Rare Kaon Decays and ϵ_K: Theory Prediction

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The Rare Kaon decays $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0\ell^+\ell^-$ and ϵ_K play an important role in the phenomenology of the Standard Model and its extensions. They are very sensitive to high energy scales and can be predicted with remarkable precision. In this talk, I give a summary of the theory prediction and present the results of our recent calculations: the NLO electroweak corrections to the top-quark contribution to the $K \to \pi\nu\bar{\nu}$ decays and the NNLO QCD corrections to the charm-top-quark contribution to ϵ_K.

1. INTRODUCTION

The aim of particle physics is the discovery of the laws that determine the interactions of the fundamental particles. It is the common belief that the Standard Model of particle physics (SM) is merely a low energy limit of a more complete theory. Kaon physics can provide information about the expected new degrees of freedom. In addition to the direct searches performed at high-energy accelerators like the LHC, which may give us information about physics beyond the SM by producing new particles directly, there is the strategy of indirect searches: precise measurements of suitable observables can set stringent constraints on the new interactions or reveal deviations from SM predictions in the experimental data. These observables should fulfill the following criteria to make them suitable for indirect searches: they should have a sizeable contribution from short-distance scales, making them sensitive to high-energy physics, and a precise theoretical prediction should be possible.

A particularly useful set of observables which fulfill these criteria are rare Kaon decays and the parameter ϵ_K, which describes indirect CP violation in the neutral Kaon system. They proceed via the quark-level flavour-changing neutral current (FCNC) $s-d$ transition, which is forbidden in the SM at tree level and induced by higher-order weak interactions.

These Kaon decays differ from corresponding B meson decays in their CKM structure. In the SM, the top- and charm-quark contribution to the $s-d$ transition is proportional to λ^5 and λ, respectively (where $\lambda = |V_{us}| \approx 0.2$), whereas for the $b-d$ transition both contributions are proportional to λ^3, and for the $b-s$ transition to λ^2. Because of the strong parametric top-quark suppression in the SM, Kaon decays can set very stringent bounds in particular on models of New Physics which deviate from the CKM pattern of the SM, i.e. beyond minimal flavour violation.

Theoretical predictions for the rare decays and ϵ_K are computed in the framework of effective field theories, which allows for a systematic separation of short-distance (SD) and long-distance (LD) scales. The information about SD scales is contained in the Wilson coefficients of the weak effective Hamiltonian; they can in principle be calculated to any desired precision within perturbation theory. The contributions of LD scales are contained in the hadronic matrix elements of the effective Hamiltonian, and are characterised by low-energy, non-perturbative QCD. The exceptional importance of the observables considered here lies in the high precision with which these matrix elements have been determined recently, thus making a reliable and accurate theory prediction possible.

The above-mentioned accurate theory prediction goes hand in hand with a strong effort to provide very precise measurements of the observables, as reflected by the talks of G. Lim, G. Ruggiero, R. Tschirhart, and others, in this conference. Whereas ϵ_K has been known experimentally with high accuracy for a long time, the rare decays are much harder to measure because of their difficult signature and branching ratios of the order of 10^{-11}. Nevertheless, there are ongoing dedicated efforts to reduce the experimental uncertainty on the branching ratios, in particular of the neutrino modes, by the planned experiments NA62 at CERN, KOTO at J-PARC, and the proposed experiment P996 at FERMILAB.

In the following we review the status of the SM predictions in more detail.

2. RARE KAON DECAYS

The exceptional theoretical cleanness of the neutrino modes $K \to \pi\nu\bar{\nu}$ originates from the quadratic Glashow-Iliopoulos-Maiani (GIM) mechanism which suppresses the up-quark contribution by a factor of $\Lambda^2_{\rm QCD}/M_W^2$, leading to the dominance of one single dimension-six operator $Q_\nu = (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ mediating the $s-d$ FCNC transition. Because it corresponds to a semileptonic interaction, the hadronic matrix elements can be extracted with high precision from $K_{\ell 3}$ decays, using isospin symmetry.

