Lepton Flavour for Hadron Flavour Physicists

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In this review talk I was asked to give an overview of recent developments in lepton flavour for an audience consisting mainly of hadron flavour physicists.

1. THE FLAVOUR PROBLEM

What does the physics of lepton flavour and hadron flavour have in common? Obviously the word "flavour". More precisely both areas of activity are concerned with different aspects of the Flavour Problem left unanswered by the Standard Model (SM). In fact the Flavour Problem is not one problem but may be regarded as several related questions which could be listed as follows:

1. Why are there three families of quarks and leptons?
2. What is the origin of quark and lepton masses?
3. Why is quark mixing so small?
4. What is the origin of quark CP violation?
5. Why is lepton mixing so large?
6. Is there leptonic CP violation?

2. THE NEUTRINO REVOLUTION

Here is a quick summary of the basic facts about what has been termed the "neutrino revolution":

- Lepton flavour is not conserved
- Neutrinos have tiny masses which are not very hierarchical
- Neutrinos mix strongly unlike quarks
- The SM parameter count is increased by at least 7 new parameters
- Neutrinos are still the least understood particles
- They provide the first new physics beyond the SM
- Neutrino mass and mixing is the most important discovery in the past dozen years

3. A BRIEF HISTORY (POST 1998)

Here are the milestones of what has been going on in the past dozen years in neutrino physics:

- 1998 - SuperKamiokande (SK) confirms that Atmospheric ν_μ are converted to another neutrino type, probably ν_τ
- 2002 - Sudbury Neutrino Observatory (SNO) confirms that Solar ν_e are converted to a linear combination of ν_μ and ν_τ
- 2002 - SK, SNO and the older neutrino experiments such as Homestake and the Gallium experiments results are combined in a global fit which points towards the large mixing angle MSW conversion of solar neutrinos in the core of the Sun
- 2004 - Reactor antineutrinos $\overline{\nu}_e$ are observed by KamLAND to oscillate with a probability consistent with the solar neutrino oscillations

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- 2006 - Accelerator neutrinos ν_μ from Fermilab are observed over a long baseline (LBL) at MINOS with a disappearance probability consistent with the atmospheric oscillation results, providing a high precision confirmation of a similar observation from KEK to SK (K2K) in 2004
- 2010 - LBL accelerator neutrinos ν_μ from CERN appear at OPERA as ν_τ (see Fig.1)
- 2010 - LBL accelerator anti-neutrinos $\overline{\nu}_\mu$ from Fermilab are observed at MINOS with lower statistics and a slightly anomalous disappearance probability
- 2010 - MiniBooNE sees weak evidence for $\overline{\nu}_\mu$ from Fermilab converting to $\overline{\nu}_e$ over a short baseline not inconsistent with a previous LSND observation, but inconsistent with MiniBooNE results using ν_μ

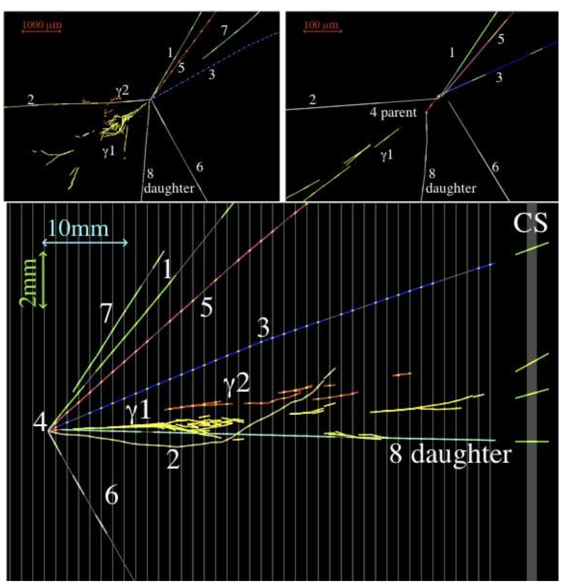

Figure 1. The observation of ν_τ at OPERA. Track 8 is from the decay of a τ produced from the charged current interaction of a ν_τ into which the ν_μ from CERN had oscillated.

4. THREE NEUTRINO MASSES AND LEPTON MIXING ANGLES

In the three active neutrino paradigm, the lepton mixing matrix can be parameterised as in Fig.2 in terms of three angles θ_{ij}, one oscillation phase δ and (if neutrinos are Majorana particles) two Majorana phases α_i.

