Precision Measurements with Pions—Review

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The branching ratio of pion decays, $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu + e^+\nu\gamma)/\Gamma(\pi^+ \to \mu^+\nu + \mu^+\nu\gamma)$, has provided the best test of electron-muon universality in weak interactions. While the Standard Model prediction is $R_{e/\mu} = (1.2353 \pm 0.0001) \times 10^{-4}$, the existing experimental results, $(1.2265 \pm 0.0056) \times 10^{-4}$ (TRIUMF) and $(1.2346 \pm 0.0050) \times 10^{-4}$ (PSI), have still orders of magnitude worse uncertainties. Since the decay $\pi^+ \to e^+\nu$ is helicity-suppressed, it is extremely sensitive to a presence of pseudo-scalar couplings arising from a wide range of new physics up to a mass scale of 1000 TeV/c^2. Two new experiments aiming at a precision level of <0.1 % for $R_{e/\mu}$, as well as related precision measurements of other pion decays, are described.

1. Introduction

The concept of lepton universality— identical coupling constants for different lepton flavours— has naturally been built in the Standard Model (SM). Many extensions of the SM, however, involve prospects that lead to seemingly violating universality for different flavours. The decay $\pi^+ \to e^+\nu$ was discovered in 1958 [1] as a confirmation of the V–A theory, and the branching ratio $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu + e^+\nu\gamma)/\Gamma(\pi^+ \to \mu^+\nu + \mu^+\nu\gamma)$ has provided the best test of electron-muon universality in weak interactions (Table 1). The present experimental branching-ratio measurements are: $(1.2265 \pm 0.0034(stat) \pm 0.0044(sys)) \times 10^{-4}$ (TRIUMF) [2] and $(1.2346 \pm 0.0034(stat) \pm 0.0044(sys)) \times 10^{-4}$ (PSI) [3] published in the early 90s, while the SM predictions have less uncertainties by an order of magnitude [8–14].

In this paper, the present status of two on-going experiments [15,16], which aim at a precision level of < 0.1 % for the measurement of $R_{e/\mu}$, will be discussed, and also precision measurements of other pion decays will be reviewed in the context of $\pi^+ \to e^+\nu$ measurements.

Table 1
Ratios of the weak coupling constants g_e/g_μ.

Mode	g_e/g_μ	Ref.
$\pi^+ \to e^+\nu/\pi^+ \to \mu^+\nu$	0.9985±0.0016	[2,3]
$K \to e\nu/K \to \mu\nu$	1.0018±0.0026	[4]
$\tau \to e\nu\overline{\nu}/\tau \to \mu\nu\overline{\nu}$	0.9998±0.0020	[5]
ν_e/ν_μ scat.	1.10±0.05	[6]
W decays	1.001±0.011	[5]
$K \to \pi e\nu/K \to \pi\mu\nu$	0.998±0.002	[7]

2. Theoretical predictions

2.1. Standard Model prediction

The decay $\pi^+ \to e^+\nu$ is helicity-suppressed in the SM by the V–A structure of charged currents. Many uncertainties such as the pion decay constant cancel in the branching ratio $R_{e/\mu}$. As indicated in the expression of $R_{e/\mu}$, the experimental result includes radiative decays with virtual and real photons. Radiative corrections were estimated to be –4 % by Berman and Kinoshita [8] assuming a point-like pion in the pre-SM era. There were some efforts to include the structure of pions in the radiative corrections [9]. Terentev [10] derived a general theorem on the leading hadronic structure dependent correction, and Marciano and Sirlin [11] showed that lepton flavour independent QCD radiative corrections cancelled in $R_{e/\mu}$. Hadron effects were classified into leading model-independent corrections and remaining corrections [12], and by scrutinizing the model dependence the uncertainty of the calculation was reduced to a 0.02 % level [13]. Recently, based on the Chiral Perturbation Theory (ChPT), Cirigliano and Rosell [14] calculated to the $O(m^2/p^4)$ terms including the helicity-unsuppressed structure dependent terms, and gave a prediction of $(1.2352 \pm 0.0001) \times 10^{-4}$.