The GIM suppression is only logarithmic for the $K_L \to \pi^0\ell^+\ell^-$ modes ($\ell = e, \mu$) because of the presence of the LD two-photon penguin, but the decays are still under good theoretical control. They are especially interesting because of their different sensitivity to helicity-suppressed contributions.

2.1. $K^+ \to \pi^+\nu\bar{\nu}$

The effective Hamiltonian for the $K^+ \to \pi^+\nu\bar{\nu}$ decay involves below the charm-quark scale to a good approximation only the operator Q_ν. Its Wilson coefficient, induced at leading order (LO) by the SM box and penguin diagrams shown in Fig. 1, contains two terms proportional to λ_t and λ_c, respectively, where $\lambda_i \equiv V_{id}V^*_{is}$, and we have used the unitarity relation $\lambda_u = -\lambda_c - \lambda_t$ to eliminate λ_u. The leading behaviour of the top-quark contribution X_t, proportional to λ_t, is given by m_t^2/M_W^2. The smallness of λ_t compensates the effect of the large top-quark mass and makes it

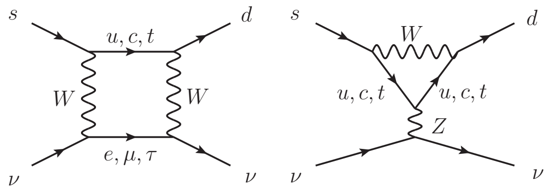

Figure 1. LO diagrams contributing to the decay amplitude for $K \to \pi\nu\bar{\nu}$ in the SM.

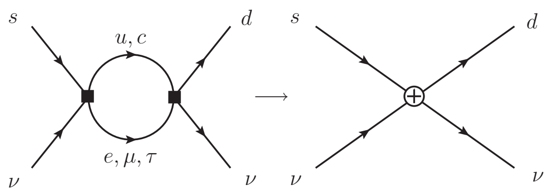

Figure 2. LO mixing of current-current and penguin operators into Q_ν.

comparable in size to the charm-quark contribution P_c, proportional to λ_c, with the leading behaviour $m_c^2/M_W^2 \ln(m_c^2/M_W^2)$. The appearance of the (large) logarithm is related to the bilocal mixing of current-current and penguin operators into Q_ν through charm-quark loops, see Fig. 2. This introduces large scale uncertainties, which have been removed by computing the next-to-next-to-leading order (NNLO) QCD corrections to P_c in renormalisation-group (RG) improved perturbation theory [1]. In addition, the electroweak corrections are known. They sum the LO and next-to-leading order (NLO) QED logarithms to all orders and fix the renormalisation scheme of the electroweak input parameters in the charm-quark sector, leading to the final prediction $P_c = 0.368(25)$ [2].

The top-quark contribution X_t does not contain a large logarithm and can be computed in fixed-order perturbation theory. The NLO QCD corrections have been known for a long time [3]. On the other hand, the two-loop electroweak corrections were known until recently only in the limit of a heavy top quark [4]. We have computed the full two-loop electroweak corrections to X_t, fixing the renormalisation scheme of the electroweak input parameters also in the top-quark sector and rendering the remaining scale and scheme dependence essentially negligible [5]. The final result is $X_t = 1.465(17)$, where the error is largely due to the remaining QCD scale uncertainty.

After integrating out the charm quark, the matrix element of Q_ν gives the dominant contribution to the decay rate. It has been determined from the $K_{\ell 3}$ decays using isospin symmetry, including NLO and partially NNLO corrections in chiral perturbation theory (ChPT) and QED radiative corrections [6].

The effects of soft charm and up quarks as well as of higher-dimensional operators have been estimated in ChPT and are contained in the phenomenological parameter $\delta P_{c,u} = 0.04(2)$, which enhances the branching ratio by roughly 6% [7]. The error on $\delta P_{c,u}$ could in principle be reduced by a lattice calculation [8].

