Measurement of $\pi\pi$ Scattering Lengths from K_{e4} and $K_{3\pi}$ Decays

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In the last few years it has become possible to study low energy $\pi\pi$ scattering with high precision in kaon decays, thanks to the high statistics measurements of $K^\pm \to \pi^\pm\pi^0\pi^0$ and $K^\pm \to \pi^+\pi^-e^\pm\nu$ (K_{e4}) decays performed by the NA48/2 experiment at the CERN SPS. In $K^\pm \to \pi^\pm\pi^0\pi^0$, at the $\pi^+\pi^-$ threshold, the $\pi^0\pi^0$ mass spectrum exhibits a Wigner-cusp, from which the scattering lengths a_0 and a_2 are extracted. From the study of K_{e4} decays a similar precision, but quite independent measurement of a_0 and a_2 can be obtained. The combination of both measurements provides one of the most stringent tests of Chiral Perturbation Theory.

Dedicated to the memory of Nicola Cabibbo

1. Introduction

In the low-energy regime, the perturbative description of the strong interaction breaks down, as the strong coupling constant becomes of $\mathcal{O}(1)$. Chiral Perturbation Theory (ChPT) is an effective theory, which circumvents this problem by making use of the chiral symmetry of the theory in the limit of vanishing quark masses. Spontaneous chiral symmetry breaking generates 8 pseudo-scalar Goldstone bosons, among them pions and kaons. They obtain their small but non-zero masses by the additional symmetry breaking of non-vanishing quark masses. The values of the iso-spin 0 and 2 S-wave $\pi\pi$ scattering lengths a_0 and a_2 are directly connected with the pion mass, as has already early been shown by Weinberg [1]. In the framework of ChPT, the scattering lengths can accurately be predicted to $a_0 m_{\pi^+} = 0.220 \pm 0.005$ and $a_2 m_{\pi^+} = -0.044 \pm 0.010$ [2]. Thus, precise measurements of the $\pi\pi$ scattering lengths are a crucial test of ChPT.

Previous measurements have traditionally been performed in the semileptonic decay $K^\pm \to \pi^+\pi^-e^\pm\nu$ (K_{e4}). An early measurement by the Geneva-Saclay experiment analyzed 30000 events [3]. More recently, the BNL experiment E865 has measured $(a_0 - a_2)m_{\pi^+} = 0.258 \pm 0.013$ from about 400000 K_{e4} events [4]. Another recent determination of the scattering lengths has been carried out by the DIRAC experiment at CERN from the lifetime of pionium atoms. They obtain $|a_0 - a_2|m_{\pi^+} = 0.264\,^{+0.033}_{-0.020}$ from an analysis of a part of their data [5].

Here, new measurements of NA48/2 are reported. In $K^\pm \to \pi^\pm\pi^0\pi^0$ decays, a Wigner-cusp from $\pi\pi$ rescattering in the decay amplitude was observed at $m(\pi^0\pi^0) = 2\,m_{\pi^+}$. This allows a very precise determination of a_0 and a_2 with a completely new method. In addition, the NA48/2 Collaboration has also determined a_0 and a_2 in K_{e4} decays, with a greatly improved precision with respect to previous measurements.

2. The NA48/2 Experiment

The NA48/2 experiment has been taking data in the years 2003 and 2004. From a 400 GeV/c proton beam, positive and negative kaons with a momentum of $p_K = (60 \pm 3)$ GeV/c were simultaneously extracted. A detailed detector description can be found in [6]. The main aim of the experiment was the search for direct CP violation in decays of charged kaons into three pions. The trigger therefore was designed to efficiently select events with three charged tracks as well as $K^\pm \to \pi^\pm\pi^0\pi^0$ events. In total, about 2×10^9 three-track events and about 90×10^6 $K^\pm \to \pi^\pm\pi^0\pi^0$ events were recorded in the two years of data-taking.

