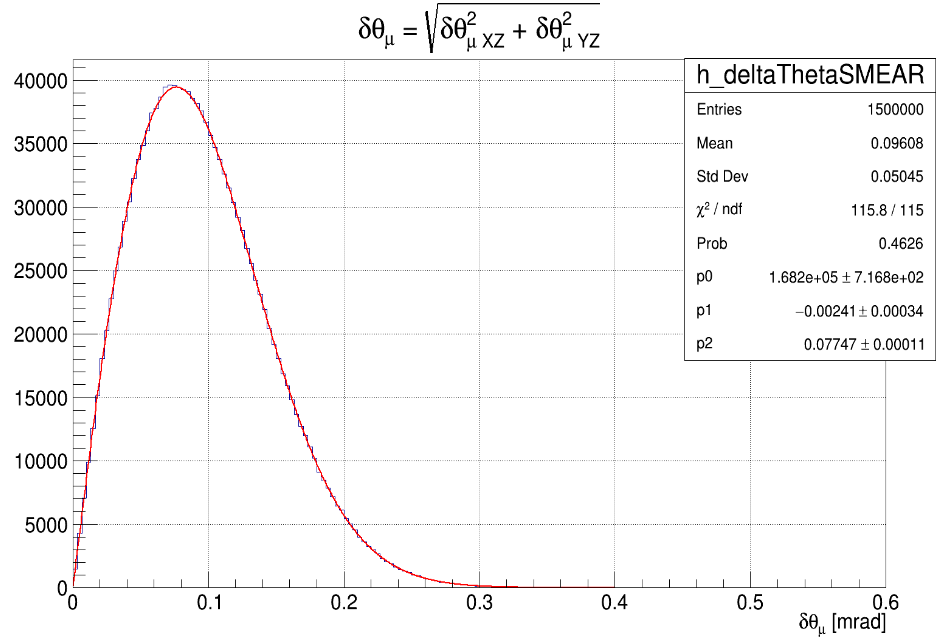
$(\delta\theta_X, \delta\theta_Y) \rightarrow (\delta\theta, \alpha)$

$$\delta\theta_X = \delta\theta\cos\alpha$$
$$\delta\theta_Y = \delta\theta\sin\alpha$$

- α has a uniform distribution
- $\delta\theta$ is distributed according to $p(\delta\theta) \propto \delta\theta f(\delta\theta_X)|_{\delta\theta_X=\delta\theta}$

For the muons:

$$f_\mu(\delta\theta_\mu) \propto \delta\theta_\mu \frac{1}{\sqrt{2\pi}\sigma_\mu} e^{-\frac{(\delta\theta_\mu-\mu_\mu)^2}{2\sigma_\mu^2}}$$



For the electrons:

A second gaussian has been added to $\delta\theta_X$ pdf, to fit the long tail of the distribution