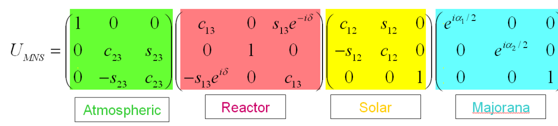

Figure 2. The lepton mixing matrix with phases factorizes into a matrix product of four matrices, associated with the physics of Atmospheric neutrino oscillations, Reactor neutrino oscillations, Solar neutrino oscillations and a Majorana phase matrix.

Ignoring the phases, the lepton mixing angles can be visualised as the Euler angles in Fig.3.

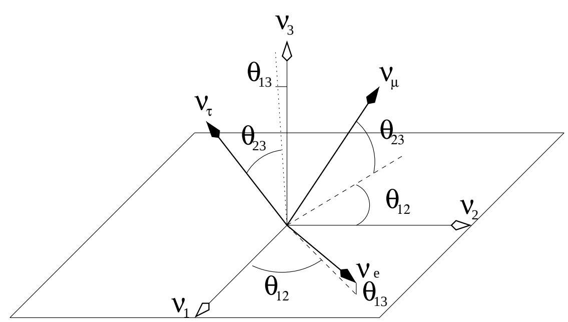

Figure 3. The relation between the neutrino weak eigenstates ν_e, ν_μ, and ν_τ and the neutrino mass eigenstates ν_1, ν_2, and ν_3 in terms of the three mixing angles θ_{12}, θ_{13}, θ_{23}. Ignoring phases, these are just the Euler angles respresenting the rotation of one orthogonal basis into another.

The mass squared ordering is not yet determined uniquely for the atmospheric mass squared splitting, but the solar neutrino data requires $m_2^2 > m_1^2$, as shown in Fig.4.

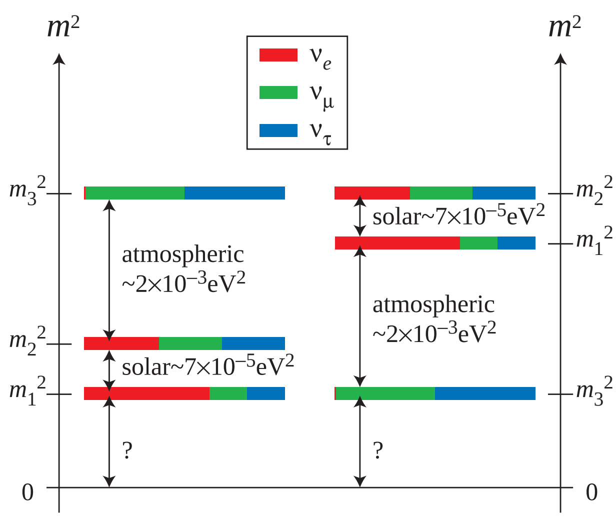

Figure 4. Alternative neutrino mass patterns that are consistent with neutrino oscillation explanations of the atmospheric and solar data. The pattern on the left (right) is called the normal (inverted) pattern. The coloured bands represent the probability of finding a particular weak eigenstate ν_e, ν_μ, and ν_τ in a particular mass eigenstate. The absolute scale of neutrino masses is not fixed by oscillation data and the lightest neutrino mass may vary from about $0.0 - 0.2$ eV where the upper limit comes from cosmology.

The current best fit values for the neutrino mass squared differences are given in Fig.5.

The current best fit values for the lepton angles are given in Fig.6.

Note that the reactor angle θ_{13} is not currently measured but its value is only inferred. The prospects for its future determination have been projected as in Fig.7.

$$\Delta m^2_{21} = 7.59 \pm 0.20 \, \left({}^{+0.61}_{-0.69}\right) \times 10^{-5} \text{ eV}^2$$
$$\Delta m^2_{31} = \begin{cases} -2.36 \pm 0.11 \; (\pm 0.37) \times 10^{-3} \text{ eV}^2 \\ +2.46 \pm 0.12 \; (\pm 0.37) \times 10^{-3} \text{ eV}^2 \end{cases}$$

Figure 5. The best fit neutrino mass squared differences with 1σ error (and 3σ error) from [1].

$$\theta_{12} = 34.4 \pm 1.0 \, \left({}^{+3.2}_{-2.9}\right)^\circ$$
$$\theta_{23} = 42.8\,{}^{+4.7}_{-2.9} \, \left({}^{+10.7}_{-7.3}\right)^\circ$$
$$\theta_{13} = 5.6\,{}^{+3.0}_{-2.7} \, (\leq 12.5)^\circ$$

Figure 6. The best fit lepton mixing angles with 1σ error (3σ error) from [1].