The branching ratio of the charged mode is given by

$$\mathrm{Br}_{\mathrm{ch}} = \kappa_+ (1 + \Delta_{\mathrm{EM}}) \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda} \left(P_c + \delta P_{c,u} \right) + \frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t \right)^2 \right], \quad (1)$$

where $\lambda = |V_{us}| = 0.2255(7)$ [9], $\kappa_+ = 0.5173(25) \times 10^{-10}$ [6] comprises the hadronic matrix element and $\Delta_{\mathrm{EM}} = -0.3\%$ [6] the effects of real soft photon emission. Using $m_t(m_t) = 163.7(1.1)\mathrm{GeV}$, $m_c(m_c) = 1.286(13)\mathrm{GeV}$, and the remaining input from Ref. [10,11], we find the following numerical prediction:

$$\mathrm{Br}_{\mathrm{ch}} = (7.84^{+0.80}_{-0.71} \pm 0.28) \times 10^{-11}\,, \quad (2)$$

where the first error is related to the uncertainties of the input parameters, and the second error quantifies the remaining theoretical uncertainty. The parametric error is dominated by the uncertainty in the CKM parameters V_{cb} (56%) and $\bar{\rho}$ (21%) and could be reduced in the future by better determinations of these parameters. The contributions to the theoretical uncertainty are ($\delta P_{c,u}$: 48%, P_c : 21%, X_t : 23%, κ_ν^+ : 8%), respectively.

The branching ratio has been measured to be $\mathrm{Br}_{\mathrm{ch}} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$ [12], consistent with the SM prediction within the (still large) experimental error.

2.2. $K_L \to \pi^0 \nu\bar{\nu}$

The neutral mode $K_L \to \pi^0 \nu\bar{\nu}$ is purely CP-violating [13,14], so only the top-quark contribution is relevant for the decay rate because of the smallness of $\mathrm{Im}\lambda_c$. It is given by the same function X_t as for the charged mode.

Again, the matrix element of Q_ν has been extracted from the $K_{\ell 3}$ decays and is contained in the parameter κ_L [6]. There are no further LD contributions, which is the reason for the exceptional theoretical cleanness of this mode.

The branching ratio is given by

$$\mathrm{Br}_{\mathrm{neutr}} = \kappa_L \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t \right)^2 , \quad (3)$$

where $\kappa_L = 2.231(13) \times 10^{-10}$ [6]. Including again the recently calculated full two-loop electroweak corrections, as well as a factor taking into account the small ($\approx -1\%$) effect of indirect CP violation [15], we find for the branching ratio, using the same input as for the charged mode,

$$\mathrm{Br}_{\mathrm{neutr}} = (2.42^{+0.40}_{-0.37} \pm 0.04) \times 10^{-11}\,. \quad (4)$$

As before, the first error corresponds to the parametric and the second to the theoretical uncertainty. Here, the parametric uncertainty is dominated by the error in the CKM parameters V_{cb} (54%) and $\bar{\eta}$ (39%) and could again be reduced in the future by better determinations of these parameters. The contributions to the second, theoretical uncertainty are (X_t : 78%, κ_ν^L : 21%, $\delta P_{c,u}$: 1%), respectively. All errors have been added in quadrature.

The neutral mode has not been observed yet; an upper bound for the branching ratio is given by $\mathrm{Br}_{\mathrm{neutr}} < 6.7 \times 10^{-8}$ (90% CL) [16].

2.3. $K_L \to \pi^0 \ell^+ \ell^-$

In contrast to the two neutrino modes, the $K_L \to \pi^0 \ell^+ \ell^-$ modes have a sizeable LD contribution. Their relevance lies in their different sensitivity to helicity-suppressed contributions, which allows to disentangle scalar/pseudoscalar from vector/axialvector contributions [17]. It can be exploited because of the good theoretical control over the individual contributions to the branching ratios, which we now consider in turn:

The direct CP-violating contribution (DCPV) is contained in two Wilson coefficients C_{7V} and C_{7A} induced by Z and γ penguins, which are known at NLO QCD [18]. The matrix elements of the corresponding operators $Q_{7V} = (\bar{s}d)_{V-A}(\ell^+\ell^-)_V$ and $Q_{7A} = (\bar{s}d)_{V-A}(\ell^+\ell^-)_A$ can be extracted from $K_{\ell 3}$ decays in analogy to the neutrino modes [6].