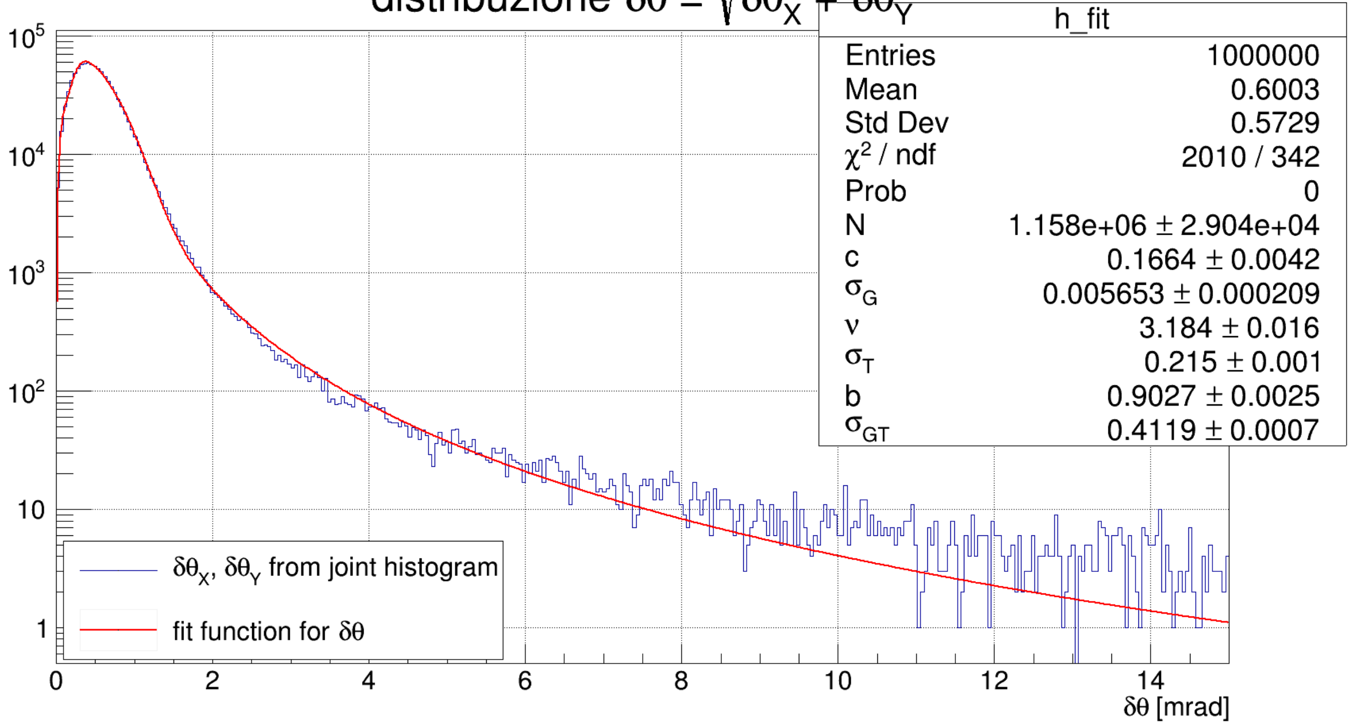
St = student distribution

$$f_e(\delta\theta_e;\ \vec{p}) = N\delta\theta_e\{c\,[St(\delta\theta_e,\nu,\sigma_T) - Gaus(\delta\theta_e,\sigma_G)] + b\;Gaus(\delta\theta_e,\sigma_G) + (1-b)Gaus(\delta\theta_e,\sigma_{GT})\}$$

Fit the distribution $\delta\theta_e$ can be difficult, so it is better to determine the parameters of the pdf using the convolution