5. WHY GO BEYOND THE STANDARD MODEL?

It has been one of the long standing goals of theories of particle physics beyond the Standard Model (SM) to predict quark and lepton masses and mixings. With the discovery of neutrino mass and mixing, this quest has received a massive impetus. Indeed, perhaps the greatest advance in particle physics over the past decade has been the discovery of neutrino mass and mixing involving two large mixing angles commonly known as the atmospheric angle θ_{23} and the solar angle θ_{12}, while the remaining mixing angle θ_{13}, although unmeasured, is constrained to be relatively small. The largeness of the two large lepton mixing angles contrasts sharply with the smallness of the quark mixing angles, and this observation, together with the smallness of neutrino masses, provides new and tantalizing clues in the search for the origin of quark and lepton flavour. However, before trying to address such questions, it is worth recalling why neutrino mass forces us to go beyond the SM.

Neutrino mass is zero in the SM for three independent reasons:

1. There are no right-handed neutrinos ν_R.

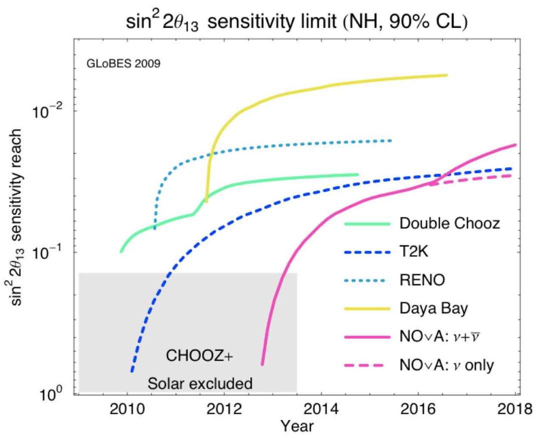

Figure 7. The future $\sin^2 2\theta_{13}$ sensitivity limit (normal hierarchy, 90% CL) from [2].

2. There are only Higgs doublets of $SU(2)_L$.

3. There are only renormalizable terms.

In the SM these conditions all apply and so neutrinos are massless with ν_e, ν_μ, ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ. Neutrinos and antineutrinos are distinguished by total conserved lepton number $L = L_e + L_\mu + L_\tau$. To generate neutrino mass we must relax one or more of these conditions. For example, by adding right-handed neutrinos the Higgs mechanism of the Standard Model can give neutrinos the same type of mass as the electron mass or other charged lepton and quark masses. It is clear that the *status quo* of staying within the SM, as it is usually defined, is not an option, but in what direction should we go?

6. NEUTRINO MASS MODELS

The rest of this talk will be organized according to the road map in Fig.8. Such a road map is clearly not unique (everyone can come up with her or his personal road map). The road map in Fig.8 contains key experimental questions (in blue) which serve as signposts along the way, leading in particular theoretical directions, starting from the top left hand corner with the question "LSND True or False?"

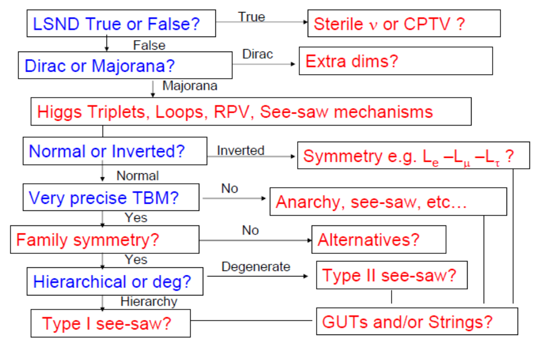

Figure 8. Mass models road map.

6.1. LSND True or False?

The antineutrino results from MiniBOONE may support the LSND result, but the neutrino results are consistent with the three active neutrino oscillation paradigm. If LSND were correct then this could imply either sterile neutrinos and/or CPT violation, or something more exotic. For the remainder of this talk we shall assume that LSND is false, and focus on models without sterile neutrinos.