The indirect CP-violating contribution (ICPV) is related via $K^0 - \bar{K}^0$ mixing to the decay $K_S \to \pi^0\ell^+\ell^-$. It is dominated by a single ChPT coupling a_S [19], whose absolute value can be extracted from the experimental $K_S \to \pi^0\ell^+\ell^-$ decay rates to give $|a_S| = 1.2(2)$ [20].

Both the ICPV and the DCPV can produce the final lepton pair in a 1^{--} state, leading to interference between the two amplitudes. Whether the interference is constructive or destructive is determined by the sign of a_S, which is unknown at the moment (see also [21–23]). It can be determined by measuring the $K_L \to \pi^0\mu^+\mu^-$ forward-backward asymmetry [17].

The purely LD CP-conserving contribution (CPC) is induced by a two-photon intermediate state $K_L \to \pi^0\gamma^*\gamma^* \to \pi^0\mu^+\mu^-$ and produces the lepton pair either in a phase-space suppressed 2^{++} or in a helicity suppressed 0^{++} state. The former is found to be negligible [22], while the latter is only relevant for the muon mode because of helicity suppression. It can be extracted within ChPT from experimental information on the $K_L \to \pi^0\gamma\gamma$ decay [24].

The prediction for the branching ratio is [17]

$$\begin{aligned} \mathrm{Br}_{e^+e^-} &= 3.54^{+0.98}_{-0.85}\,(1.56^{+0.62}_{-0.49}) \times 10^{-11}\,, \\ \mathrm{Br}_{\mu^+\mu^-} &= 1.41^{+0.28}_{-0.26}\,(0.95^{+0.22}_{-0.21}) \times 10^{-11}\,, \end{aligned} \qquad (5)$$

for constructive (destructive) interference. The error of the prediction is completely dominated by the uncertainty in a_S and could be reduced by better measurements of the $K_S \to \pi^0\ell^+\ell^-$ modes [17].

Experimentally, upper limits are known for the two decays [25,26]: $\mathrm{Br}_{e^+e^-} < 28 \times 10^{-11}$ (90% CL), $\mathrm{Br}_{\mu^+\mu^-} < 38 \times 10^{-11}$ (90% CL), which lie still one order of magnitude above the SM prediction.

3. THE PARAMETER ϵ_K

The parameter ϵ_K describes indirect CP violation in the neutral Kaon system and plays a prominent role as an ingredient to the global fit of the unitarity triangle and as a constraint on models of new physics. It is defined as the ratio of the respective decay amplitudes of a K_L and a K_S decaying into two pions in an isospin-zero state (see, for instance, Ref. [27] for a thorough discussion and definition), and can be expressed by the following formula [28]:

$$\epsilon_K = e^{i\phi_\epsilon}\sin\phi_\epsilon\left(\frac{\mathrm{Im}(M_{12}^*)}{\Delta M_K} + \xi\right)\,. \qquad (6)$$

Here M_{12} is the transition matrix element between a $\bar{K}^0$ and a K^0, and ξ is related to the isospin zero amplitude of $K \to \pi\pi$ [28]. The Kaon mass difference ΔM_K and the phase $\phi_\epsilon = \arctan(2\Delta M_K/\Delta\Gamma_K)$ ($\Delta\Gamma_K$ being the difference of the K_S and K_L decay widths) are taken from experiment. A theoretical prediction is then obtained by computing $\mathrm{Im}(M_{12}^*)$ and ξ.

The dominant contribution to the $|\Delta S| = 2$ effective Hamiltonian $\mathcal{H}_{\text{eff}}^{|\Delta S|=2}$ below the charm-quark scale is proportional to the dimension-six operator $Q_{S2} = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ (higher-dimensional operators are estimated to contribute less than 1% to $\mathrm{Im}(M_{12}^*)$ [29]). Its LD matrix elements are parameterised by the bag factor B_K, which can be evaluated by lattice QCD and was the dominant source of uncertainty until a couple of years ago. Recent progress in lattice calculations has greatly reduced the error, yielding a value of $B_K = 0.725(26)$ [30].