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3. Measurement of $K_{3\pi}$ Decays

As first reported from an analysis of a partial data set [7], the $\pi^0\pi^0$ invariant mass (M_{00}) spectrum of $K^\pm \to \pi^\pm\pi^0\pi^0$ decays shows a cusp-like behaviour in the region around $M_{00} = 2\,m_{\pi^+}$. The existence of this threshold anomaly had originally been predicted in 1961 by Budini and Fonda [8]. More recently, Cabibbo [9] has proposed an interpretation of the anomaly, expressing the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay amplitude in terms of the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude at threshold.

The scattering lengths a_0 and a_2 were determined from fits of the full NA48/2 data set to the M_{00} spectrum (Fig. 1(a)). The free fit parameters were the scattering lengths a_0 and $a_0 - a_2$, the Dalitz plot parameters, the normalization, and the fraction $f_{\rm atom}$ of pionium formation at the cusp point. The fit residuals are shown in Fig. 1(b) and (c) without and with taking pionium formation into account.

Two theoretical approaches were used: the Cabibbo-Isidori (CI) formulation [10], and the more recent Bern-Bonn (BB) formulation [11]. The BB approach uses a non-relativistic Lagrangian framework, which automatically satisfies unitarity and analiticity constraints, and allows one to include electromagnetic contributions in a standard way. In the CI approach, the structure of the cusp singularity is treated using unitarity, analiticity and cluster decomposition properties of the S-matrix. The decay amplitude is expanded in powers of $\pi\pi$ scattering lengths up to order (scattering length)^2, and electromagnetic effects are omitted. To take them into account, the BB electromagnetic effect is used to correct the CI amplitude.

The final NA48/2 result [12] uses the BB approach with radiative corrections and the pionium fraction left floating in the fit:

$$\begin{aligned}(a_0 - a_2)\, m_{\pi^+} &= 0.257\,(5)_{\rm stat}\,(3)_{\rm syst}\,(1)_{\rm ext}, \\ a_2\, m_{\pi^+} &= -0.024\,(13)_{\rm stat}\,(9)_{\rm syst}\,(2)_{\rm ext} \qquad (1)\end{aligned}$$

with a correlation coefficient of -0.839 (Fig. 2). Imposing the ChPT constraint between a_0 and a_2 results in the very precise value of $a_0 - a_2 = 0.2633\,(24)_{\rm stat}\,(14)_{\rm syst}\,(19)_{\rm ext}$, which is in perfect agreement with the prediction [2].

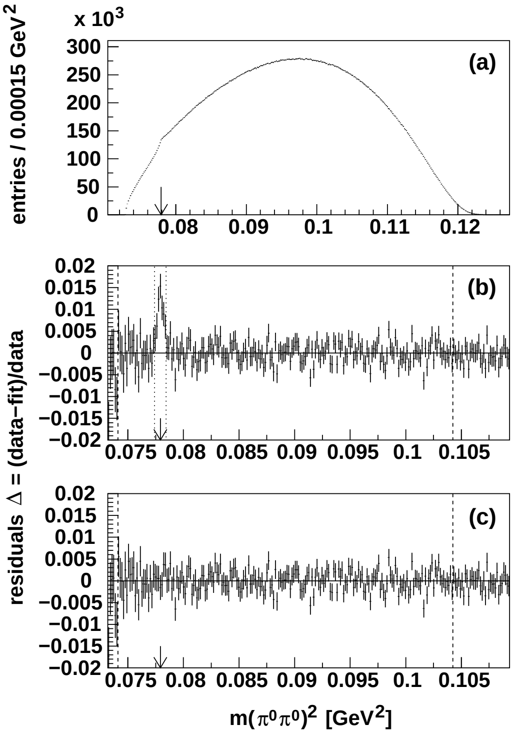

Figure 1. (a) M_{00}^2 spectrum of $K^\pm \to \pi^\pm\pi^0\pi^0$ events. The arrow indicates the cusp point at $4m_{\pi^+}^2$. (b)+(c): Fit residuals without and with pionium contribution included.