6.2. Dirac or Majorana?

Majorana neutrino masses are of the form $m^\nu_{LL}\overline{\nu_L}\nu^c_L$ where ν_L is a left-handed neutrino field and ν^c_L is the CP conjugate of a left-handed neutrino field, in other words a right-handed antineutrino field. Such Majorana masses are possible since both the neutrino and the antineutrino are electrically neutral. Such Majorana neutrino masses violate total lepton number L conservation, so the neutrino is equal to its own antiparticle. If we introduce right-handed neutrino fields then there are two sorts of additional neutrino mass terms that are possible. There are additional Majorana masses of the form $M^\nu_{RR}\overline{\nu_R}\nu^c_R$. In addition there are Dirac masses of the form $m^\nu_{LR}\overline{\nu_L}\nu_R$. Such Dirac mass terms conserve total lepton number L, but violate separate lepton numbers L_e, L_μ, L_τ. The question of "Dirac or Majorana?" is a key experimental question which could be decided by the experiments which measure neutrino masses directly.

6.3. What if Neutrinos are Majorana?

We have already remarked that neutrinos, being electrically neutral, allow the possibility of Majorana neutrino masses. However such masses are forbidden in the SM since neutrinos form part of a lepton doublet L, and the Higgs field also forms a doublet H, and $SU(2)_L \times U(1)_Y$ gauge invariance forbids a Yukawa interaction like HLL. So, if we want to obtain Majorana masses, we must go beyond the SM.

One possibility is to introduce Higgs triplets Δ such that a Yukawa interaction like ΔLL is allowed. However the limit from the SM ρ parameter implies that the Higgs triplet should have a VEV $< \Delta >< 8$ GeV. One big advantage is that the Higgs triplets may be discovered at the LHC and so this mechanism of neutrino mass generation is directly testable [6].

Another possibility, originally suggested by Weinberg, is that neutrino Majorana masses originate from operators $HHLL$ involving two Higgs doublets and two lepton doublets, which, being higher order, must be suppressed by some large mass scale(s) M. When the Higgs doublets get their VEVs Majorana neutrino masses result: $m^{\nu}_{LL} = \lambda_\nu < H >^2 /M$. This is nice because the large Higgs VEV $< H >\approx 175$ GeV can lead to small neutrino masses providing that the mass scale M is high enough. E.g. if M is equal to the GUT scale $1.75.10^{16}$ GeV then $m^{\nu}_{LL} = \lambda_\nu 1.75.10^{-3}$ eV. To obtain larger neutrino masses we need to reduce M below the GUT scale (since we cannot make λ_ν too large otherwise it becomes non-perturbative).

Typically in physics whenever we see a large mass scale M associated with a non-renormalizable operator we tend to associate it with tree level exchange of some heavy particle or particles of mass M in order to make the high energy theory renormalizable once again. This idea leads directly to the see-saw mechanism where the exchanged particles can either couple to HL, in which case they must be either fermionic singlets (right-handed neutrinos) or fermionic triplets, or they can couple to LL and HH, in which case they must be scalar triplets. These three possibilities have been called the type I, III and II see-saw mechanisms, respectively. If the coupling λ_ν is very small (for some reason) then M could even be lowered to the TeV scale and the see-saw scale could be probed at the LHC [7], however the see-saw mechanism then no longer solves the problem of the smallness of neutrino masses.

There are other ways to generate Majorana neutrino masses which lie outside of the above discussion. One possibility is to introduce additional Higgs singlets and triplets in such a way as to allow neutrino Majorana masses to be generated at either one [8] or two [9] loops. Another possibility is within the framework of R-parity violating Supersymmetry in which the sneutrinos $\tilde{\nu}$ get small VEVs inducing a mixing between neutrinos and neutralinos χ leading to Majorana neutrino masses $m_{LL} \approx < \tilde{\nu} >^2 /M_\chi$, where for example $< \tilde{\nu} >\approx$ MeV, $M_\chi \approx$ TeV leads to $m_{LL} \approx$ eV. A viable spectrum of neutrino masses and mixings can be achieved at the one loop level [10].

6.4. Normal or Inverted?

If the mass ordering is inverted then this may indicate a new symmetry such as $L_e - L_\mu - L_\tau$ [11] or a $U(1)$ family symmetry [12]. However let us assume that the hierarchy is normal and proceed down the road map to the next experimental question.