The SD contributions are contained in the Wilson coefficents of $\mathcal{H}_{\text{eff}}^{|\Delta S|=2}$, which are induced at LO in the SM by the box diagrams of Fig. 3. Using again the unitarity relation $\lambda_u = -\lambda_c - \lambda_t$, $\mathcal{H}_{\text{eff}}^{|\Delta S|=2}$ can be split into three parts of the form $\lambda_i\lambda_j\eta_{ij}S(m_i^2/M_W^2, m_j^2/M_W^2)$, $i,j = c,t$, where the loop functions S denote the contribution of

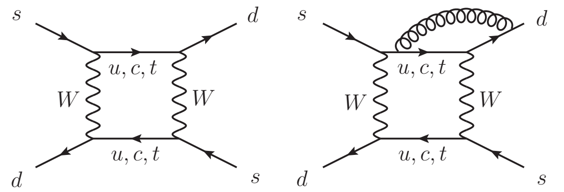

Figure 3. SM box diagrams contributing to $\mathcal{H}_{\text{eff}}^{|\Delta S|=2}$.

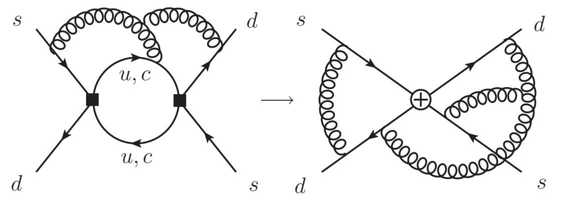

Figure 4. SM Diagrams contributing to the mixing of $|\Delta S| = 1$ operators into Q_{S2} at NNLO QCD.

the LO SM box diagrams, and the coefficients η_{ij} comprise the higher-order QCD corrections.

The coefficient $\eta_{tt} = 0.5765(65)$ of the dominant top-quark contribution, proportional to λ_t^2, can be computed in fixed-order perturbation theory and includes the NLO QCD corrections [31].

The smallest contribution is proportional to λ_c^2. It arises from a matching calculation at the charm-quark scale, and the NLO QCD calculation yields $\eta_{cc} = 1.43(23)$ [32].

The calculation of the sizeable mixed charm-top-quark contribution, proportional to $\lambda_c\lambda_t$, requires a full RG analysis with double insertions of $|\Delta S| = 1$ operators (see Fig. 4). We have performed a NNLO QCD calculation and find $\eta_{ct} = 0.496(47)$ [33].

The parameter ξ is given by the absorptive part of the $|\Delta S| = 2$ amplitude. It is a purely LD contribution and has been calculated by relating it to the ratio ϵ'/ϵ [27,28]. $\mathrm{Im}(M_{12}^*)$ is obtained from the matrix elements of the $|\Delta S| = 2$ effective Hamiltonian $\mathcal{H}_{\text{eff}}^{|\Delta S|=2}$. The double insertion of two $|\Delta S| = 1$ operators generates LD contributions which have been estimated in ChPT in [34]. These two corrections, together with the experimental value of ϕ_ϵ, are combined into the phenomenological parameter $\kappa_\epsilon = 0.94(2)$, which multiplies (6), simultaneously setting $\xi = 0$ and $\phi_\epsilon = 45°$.

The parameter ϵ_K is measured with high accuracy: the value quoted by the Particle Data Group is $\epsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i(43.5\pm0.7)°}$ [11]. The inclusion of the NNLO QCD corrections to η_{ct} leads to the prediction $|\epsilon_K| = (1.90 \pm 0.26) \times 10^{-3}$, increasing the central value by a few per cent and thus releasing the slight discrepancy with the experimental result.

4. CONCLUSION

In this talk I reviewed the status of the SM prediction for the rare Kaon decays $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0\ell^+\ell^-$, and the parameter ϵ_K. Each of those observables is a highly sensitive probe of physics in the SM and its extensions: the neutrino decay modes can be predicted theoretically with exceptional precision, resulting in a theory uncertainty of 3.6% for the charged mode and 1.7% for the neutral mode. The $K_L \to \pi^0\ell^+\ell^-$ decays allow to disentangle scalar/pseudoscalar from vector/axialvector contributions, and ϵ_K is an important ingredient to the global fit of the unitarity triangle and a prominent constraint on models of new physics. Together with the planned high-precision experiments measuring the rare Kaon decays, these observables will play a decisive role in the next decade of particle physics.

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