4. Measurement of K_{e4} Decays

An independent and complementary method for the determination of $\pi\pi$ scattering lengths is the analysis of $K^\pm \to \pi^+\pi^- e^\pm\nu$ (K_{e4}) decays. The K_{e4} decay is a rare decay with a branching fraction of about 4×10^{-5} [13]. Its amplitude depends on the two complex phases δ_0 and δ_1, which are the S and P wave $\pi\pi$ scattering phases for isospin $I = 0$ and 1, respectively. In K_{e4} decays, their difference $\delta = \delta_0 - \delta_1$ can be measured as a function of $m_{\pi\pi}$.

A first K_{e4} analysis has been carried out by NA48/2 on a partial data set [14]. The full NA48/2 data set comprises 1.1 million K_{e4} decays [15]. They were selected with a very small background contamination of 0.6%, deter-

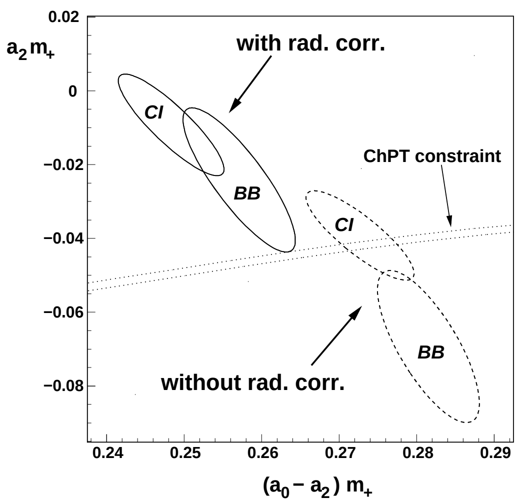

Figure 2. Fit results of the $\pi^0\pi^0$ spectrum, using the CI and BB models with and without radiative corrections. The ellipses correspond to 68% CL of the statistical uncertainties.

mined from data by using "wrong-sign" combinations $\pi^\pm\pi^\pm e^\mp$. The analysis was carried out in the five independent Cabibbo-Maksymowicz variables: these are the squared invariant dipion and dilepton masses s_π and s_e, the angles θ_π and θ_e of π^+ and e^+ with respect to the $\pi\pi$ and $e\nu$ directions in the $\pi\pi$ and $e\nu$ rest frames, respectively, and the angle ϕ between the $\pi\pi$ and $e\nu$ decay planes (Fig. 3).

A combined fit to the decay form factors and the phase shift difference δ as a function of $m_{\pi\pi}$ was performed. The fit results are shown in Fig. 4 together with the earlier measurements of the Geneva-Saclay [3] and BNL E865 [4] experiments. Effects from isospin symmetry breaking ($m_d \neq m_u$) do have a significant impact on the value of the phase shift difference δ. A ChPT calculation gives a shift of δ between 12 and 15 mrad int the region relevant for the fit [16]. Note, that these corrections had also to be applied on the data of the previous K_{e4} measurements.

From the phase shift measurements, the $\pi\pi$ scattering lengths can be extracted. Using dispersion relations (Roy equations) and data above

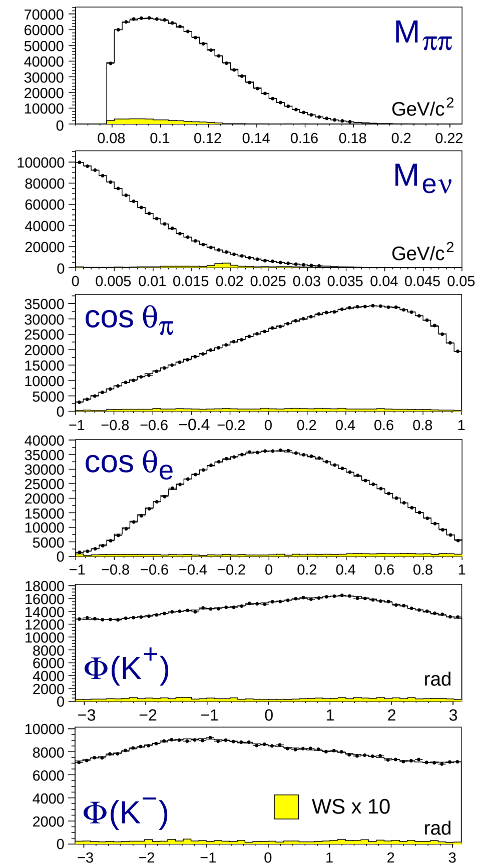

Figure 3. Data-MC comparison of the kinematic variables after the fit. The background is indicated in yellow and scaled by 10 to be visible.