2.2. Roles of other pion decays

Precision measurements of other pion decay modes were used for inputs of ChPT calculations as well as for the confirmation of the ChPT predictions.

Pure structure-dependent radiative corrections, which are not helicity-suppressed, are potentially important for the $R_{e/\mu}$ calculation. The amplitudes are parameterized by the vector and axial-vector form factors, F_V and F_A, respectively. The PIBETA group [17], using a CsI crystal calorimeter measured $F_V = 0.0258(17)$ and $F_A = 0.0117(17)$—they also set a tight bound for the presence of the tensor form factor, $F_T = (-0.6 \pm 2.8) \times 10^{-4}$. The structure dependent radiative correction for $R_{e/\mu}$ was estimated to be ∼0.03 % from the F_A/F_V ratio. In this experiment $\pi^+ \to e^+\nu$ was one of the sources of the backgrounds.

The ChPT predicts the rate of the decay $\pi^+ \to \pi^0 e^+\nu$, which is a potential source of precise measurements of V_{ud}. It also provides the best test of Conserved-Vector-Current (CVC) theory in a meson. The PIBETA experiment [18] measured the branching ratio to be $(1.036 \pm 0.004 \pm 0.005) \times 10^{-8}$, providing an independent measurement of $V_{ud} = 0.9728(30)$. The decay $\pi^+ \to e^+\nu$ was used as a normalization source in this experiment.

The lifetime of π^0 is related to the vector form factor F_V in the $\pi^+ \to e^+\nu\gamma$ decay through the CVC theory. A new result expected to be published by the PrimEx group at JLAB [19] is based on the Primakoff effect in which the π^0 production rate with a tagged photon is related to the π^0 lifetime. Measuring the cross sections with various nuclei, the decay width of π^0 was measured to be consistent with the ChPT prediction with three times improvement to the previous measurements [20].

The lifetime of π^+ is directly related to the pion decay constant f_π, which is one of the input parameters to the ChPT calculations. The precision of the pion lifetime measurement was already very good for that purpose. However, because the pion lifetime uncertainty was one of the error sources for the $R_{e/\mu}$ measurement, an improved measurement was necessary. The lifetime was measured to be 26.023±0.002 ns (TRIUMF) and 26.033±0.001 ns (Koptev) [21] both using the time-dependent yield of 30 MeV/c surface muons from the production target with the beam channel as a momentum analyzer.

2.3. Physics beyond the Standard Model

Since the uncertainty of the SM calculation for $R_{e/\mu}$ is very small, a measurement in disagreement with the universal coupling constant would imply the existence of new physics beyond the SM. Because of the helicity-suppression mechanism in the $\pi^+ \to e^+\nu$ decay, it is extremely sensitive to helicity-unsuppressed couplings such as the pseudo-scalar coupling. Since the pseudo-scalar contribution comes as an interference term with the dominant axial-vector term, the contribution is proportional to $1/m_H^2$, where m_H is the mass of a hypothetical particle. This is in contrast to $1/m_H^4$ dependence in lepton flavour violating decays.

At the level of 0.1 % measurements, new physics beyond the SM up to the mass scale of 1000 TeV/c^2 may be accessed by a deviation from the precise SM expectation [22,23]. Examples of the new physics probed include SUSY models with and without R-parity violation [24]. Also, lepton flavor violating terms [25] in SUSY could cause a deviation from the SM prediction by ∼0.05 %. Other possible sources of deviation are: heavy neutrino mixing, Majorons, charged Higgs particles arising from extended symmetries, effects of extra dimensions, leptoquarks, and compositeness.

3. Experiments

3.1. Method

Early $\pi^+ \to e^+\nu$ experiments [1] used a range telescope, and a bubble chamber. The first precision experiment of $R_{e/\mu}$ was performed with a magnetic spectrometer [26]. The $\pi^+ \to e^+\nu$ decay was normalized with the $\mu^+ \to e^+\nu\overline{\nu}$ decay ($T_{e^+} = 0 - 52.3$ MeV) following the decay

$\pi^+ \to \mu^+\nu$ (the $\pi^+ \to \mu^+ \to e^+$ decay), and inner radiative corrections were added afterward.