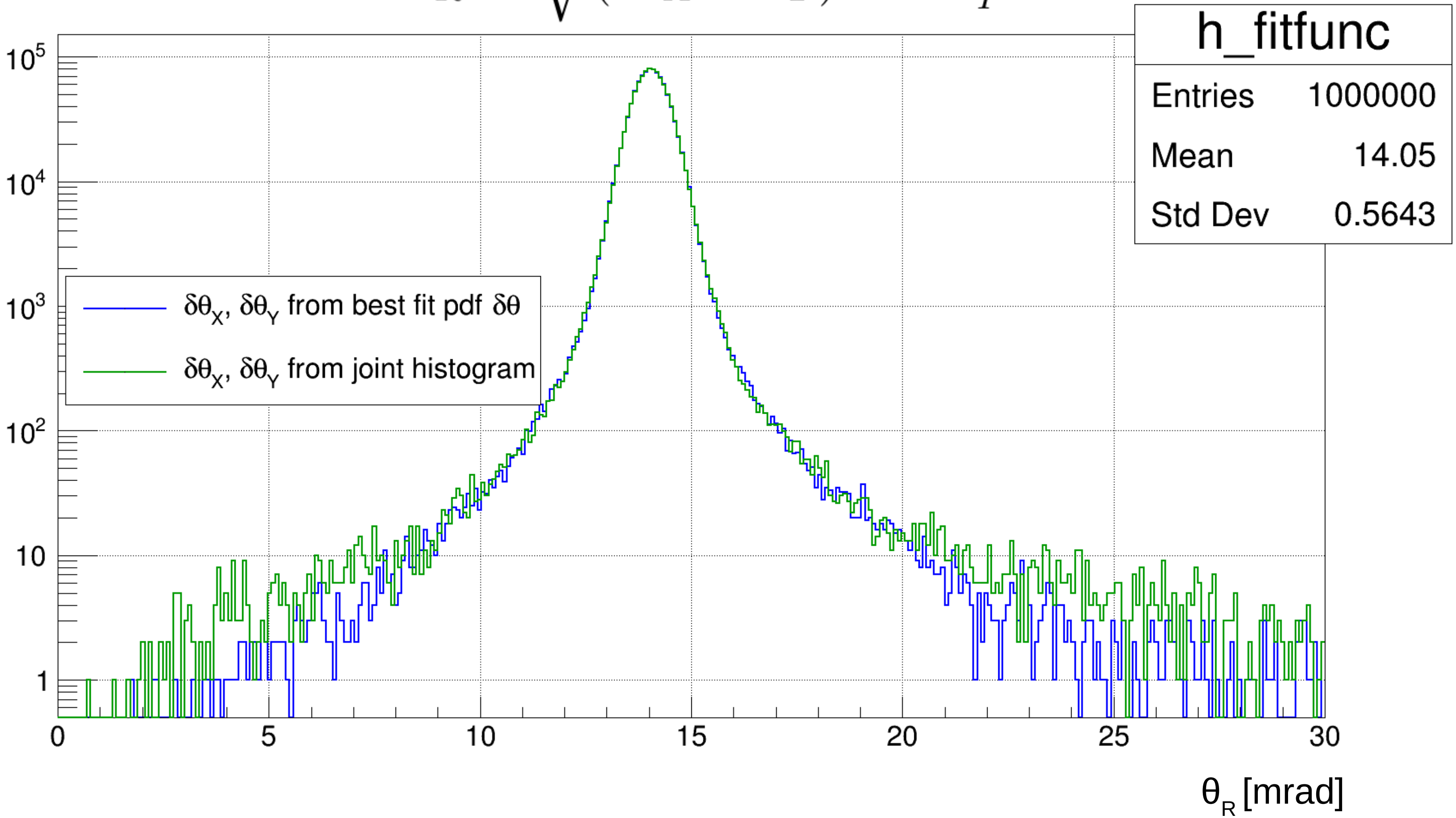
$$p(\delta\theta;\ \vec{p}) = \int^{\delta\theta} f_\mu(\delta\theta_\mu) f_e(\delta\theta - \delta\theta_\mu;\ \vec{p})\ d\delta\theta_\mu$$

distribuzione $\delta\theta = \sqrt{\delta\theta_X^2 + \delta\theta_Y^2}$



Even if the fit is not precise, because of the high correlation between the parameters, in this way the distribution of θ_R is reproduced with more accuracy