800 MeV, a_2 is related to a_0 (the so-called Universal Band). From a one-parameter fit, NA48/2 obtains [15]

$$a_0\, m_{\pi^+} = 0.221\,(5)_{\rm stat}\,(2)_{\rm syst}(6)_{\rm theo}, \qquad (2)$$

where the theoretical error corresponds to the width of the universal band, which is given by the experimental uncertainties of the measurements above 800 MeV. This result implies $a_2\, m_{\pi^+} = -0.0442$. When both a_0 and a_2 were left free in the fit, the results were

$$\begin{aligned} a_0\, m_{\pi^+} &= 0.222\,(13)_{\rm stat}\,(5)_{\rm syst}\,(4)_{\rm theo}, \\ a_2\, m_{\pi^+} &= -0.043\,(9)_{\rm stat}\,(3)_{\rm syst}\,(3)_{\rm theo}. \qquad (3) \end{aligned}$$

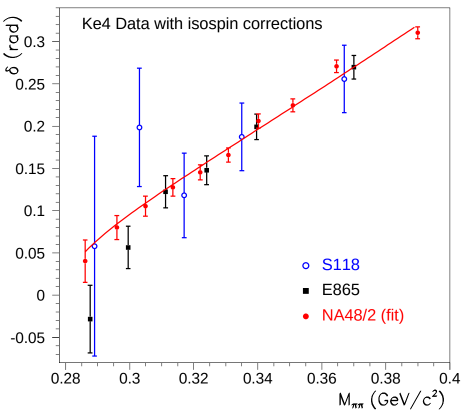

Figure 4. Measurements of the phase difference $\delta \equiv \delta_s - \delta_p$ as function of $m_{\pi\pi}$ in K_{e4} decays. The line shows the best fit to a_0 and a_2 of NA48/2.

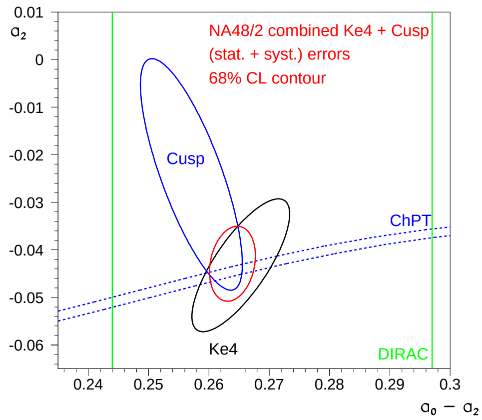

Figure 5. NA48/2 measurements of a_2 versus $a_0 - a_2$ from $K_{3\pi}$ and K_{e4} decays and combined.

5. Conclusion

With the analyses of $K_{3\pi}$ and K_{e4}, NA48/2 dominates the measurements of S-wave $\pi\pi$ scattering lengths. The two channels $K_{3\pi}$ and K_{e4} provide complementary information on the scattering lengths a_0 and a_2, with independent experimental uncertainties and very different theoretical inputs. They therefore can easily be combined, as illustrated in Fig. 5, to

$$\begin{aligned} a_0\, m_{\pi^+} &= 0.2210\,(47)_{\rm stat}\,(40)_{\rm syst}, \\ a_2\, m_{\pi^+} &= -0.0429\,(44)_{\rm stat}\,(28)_{\rm syst}, \quad \text{and} \\ (a_0 - a_2)\, m_{\pi^+} &= 0.2639\,(20)_{\rm stat}\,(15)_{\rm syst}. \end{aligned} \tag{4}$$

With these new results, the experimental precision finally matches the one of the theoretical prediction [2]. The agreement between measurement and theory is outstanding, and provides one of the most stringent tests of ChPT to date.

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