The following two experiments [27,28] in 60s and 70s used a large NaI(Tℓ) crystal as a positron detector. Positrons from $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu^+ \to e^+$ decays in two equal time bins separated by several pion lifetimes were counted to deduce the branching ratio. Many systematic effects, such as the solid angle of positron detection and the number of incident pion stops, cancelled in the first order, and small energy-dependent effects were corrected for based on the Monte Carlo (MC) calculations. Radiative decays were included in the measurements. The largest systematic uncertainty was in the correction of the low energy positrons from the decay $\pi^+ \to e^+\nu$, which was obtained from MC calculations with a measured resolution function using positron beams. The uncertainty of $R_{e/\mu}$ was reduced to a percent level.

There were two experiments in early 90s [2,3]. The previous TRIUMF [2] experiment, which was a refinement of the earlier experiments, used a large NaI(Tℓ) with a better energy resolution ($\sim$ 5 % FWHM for 70 MeV positrons) as the primary component of the detector. Based on 2×10^5 clean $\pi^+ \to e^+\nu$ events, a simultaneous fit of $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu^+ \to e^+$ time spectra, which allowed more careful studies of systematic uncertainties, provided a raw branching ratio. Since the time spectra are different (inverse) for $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu^+ \to e^+$ decays, the measurement was sensitive to the uncertainties in the pion lifetime and the time distortion due to non-linearity. The low-energy tail correction was empirically obtained from the $\pi^+ \to e^+\nu$ data. The previous PSI experiment [3] used BGO crystals with a solid angle of 99.8 %, collecting 3×10^5 $\pi^+ \to e^+\nu$ events. The experiment measured positrons from $\pi^+ \to e^+\nu$ decays and muons from $\pi^+ \to \mu^+\nu$ decays in the target. This made the experiment insensitive to the pion lifetime but vulnerable to different systematic errors related to the time measurements and acceptances for positrons and muons. The yields of high energy positrons and muons in the time window of 7.5–200 ns were used in the analysis (the $\pi^+ \to e^+\nu$ events were selected by the total energy in the target and the calorimeter). The amount of the low energy tail was estimated purely from MC calculations.

3.2. Previous TRIUMF experiment

Since the new experiments PIENU [15] and PEN [16] employ the positron detection method for normalization, the previous TRIUMF experiment [2] is described in detail as the base for improvements (*indicated in italic*).

The experiment was carried out using a π^+ beam of momentum $P = 83 \pm 1$ MeV/c. (*There were neutron pulses at a rate of 30 k/s in the NaI originated from the pion production target that caused background in the time spectrum through pile-up.*) The incoming beam was stopped at a rate of 7×10^4 s^{-1} in a 1.2-cm-thick plastic-scintillator target tilted by 45° (*long path lengths of positrons for a certain angle and a path length variation in the target*). Thin plastic scintillators surrounding the target confined the stopping region of the pions, and contained the muon from the $\pi^+ \to \mu^+\nu$ decay. Positrons from the decay of stopped pions in the target were detected at 90° to the beam by a 2.9 % solid-angle telescope consisting of two planar wire chambers, four thin plastic scintillators, and a 46-cm-diameter × 51-cm-long NaI(Tl) crystal (*small solid angle*).