6.5. Very precise tri-bimaximal mixing?

It is a striking fact that current data on lepton mixing is (approximately) consistent with the so-called tri-bimaximal (TB) mixing pattern [13],

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P_{Maj}, \tag{1}$$

where P_{Maj} is the diagonal phase matrix involving the two observable Majorana phases. However there is no convincing reason to expect exact TB mixing, and in general we expect deviations. These deviations can be parametrized by three parameters r, s, a defined as [14]:

$$\sin\theta_{13} = \frac{r}{\sqrt{2}}, \quad \sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s),$$
$$\sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a). \tag{2}$$

Global fits of the conventional mixing angles can be translated into the 1σ ranges

$$0.14 < r < 0.24, \quad -0.05 < s < 0.02,$$
$$-0.04 < a < 0.10. \tag{3}$$

Clearly a non-zero value of r, if confirmed, would rule out TB mixing. However it is possible to preserve the good predictions that $s = a = 0$, by postulating a modified form of mixing matrix called tri-bimaximal-reactor (TBR) mixing [15], where only r is allowed to be non-zero.

6.6. Family Symmetry?

Let us expand the neutrino mass matrix in the diagonal charged lepton basis, assuming exact TB mixing, as $M^{\nu}_{TB} = U_{TB}\mathrm{diag}(m_1, m_2, m_3)U^T_{TB}$ leading to (absorbing the Majorana phases in m_i):

$$M^{\nu}_{TB} = m_1\Phi_1\Phi_1^T + m_2\Phi_2\Phi_2^T + m_3\Phi_3\Phi_3^T \tag{4}$$

where $\Phi_1^T = \frac{1}{\sqrt{6}}(2, -1, 1)$, $\Phi_2^T = \frac{1}{\sqrt{3}}(1, 1, -1)$, $\Phi_3^T = \frac{1}{\sqrt{2}}(0, 1, 1)$, are the respective columns of U_{TB} and m_i are the physical neutrino masses. In the neutrino flavour basis (i.e. diagonal charged lepton mass basis), it has been shown that the above TB neutrino mass matrix is invariant under S, U transformations:

$$M^{\nu}_{TB} = SM^{\nu}_{TB}S^T = UM^{\nu}_{TB}U^T \ . \tag{5}$$

A very straightforward argument [16] shows that this neutrino flavour symmetry group has only four elements corresponding to Klein's four-group $Z_2^S \times Z_2^U$. By contrast the diagonal charged lepton mass matrix (in this basis) satisfies a diagonal phase symmetry T. The matrices S, T, U form the generators of the group S_4 in the triplet representation, while the A_4 subgroup is generated by S, T.

As discussed in [17], the flavour symmetry of the neutrino mass matrix may originate from two quite distinct classes of models. The first class of models, which we call direct models, are based on a family symmetry $G_f = S_4$, or a closely related family symmetry as discussed below, some of whose generators are directly preserved in the lepton sector and are manifested as part of the observed flavour symmetry. The second class of models, which we call indirect models, are based on some more general family symmetry G_f which is completely broken in the neutrino sector, while the observed neutrino flavour symmetry $Z_2^S \times Z_2^U$ in the neutrino flavour basis emerges as an accidental symmetry which is an indirect effect of the family symmetry G_f. In such indirect models the flavons responsible for the neutrino masses break G_f completely so that none of the generators of G_f survive in the observed flavour symmetry $Z_2^S \times Z_2^U$.

In the direct models, the symmetry of the neutrino mass matrix in the neutrino flavour basis (henceforth called the neutrino mass matrix for brevity) is a remnant of the $G_f = S_4$ symmetry of the Lagrangian, where the generators S, U are preserved in the neutrino sector, while the diagonal generator T is preserved in the charged lepton sector. For direct models, a larger family symmetry G_f which contains S_4 as a subgroup is also possible e.g. $G_f = PSL(2, 7)$ [16]. Typically direct models satisfy form dominance [23], and require flavon F-term vacuum alignment, permitting an $SU(5)$ type unification [20]. Such minimal A_4 models lead to neutrino mass sum rules between the three masses m_i, resulting in/from a simplified mass matrix in Eq.4. A_4 may result from 6D orbifold models [22] and recently an $A_4 \times SU(5)$ SUSY GUT model has been constructed in 6D [18], while a similar model in 8D enables vacuum alignment to be elegantly achieved by boundary conditions [19].