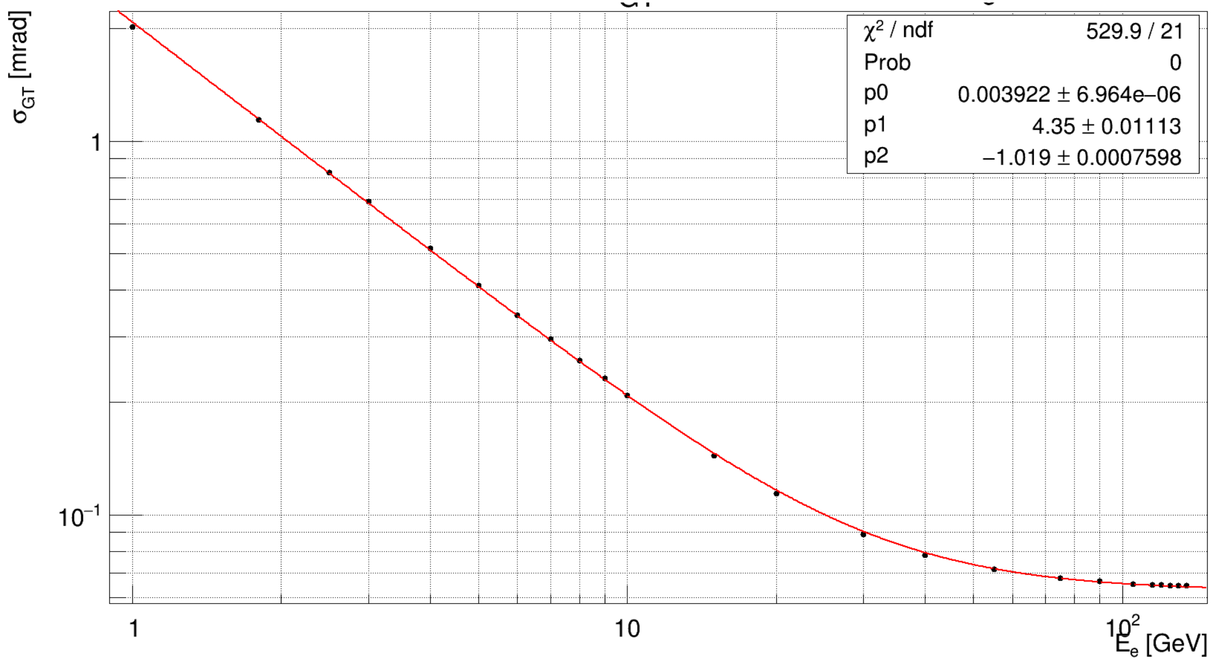
$$\theta_R = \sqrt{(\delta\theta_X + \theta_T)^2 + \delta\theta_Y^2}$$



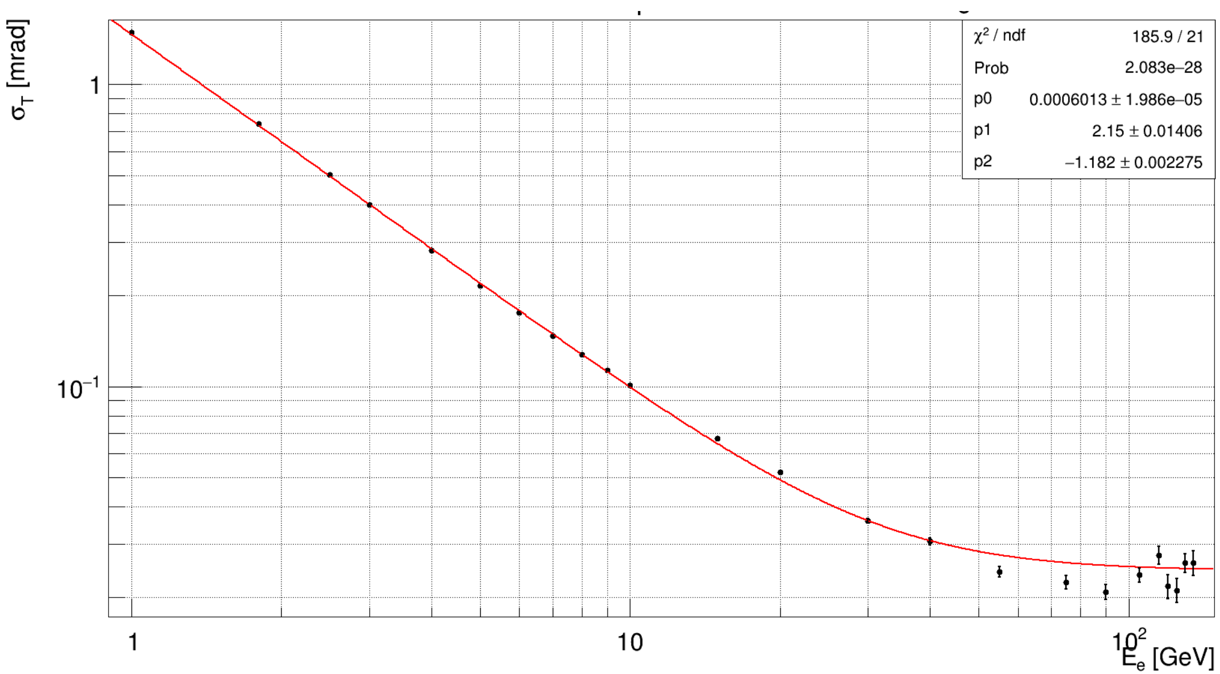
In order to obtain a complete parameterization of MS effects, it is necessary determine how the parameters that define the shape of $f_e(\delta\theta)$ evolve with electron's energy.