The raw branching ratio was determined by simultaneously fitting, to the expected time evolution in the time window of –150 to 250 ns with respect to the pion stop, the measured decay-time spectra for positron events above and below the threshold energy in the NaI crystal at 56.4 MeV, which divided the energy spectrum into the $\pi \to e\nu$ and $\pi \to \mu \to e$ decay regions, respectively. (*The time window was too narrow to correctly evaluate the amplitudes of the backgrounds with different time dependences.*)

In order to empirically determine the fraction of the $\pi^+ \to e^+\nu$ events below the cut-off energy 56.4 MeV, the dominant $\pi \to \mu \to e$ component was suppressed by using energy and pulse-shape information from the target counter as well as the decay time information; in the $\pi \to e\nu$ decay events, the total energy in the target included the kinetic energy of the stopping pion plus a small contribution from the exiting decay positron, while in $\pi \to \mu \to e$ decay events there was an additional 4-MeV pulse from the kinetic energy of the decay muon. A suppression factor of 10^5 for $\pi \to \mu \to e$ events was obtained. The residual background in the low-energy positron energy spectrum, due mostly to decay-in-flight (DIF) pions, was subtracted using the energy spectrum of events occurring ~8 pion lifetimes after the pion stop. The overall tail correction 1.93 ± 0.25 % was obtained after adding a 0.4 % correction (*due to the long e^+ path length*) for the bias that was introduced by the selection criteria of the $\pi^+ \to e^+\nu$ events. (*The uncertainty of the tail correction was limited by statistics.*)

MC calculations were used to correct for systematic effects related to positron annihilation in flight, multiple Coulomb scattering, and the fraction of very low-energy $\pi \to \mu \to e$ positrons (below 5 MeV) lost in the trigger. The combined correction from the MC studies was 0.27 ± 0.11 % (*due mainly to the uncertainties in the detector geometry and the beam distribution*).

The corrections applied to the raw branching ratio are summarized in Table 2.

Table 2
Summary of the corrections in Ref. [2].

Corrections	Values(%)		Errors
Tail correction	1.93	±	0.25
Acceptance difference	0.27	±	0.11
π lifetime	0.00	±	0.09
Others	0.05	±	0.11

3.3. New TRIUMF experiment

The concept of the new experiment PIENU [15] is based on the previous TRIUMF experiment. The branching ratio $R_{e/\mu}$ is obtained from the ratio of positron yields from the $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu^+ \to e^+$ decays.

A 75 MeV/c π^+ beam is degraded by two thin plastic scintillators (6.35 mm and 3 mm thick) and stopped in an 8-mm thick active target at a rate of 5×10^4 pions/s. The new extension of the beam line [29], which suppressed positrons in the beam to < 2 % of pions, also reduced (together with more shielding) the neutron rate from the production target by an order of magnitude.

In order to gain a large solid angle while reducing the variation of the amount of material along the positron path, the primary calorimeter of a 48-cm (diam.) × 48-cm (length) single-crystal NaI(Tℓ) is placed in the beam axis. Two rings of 97 pure CsI crystals (9 radiation length) surround the NaI to reduce the shower leakage. The improvement factor of 30 in statistics is therefore expected to come from a larger solid angle by an order of magnitude and a longer running period.

Pion tracking is provided by six planes of wire chambers at the exit of the beam line, and two sets of X-Y Si-strip counters immediately upstream of the target. Positron tracking comes from one set of X-Y Si-strip counters immediately downstream of the target, and three layers of wire chambers in front of the NaI.

In the empirical low-energy tail evaluation, the major low-energy background in the background-suppressed spectrum came from DIF of pions near the target which had the same signature as that of the decay $\pi^+ \to e^+\nu$ [2]. The new upstream tracking detectors provide detection of a kink when DIF happens, and reduce the background by a factor of two. The overall improvement by a factor of 5–7 in the uncertainty of the tail correction is expected.

As the response function of the NaI system to the 70 MeV positrons was not known in the previous experiment due to the uncertainty of the contamination of low momentum beam components, careful beam studies as well as the response-function measurements were performed in the engineering run. The measurements using positron beams at various entrance angles and energies confirmed effectiveness of the CsI rings to lower the fraction of the tail below 1 %. However, measurements with the 70 MeV positron beam indicated a presence of two bumps at 60 MeV (1 % of the main peak) and 53 MeV (0.2 %) [30] in addition to the main 70 MeV peak. So far, GEANT4-based MC can reproduce the structure but not the correct amplitudes. The disagreements may be due to uncertainties in the neutron production cross sections by photo-nuclear effects, and amplified uncertainties in the neutron-scattering cross sections at low energy since the number of neutron scatterings in those events is as high as ∼15. The uncertainty arising from this could be ∼0.1 % if the tail correction purely comes from MC.