In the indirect models [17] the idea is that the three columns of U_{TB} Φ_i are promoted to new Higgs fields called "flavons" whose VEVs break the family symmetry, with the particular vacuum alignments along the directions Φ_i. In the indirect models the underlying family symmetry of the Lagrangian G_f is completely broken, and the flavour symmetry of the neutrino mass matrix $Z_2^S \times Z_2^U$ emerges entirely as an accidental symmetry, due to the presence of flavons with particular vacuum alignments proportional to the columns of U_{TB}, where such flavons only appear quadratically in effective Majorana Lagrangian [17]. Such vacuum alignments can be elegantly achieved using D-term vacuum alignment, which allows the large classes of discrete family symmetry G_f, namely the $\Delta(3n^2)$ and $\Delta(6n^2)$ groups [17]. The indirect models satisfy natural form dominance since each column of the Dirac mass matrix corresponds to a different flavon VEV. In the limit $m_1 \ll m_2 < m_3$ FD reduces to constrained sequential dominance (CSD)[24].

6.7. Hierarchical or Degenerate?

This key experimental question may be decided by the same experiments as will also determine the nature of neutrino mass (Dirac or Majorana). Although not a theorem, it seems that a hierarchical spectrum could indicate a type I see-saw mechanism, while a (quasi) degenerate spectrum could imply a type II see-saw mechanism. It is possible that a type II see-saw mechanism could naturally explain the degenerate mass scale with the degeneracy enforced by an $SO(3)$ family symmetry, while the type I see-saw part could be responsible for the small neutrino mass splittings and the (TB) mixing [28]. An A_4 model of quasi-degenerate neutrinos with TB mixing, working at the effective neutrino mass operator level, was considered recently in [29].

6.8. GUTs and/or Strings?

Finally we have reached the end of the decision tree, with the possibility of an all-encompassing unified theory of flavour based on GUTs and/or strings. Such theories could also include a family symmetry in order to account for the TB mixing. There are many possibilities for the choice of family symmetry and GUT symmetry. Examples include the Pati-Salam gauge group $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ in combination with $SU(3)$ [26], $SO(3)$ [24,25], A_4 [30] or Δ_{27} [31]. Other examples are based on $SU(5)$ GUTs in combination with A_4 [32], T' [33] or S_4 [34].

In typical Family Symmetry $\otimes$ GUT models the origin of the quark mixing angles derives predominantly from the down quark sector, which in turn is closely related to the charged lepton sector. In order to reconcile the down quark and charged lepton masses, simple ansatze, such as the Georgi-Jarlskog hypothesis [35], lead to very simple approximate expectations for the charged lepton mixing angles such as $\theta^e_{12} \approx \lambda/3$, $\theta^e_{23} \approx \lambda^2$, $\theta^e_{13} \approx \lambda^3$, where $\lambda \approx 0.22$ is the Wolfenstein parameter from the quark mixing matrix. If the family symmetry enforces accurate TB mixing in the neutrino sector, then $\theta^e_{12} \approx \lambda/3$ charged lepton corrections will cause deviations from TB mixing in the physical lepton mixing angles, and lead to a sum rule relation [24,36,37], which can be conveniently expressed as [14] $s \approx r\cos\delta$ where $r \approx \lambda/3$ and δ is the observable CP violating oscillation phase, with RG corrections of less than one degree [38]. Such sum rules can be tested in future high precision neutrino oscillation experiments [39].

Note that in such a GUT-flavour framework, one expects the charged lepton corrections to the neutrino mixing angles to be less than of order $\theta^e_{12}/\sqrt{2}$ (where typically θ^e_{12} is a third of the Cabibbo angle) plus perhaps a further 1^o from renormalization group (RG) corrections. Thus such theoretical corrections cannot account for an observed reactor angle as large as 8^o, corresponding to $r = 0.2$, starting from the hypothesis of exact TB neutrino mixing.

7. CONCLUSION

Neutrino mass and mixing clearly requires new physics beyond the SM, but in which direction should we go? There are many roads for model building, but we have seen that answers to key experimental questions will provide the sign posts *en route* to a unified theory of flavour.

In particular we would like to emphasize that a measurement of a large reactor angle, consistent with the present 2σ indication for $r = 0.2$, can still be consistent with tri-bimaximal solar and atmospheric mixing, corresponding to $s = a = 0$, according to the tri-bimaximal-reactor mixing hypothesis. By contrast, tri-maximal mixing predicts $s = 0$ but $a = -(r/2)\cos\delta$.

The common feature of all these approaches is the presence of an underlying family symmetry, even though the reactor angle may be quite large. The existence of such disparate approaches only underlines the need for further high precision data from the neutrino experiments in order to resolve which approach is correct.

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