Since the fit procedure is not very precise at the moment, some parameters show a well defined behaviour

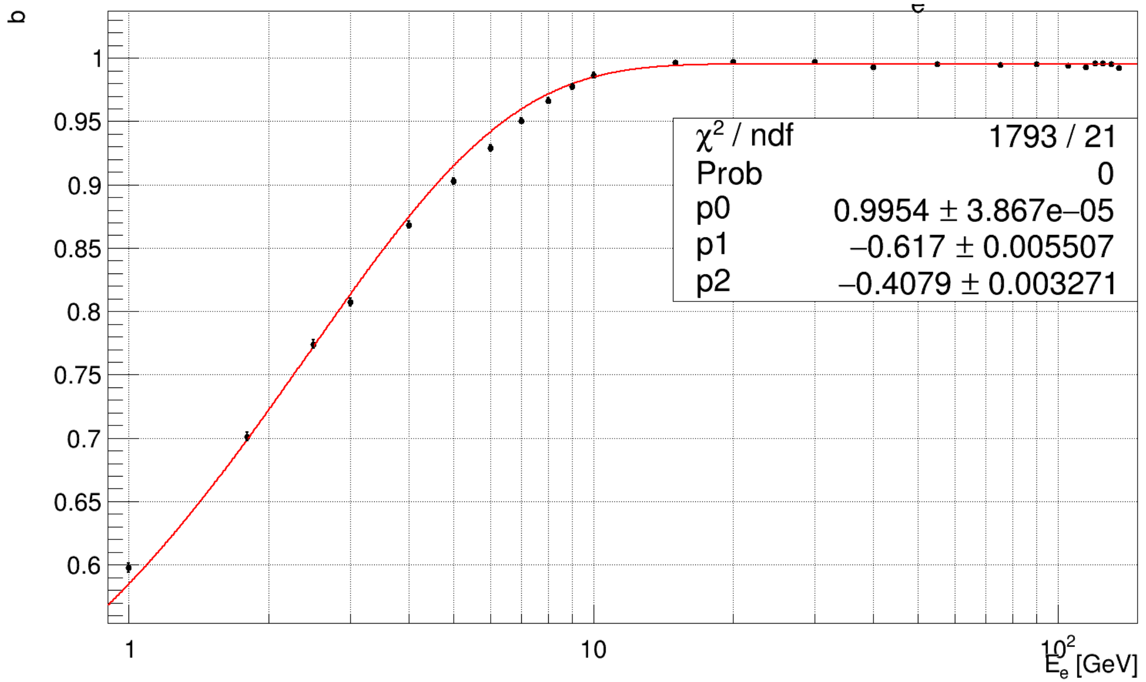
$$\sigma_{GT}(E_e) = \sqrt{p_0 + p_1 E_e^{2 \cdot p_2}}$$