The second largest systematic uncertainty came from energy dependence of multiple- and Bhabha-scattering cross sections in the MC calculations; there is a slight difference in the numbers of events going out (coming in) from (to) the solid angle. This energy dependent effect is expected to be roughly proportional to the ratio of the circumference of the solid-angle defining counter and the solid angle. The new geometry is expected to reduce the uncertainty of this correction by more than a factor of 3 from the previous level of 0.1 %.

The contribution from the pion lifetime uncertainty has been reduced to the level of 0.02 % [21] since the previous experiments.

Table 3
Expected uncertainties in PIENU [15].

Sources	Uncertainties (%)
Statistical error	0.05
Tail correction	0.03
Acceptance difference	0.03
Pion lifetime	0.02
Others	0.03
Total	0.06

The PIENU experiment had engineering runs in early 2009 and the production run started in late 2009. The experiment accumulated about 3×10^6 clean $\pi^+ \to e^+\nu$ events by the middle of 2010, and is expected to run till the end of 2011 for another 4×10^6 clean $\pi^+ \to e^+\nu$ events.

3.4. New PSI experiment

In the new PSI experiment (PEN) [16], ∼70 MeV/c pions are tagged by a counter in the beam line, degraded by a 5-mm thick active plastic-scintillator degrader, tracked by a mini-TPC, collimated by a passive Al collimator, and stopped in a 15-mm thick fast-scintillator target at a rate of 20 k/s. Positrons from the $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu^+ \to e^+$ decays are measured by two sets of cylindrical wire chambers, a thin 20-element plastic-scintillator hodoscope, and 240 12-radiation-length thick pure CsI detectors that surround the target with a solid angle of $\sim 3\pi$ sr.

Fast wave-form digitizers (2 GHz) are employed for the beam counters, which are effective for suppressing the background in the low energy region by identifying two-pulse events ($\pi^+ \to e^+\nu$) from three-pulse events ($\pi^+ \to \mu^+ \to e^+$), and reduction of detection acceptance variation near the time zero.

The 40-mm × 40-mm × 50-mm mini-TPC introduced in 2009 for pion tracking (replacing the wedge degrader) provides information on the pion stopping position, and detection of DIF events upstream of the target, which are the source of the major remaining background in the suppressed spectrum.

Table 4
Expected uncertainties in PEN [16].

Sources	Uncertainties (%)
Statistical	0.02
Systematic	0.03
π/μ discri.	0.01
$E_e < E_{threshold}$	0.01
Acceptance	
Radiative decay	<0.01
Photo-nuclear int.	0.01
T0 variation	<0.02
Total	0.05

During the engineering runs in 2007 and 2008, 5×10^6 $\pi^+ \to e^+\nu$ events (before cuts) were accumulated. About 10×10^6 $\pi^+ \to e^+\nu$ events (before cuts) were accumulated during the production run in 2009, and a similar statistics is expected in the 2010 run. The expected uncertainties are summarized in Table 4, which include corrections for pion and muon event discrimination, positrons below the detection threshold from $\mu^+ \to e^+\nu\overline{\nu}$ decays, acceptance difference between $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu^+ \to e^+$ events, effects of radiative decays, photo-nuclear interactions, and the knowledge of the time zero for both decay modes.

4. Conclusion

Although the decay $\pi^+ \to e^+\nu$ is considered as a normalization source or even a source of background for some measurements, precision studies of $\pi^+ \to e^+\nu$ decays still remain one of the topical subjects. Precision measurements of $R_{e/\mu}$ provide the best test of μ-e universality in weak interactions. It is also sensitive to the presence of pseudo-scalar interactions arising from physics beyond the SM.

The two on-going experiments at TRIUMF and PSI are expected produce measurements at a level of <0.1 % uncertainty in a few years. At this level of precision, new physics up to 1000 TeV/c^2 could be surveyed.

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