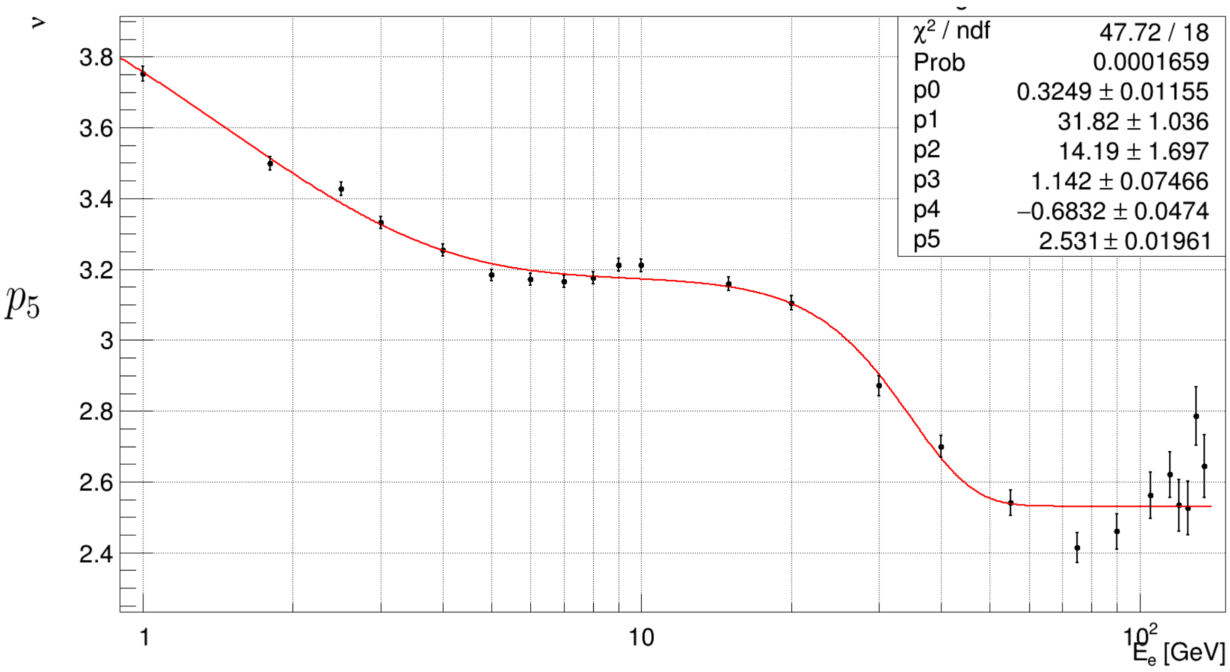
$$\sigma_{T}(E_e) = \sqrt{p_0 + p_1 E_e^{2 \cdot p_2}}$$



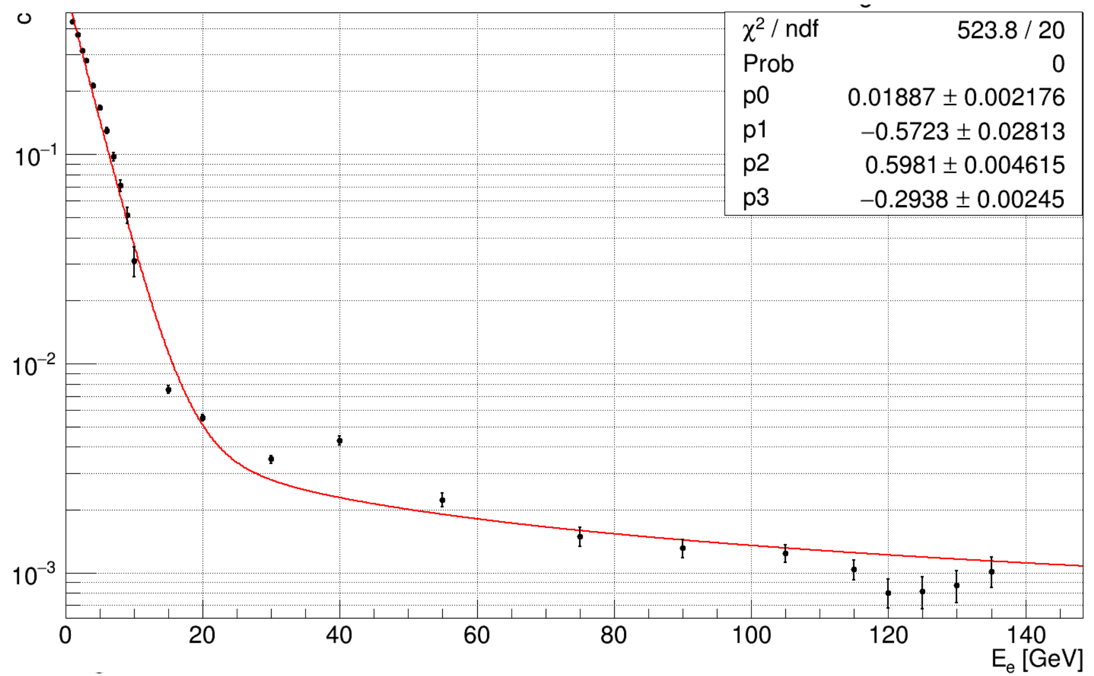
$$b(E_e) = p_0 + p_1\ e^{p_2 \cdot E_e}$$

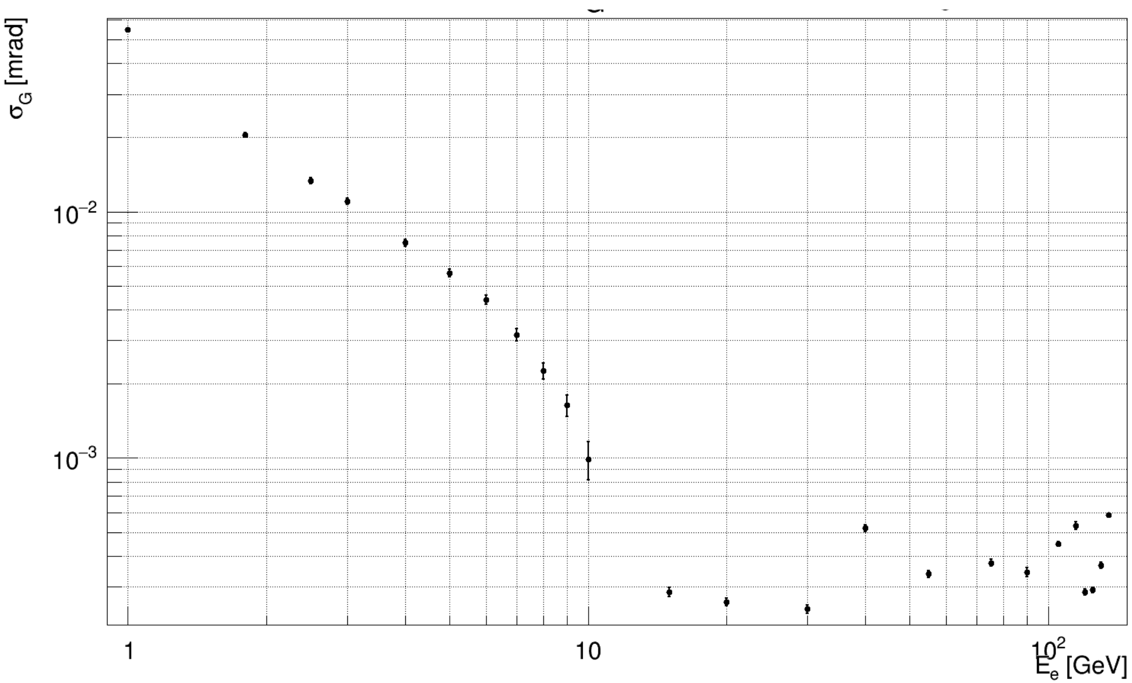


$$\nu(E_e) = p_0\ erfc\left(\frac{E_e - p_1}{p_2}\right) + p_3 e^{p_4 \cdot E_e} + p_5$$



$$c(E_e) = p_0 E^{p_1} + p_1\ e^{p_2 \cdot E_e}$$





Conclusions

- The work done on the Multiple Scattering shows that the approximation of independency between $\delta\theta_X$ and $\delta\theta_Y$ is not sufficient to determine properly the distribution of θ_R, because of a different behaviour in the signal region ($\theta_R \sim 0$ rad).

- Further studies will be exploited to determine the systematic effect of a miscalibration on the width and on the tails of the MS distribution. The procedure is very complex because of the multiple integrals involved. Studies are ongoing on this topic.